LINE LOADS IN CYLINDRICAL SHELLS

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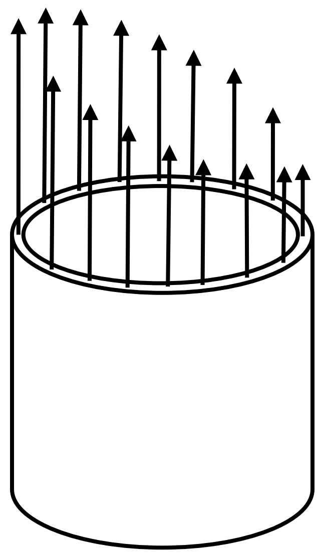

Figure 1. Axial Line load at an Instant in Time

The axial force is constant around the edge. The moment varies around the edge. The combined effect is shown in Figure 1.

Variables

A	=	Area	T	=	Torque
I	=	Area moment of inertia	V	=	Shear force
J	=	Polar moment of inertia	r	=	Inner radius
M	=	Bending moment	dr	=	Thickness
P	=	Axial Force	σ	=	Normal Stress
R	=	Outer radius	τ	=	Shear stress

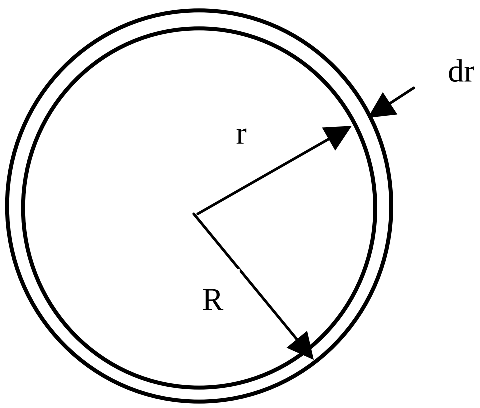

Figure 2.

Area

$$A = \pi\left(R^2 - r^2\right) \tag{1}$$

$$R = r + dr \tag{2}$$

$$r = R - dr \tag{3}$$

$$A = \pi\left(R^2 - (R - dr)^2\right) \tag{4}$$

$$A = \pi\left(R^2 - R^2 + 2Rdr - dr^2\right) \tag{5}$$

$$A = \pi\left(2Rdr - dr^2\right) \tag{6}$$

$$A = \pi\, dr\,(2R - dr) \tag{7}$$

For a thin-wall cylinder,

$$dr << R$$

$$A = 2\pi R\, dr \tag{8}$$

Area Moment of Inertia

$$I = \frac{\pi}{4}\left(R^4 - r^4\right) \tag{9}$$

$$I = \frac{\pi}{4}\left(R^4 - (R - dr)^4\right) \tag{10}$$

$$I = \frac{\pi}{4}\left(R^4 - \left(R^2 - 2Rdr + dr^2\right)^2 \right) \tag{11}$$

$$I = \frac{\pi}{4}\left(R^4 - \left(R^4 - 4R^3dr + 2R^2dr^2\right)\right)$$
$$-\frac{\pi}{4}\left(4R^2dr^2 - 4Rdr^3 + dr^4\right) \tag{12}$$

$$I = \frac{\pi}{4}\left(4R^3dr - 2R^2dr^2 - 4R^2dr^2 + 4Rdr^3 - dr^4\right) \tag{13}$$

$$I = \frac{\pi dr}{4}\left(4R^3 - 2R^2dr - 4R^2dr + 4Rdr^2 - dr^3\right) \tag{14}$$

$$I = \frac{\pi dr}{4}\left(4R^3 - 6R^2dr + 4Rdr^2 - dr^3\right) \tag{15}$$

For a thin-wall cylinder,

$$dr << R$$

$$I = \pi R^3 dr \tag{16}$$

<u>Polar Moment of Inertia</u>

$$J = I_y + I_z = I + I \tag{17}$$

$$J = 2\pi R^3 dr \tag{18}$$

<u>Axial Line Load</u>

The maximum normal stress is the combination due to axial and moment loads.

$$\sigma = \frac{P}{A} + \frac{MR}{I} \tag{19}$$

Substituting for the area A and moment of inertia I from the above equations gives

$$\sigma = \frac{P}{2\pi R\, dr} + \frac{MR}{\pi R^3 dr} \tag{20}$$

Multiplying both sides of the equation by dr gives the maximum axial line load defined as the normal stress per section thickness.

$$\sigma\, dr = \frac{P}{2\pi R} + \frac{MR}{\pi R^3} \tag{21}$$

The dimension is [force/length].

Shear Line Load

The maximum direct shear stress for a hollow circular section is given by Reference 1.

$$\tau_{d,max} = \frac{2V}{A} \tag{22}$$

The maximum shear stress is the combination of direct shear and torsional shear.

$$\tau = \frac{2V}{A} + \frac{TR}{J} \tag{23}$$

Substituting for the area A and polar moment of inertia J from the above equations gives

$$\tau = \frac{V}{\pi R\, dr} + \frac{T}{\pi R^2\, dr} \tag{24}$$

Multiplying both sides of the equation by dr gives the maximum shear line load defined as the shear stress per section thickness.

$$\tau\, dr = \frac{V}{\pi R} + \frac{T}{\pi R^2} \tag{25}$$

Reference

1. R. Huston and H. Josephs, Practical Stress Analysis in Engineering Design, Dekker, CRC Press, 2008. See Table 13.1.