AN INTRODUCTION TO AIRCRAFT WING FLUTTER Revision A

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Introduction

Certain aircraft wings have experienced violent oscillations during high speed flight. Flutter is a particular form of oscillation.

Rudders, elevators, and other airfoil sections may also experience flutter.

Wing flutter occurs under combined torsional and bending motion. Thus, there are two-degrees-of-freedom. The system may have additional degrees-of-freedom, but only "binary" flutter is considered in this report.

The bending motion is described in terms of displacement. The torsional motion is described in terms of rotation.

The flutter is driven by airflow. The airflow is considered to be steady. Thus, the airflow lacks frequency content. Unsteady airflow, or gust, is not considered in this report.

The airflow exerts a force and moment upon the wing. Both the force and moment vary with the wing's position and motion. Effectively, the wing modifies the pressure field which surrounds it. Thus, flutter is considered to be a form of "self-excited" vibration.

Aerodynamic Instability

The wing can become unstable under certain conditions. Assume the following:

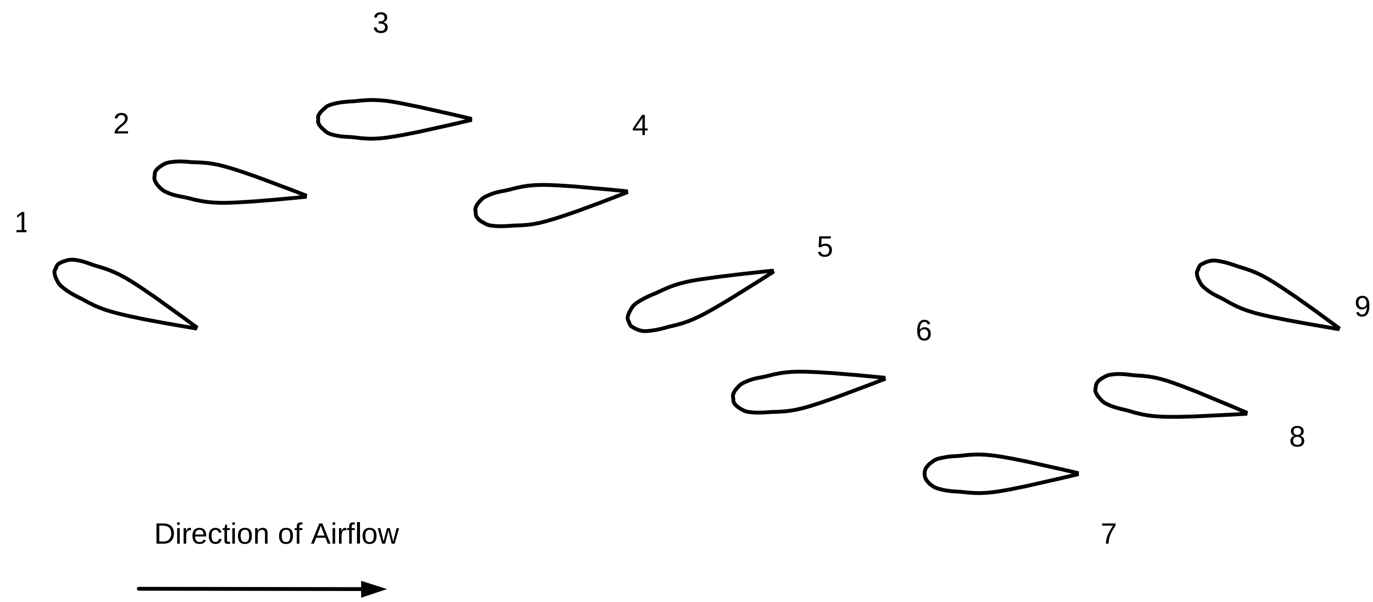
1. Both the bending and torsional motions vary in sinusoidal manner with time.
2. Both the bending and torsional bending motions occur at the same frequency.
3. The rotation leads the displacement by a quarter of a cycle (90 degrees).

