FLAT PANEL FLUTTER IN SUPERSONIC FLOW

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Introduction

Panel flutter is a self-excited, dynamic, aeroelastic instability.

It occurs most frequently in supersonic flow. The instability at subsonic speeds takes the form of a static divergence or aeroelastic buckling.

Initially, the amplitude of motion on an unstable panel increases exponentially with time. Then, structural nonlinearities associated with the lateral deformations of the panel limit the flutter amplitude.

Panel flutter may result in a fatigue failure. It may also cause functional failure of equipment attached to the panel.

Variables

c	is the speed of sound	γ	is the adiabatic constant of air
L_x	is the length along the flow vector	ν	is the Poisson ratio
L_y	is the width	μ	is the air density to plate density ratio
D	is the panel stiffness parameter	ρ_∞	is the air density
E	is the elastic modulus	ω	is the angular frequency
h	is the panel thickness		
j	$= \sqrt{-1}$		
p_∞	is the free stream static pressure		
W	is the out-of-plane displacement		
M	is the Mach number		
M_{cr}	is the critical Mach number		
U	is the aero flow speed		

Governing Equation

Assume that linear piston theory can be used, and that the Mach number $M > 1.7$.

The panel flutter eigenvalue problem is

$$D\left(\frac{\partial^4}{\partial x^4} + 2\frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4}\right)W(x,y) - \omega^2 W(x,y)$$
$$+ \frac{\mu M}{\sqrt{M^2 - 1}}\left(-j\omega W(x,y) + M\frac{\partial W(x,y)}{\partial x}\right) = 0 \tag{1}$$

Note that the plate stiffness factor D is given by

$$D = \frac{Eh^3}{12\,(1-\nu^2)} \tag{2}$$

The eigenvalue problem is solved by applying the appropriate boundary conditions.

Simply-Supported Panel

Consider a panel that is simply supported along all sides. The theoretical formula for this case is taken from Reference 1.

Panel flutter occurs when the Mach number M is greater than the critcal Mach number M_{cr}.

$$M > M_{cr} = \frac{D}{\gamma p_\infty\ L_x^3}\frac{8\pi^3}{3\sqrt{3}}\left(5 + \frac{L_x^2}{L_y^2}\right)\sqrt{2 + \frac{L_x^2}{L_y^2}} \tag{3}$$

Note that

$$\gamma p_\infty = \rho_\infty c^2 \tag{4}$$

Thus

$$M > M_{cr} = \frac{D}{\rho_\infty c^2 \; L_x^3} \frac{8\pi^3}{3\sqrt{3}} \left(5 + \frac{L_x^2}{L_y^2} \right) \sqrt{2 + \frac{L_x^2}{L_y^2}} \tag{5}$$

Again, equation (5) is for the case of a panel simply-supported along all edges.

Reference (9) gives a formula equivalent to equation (5) for the case of a semi-infinite simply-supported panel. This is a panel which has simply-supported boundary conditions along each edge perpendicular to the air flow. The two edges parallel to the air flow are free.

Further consideration is needed to determine how the same critical Mach number formula can apply to two boundary cases.

Example 1

Equation (5) is implemented via a Matlab script for the following example.

```
>> ss_panel_flutter

 ss_panel_flutter.m  ver 1.0   August 24, 2010
 by Tom Irvine

 Movchan (157) formula for a panel simply supported on all edges.

 Enter material:
 1=aluminum  2=steel  3=other 1

 Enter length (in) 36
 Enter width (in) 24
 Enter thickness (in) 0.125

 Enter the altitude (feet) 14000

  Plate Stiffness Factor =     1789 lbf in

  Speed of sound = 1.273e+004 in/sec
                 =     1061 ft/sec

     Air density = 7.463e-008 lbf sec^2/in^4
                 = 2.881e-005 lbm/in^3

          rho_c2 =     12.1 lbf/in^2

 Mcr =    2.261
```

Conclusion

Equation (5) does not account for membrane stress and thermoelastic effects.

