ROCKET VEHICLE LIFTOFF ACOUSTICS
Part I: Acoustic Pressure Level near Nozzle Exit Plane Revision H

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November 9, 2000

Introduction

Rocket motors generate tremendous acoustic energy at liftoff.

Turbulent mixing of the hot exhaust gas with the surrounding air is the dominant acoustic source. The exhaust gas may also have aerodynamic shock waves, which further add to the noise. Combustion instability and rough burning may also contribute to the noise

Consider a rocket vehicle which has a payload enclosed in a nosecone fairing.

The acoustic energy propagates to the payload fairing. The energy is then transmitted through the fairing wall to the enclosed air volume. The payload may be sensitive to the transmitted acoustic excitation, especially if the payload has solar panels or delicate instruments.

The purpose of this report is to present a method for liftoff acoustic analysis. Specifically, this report derives an empirical method for predicting the acoustic pressure level near the nozzle exit plane.

Equations for both the acoustic power and the acoustic pressure are derived. The exhaust flow is assumed to be undeflected.

Note that pressure depends on the distance from the source. On the other hand, power is independent of distance.

Furthermore, there is no reliable equation for relating the acoustic power to the pressure in the near field surrounding the nozzle. The established relationship between acoustic power and pressure is only valid for the free field.

Rocket Power

The power of the exhaust jet P_{jet} is

$$P_{jet} = \frac{1}{2}\dot{m}v^2 \tag{1}$$

$$P_{jet} = \frac{1}{2}Fv \tag{2}$$

$$P_{jet} = \frac{1}{2}\frac{F^2}{\dot{m}} \tag{3}$$

where

$\dot{m}$ is the propellant mass flow rate
v is the constant gas ejection velocity
F is the thrust force

The power transmitted to the vehicle is $P_{vehicle}$ is

$$P_{vehicle} = Fu \tag{4}$$

where u is the vehicle velocity.

Equations (1) through (4) are taken from Reference 1.

Energy Loss Mechanisms

Unfortunately, the combustion process does not occur in an ideal thermodynamic manner. Energy is lost through a variety of mechanisms:

1. Combustion loss due to poor mixing and incomplete burning
2. Heat loss to walls
3. Unavailable thermal energy of exhaust jet
4. Residual kinetic energy of exhaust gases
5. Nozzle liner erosion
6. Mechanical vibration
7. Acoustic energy radiation

The first four loss mechanisms are taken from Reference 1. The fifth through seventh are contributed by the author of this report.

Less than 50% of the total chemical potential energy may be transformed into useful propulsion energy for a given vehicle due to these loss mechanisms.

Acoustic Power

The task is to determine the portion of power transformed into acoustic radiation.

Nigam and Narayanan (Reference 2) cite empirical studies which show that the acoustic power radiation Π_{rad} for a supersonic jet exhaust flow is

$$\Pi_{rad} \cong 0.0025 \left[\frac{\rho_o A}{2} \right] V_c^{\,3} \tag{5}$$

where

ρ_o is the ambient density
A is the nozzle exit area
V_c is the characteristic exhaust velocity

Temporarily alter equation (5) by replacing the scale factor with an unknown factor.

$$\Pi_{rad} \cong \alpha \left[\frac{\rho_o A}{2} \right] v_e^{\,3} \qquad (6)$$

where

α	is a nondimensional empirical factor
v_e	is the actual exhaust velocity

For simplicity, assume that the pressure at the nozzle exit plane equals the ambient pressure so that the effective exhaust velocity is equal to the actual exhaust velocity.

The next task is to derive the empirical factor α. Fulfillment of this task requires a consideration of intensity and pressure. The reason is that existing empirical data is given in terms of pressure rather than acoustical power.

Free-Field Sound Power and Intensity

A free field is a volume in which there are no reflections. Free field propagation is characterized by a 6 dB drop in the sound pressure level and in the intensity level for each doubling of distance. This is the "inverse-square law."

Consider a point source which radiates sound in spherical manner in a free field. The intensity magnitude I is related to the sound power Π_{rad} by

$$|\,I\,| = \frac{\Pi_{rad}}{4\pi r^2} \qquad (7)$$

where r is the radius from the source to the measurement location.

The magnitude symbol on the left side of equation (7) is necessary because intensity is actually a vector.

Note that the denominator in equation (7) is the surface area of a sphere.

Free-Field Pressure and Intensity

The sound intensity is related to the root-mean-square pressure P_{rms} by

$$|\, I \,| = \frac{(P_{rms})^2}{\rho_o\, c} \tag{8}$$

where

ρ_o is the mass density of medium
c is the speed of sound in the medium

Source Sound Pressure

The sound field surrounding a rocket nozzle at liftoff is a near field rather than a free field. Ground reflections may be significant. The reflections depend on the type of exhaust deflector, if any. Furthermore, the radiated acoustic power may vary with direction.

