PLATE POINT MOBILITY

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Variables

f	Excitation frequency
f_{mn}	Natural frequency for mode *m,n*
$H_{v,ij}(f)$	The steady state velocity at coordinate *i* due to a harmonic force excitation only at coordinate *j*. This is also known as the mobility function.
ξ_{mn}	Damping ratio for mode *m,n*
ϕ_{imn}	Mass-normalized eigenvector for physical coordinate *i* and mode number *m,n*
ω	Excitation frequency (rad/sec)
ω_{mn}	Natural frequency (rad/sec) for mode *m,n*

Introduction

Consider a flat plate excited by a harmonic point force.

The steady-state velocity at coordinate *i* due to a harmonic force excitation only at coordinate *j* is

$$H_{v,ij}(f) = j\omega \sum_{m}^{M} \sum_{n=1}^{N} \left\{ \frac{\phi_{imn}\ \phi_{jmn}}{\omega_{mn}^{2}} \frac{1}{\left[\left(1-\rho_{mn}^{2}\right) + j\left(2\xi_{mn}\rho_{mn}\right)\right]} \right\} \quad (1)$$

where

$$\rho_{mn} = f / f_{mn}$$

$$j = \sqrt{-1}$$

Note that j is used both as an index and as an imaginary number in equation (1).

Equation (1) is derived from Reference 1.

Now consider a plate simply-supported on all sides. The steady-state velocity at coordinate *i* due to a harmonic force excitation only at coordinate *j* is

$$H_{v,ij}(f) =$$

$$j\omega\left(\frac{4}{\rho a b h}\right)\sum_{m}^{M}\sum_{n=1}^{N}\left\{\frac{1}{\omega_{mn}{}^{2}}\frac{\left[\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)\right]\left[\sin\left(\frac{m\pi x_o}{a}\right)\sin\left(\frac{n\pi y_o}{b}\right)\right]}{\left[\left(1-\rho_{mn}^{2}\right)+j\left(2\xi_{mn}\rho_{mn}\right)\right]}\right\} \tag{2}$$

where

(x,y)	is the coordinate for the input index *i*
(x_o,y_o)	is the coordinate for the input index *j*

Equation (2) is derived from References 1, 2 and 3.

Example

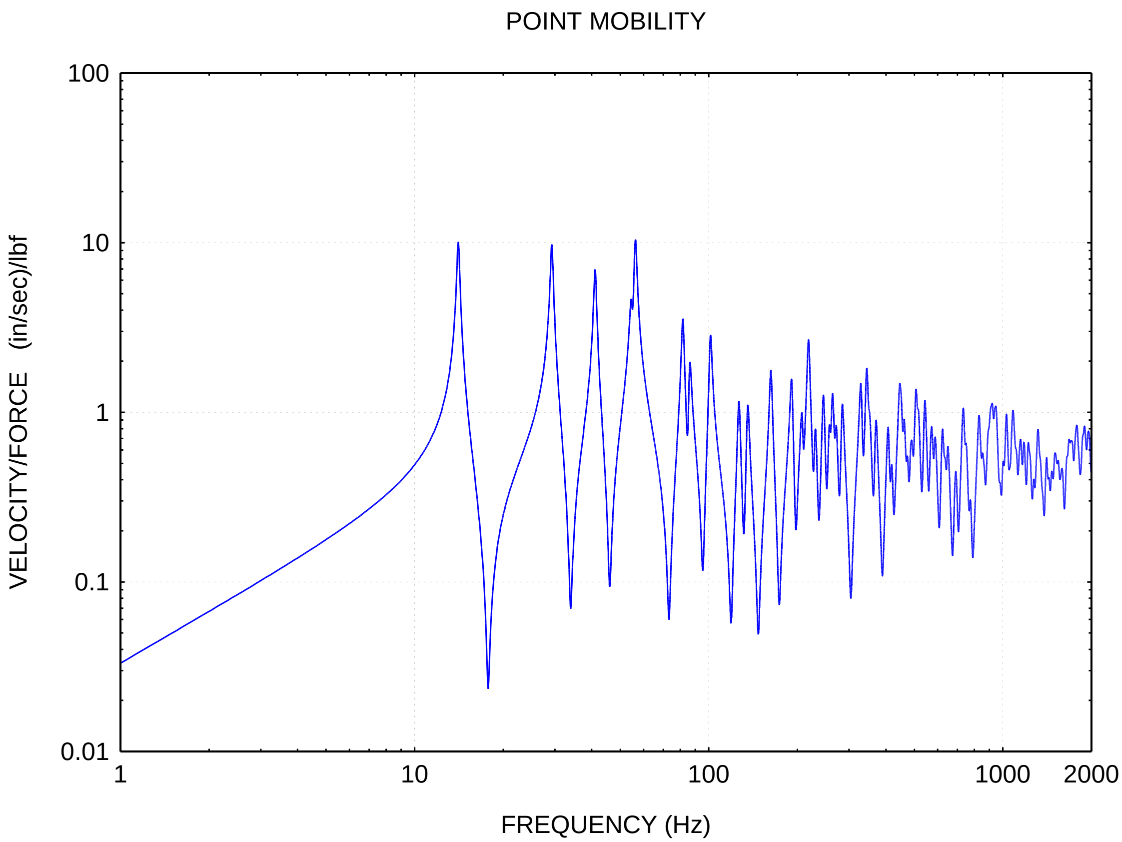

Figure 1.

The point mobility function is shown for an aluminum plate with dimensions (48" x 36" x 0.125").

The input and response are each located at the quarter point along both the length and the width.

The loss factor is 0.02 for all modes.

The theoretical mobility for a 0.125" thick aluminum plate with infinite length and width is 0.55 (in/sec)/lbf, based on Reference 4.

The finite, simply-supported plate in this example has a compliance which roughly converges to this value at frequencies beyond those shown in Figure 1. Note that exact convergence is not expected.

References

1. T. Irvine, Calculating Transfer Functions from Normal Modes, Revision B, Vibrationdata, 2010.

2. Dr. S. Hambric, Structural-Acoustic Tutorial Part 1- Fundamentals, ASME IMECE, Orlando, Florida, 2009.

3. T. Irvine, Natural Frequencies of Rectangular Plate Bending Modes, Revision B, Vibrationdata, 2011.

4. T. Irvine, Radiation & Driving Point Impedance of a Thin, Isotropic Plate, Vibrationdata, 2008.