These conditions would cause an aerodynamic instability as shown in Figure 1, taken from Reference 1.

Note that the leading edge is above the trailing edge during the entire upward stroke, steps 1 to 3 and steps 7 to 9.

Furthermore, the leading edge is below the trailing edge during the entire downward stroke, steps 3 to 7.

Thus, the lift force acts in the direction of motion during each stroke. The system is unstable.



The maximum postive rotation is at points 1 and 9.

The maximum positive displacement is at point 3.

Figure 1. Torsion and Bending of an Wing

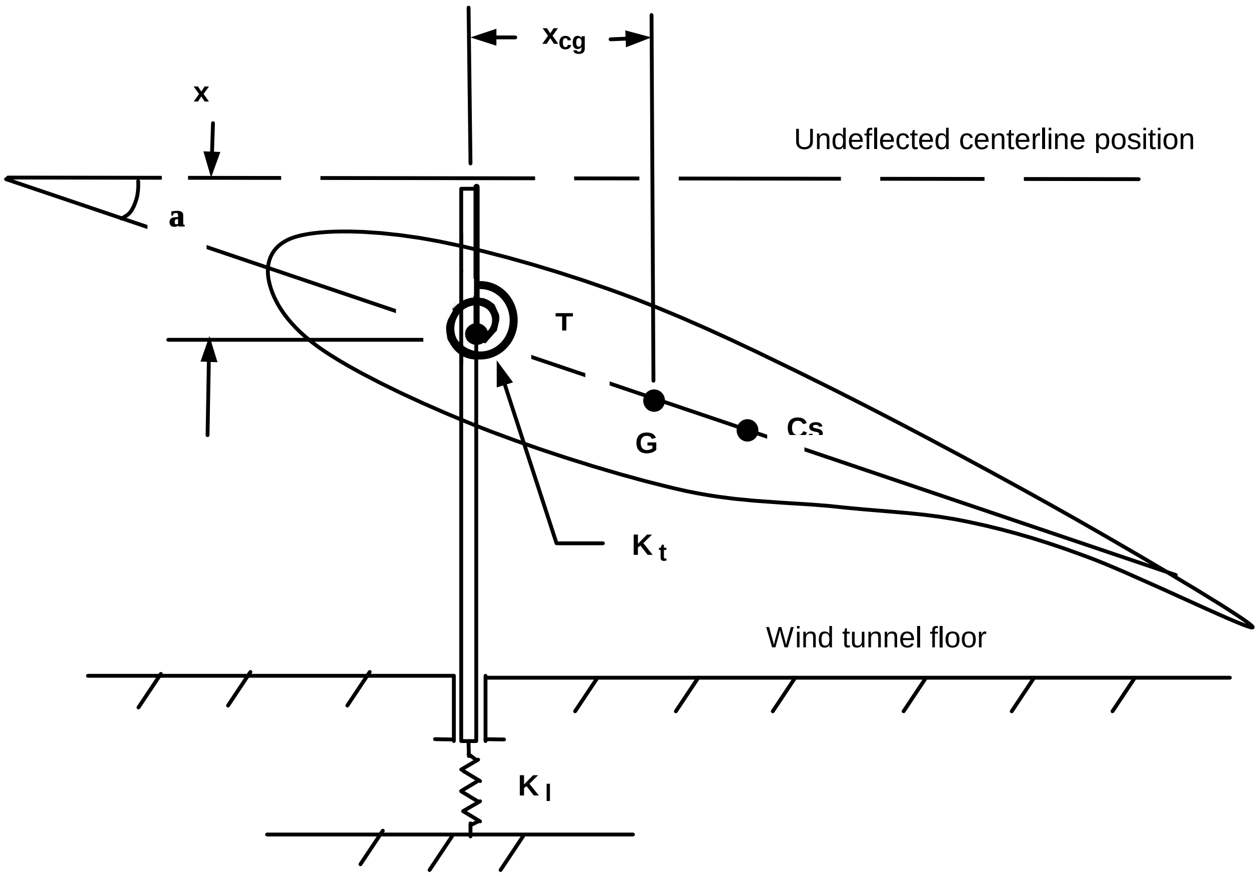
<u>Wing Diagrams</u>

The diagrams in Figures 2 and 3 shown the parameters used in the derivation of the equations of motion. These figures are based on similar figures in References 2 and 3.