Nevertheless, assume a free field for simplicity. Note that this assumption is only temporary. Equations (6) through (8) can be used to estimate the source sound pressure.

Substitute equation (8) into equation (7).

$$\frac{(P_{rms})^2}{\rho_o c} = \frac{\Pi_{\,rad}}{4\pi r^2} \tag{9}$$

$$(P_{rms})^2 = \frac{\rho_o\, c\, \Pi_{\,rad}}{4\pi r^2} \tag{10}$$

$$P_{rms} = \sqrt{\frac{\rho_o\, c\, \Pi_{\,rad}}{4\pi r^2}} \tag{11}$$

Substitute equation (6) into equation (11).

$$P_{rms} = \sqrt{\frac{\alpha\left[\frac{\rho_o\, A}{2}\right] {v_e}^3\, \rho_o\, c}{4\pi r^2}} \tag{12}$$

$$P_{rms} = \sqrt{\frac{\alpha}{8\pi}}\sqrt{\frac{\rho_o^2\ A\ v_e^3\ c}{r^2}} \tag{13}$$

$$P_{rms} = \sqrt{\frac{\alpha}{8\pi}}\left[\frac{\rho_o}{r}\right]\sqrt{A\,c\,v_e^3} \tag{14}$$

$$P_{rms} = \hat{\alpha}\left[\frac{\rho_o}{r}\right]\sqrt{A\,c\,v_e^3} \tag{15}$$

where

$$\hat{\alpha} = \sqrt{\frac{\alpha}{8\pi}}$$

Solve for the scale factor.

$$\hat{\alpha} = \frac{P_{rms}}{\left[\frac{\rho_o}{r}\right]\sqrt{A\,c\,v_e^3}} \tag{16}$$

<u>Empirical Data</u>

Consider a rocket vehicle launched at sea level. The atmospheric properties are given in Table 1.

Table 1. Atmospheric Parameters	
Speed of Sound in Air	c = 1130 ft / sec
Density of Air at Sea Level	ρ_o = 0.00235 slugs / ft^3
Acoustic Impedance	ρ_o c = 2.656 slugs / (ft^2 sec)

Acoustic performance data for four rocket engines are given in Table 2. In addition, $\hat{\alpha}$ is calculated for each engine.

Table 2. Rocket Engine Performance Values, Single Engine Configuration, Sea Level						
Engine	Acoustic Pressure at 10 inches from Nozzle Exit Plane			Nozzle Exit Area	Actual Exhaust Velocity	$\hat{\alpha}$
	SPL (dB)	Prms (psi)	Prms (psf)	A (ft^2)	Ve (ft/sec)	(per eq. 16)
F-1	164	0.44	63.36	115.8	8533	7.88e-05
J-2	161	0.32	46.08	33.8	11,900	6.44e-05
H-1	161	0.32	46.08	37.3	8310	1.07e-04
PK	159	0.26	37.22	20.2	8940	1.03e-04

The SPL (dB) in Table 2 is referenced to 20 micro Pascals. The equivalent value is 2.9e-09 psi.

The F-1, J-2, and H-1 engines are liquid fuel engines. Data for this set is taken from Reference 3.

The PK is a Peacekeeper solid rocket motor. Data for this motor is taken from Reference 4.

Additional performance values are given in Table 3.

Table 3. Additional Rocket Engine Performance Values, Single Engine Configuration, Sea Level (Reference 1)				
Engine	Average Thrust (lbf)	Specific Impulse (sec)	Weight Flow (lbm/sec)	Exhaust Velocity (ft/sec)
F-1	1,500,000	265	5736	8533
J-2	200,000	370	541	11,900
H-1	165,000	258	640	8310
PK	500,000	278	1800	8940

Note the J-2 is an upper stage engine. The sea level thrust, specific impulse, and exhaust velocity for this engine are estimates.

Statistics for $\hat{\alpha}$ are given in Table 4.

Table 4. Statistics for $\hat{\alpha}$	
Parameter	Value
Average	8.83e-05
Sample Standard Deviation	2.02e-05
P95/50 Upper Limit	1.25e-04

The P95/50 calculation is performed in Appendix A.

The equation for the P95/50 acoustic pressure near the nozzle exit plane is thus

$$P_{rms} = [1.25e-04] \left[\frac{\rho_o}{r}\right] \sqrt{A\, c\, v_e^{\,3}} \qquad \text{for } r = 10 \text{ inches} \tag{17}$$

Again, the variables are:

ρ_o is the mass density of surrounding medium
c is the speed of sound in the surrounding medium
A is the nozzle exit area
v_e is actual exhaust velocity
r is the distance from the nozzle exit plane

Equation (17) assumes an undeflected exhaust flow.

