MATLAB ODE45 FOR COUPLED DYNAMIC SYSTEMS
Revision A

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Introduction

Matlab has a function ODE45 for solving initial value problems for ordinary differential equations. This function implements a Runge-Kutta method with a variable time step for efficient computation.

Consider a general two-degree-of-freedom system.

$$\begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} \quad (1)$$

$$\begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} - \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad (2)$$

$$\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} = \left\{ \mathrm{inv}\begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \right\} \left\{ \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} - \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \right\} \quad (3)$$

Let

$$y_1 = \dot{x}_1$$
$$y_2 = \dot{x}_2$$

$$\begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{bmatrix} = \left\{ \mathrm{inv}\begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \right\} \left\{ \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} - \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} + \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \right\} \quad (4)$$

Let

$$M = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \ , \quad C = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} \ , \quad K = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \ , \quad F = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}$$

$$\begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{bmatrix} = M^{-1} \left\{ F - C \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} + K \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \right\} \tag{5}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ \dot{y}_1 \\ \dot{y}_2 \end{bmatrix} = \begin{bmatrix} 0 \\ M^{-1}F \end{bmatrix} + \begin{bmatrix} 0 & I \\ M^{-1}K & M^{-1}C \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ y_1 \\ y_2 \end{bmatrix} \tag{6}$$

<u>References</u>

1. T. Irvine, The State Space Method for Solving Shock and Vibration Problems, Revision A, Vibrationdata, 2005.

2. T. Irvine, Free Vibration of a Two-Degree-of-Freedom System Subjected to Initial Velocity and Displacement, Revision B, Vibrationdata, 2010.

APPENDIX A

Example

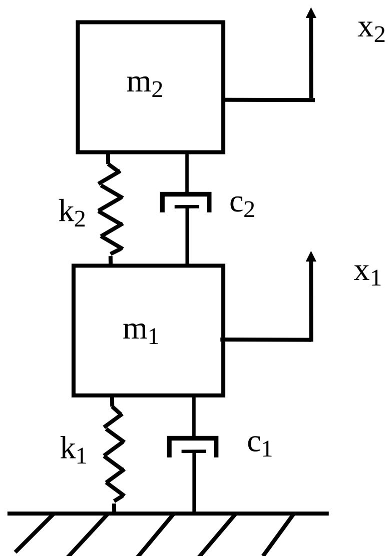


Figure A-1.

m_1	=	2.0 lbm
m_2	=	1.0 lbm
c_1	=	0.4 lbf sec/in
c_2	=	0.6 lbf sec/in
k_1	=	15,000 lbf/in
k_2	=	10,000 lbf/in

The initial displacements are

x_1	=	0.002 in
x_2	=	0.001 in

The initial velocity is zero for each mass.