For simplicity, model the wing as a two-dimensional structure, using lumped-parameter elements. Also consider that it is being tested in a wind tunnel. Assume an incompresible airflow.

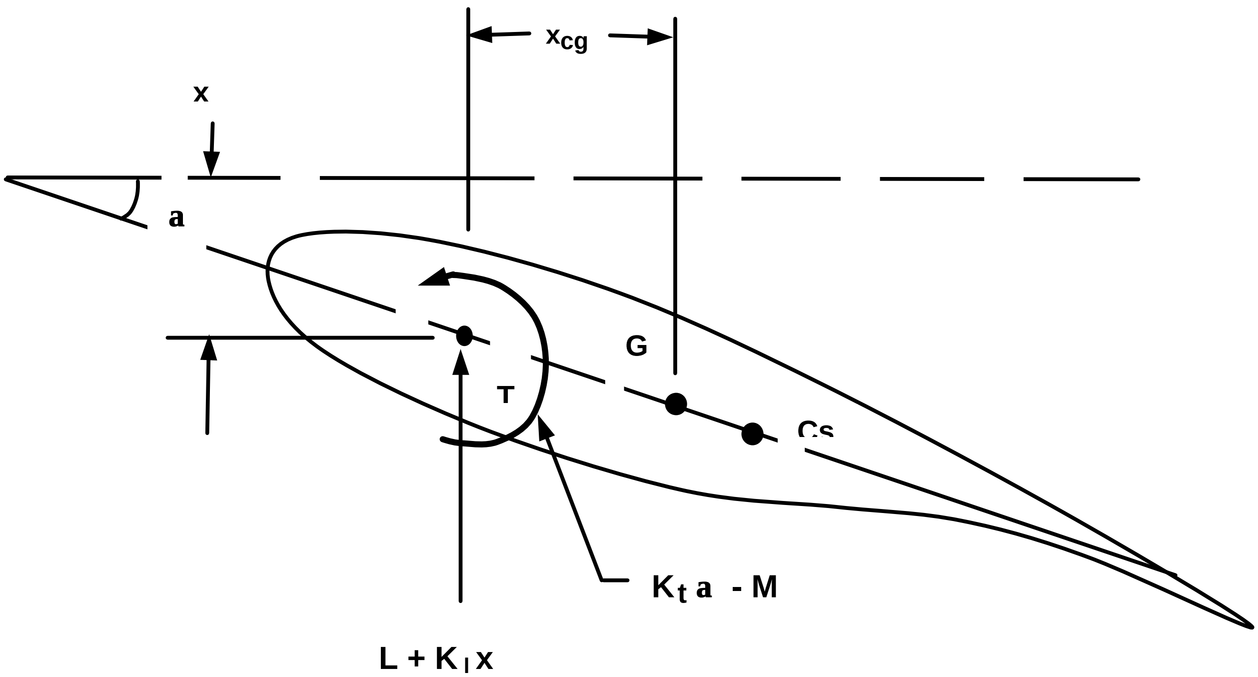
Note that the stiffness elements in Figures 2 and 3 could also represent the stiffness from a fuselage.

The center of twist T in Figure 2 is the point on the wing where a vertical force causes only a vertical displacement and no rotation. T is also that point of the wing which does not displace itself if the wing is subject to a pure torque causing a rotation of the section. The "elastic axis" passes through the center of twist. The reaction force and moment are referenced to this point.



T = center of twist
G = center of gravity
Cs = center of the span
x = displacement
α = rotation or angle-of-attack
K_l = linear stiffness
K_t = torsional stiffness

Figure 2. Wing in Wind Tunnel Test



L is the aerodynamic lift force acting through the center of twist.

M is the aerodynamic moment acting about the center of twist.