Equation (17) overcomes the free field assumption because it was derived using measured data at a distance of 10 inches from the exit plane. Equation (17) is thus a near field equation at a fixed distance.

Equation (17) may be used to estimate the source acoustic pressure level for a new engine or motor at a distance of 10 inches from the exit plane. Static fire data should be collected to verify the expected value.

The next possible step would be to calculate α from $\hat{\alpha}$. Then, the acoustic power equation, equation (6), could be completed. This step will not be performed, however, since α was derived in the near field. The established relationship between acoustic power and pressure is only valid for the free field, at great distances from the nozzle exit plane.

Acoustic Power and Pressure Relationship

The pressure in a free-field given spherical radiation is

$$P_{rms} = \sqrt{\frac{\rho_o \, c \, \Pi_{rad}}{4\pi r^2}} \tag{18}$$

The acoustic power is thus

$$\Pi_{rad} = \frac{4\pi r^2}{\rho_o \, c} \left(P_{rms}\right)^2 \tag{19}$$

H-1 Engine Acoustic Power

Calculate the acoustic power of the H-1 engine. Reference 3 gives the pressure of the H-1 engine as 0.0048 psi at 10,000 inches. This is equal to 0.69 psf at 833 ft. Note that a free field exists at this distance.

$$\Pi_{rad} = \frac{4\pi\left(833 \text{ ft}\right)^2}{2.656 \dfrac{\text{slug}}{\text{ft}^2 \text{ sec}}} \left(0.69 \frac{\text{lbf}}{\text{ft}^2}\right)^2 \tag{20}$$

$$\Pi_{rad} = 1{,}563{,}000 \ \frac{\text{ft lbf}}{\text{sec}} \tag{21}$$

$$\Pi_{rad} = 2.12 \left(10^6\right) \text{ W} \tag{22}$$

The sound power level PWL is

$$\text{PWL} = 183 \ \text{dB} \tag{23}$$

Referenced to 10e-12 W.

Recall the Nigam and Narayanan method, equation (5), restated as equation (24).

$$\Pi_{rad} \cong 0.0025 \left[\frac{\rho_o \, A}{2}\right] U^3 \tag{24}$$

Solve this equation for the H-1 engine.

$$\Pi_{\text{rad}} \cong 0.0025 \left[\frac{\left(0.00235 \dfrac{\text{slugs}}{\text{ft}^3}\right)\left(37.3 \text{ ft}^2\right)}{2} \right] \left(8310 \frac{\text{ft}}{\text{sec}}\right)^3 \tag{25}$$

$$\Pi_{\text{rad}} = 6.29 \left(10^7\right) \frac{\text{ft lbf}}{\text{sec}} \tag{26}$$

$$\Pi_{\text{rad}} = 8.53\left(10^7\right) \text{ W} \tag{27}$$

The sound power level PWL is

PWL = 199 dB (per Nigam and Narayanan method, Reference: 10e-12 W.)

(28)

Thus, the power level per the Nigam and Narayanan method is 16 dB higher than the power level calculated from measured pressure data.

There are three candidate explanations for the difference.

1. The Nigam and Narayanan method is very conservative.
2. The measured pressure values in Reference 8 were lower than reality.
3. The relationship between acoustic power and pressure for a rocket vehicle environment is more complex than the free field relationship in equation (18).

Alternate Method

Alternate methods for calculating the acoustic power level are given in Appendices B through F. A comparison of these methods is given in Appendix G.

Spectral Shape

Equation (17) does not give the spectral shape of the acoustic pressure, however. An example of an acoustic test spectrum is given in Appendix H for reference.

An acoustic power spectrum as a function of frequency is given in Figure 2 of Reference 6. The spectrum can be adapted for a pressure spectrum. Furthermore, the spectrum is intended for subsonic jets. Nevertheless, its spectral shape qualitatively matches published data for rocket vehicles, including the spectrum given in Appendix E.

The adapted pressure spectrum is shown in Figure 1.