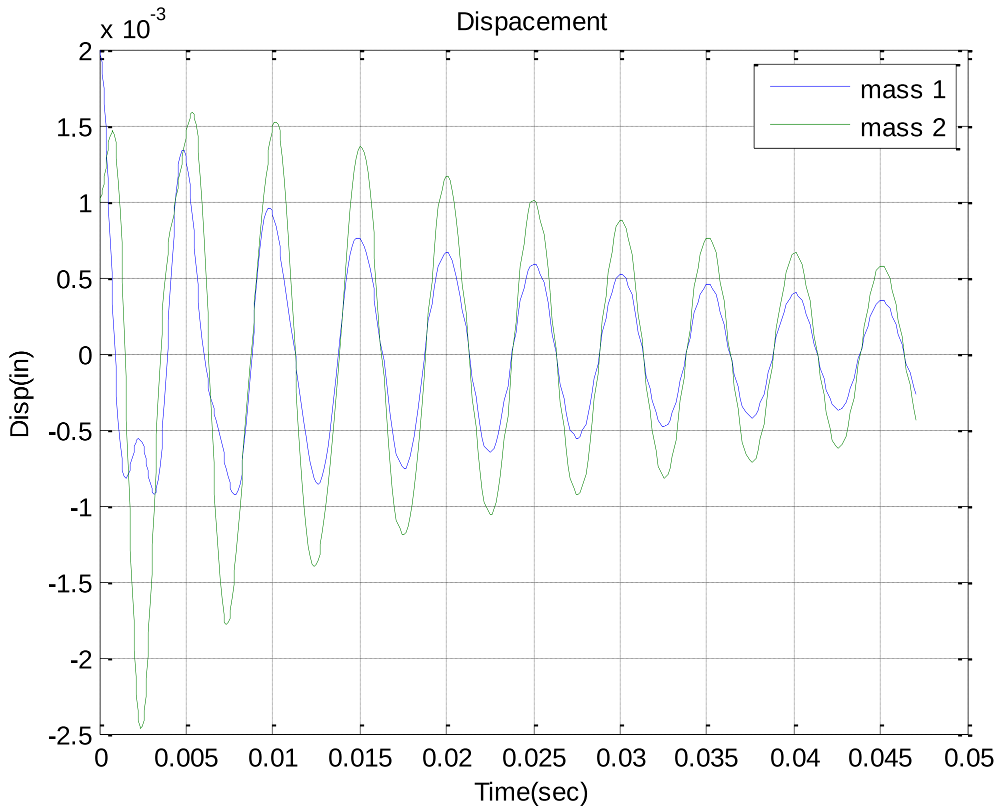
Assemble the equations in matrix form.

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix}+\begin{bmatrix} c_1+c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix}\begin{bmatrix} \dot{x}_1 \\ \ddot{x}_2 \end{bmatrix}+\begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{A-1}$$

$$\begin{bmatrix} 2/386 & 0 \\ 0 & 1/386 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix}+\begin{bmatrix} 1.0 & -0.6 \\ -0.6 & 0.6 \end{bmatrix}\begin{bmatrix} \dot{x}_1 \\ \ddot{x}_2 \end{bmatrix}+\begin{bmatrix} 25{,}000 & -10{,}000 \\ -10{,}000 & 10{,}000 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{A-2}$$

The system is solved using Matlab script: mdof_ode45.m.

The results are shown in the following figures.



Dispacement
x 10^-3
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
Disp(in)
0
0.005
0.01
0.015
0.02
0.025
0.03
0.035
0.04
0.045
0.05
Time(sec)
mass 1
mass 2

Figure A-2.

