LINEAR PISTON THEORY FOR FLAT PANELS & CYLINDRICAL SHELLS

By Tom Irvine
Email: tomirvine@aol.com

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Cylindrical Shell

Consider a cylindrical shell subjected to external supersonic axial flow.

The first order quasi-steady piston theory for supersonic flow from Reference 1 gives the aerodynamic pressure P(x, t) as

$$P(x,\theta,t)=\frac{\gamma P_{\infty} M^2}{\sqrt{M^2-1}}\left[\frac{\partial w}{\partial x}+\frac{1}{M a_{\infty}}\left(\frac{M^2-2}{M^2-1}\right)\frac{\partial w}{\partial t}-\frac{w}{2R\sqrt{M^2-1}}\right], \quad \text{for } M>1 \tag{1}$$

where

γ	is the adiabatic exponent
P_{∞}	is the free stream pressure
M	is the Mach number
a_{∞}	is the free stream speed of sound
R	is the radius
w	is the radial displacement

Also note that the x-axis is longitudinal.

Flat Panel

The flat panel formula can be determined from the cylindrical shell formula by letting $R \rightarrow \infty$.

$$P(x,t) = \frac{\gamma P_{\infty} M^2}{\sqrt{M^2 - 1}} \left[\frac{\partial w}{\partial x} + \frac{1}{M a_{\infty}} \left(\frac{M^2 - 2}{M^2 - 1} \right) \frac{\partial w}{\partial t} \right], \quad \text{for } M > 1 \tag{2}$$

An alternate formula is given in Appendix A.

References

1. M. Amabili and F. Pellicano, Supersonic Flutter of a Circular Cylindrical Shell with Structural Nonlinearity, Computational fluid and solid mechanics: proceedings, First MIT, Volume 2, By Klaus-Jürgen Bathe, 2003.

2. W. Anderson, Studies in Panel Flutter at Higher Mach Numbers, Thesis, California Institute of Technology, 1963.

APPENDIX A

Flat Panel, Alternate Formula

Consider a panel subjected to supersonic aerodynamic flow exicitation.

The aerodynamic pressure against the panel from linear piston theory from Reference 2 is

$$\bar{p}(x,t) = \frac{\rho U^2}{M}\frac{\partial}{\partial x} w(x,t) + \frac{\rho U}{M}\frac{\partial}{\partial t} w(x,t) , \quad \text{for } M > 1 \tag{A-1}$$

where

M	is the Mach number
U	is the aero flow speed
$w(x,t)$	is the panel displacement
x	is the dimension along the flow
ρ	is the air density