PLATE POINT MOBILITY

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Variables

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f$</td>
<td>Excitation frequency</td>
</tr>
<tr>
<td>$f_{mn}$</td>
<td>Natural frequency for mode $m,n$</td>
</tr>
<tr>
<td>$H_{v,ij}(f)$</td>
<td>The steady state velocity at coordinate $i$ due to a harmonic force excitation only at coordinate $j$. This is also known as the mobility function.</td>
</tr>
<tr>
<td>$\xi_{mn}$</td>
<td>Damping ratio for mode $m,n$</td>
</tr>
<tr>
<td>$\phi_{imn}$</td>
<td>Mass-normalized eigenvector for physical coordinate $i$ and mode number $m,n$</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Excitation frequency (rad/sec)</td>
</tr>
<tr>
<td>$\omega_{mn}$</td>
<td>Natural frequency (rad/sec) for mode $m,n$</td>
</tr>
</tbody>
</table>

Introduction

Consider a flat plate excited by a harmonic point force.

The steady-state velocity at coordinate $i$ due to a harmonic force excitation only at coordinate $j$ is

$$H_{v,ij}(f) = j\omega \sum_{m} \sum_{n=1}^{N} \frac{\phi_{imn} \phi_{jmn}}{\omega_{mn}^2} \frac{1}{\left(1 - \rho_{mn}^2\right) + j\left(2\xi_{mn}\rho_{mn}\right)}$$

(1)
where
\[
\rho_{mn} = \frac{f}{f_{mn}} \\
j = \sqrt{-1}
\]

Note that \( j \) is used both as an index and as an imaginary number in equation (1).

Equation (1) is derived from Reference 1.

Now consider a plate simply-supported on all sides. The steady-state velocity at coordinate \( i \) due to a harmonic force excitation only at coordinate \( j \) is

\[
H_{v,ij}(f) = j \omega \left( \frac{4}{\rho_{ab} h} \right) \sum_{m=1}^{M} \sum_{n=1}^{N} \frac{1}{\omega_{mn}^2} \left\{ \sin \left( \frac{m \pi x}{a} \right) \sin \left( \frac{n \pi y}{b} \right) \left[ \sin \left( \frac{m \pi x_o}{a} \right) \sin \left( \frac{n \pi y_o}{b} \right) \right] \right\} \\
\left( 1 - \rho_{mn}^2 \right) + j \left( 2 \xi_{mn} \rho_{mn} \right)
\]

(2)

where

\((x,y)\) is the coordinate for the input index \( i \)

\((x_o,y_o)\) is the coordinate for the input index \( j \)

Equation (2) is derived from References 1, 2 and 3.
Example

![Graph of Point Mobility](image)

**Figure 1.**

The point mobility function is shown for an aluminum plate with dimensions (48” x 36” x 0.125”).

The input and response are each located at the quarter point along both the length and the width.

The loss factor is 0.02 for all modes.

The theoretical mobility for a 0.125” thick aluminum plate with infinite length and width is 0.55 (in/sec)/lbf, based on Reference 4.

The finite, simply-supported plate in this example has a compliance which roughly converges to this value at frequencies beyond those shown in Figure 1. Note that exact convergence is not expected.
References


