FLAT PANEL FLUTTER IN SUPersonic FLOW

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Introduction

Panel flutter is a self-excited, dynamic, aeroelastic instability.

It occurs most frequently in supersonic flow. The instability at subsonic speeds takes the form of a static divergence or aeroelastic buckling.

Initially, the amplitude of motion on an unstable panel increases exponentially with time. Then, structural nonlinearities associated with the lateral deformations of the panel limit the flutter amplitude.

Panel flutter may result in a fatigue failure. It may also cause functional failure of equipment attached to the panel.

Variables

- $c$ is the speed of sound
- $\gamma$ is the adiabatic constant of air
- $L_x$ is the length along the flow vector
- $\nu$ is the Poisson ratio
- $L_y$ is the width
- $\mu$ is the air density to plate density ratio
- $D$ is the panel stiffness parameter
- $\rho_{\infty}$ is the air density
- $h$ is the panel thickness
- $\omega$ is the angular frequency
- $j = \sqrt{-1}$
- $p_{\infty}$ is the free stream static pressure
- $W$ is the out-of-plane displacement
- $M$ is the Mach number
- $M_{cr}$ is the critical Mach number
- $U$ is the aero flow speed
Governing Equation

Assume that linear piston theory can be used, and that the Mach number $M > 1.7$.

The panel flutter eigenvalue problem is

$$D \left( \frac{\partial^4}{\partial x^4} + 2 \frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4} \right) W(x, y) - \omega^2 W(x, y)$$

$$+ \frac{\mu M}{\sqrt{M^2 - 1}} \left( -j \omega W(x, y) + M \frac{\partial W(x, y)}{\partial x} \right) = 0$$

(1)

Note that the plate stiffness factor $D$ is given by

$$D = \frac{Eh^3}{12 (1 - v^2)}$$

(2)

The eigenvalue problem is solved by applying the appropriate boundary conditions.

Simply-Supported Panel

Consider a panel that is simply supported along all sides. The theoretical formula for this case is taken from Reference 1.

Panel flutter occurs when the Mach number $M$ is greater than the critical Mach number $M_{cr}$.

$$M > M_{cr} = \frac{D}{\gamma p_{\infty} L_x^3} \frac{8 \pi^3}{3 \sqrt{3}} \left( \left( 5 + \frac{L_x^2}{L_y^2} \right) \sqrt{2 + \frac{L_x^2}{L_y^2}} \right)$$

(3)

Note that

$$\gamma p_{\infty} = \rho_{\infty} c^2$$

(4)
Thus

\[
M > M_{cr} = \frac{D}{\rho_\infty c^2} \frac{8\pi^3}{L_x^3} \frac{\left(5 + \frac{L_x}{L_y} \right)^2}{3\sqrt{3}} \sqrt{2 + \frac{L_x^2}{L_y^2}}
\]  \tag{5}

Again, equation (5) is for the case of a panel simply-supported along all edges.

Reference (9) gives a formula equivalent to equation (5) for the case of a semi-infinite simply-supported panel. This is a panel which has simply-supported boundary conditions along each edge perpendicular to the air flow. The two edges parallel to the air flow are free.

Further consideration is needed to determine how the same critical Mach number formula can apply to two boundary cases.

**Example 1**

Equation (5) is implemented via a Matlab script for the following example.

```matlab
>> ss_panel_flutter

ss_panel_flutter.m ver 1.0 August 24, 2010
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Movchan (157) formula for a panel simply supported on all edges.

Enter material:
1=aluminum  2=steel  3=other 1

Enter length (in) 36
Enter width (in) 24
Enter thickness (in) 0.125

Enter the altitude (feet) 14000

Plate Stiffness Factor = 1789 lbf in

Speed of sound = 1.273e+004 in/sec
    = 1061 ft/sec

Air density = 7.463e-008 lbf sec^2/in^4
    = 2.881e-005 lbm/in^3

\[\rho_c2 = 12.1 \text{ lbf/in}^2\]

Mcr = 2.261
Conclusion

Equation (5) does not account for membrane stress and thermoelastic effects. An empirical method which appears to be more conservative is given in Appendix A.

References

APPENDIX A

Empirical Method

The following method is taken from References 4 and 5.

Figure A-1. Flight Test Results

The curve in Figure A-1 was based on experimental and flight data. Reference 4 noted:

1. The curve becomes meaningless near Mach one.
2. The boundary layer was relatively thin.
3. Differential pressure across the panel was zero.

The flutter parameter effectively requires $M > \sqrt{2}$
Example 1

The method in Figure A-1 is applied via a Matlab script.

Figure A-2.

```matlab
>> vsa_panel_flutter
vsa_panel_flutter.m   ver 1.0   August 23, 2010
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NASA TN D-1058 Method
Enter Length (inch) 36
Enter Width (inch) 24
Enter Thickness (inch) 0.125
Enter Mach number 1.5
Enter dynamic pressure (psf) 1500

Enter material:
1=aluminum  2=steel  3=other 1

Flutter Parameter = 0.3555   Upper Limit = 0.4245
Flutter Zone
```
Example 2

Figure A-3.

>> vsa_panel_flutter

vsa_panel_flutter.m  ver 1.0  August 23, 2010
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NASA TN D-1058 Method

Enter Length (inch) 36
Enter Width (inch) 24
Enter Thickness (inch) 0.1875
Enter Mach number 1.5
Enter dynamic pressure (psf) 1500
Enter material: 1=aluminum  2=steel  3=other 1

Flutter Parameter =  0.5333  Upper Limit =  0.4245

No Flutter Zone