HORIZONTAL IMPACT SHOCK

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Derivation

Consider a single-degree-of-freedom system as shown in Figure 1.

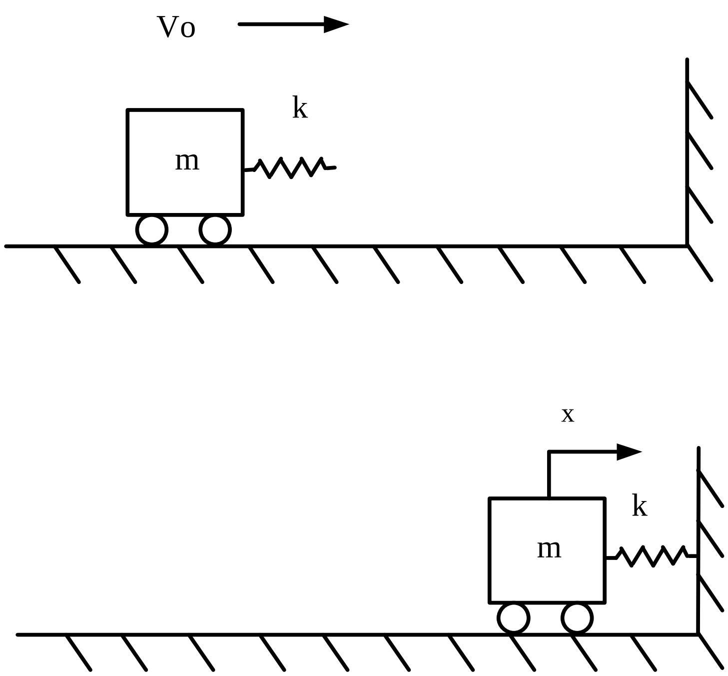

Figure 1.

where

- m is the mass,
- k is the spring stiffness,
- x is the absolute displacement of the mass,
- Vo is the initial velocity.

Assume

1. The object can be modeled as a single-degree-of-freedom system.
2. There is no energy dissipation. The collision is perfectly elastic.
3. The object remains attached to the wall via the spring after initial contact.
4. The object freely vibrates at its natural frequency after contact.
5. The system has a linear response.

The energy method is used to derive the equation of motion.

The total energy of a conservative system is constant. Thus,

$$\frac{d}{dt}\left(KE+PE\right)=0 \tag{1}$$

where

KE = kinetic energy
PE = potential energy

$$KE=\frac{1}{2}m\dot{x}^2 \tag{2}$$

$$PE=\frac{1}{2}kx^2 \tag{3}$$

$$\frac{d}{dt}\left(\frac{1}{2}m\dot{x}^2+\frac{1}{2}kx^2\right)=0 \tag{4}$$

$$m\dot{x}\ddot{x}+kx\dot{x}=0 \tag{5}$$

$$m\ddot{x}+kx=0 \tag{6}$$

Thus, the initial velocity when the mass encounters the wall is

$$\dot{x}(0)=Vo \tag{7}$$

Furthermore, the initial displacement is taken as zero.

$$x(0)=0 \tag{8}$$

Assume a displacement equation with constant coefficients a and b.

$$x(t) = a \sin \omega_n t + b \cos \omega_n t \tag{9}$$

The velocity is

$$\dot{x}(t) = a\omega_n \cos \omega_n t - b\omega_n \sin \omega_n t \tag{10}$$

The acceleration is

$$\ddot{x}(t) = -a\,\omega_n{}^2 \sin \omega_n t - b\,\omega_n{}^2 \cos \omega_n t \tag{11}$$

Substitute equations (9) and (11) into (6).

$$m\left[-a\,\omega_n{}^2 \sin \omega_n t - b\,\omega_n{}^2 \cos \omega_n t\right] + k\left[a \sin \omega_n t + b \cos \omega_n t\right] = 0 \tag{12}$$

$$-m\omega_n{}^2\left[a \sin \omega_n t + b \cos \omega_n t\right] + k\left[a \sin \omega_n t + b \cos \omega_n t\right] = 0 \tag{13}$$

$$\left[k - m\omega_n{}^2\right]\left[a \sin \omega_n t + b \cos \omega_n t\right] = 0 \tag{14}$$

$$k - m\omega_n{}^2 = 0 \tag{15}$$

$$m\omega_n{}^2 = k \tag{16}$$

$$\omega_n{}^2 = \frac{k}{m} \tag{17}$$

Thus the proposed solution is valid as long as

$$\omega_n = \sqrt{\frac{k}{m}} \tag{18}$$

Note that ω_n is the natural frequency.

The zero displacement initial condition requires b = 0. Thus

$$x(t) = a \sin \omega_n t \tag{19}$$

The velocity is

$$\dot{x}(t) = a \omega_n \cos \omega_n t \tag{20}$$

Recall

$$\dot{x}(0) = Vo \tag{21}$$

Thus

$$a = \left[\frac{Vo}{\omega_n}\right] \tag{22}$$

Substitute equation (22) into (19). The resulting displacement is

$$x(t) = \left[\frac{Vo}{\omega_n}\right] \sin \omega_n t \tag{23}$$

The velocity equation is

$$\dot{x}(t) = Vo \cos \omega_n t \tag{24}$$

The acceleration equation is

$$\ddot{x}(t) = -\omega_n Vo \sin \omega_n t \tag{25}$$

The force transmitted through the spring f(t) is

$$f(t) = -\omega_n m Vo \sin \omega_n t \tag{26}$$

The shock analysis is only concerned with the maximum values. These are summarized in Table 1.

Table 1. Maximum Absolute Values			
Parameter	Symbol	Maximum	Equivalent Form
Displacement	x	$\frac{V_o}{\omega_n}$	$V_o \sqrt{\frac{m}{k}}$
Velocity	$\dot{x}$	V_o	V_o
Acceleration	$\ddot{x}$	$\omega_n V_o$	$V_o \sqrt{\frac{k}{m}}$
Transmitted Force	f	$m\omega_n V_o$	$m V_o \sqrt{\frac{k}{m}}$
Total Impulse over a Half-sine Period	I	$2m V_o$	$2m V_o$