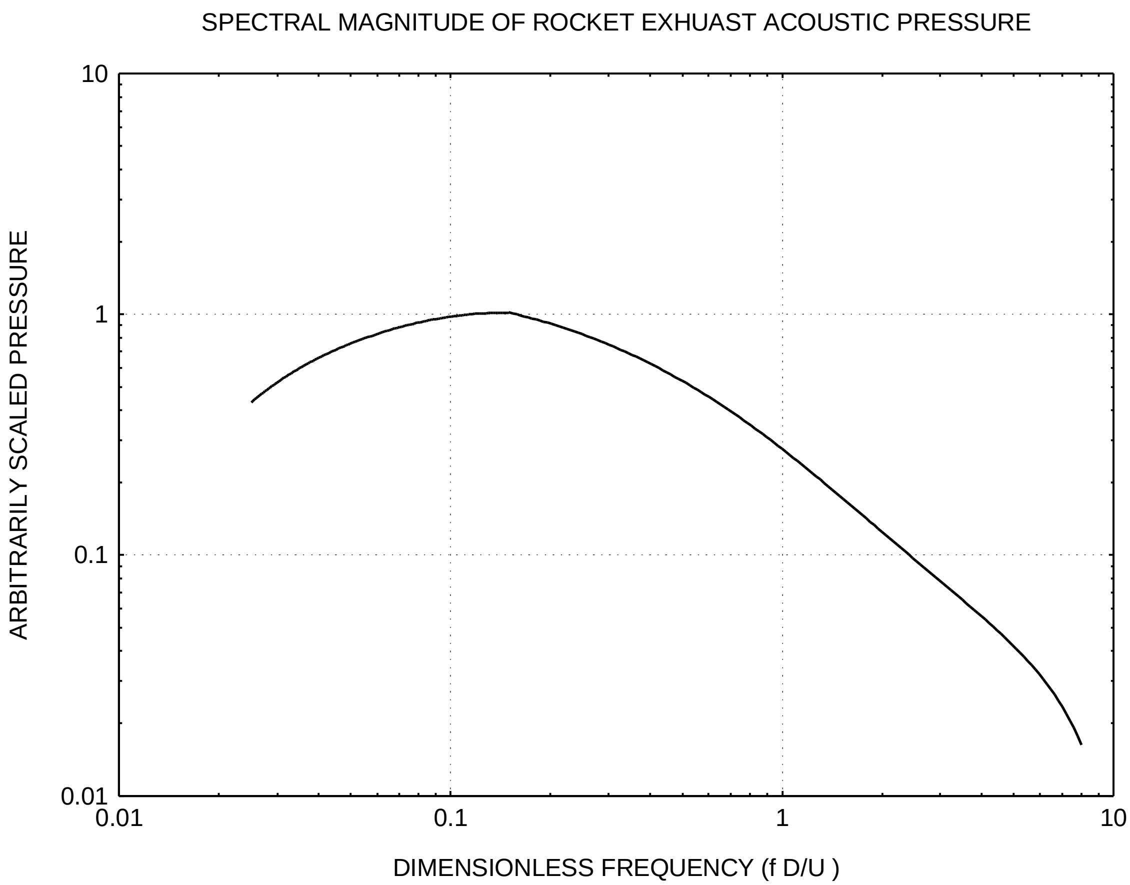

Figure 1.

The curve in Figure is represented by equations (29) and (30).

Define a non-dimensional frequency parameter $\hat{f}$.

$$\hat{f} = \frac{f\,D}{U} \tag{29}$$

where

U is the exhaust velocity
D is the nozzle diameter

Let $\hat{P}$ be an arbitrarily scaled spectral pressure magnitude.

$$\hat{P}(\hat{f}) = \begin{cases} -11.66 - 4.89\,\hat{f} + 14.8\,\hat{f}^{\;0.0521} & , \text{ for } \hat{f} < 0.15 \\ 0.0541 - 0.00543\,\hat{f} + \left(\hat{f} + 0.865\right)^{-2.38} & , \text{ for } \hat{f} \geq 0.15 \end{cases} \tag{30}$$

A scale factor should be applied to $\hat{P}$ so that the overall level matches the expected P_{rms} value. Again, P_{rms} can be calculated from either equation (17) or (18), depending on the field type and know parameters.

The qualitative shape of a pressure spectrum should vary with distance. Specifically, the higher frequency energy should be attenuated by a greater factor than the lower frequency energy. Nevertheless, reference 5 states that "the spectrum does not undergo, in the same progression, as large a shift to lower frequencies as anticipated."

Thus, the spectral curve in Figure 1 may be used for all locations, until attenuation functions can be derived as function of frequency.

PK Pressure Spectrum

Determine the pressure spectrum near the Peacekeeper nozzle exit plane using equation (30). The input parameters are given in Table 4.

Table 4. PK Parameters	
Parameter	Value
nozzle diameter	D = 5.1 ft
exhaust velocity	U = 8940 ft/sec
overall pressure level	Prms = 0.26 psi

The resulting spectrum is shown in Figure 2. Note that it is scaled to give an overall level of 0.26 psi (159 dB).