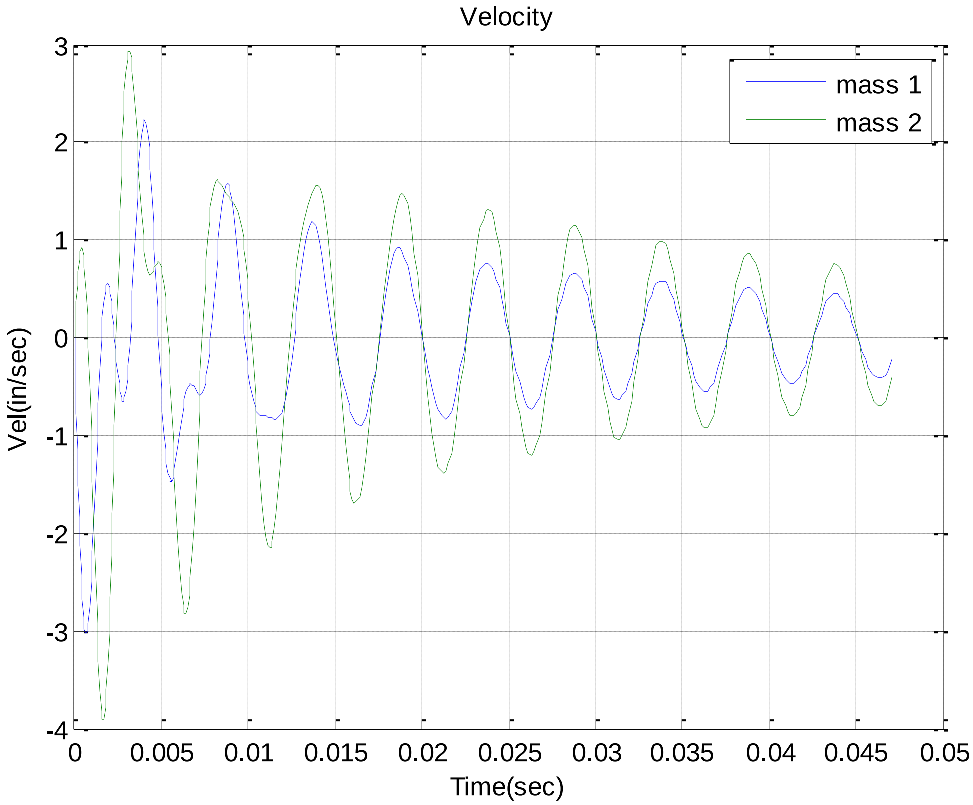

Velocity
mass 1
mass 2
Vel(in/sec)
Time(sec)
3
2
1
0
-1
-2
-3
-4
0
0.005
0.01
0.015
0.02
0.025
0.03
0.035
0.04
0.045
0.05

Figure A-3.

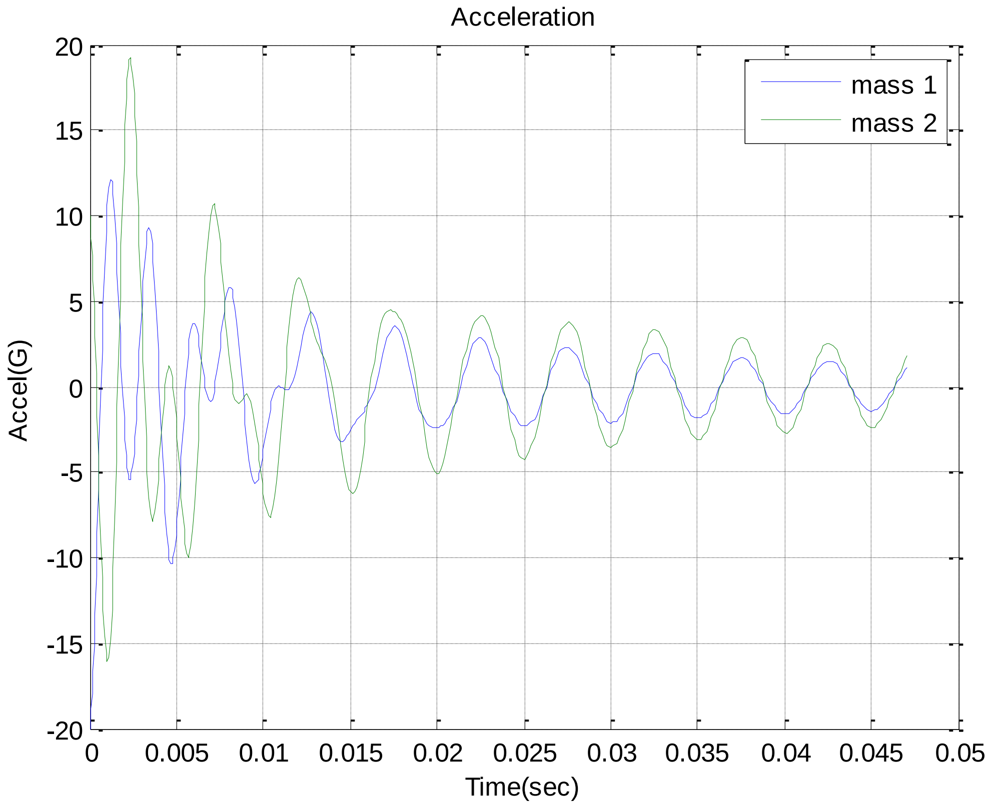

Acceleration
mass 1
mass 2
Accel(G)
Time(sec)
20
15
10
5
0
-5
-10
-15
-20
0
0.005
0.01
0.015
0.02
0.025
0.03
0.035
0.04
0.045
0.05

Figure A-4.