SAMPLE LATERAL NATURAL FREQUENCY CALCULATIONS FOR A SPACE VEHICLE/DISPENSER ANALYSIS

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Acronyms

EELV - Evolved Expendable Launch Vehicle
IPC - Intermediate Payload Class
LV - Launch Vehicle
MPC - Medium Payload Class
SIS - Standard Interface Specification
SV - Space Vehicle (satellite)

Background

The following analysis was performed for a pair of identical space vehicles that were to have been launched on an EELV, such as a Delta 4 or Atlas 5. The satellites would have been mounted to the launch vehicle via a dispenser. The combined space vehicle and dispenser system was required to meet the frequency and stiffness requirements in the EELV SIS document.

Introduction

The SV/dispenser system must have a sufficiently high stiffness-to-mass ratio that the fundamental axial and lateral natural frequencies meet the minimum requirements stated in the SIS. Specifically, the minimum lateral frequency is 10 Hz for IPC and MPC classes, per the EELV SIS ver 6.0.

A consequence of the "law of practical physics" is that the fundamental frequency of an assembly of components is lower than the lowest component frequency. Thus, the dispenser and each SV must have a lateral frequency greater than 10 Hz individually, so that the SV/dispenser system will meet the 10 Hz limit.

An extreme case would be a perfectly rigid dispenser. In this case, each SV could have an individual lateral natural frequency of exactly 10 Hz.

The dispenser, however, will be elastic. Thus, further consideration of the structural dynamics of the SV/dispenser system is needed. This report provides some simple calculations in order to clarify the problem.

Lateral Analysis

Assume

1. Two SVs are mounted to a dispenser.
2. Each SV can be modeled as an individual single-degree-of-freedom system (SDOF).
3. Each SV has a mass of 2367 kg.
4. The dispenser has a mass of 379 kg.
5. The dispenser can be modeled as an SDOF.
6. The lateral modes are independent of the axial modes.

Let

m_1 be the mass of the dispenser

m_2 and m_3 be the mass of the respective SVs

k_1 be the lateral stiffness of the dispenser

k_2 and k_3 be the lateral stiffness values of the respective SVs

A diagram is of the SV/dispenser model is shown in Figure 1. Note that is essentially the same model that was used for the axial case. Again, the lateral motion is assumed to be independent of the axial motion.

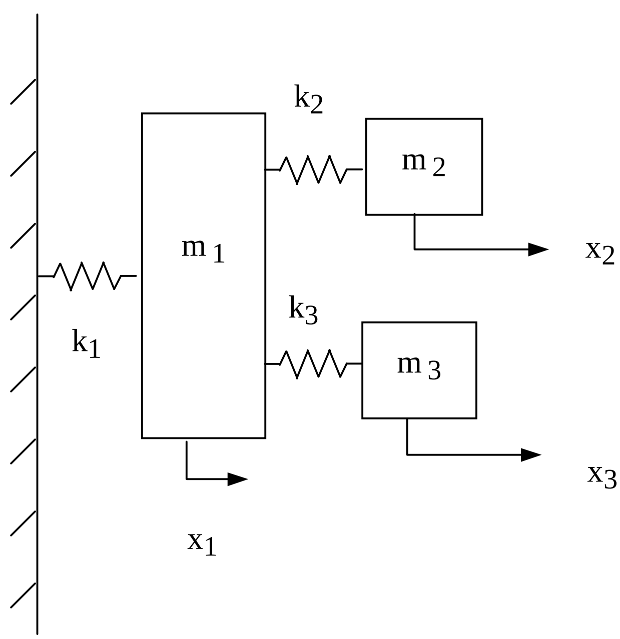

Figure 1. SV/dispenser Model

The equations of motion are

$$\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{bmatrix} + \begin{bmatrix} (k_1 + k_2 + k_3) & -k_2 & -k_3 \\ -k_2 & k_2 & 0 \\ -k_3 & 0 & k_3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{1}$$

Now assume that the mass of the dispenser is 379 kg and that its lateral natural frequency is 100 Hz. Calculate the dispenser stiffness k_1.

$$fn = \frac{1}{2\pi}\sqrt{\frac{k_1}{m_1}} \tag{2}$$

$$k_1 = [2\pi\ fn]^2\ m_1 \tag{3}$$

$$k_1 = [2\pi\ (100\ \text{Hz})\ ]^2\ [379\ \text{kg}\ ] \tag{4}$$

The dispenser stiffness is thus

$$k_1 = 0.148\text{e} + 09\ \ \text{N/m} \tag{5}$$

Now assume the sample values as shown in Table 1.

Table 1. Sample Values

Variable	Value
m1	379 kg
m2	2367 kg
m3	2367 kg
k1	0.148e+09 N/m
k2	k
k3	k

Notes for Table 1.

1. The SVs are assumed to be identical.
2. The dispenser mass is taken as 8% of the total SV mass.
3. The stiffness value k is a dependent variable which must be calculated.

Apply the sample values to the matrix equation.

$$\begin{bmatrix} 379 & 0 & 0 \\ 0 & 2367 & 0 \\ 0 & 0 & 2367 \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{bmatrix} + \begin{bmatrix} [(0.148e+09)+2k] & -k & -k \\ -k & k & 0 \\ -k & 0 & k \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{6}$$

Equation (6) can be transformed into a generalized eigenvalue problem per the method shown in Reference 1. Each eigenvalue represents the square of a natural frequency.