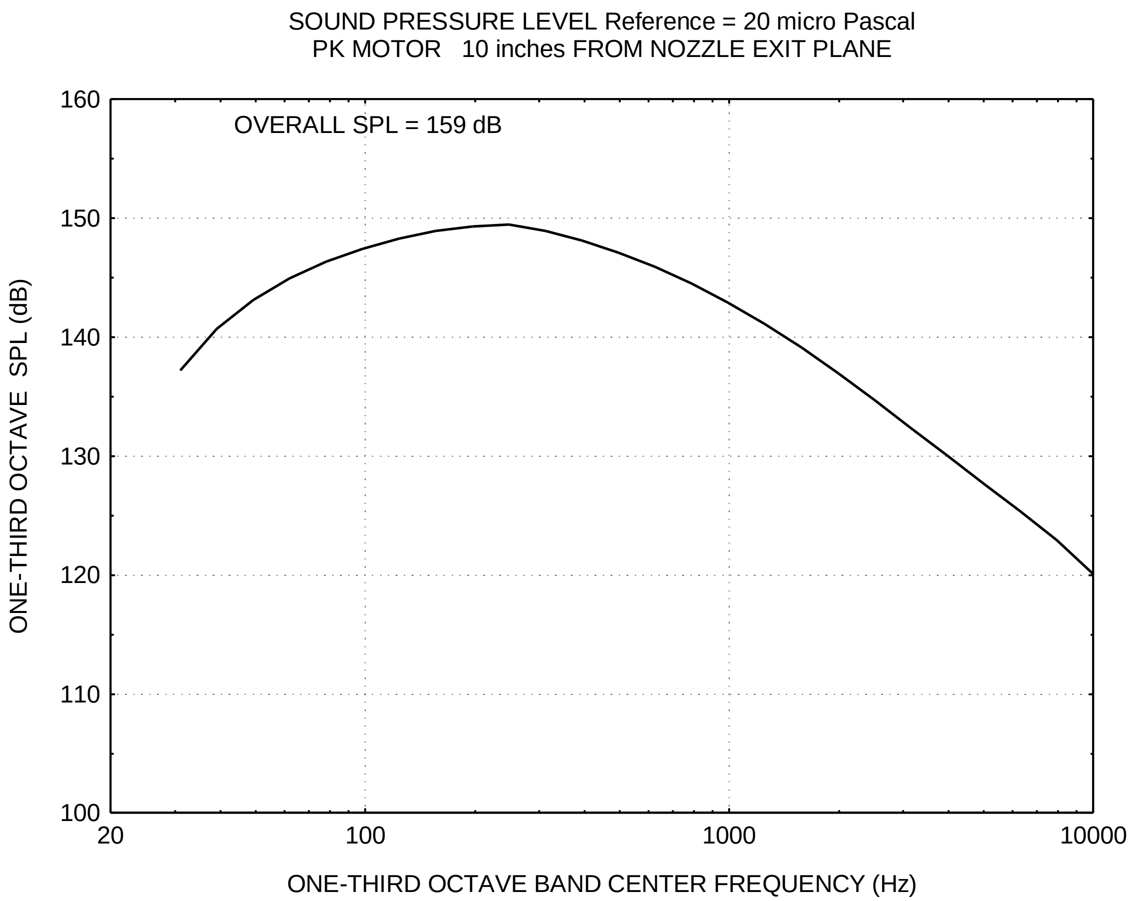

Figure 2.

Intensification

The source energy may be intensified by ground reflections. The amount of intensification depends on the type of flame deflector used, if any. Intensification factors can be estimated using the empirical data in Reference 6.

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APPENDIX A

<u>P95/50 Rule</u>

The P95/50 rule yields the maximum predicted level, which is equal to or greater than the value at the ninety-fifth percentile at least 50 percent of the time.

The "95" in the P95/50 rule is taken as the 95% probability in the Normal distribution. The "50" is the 50% confidence value in the Chi-square distribution.

The P95/50 tolerance value for n = 4 is k = 1.83, from Table A-1.

The tolerance value is applied to the sample standard deviation to yield an estimate of the upper limits as follows:

$$\text{Limit} = \overline{x} + ks \tag{A-1}$$

The mean value $\overline{x}$ is

$$\overline{x} = 8.83\text{e-}05 \tag{A-2}$$

The sample standard deviation s is

$$s = 2.02\text{e-}05 \tag{A-3}$$

The P95/50 limit is

$$\text{Limit} = 8.83\text{e-}05 + (1.83)(2.02\text{e-}05) \tag{A-4}$$

$$\text{Limit} = 1.25\text{e} - 04 \tag{A-5}$$

This statistical method is based on References 7 and 8.

Table A-1. Tolerance Factors from References 7 and 8.