In addition, the wing has:

m = mass
J_G = polar moment of inertia about center of gravity

Sign Convention:

Downward Displacement is positive.
Clockwise Rotation is positive.

The displacement is measured with respect to location T, the center of twist.

Figure 3. Free-body Diagram

Derivation of Equations of Motion

The following derivation is based on Reference 3.

Lagrange's equations of motion for this system are

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}}\right) - \frac{\partial T}{\partial x} + \frac{\partial U}{\partial x} = Q_x \tag{1}$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\alpha}}\right) - \frac{\partial T}{\partial \alpha} + \frac{\partial U}{\partial \alpha} = Q_\alpha \tag{2}$$

The kinetic energy is

$$T = \frac{1}{2} m \left\{ \frac{d}{dt}\left[x + x_{cg} \sin\alpha \right] \right\}^2 + \frac{1}{2} J_G \, \dot{\alpha}^2 \tag{3a}$$

$$T = \frac{1}{2} m \left\{ \dot{x} + \dot{\alpha}\, x_{cg} \cos\alpha \right\}^2 + \frac{1}{2} J_G \dot{\alpha}^2 \tag{3b}$$

For small rotations, $\cos\alpha \approx 1$. Thus,

$$T = \frac{1}{2} m \left\{ \dot{x} + \dot{\alpha}\, x_{cg} \right\}^2 + \frac{1}{2} J_G \dot{\alpha}^2 \tag{3c}$$

$$T = \frac{1}{2} m \left\{ \dot{x}^2 + 2(\dot{x}\,\dot{\alpha}\, x_{cg}) + \left(\dot{\alpha}\, x_{cg}\right)^2 \right\} + \frac{1}{2} J_G \dot{\alpha}^2 \tag{3d}$$

Consider the kinetic energy with respect to the translational displacement.

$$\frac{\partial T}{\partial x} = 0 \tag{4}$$

$$\frac{\partial T}{\partial \dot{x}} = m\left\{ \dot{x} + \dot{\alpha}\, x_{cg} \right\} \tag{5}$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}}\right) = m\left\{ \ddot{x} + \ddot{\alpha}\, x_{cg} \right\} \tag{6}$$

Consider the kinetic energy with respect to the angular displacement.

$$\frac{\partial T}{\partial \alpha} = 0 \tag{7}$$

$$\frac{\partial T}{\partial \dot{\alpha}} = m x_{cg} \left\{ \dot{x} + \dot{\alpha}\, x_{cg} \right\} + J_G \dot{\alpha} \tag{8}$$

$$\frac{\partial T}{\partial \dot{\alpha}} = m x_{cg} \left\{ \dot{x} + \dot{\alpha}\, x_{cg} \right\} + J_G \dot{\alpha} \tag{8}$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\alpha}}\right) = m x_{cg} \left\{ \ddot{x} + \ddot{\alpha}\, x_{cg} \right\} + J_G \ddot{\alpha} \tag{9a}$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\alpha}}\right) = m x_{cg} \ddot{x} + \left\{ J_G + m\, x_{cg}^{\;2} \right\} \ddot{\alpha} \tag{9b}$$

The potential energy is

$$U = \frac{1}{2} k_l x^2 + \frac{1}{2} k_t \alpha^2 \tag{10}$$

Consider the potential energy with respect to the translational displacement.

$$\frac{dU}{dx} = k_l x \tag{11}$$

Consider the potential energy with respect to the angular displacement.

$$\frac{dU}{d\alpha} = k_t \alpha \tag{12}$$

The generalized force for the translation coordinate is

$$Q_x = -L \tag{13}$$

The generalized moment for the translation coordinate is

$$Q_\alpha = M \tag{14}$$

The two Lagrange equations are thus

$$m\left(\ddot{x} + \ddot{\alpha}\, x_{cg}\right) + k_l x = -L \tag{15}$$

$$m x_{cg} \ddot{x} + \left\{ J_G + m x_{cg}^2 \right\} \ddot{\alpha} + k_t \alpha = M \tag{16}$$