The calculation is carried out via a software program, which implements a polynomial solution.

A trial-and-error approach is used to select the value of k that yields a fundamental natural frequency of 10 Hz.

The resulting lateral stiffness is

$$k = 1.10e+07 \text{ N/m} \tag{7}$$

Thus the SV lateral stiffness requirement is

$$k_2 = 1.10e+07 \text{ N/m} \tag{8}$$

$$k_3 = 1.10e+07 \text{ N/m} \tag{9}$$

Each SV must have a lateral natural frequency as follows.

$$fn = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \tag{10}$$

$$fn = \frac{1}{2\pi}\sqrt{\frac{1.10e+07 \quad N/m}{2367 \quad kg}} \tag{11}$$

$$fn = 10.8 \text{ Hz} \tag{12}$$

Again, the value in equation (12) is the minimum value assuming that the dispenser has a lateral natural frequency of 100 Hz.

This calculation is repeated for a number of frequency cases. The results are summarized in Table 2. The detailed results are given in Table 3.

Table 2.

Individual Lateral Natural Frequencies to Meet SV/dispenser Limit of 10 Hz (Two SV Configuration)

Dispenser fn (Hz)	Individual SV fn (Hz)
40	25.4
50	14.4
60	12.7
70	11.8
80	11.3
100	10.8
200	10.2
∞	10.0

A good design goal for the dispenser would thus be to have a minimum lateral natural frequency of 100 Hz.

Table 3. Parameters to Meet SV/dispenser Lateral Limit of 10 Hz (Two SV Configuration)					
Dispenser			Individual SV		
fn (Hz)	Mass (kg)	Lateral Stiffness (N/m)	fn (Hz)	Mass (kg)	Lateral Stiffness (N/m)
40	379	2.39e+07	25.4	2367	6.00e+07
50	379	3.74e+07	14.4	2367	1.95e+07
60	379	5.39e+07	12.7	2367	1.50e+07
70	379	7.33e+07	11.8	2367	1.30e+07
80	379	9.58e+07	11.3	2367	1.20e+07
100	379	1.48e+08	10.8	2367	1.10e+07
200	379	5.99e+08	10.2	2367	9.80e+06
∞	379	∞	10.0	2367	9.34e+06

Three SV Configuration

The analysis is repeated for a configuration of three SVs mounted to a dispenser in Appendix A.

Reference

1. T. Irvine, The Generalized Coordinate Method for Discrete Systems, Vibrationdata.com Publications, 2000.

APPENDIX A

<u>Three SV Configuration</u>

A diagram is of the SV/dispenser model is shown in Figure A-1.

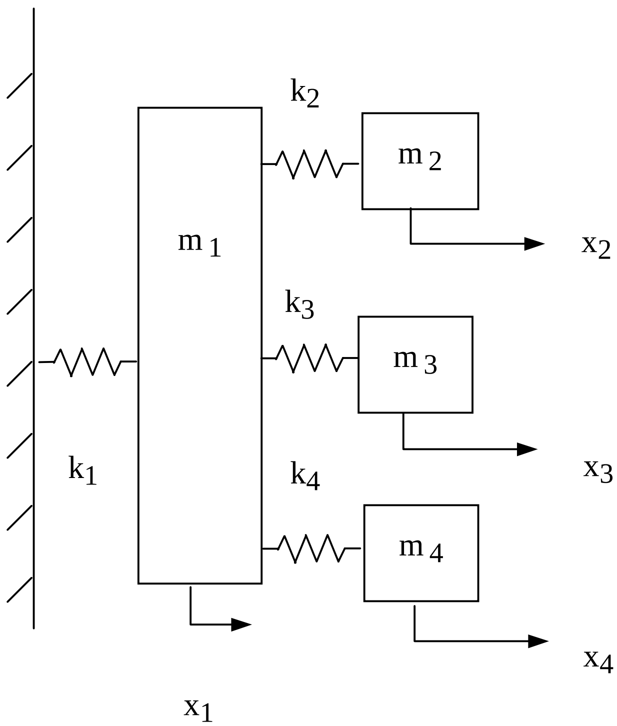

Figure A-1.

The results for the three SV configuration are summarized in Table A-1. Detailed results are given in Table A-2.

Table A-1.

Individual Lateral Natural Frequencies to Meet SV/dispenser Limit of 10 Hz
(Three SV Configuration)

Dispenser fn (Hz)	Individual SV fn (Hz)
40	25.3
50	13.4
60	12.7
70	11.8
80	11.3
100	10.8
200	10.2
∞	10.0

A good design goal for the dispenser would thus be to have a minimum lateral natural frequency of 200 Hz.

Table A-2.

Parameters to Meet SV/dispenser Lateral Limit of 10 Hz (Three SV Configuration)

Dispenser			Individual SV		
fn (Hz)	Mass (kg)	Lateral Stiffness (N/m)	fn (Hz)	Mass (kg)	Lateral Stiffness (N/m)
40	568	3.59e+07	25.3	2367	6.00e+07
50	568	5.61e+07	13.4	2367	1.95e+07
60	568	8.07e+07	12.7	2367	1.50e+07
70	568	1.10e+08	11.8	2367	1.30e+07
80	568	1.44e+08	11.3	2367	1.20e+07
100	568	2.24e+08	10.8	2367	1.10e+07
200	568	8.97e+08	10.2	2367	9.80e+06
∞	568	∞	10.0	2367	9.34e+06