HYDRODYNAMIC CONICIDENCE
Revision A

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Variables

c , c_o	=	speed of sound in air
C_B	=	bending wave speed
C_L	=	longitudinal wave speed
D	=	plate flexure rigidity
E	=	elastic modulus
f	=	frequency (Hz)
G_{pp}	=	pressure autospectrum
h	=	plate thickness or core thickness
k_1	=	wavenumber along the flow
k_2	=	wavenumber across the flow
$\vec{k}$	=	vector having k_1 and k_2 components in the plane
k_ξ	=	wavenumber along the flow
k_η	=	wavenumber across the flow
k_b	=	free structural wavenumber in a plate
U_c	=	convection velocity
U_o	=	freestream flow velocity

α	=	frequency-dependent function
β	=	frequency-dependent function
ν	=	Poisson ratio
ω	=	frequency (rad/sec)
σ	=	scale value

Introduction

Consider a thin rectangular panel excited by a turbulent boundary layer.

Hydrodynamic coincidence occurs at the frequency at which the structural bending and fluctuating pressure wavenumbers match.

An equivalent statement is that hydrodynamic coincidence occurs at the frequency at which the phase speed of the fluctuating pressure matches the structural bending wave speed.

As a result, the structural vibration level increases significantly.

Turbulent Boundary Layer

This section is taken from Reference 1.

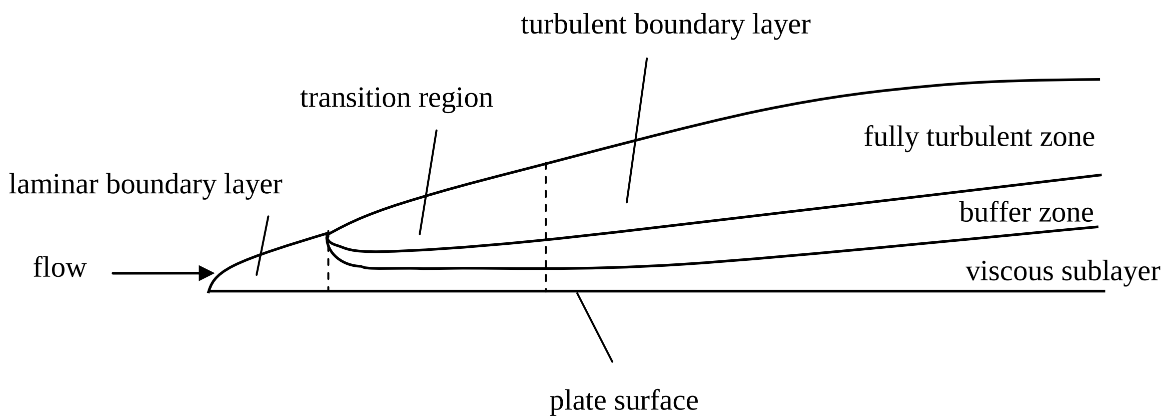

Figure 1. Flow Generating a Turbulent Boundary Layer on a Plate

The majority of the turbulence energy is produced in the viscous sublayer and the buffer zone.

Most of the energy in turbulent flow is contained in large eddies with relatively long life spans. Smaller eddies have less energy and shorter life spans.

Note that the viscous sublayer has sometimes been referred to as the laminar sublayer, but recent experimental and numerical investigations have shown that this region is not laminar.