Number	Tolerance Factor
2	2.339
3	1.939
4	1.830
5	1.779
6	1.750
7	1.732
8	1.719
9	1.709
10	1.702
11	1.696
12	1.691
13	1.687
14	1.684
15	1.681

Number	Tolerance Factor
16	1.678
17	1.676
18	1.674
19	1.673
20	1.671
21	1.670
22	1.669
23	1.668
24	1.667
25	1.666
30	1.662
35	1.659
40	1.658
∞	1.64485

APPENDIX B

Condos and Butler Method

Condos and Butler (Reference 9 and 10) give the following empirical formula for the sound power level PWL in dB.

$$PWL = 78 + 13.5 \log \left[\frac{21.8\, T^2}{W} \right] \qquad \text{(B-1)}$$

where

T is the thrust in lbf
W is the exhaust weight flow rate in lbm/sec

Condos and Butler failed to give a zero dB reference value. There are two candidate reference values: 10e-12 W and 10e-13 W. A reference of 10e-13 W is assumed based on a similar formula in Appendix C.

Apply equation (B-1) to the H-1 engine.

Table B-1 H-1 Engine Parameters				
Engine	Average Thrust (lbf)	Specific Impulse (sec)	Weight Flow (lbm/sec)	Exhaust Velocity (ft/sec)
H-1	T = 165,000	Isp = 258	W = 640	Ve = 8310

$$PWL = 78 + 13.5 \log \left[\frac{21.8\, [\, 165{,}000\,]^2}{640} \right] \qquad \text{(B-2)}$$

$$PWL = 199 \ \text{dB}, \qquad \text{Ref} = 10\text{e-}13 \text{ W} \qquad \text{(B-3)}$$

The acoustic power is

$$\Pi_{rad} = 7.94\, (10^6)\ \text{W} \qquad \text{(B-4)}$$

Recall that the Nigam and Narayanan method gave a power level of PWL = 199 dB in the main text. This value agrees with the Condos and Butler PWL.

APPENDIX C

Richards and Mead Method

Richards and Mead (Reference 11) give the following formula for the acoustic power radiated by a supersonic exhaust stream.

$$PWL = 78 + 13.5 \log\left[0.67 \frac{W V^2}{G}\right] \tag{C-1}$$

where

W is the the exhaust weight flow rate in lbm/sec

V is the exhaust velocity in feet/sec

G is the acceleration of gravity

Furthermore, the zero dB reference is given as 10e-13 W.

Apply equation (C-1) to the H-1 engine.

Table C-1 H-1 Engine Parameters

Engine	Average Thrust (lbf)	Specific Impulse (sec)	Weight Flow (lbm/sec)	Exhaust Velocity (ft/sec)
H-1	T = 165,000	Isp = 258	W = 640	Ve = 8310

$$PWL = 78 + 13.5 \log\left[0.67 \frac{(640)(8310)^2}{(32.2)}\right] \tag{C-2}$$

$$PWL = 199 \text{ dB}, \qquad \text{Ref} = 10\text{e-}13 \text{ W} \tag{C-3}$$

The acoustic power is

$$\Pi_{rad} = 7.94\ (10^6)\ W \tag{C-4}$$

Let $G = 32.2\ \mathrm{ft/sec}^2$. Equation (C-1) becomes.

$$PWL = 78 + 13.5 \log \left[0.0208\ W\ V^2 \right] \qquad \text{(C-5)}$$

The Richards and Mead equation (C-5) is equivalent to Condos and Butler equation (B-1).

Note that

$$0.0208\ W\ V^2 \cong \frac{21.8\ T^2}{W} \qquad \text{(C-6)}$$

$$0.0208\ W^2\ V^2 \cong 21.8\ T^2 \qquad \text{(C-7)}$$

$$\frac{1}{1048}\ W^2\ V^2 \cong T^2 \qquad \text{(C-8)}$$

$$\frac{1}{32.3}\ W\ V \cong T \qquad \text{(C-9)}$$

$$T \cong \frac{1}{32.3}\ W\ V \qquad \text{(C-10)}$$

$$T = \frac{1}{G}\ W\ V \qquad \text{(C-11)}$$

Again, the variables are in terms of customary English units.

APPENDIX D

McDonnell Douglas Astronautics

The following is taken from a McDonnell Douglas Astronautics document, Reference 12.

A free jet from a single nozzle has an acoustics power ≈ 0.5% of the mechanical power.