An empirical method which appears to be more conservative is given in Appendix A.

References

1. Vedeneev, V.V., et al., Experimental Observation of Single Mode Panel Flutter in Supersonic Gas Flow, Journal of Fluids and Structures, 2010.
2. Bisplinghoff and Ashley, Principles of Aeroelasticity, Dover, New York, 1975.
3. NASA SP-8004, Panel Flutter, 1972.
4. Vibration, Shock, and Acoustics; McDonnell Douglas Astronautics Company, Western Division, 1971.
5. NASA TN D-1058, Flight Flutter Results for Flat Rectangular Panels, 1962.
6. J. Wright and J. Cooper, Introduction to Aircraft Aeroelasticity, AIAA Education Series, 2007.
7. T. Irvine, Lagrange's Equations, Vibrationdata, 1999.
8. T. Irvine, Linear Piston Theory for Flat Panels & Cylindrical Shells, Vibrationdata, 2010.
9. Hassan Kamal, Linear Solution of the Problem of Panel Flutter, Mechanics Based Design of Structures and Machines, 22: 1, 37 - 47, 1994.

APPENDIX A

Empirical Method

The following method is taken from References 4 and 5.

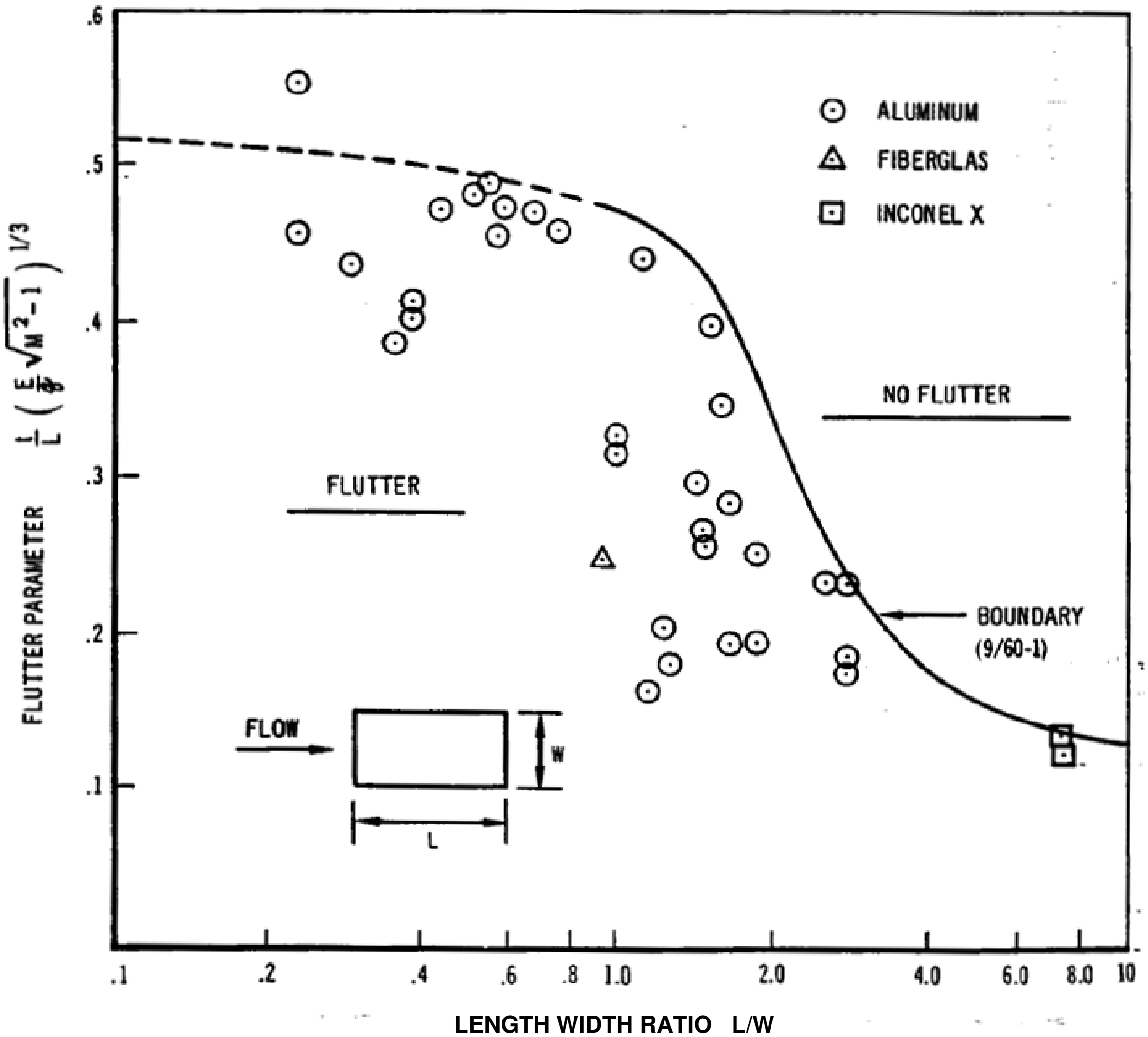

Figure A-1. Flight Test Results

The curve in Figure A-1 was based on experimental and flight data. Reference 4 noted:

1. The curve becomes meaningless near Mach one.
2. The boundary layer was relatively thin.
3. Differential pressure across the panel was zero.

The flutter parameter effectively requires $M > \sqrt{2}$

Example 1

The method in Figure A-1 is applied via a Matlab script.