Autospectrum

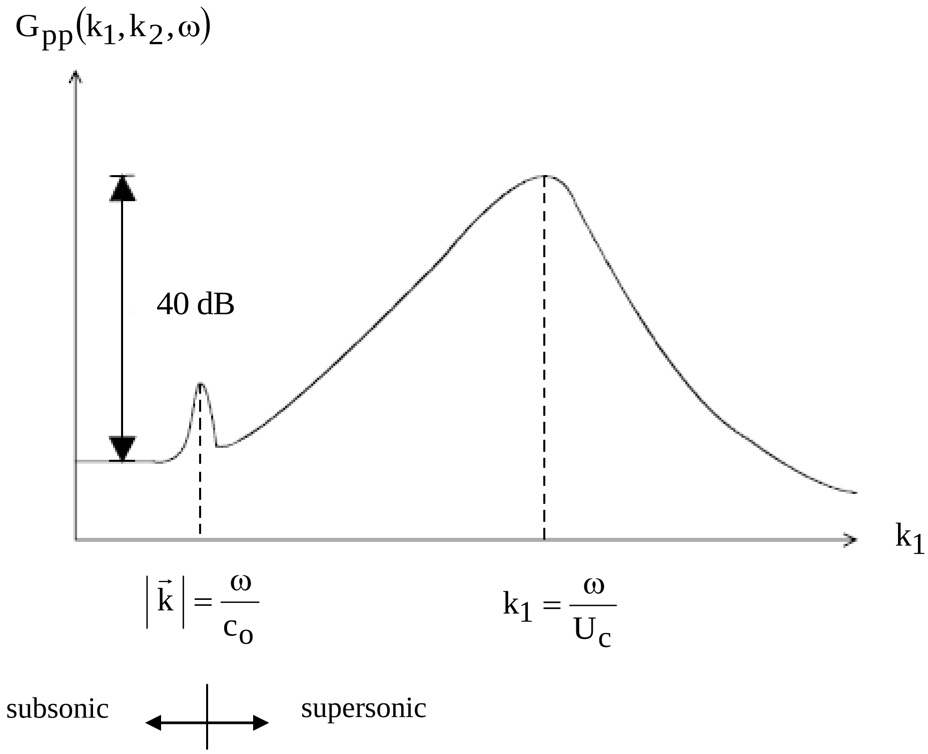

Figure 2. Typical Autospectrum in the Wavenumber Domain of the Flow at a Plane, Rigid Surface

The autospectrum graph is taken from Reference 1. The first peak is due to acoustic coincidence. The second peak is due to hydrodynamic coincidence. The pressure energy at hydrodynamic coincidence can be significantly greater than that at the acoustic coincidence.

Plate Wavenumber

The bending wave speed in a thin plate from Reference 2, chapter 3 is

$$C_B \approx \sqrt{1.8 C_L h f} \tag{1}$$

The bending wavenumber relationship from Reference 3 is

$$k_b{}^2 = \sqrt{\frac{\omega^2 \rho}{D}} \tag{2}$$

where

$$D = \frac{Eh^3}{12\left(1-\nu^2\right)} \tag{3}$$

Convection Velocity

The convection velocity for a turbulent boundary layer against a rocket vehicle in transonic or supersonic flight is:

$$U_c = \sigma U_o \tag{4}$$

The scale value σ can vary, as shown in the following table.

Typical σ Range	Zone
0.6 to 0.8	Attached Flow
0.2 to 0.8	Separated Flow

Note that separated flow may occur near protuberances.

As an aside, the convection velocity for the eddies along a rocket exhaust plume is:

$$U_c \sim 0.8\ U_o \tag{5}$$

The convection velocity is the rate at which the eddies propagate along the exhaust flow.

The above formulas for estimating convection velocity are simplified. Reference 4 notes:

> The convection velocity is found to be a function of both frequency and spatial separation. This variation with frequency is due to the vary speed of the different size eddies in the boundary layer. The variation is thought to be due primarily to the typical accelerated eddy trajectories in the boundary layer together with the effects of varying coherence lengths of eddies moving at different speeds.

<u>Fluctuating Pressure Wavenumber</u>

The following wavenumbers are taken from References 5 and 6.

The wavenumber along the flow is

$$k_{\xi} = \alpha(\omega)\frac{\omega}{U_c} \tag{6}$$

The wavenumber across the flow is

$$k_{\eta} = \beta(\omega)\frac{\omega}{U_c} \tag{7}$$

A default assumption is that

$$\alpha(\omega) = 1 \tag{8}$$

$$\beta(\omega) = 0 \tag{9}$$

<u>References</u>

1. Seppo Uosukainen, Turbulences as Sound Sources, VTT Publications 513, Finland, ESPOO 2003.
2. L. Cremer and M. Heckl, Structure-Borne Sound, Springer-Verlag, New York, 1988.
3. F. Fahy, Sound and Structural Vibration, Radiation, Transmission and Response, Academic Press, New York, 1971.
4. M. Lowson, Prediction of Boundary Layer Pressure Fluctuations, AFFDL-TR-67-167, Wright-Patterson AFB.

5. B. Prock, Vibro-Acoustic Hybrid Modeling and Analysis of the ARES IX Roll Control System, 2008.
6. M. Yang & J. Wilby, Derivation of Aero-Induced Fluctuating Pressure Environments for Ares I-X, AIAA, 2007.