The sound power level PWL is

$$\mathrm{PWL} = 120.4 + 10 \log\left[T \; Isp \right] \tag{D-1}$$

Referenced to 10e-13 W.

where

T is the thrust in lbf

Isp is the specific impulse in seconds

Table D-1 H-1 Engine Parameters				
Engine	Average Thrust (lbf)	Specific Impulse (sec)	Weight Flow (lbm/sec)	Exhaust Velocity (ft/sec)
H-1	T = 165,000	Isp = 258	W = 640	Ve = 8310

$$\mathrm{PWL} = 120.4 + 10 \log\left[(165{,}000)(258) \right] \tag{D-2}$$

$$\mathrm{PWL} = 197 \text{ dB}, \quad \text{Ref} = 10\text{e-}13 \text{ W} \tag{D-3}$$

The acoustic power is

$$\Pi_{rad} = 4.67\,(10^{6}) \text{ W} \tag{D-4}$$

An equivalent formula is

$$PWL = 120.4 + 10 \log\left[T \frac{Ve}{G} \right] \qquad \text{(D-5)}$$

where

Ve is the exhaust velocity

G is the acceleration of gravity

APPENDIX E

Potter and Crocker Method

The following is taken from Reference 13.

The sound power level PWL for a rocket engine is

$$PWL = 68 + 13.5 \log\left[0.91\, T \; Ve \right] \quad \text{(E-1)}$$

Referenced to 10e-12 W.

where

T is the thrust in N

Ve is the exhaust velocity in m/sec

Apply equation (E-1) to the H-1 engine.

Table E-1 H-1 Engine Parameters				
Engine	Average Thrust (lbf)	Specific Impulse (sec)	Weight Flow (lbm/sec)	Exhaust Velocity (ft/sec)
H-1	T = 165,000	Isp = 258	W = 640	Ve = 8310

T = 165,000 lbf (7.34e+05 N)

Ve = 8310 ft/sec (2533 m/sec)

The sound power level is

$$PWL = 68 + 13.5 \log\left[0.91\,(7.34e+05)(2533) \right] \quad \text{(E-2)}$$

$$PWL = 193 \text{ dB}, \qquad \text{Ref} = 10\text{e-}12 \text{ W} \quad \text{(E-3)}$$

$$\Pi_{rad} = 2.00\,(10^{7})\text{ W} \quad \text{(E-4)}$$

Eldred Method

Reference 13 also states that equation (E-1) is known to over-predict the sound power level for rocket engines with thrust F > 500 kN (112,000 lbf). An alternate equation is recommend for engines with higher thrust levels.

The sound power level PWL for larger rocket engines is

$$\text{PWL} = 120 + 10 \log\left[\, 0.005\ \text{T Ve} \,\right] \tag{E-5}$$

Referenced to 10e-12 W.

where

T is the thrust in N

Ve is the exhaust velocity in m/sec

Apply equation (E-1) to the H-1 engine.

T = 165,000 lbf (7.34e+05 N)

Ve = 8310 ft/sec (2533 m/sec)

The acoustic power level is

$$\text{PWL} = 120 + 10 \log\left[\, 0.005\,(7.34\text{e}+05)(2533) \,\right] \tag{E-6}$$

$$\text{PWL} = 190 \text{ dB}, \qquad \text{Ref} = 10\text{e-}12 \text{ W} \tag{E-7}$$

$$\Pi_{rad} = 9.30\ (10^{6})\ \text{W} \tag{E-8}$$

APPENDIX F

<u>Gierke Method</u>

The following is taken from Reference 14.

The sound power level PWL for a rocket engine is

$$PWL = 68 + 13.5 \log\left[0.676 \frac{W\,Ve^2}{G} \right] \qquad \text{(F-1)}$$

Referenced to 10e-12 W.

where

W is the weight flow in lbm/sec

Ve is the exhaust velocity in ft/sec

G is the gravitational acceleration (32.2 ft/sec^2)

Table F-1 H-1 Engine Parameters				
Engine	Average Thrust (lbf)	Specific Impulse (sec)	Weight Flow (lbm/sec)	Exhaust Velocity (ft/sec)
H-1	T = 165,000	Isp = 258	W = 640	Ve = 8310

$$PWL = 68 + 13.5 \log\left[0.676 \frac{(640)(8310)^2}{(32.2)} \right] \qquad \text{(F-2)}$$

$$PWL = 189 \text{ dB}, \quad \text{Ref} = 10\text{e-}12 \text{ W} \qquad \text{(F-3)}$$

The acoustic power is

$$\Pi_{rad} = 7.94\left(10^6\right) W \qquad \text{(F-4)}$$

APPENDIX G

Comparison of Methods

A comparison of the methods for predicting acoustic power is given in Table G-1. The methods are ranked in order of highest power prediction.

Table G-1. Comparison of Methods using H-1 Engine Data

Method	Π_{rad} (MW)	PWL (dB) Ref = 10e-12 W
Nigam and Narayanan	85.3	199
Potter and Crocker	20.0	193
Eldred	9.30	190
Condos and Butler	7.94	189
Gierke	7.94	189
Richards and Mead	7.94	189
McDonnell Douglas	4.67	187
Power calculated from Barrett's Pressure Data	2.12	183

The Nigam and Narayanan method is excessively high. Further effort is needed to understand the basis and assumptions of this method.