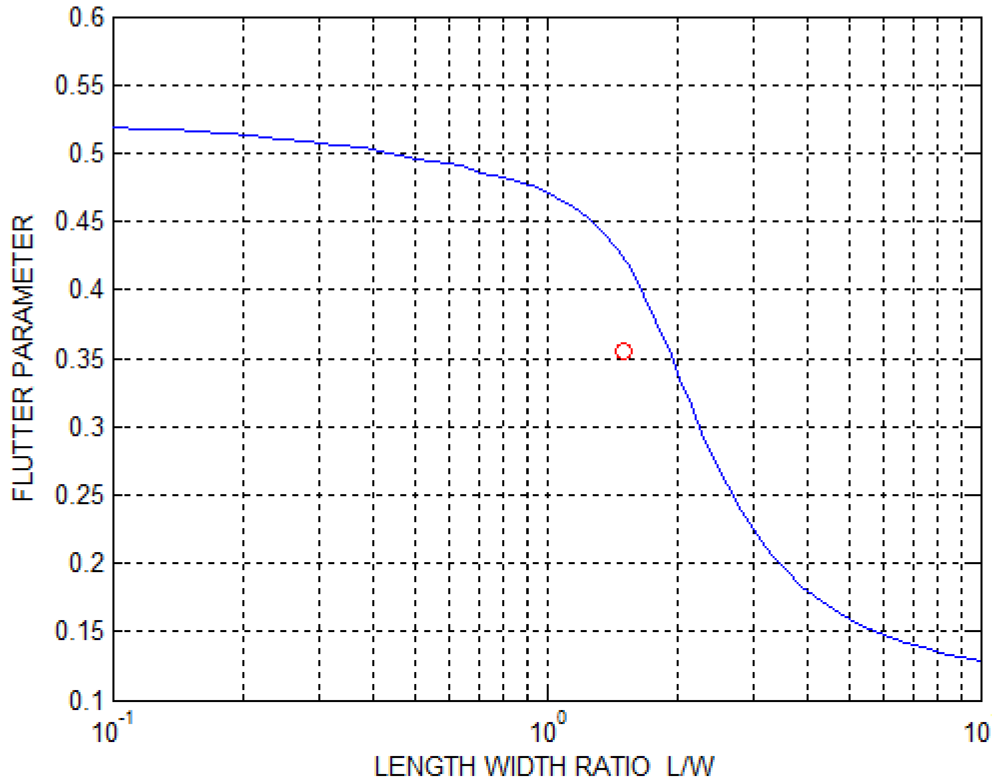

Figure A-2.

```
>> vsa_panel_flutter

  vsa_panel_flutter.m   ver 1.0   August 23, 2010
  by Tom Irvine

  NASA TN D-1058 Method

 Enter Length (inch) 36
 Enter Width  (inch) 24
 Enter Thickness (inch) 0.125
 Enter Mach number 1.5
 Enter dynamic pressure (psf) 1500

 Enter material:
 1=aluminum  2=steel  3=other 1

 Flutter Parameter =   0.3555   Upper Limit =   0.4245

 Flutter Zone
```

Example 2