The two equations can be expressed in matrix form.

$$\begin{bmatrix} m & m x_{cg} \\ m x_{cg} & J_G + m x_{cg}^2 \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{\alpha} \end{bmatrix} + \begin{bmatrix} k_l & 0 \\ 0 & k_t \end{bmatrix} \begin{bmatrix} x \\ \alpha \end{bmatrix} = \begin{bmatrix} -L \\ M \end{bmatrix} \tag{17}$$

Now let J_T be the polar moment of inertia about the center of twist. Thus,

$$J_T = J_G + m x_{cg}^2 \tag{18}$$

The equations of motion become

$$\begin{bmatrix} m & m x_{cg} \\ m x_{cg} & J_T \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{\alpha} \end{bmatrix} + \begin{bmatrix} k_l & 0 \\ 0 & k_t \end{bmatrix} \begin{bmatrix} x \\ \alpha \end{bmatrix} = \begin{bmatrix} -L \\ M \end{bmatrix} \tag{19}$$

Note that the equations have a coupled mass matrix. This is also called dynamic coupling.

The stiffness matrix is uncoupled, however. Thus, the system is not statically coupled.

The spring constants in equation (20) are often replaced in terms of the natural frequencies of the uncoupled bending and torsional oscillations, defined by

$$\omega_h = \sqrt{\frac{k_l}{m}} \tag{20}$$

$$\omega_\alpha = \sqrt{\frac{k_t}{J_T}} \tag{21}$$

The equations of motion thus become

$$\begin{bmatrix} m & m x_{cg} \\ m x_{cg} & J_T \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{\alpha} \end{bmatrix} + \begin{bmatrix} m \omega_h^{2} & 0 \\ 0 & J_T \, \omega_\alpha^{2} \end{bmatrix} \begin{bmatrix} x \\ \alpha \end{bmatrix} = \begin{bmatrix} -L \\ M \end{bmatrix} \tag{22}$$

Furthermore, the $m x_{cg}$ term is regarded as the static unbalance of the wing about the center of twist. Note that the wing has initial displacement and rotation due to the offset between the center of gravity and the center of twist.

Aerodynamic Lift Force and Moment

The lift force L and moment M in equation (22) are functions of the wing's position and motion. The lift and moment terms are also functions of the airspeed and air density.

Regretfully, the References collectively fail to agree upon the lift and moment formulas. Reference 1 only gives a qualitative explanation. References 3 and 4 seem to agree upon the lift and moment equations. Reference 2, however, gives a rather different set of formulas.

The author of this report is thus continuing his search for a set of lift and moment equations suitable for teaching purposes.

Design Concerns

The onset of flutter depends on the fulfillment of the aerodynamic instability conditions, given previously. The air speed and the air density are relevant variables.

The energy input per cycle is dependent upon the air speed. The energy dissipated per cycle by mechanical and aerodynamic damping action also depends on the air speed.

In particular, the ratio of the energy input to the energy dissipated will depend on the speed. A steady oscillation may occur when this ratio is unity. The air speed for this case is called the "critical air speed." An aircraft may have various possible flutter modes. Ideally, the lowest critical speed exceeds the highest possible flying speed by a reasonable safety margin.

There are several additional measures to prevent flutter.

One method is to uncouple the torsional and bending motion. This is done by modifying the mass distribution to minimize the x_{cg} term. In other words, the center of gravity should be moved closer to the center of twist.

Another method is to increase the stiffness/mass ratios within the structure. This would increase the natural frequencies. Note that the energy input per cycle during flutter is nearly independent of frequency. The energy dissipated per cycle is proportional to frequency, however.

References

1. R. Bishop, Vibration, Cambridge University Press, London, 1979.
2. Den Hartog, Mechanical Vibrations, McGraw-Hill, New York, 1940.
3. Bisplinghoff and Ashley, Principles of Aeroelasticity, Dover, New York, 1962.
4. Y.C. Fung, An Introduction to the Theory of Aeroelasticity, Dover, New York, 1993.