The Condos and Butler, the Gierke, and the Richards and Mead methods gave the same result.

The "exact" power value is unknown. Acoustic power can only be calculated by indirect means.

The author's opinion is that the most accurate acoustic power value is the value calculated from Barrett's pressure data. The McDonnell Douglas method gives the closest power level relative to the Barrett level.

The McDonnell Douglas method thus tentatively appears to be the best method for calculating acoustic power based on engine performance.

Again, this conclusion is based on data from only one engine, the H-1. Further research is needed.

Note that some of the methods may include unspecified safety factors.

APPENDIX H

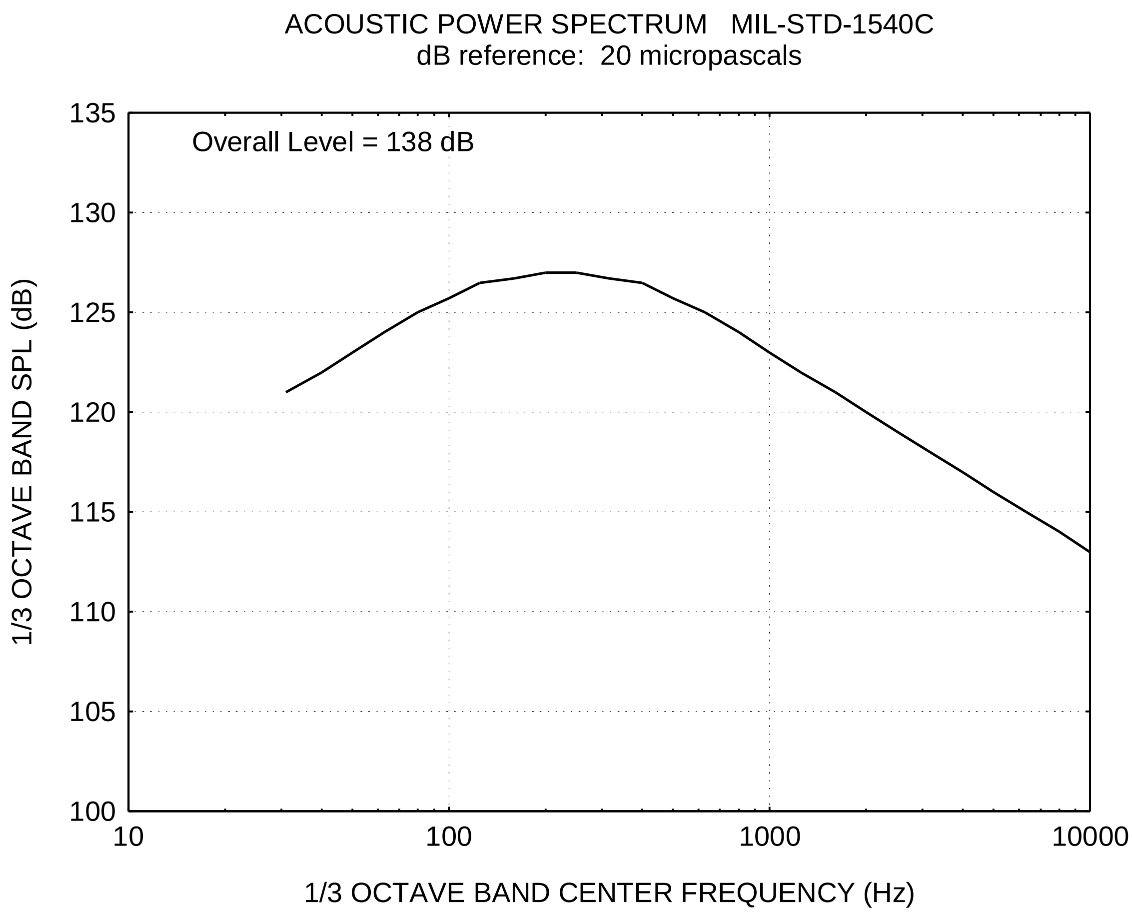

Figure F-1.

Table F-1. Coordinates for Acoustic Power Spectrum

1/3 Octave Center Frequency (Hz)	Sound Pressure Level (dB)
31	121
40	122
50	123
63	124
80	125
100	125.7
125	126.5
160	126.7
200	127
250	127
315	126.7
400	126.5
500	125.7
630	125
800	124
1000	123
1250	122
1600	121
2000	120
2500	119
3150	118
4000	117
5000	116
6300	115
8000	114
10000	113