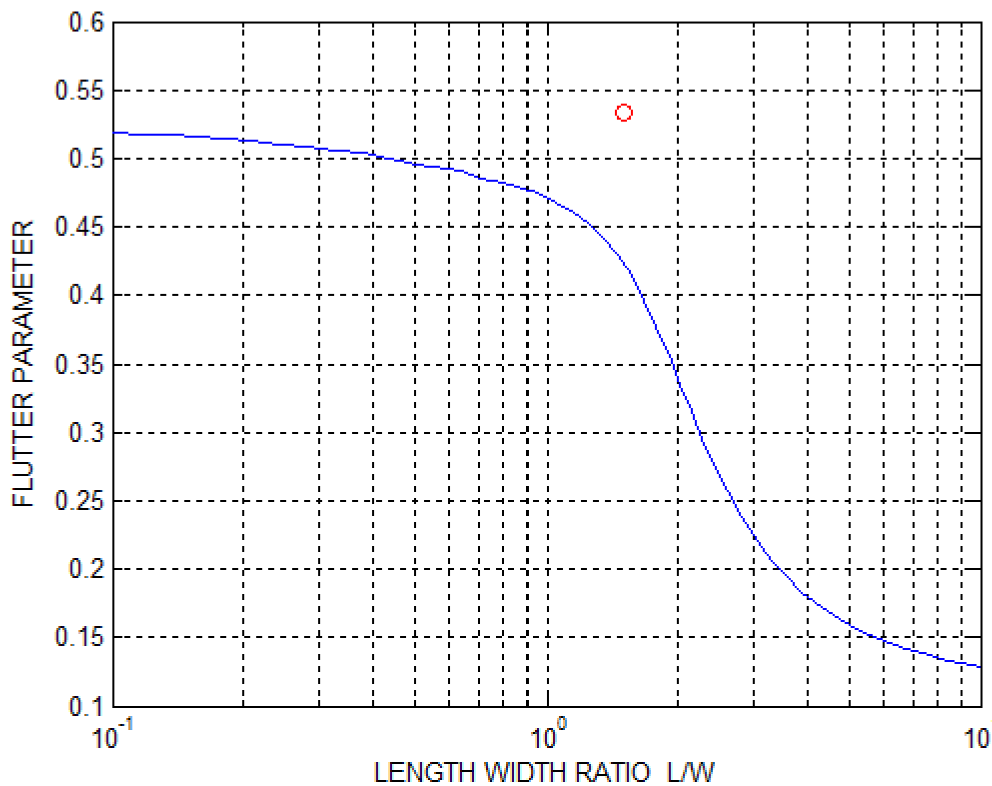

Figure A-3.

```
>> vsa_panel_flutter

 vsa_panel_flutter.m   ver 1.0   August 23, 2010
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 NASA TN D-1058 Method

 Enter Length (inch) 36
 Enter Width  (inch) 24
 Enter Thickness (inch) 0.1875
 Enter Mach number 1.5
 Enter dynamic pressure (psf) 1500
 Enter material:
 1=aluminum  2=steel  3=other 1

 Flutter Parameter =   0.5333   Upper Limit =   0.4245

 No Flutter Zone
```