METHODS FOR CONVERTING A POWER SPECTRAL DENSITY
TO A SHOCK RESPONSE SPECTRUM

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## Introduction

Engineers have an occasional need to convert a power spectral density (PSD) into an "equivalent" shock response spectrum (SRS).

One case would be where the goal is to show that a PSD test level covers an SRS requirement.

Another case would a vibroacoustic analysis whereby a sound pressure level (SPL) is converted to an SRS with a PSD as intermediate step.

The conversion methods assume that the only damage mode of interest is the peak response of the component. Furthermore, the component is assumed to behave as a single-degree-of-freedom system.

The conversions methods do not consider fatigue.

## Method 1, VRS

The first method is based on the corresponding vibration response spectrum (VRS) of a PSD. The VRS is typically expressed in terms of the GRMS or 3σ response. Note that the GRMS value is equal to 1σ assuming zero mean.

This method, however, represents the VRS in terms of its nσ value. The equivalent SRS is assumed to be equal to the nσ VRS.

The n value is calculated as

$$n = \sqrt{2 \ln (fn\, T)} \tag{1}$$

where fn is the natural frequency and T is the duration.

Note that n is value for which exactly one peak is expected to occur for a given fn and T.

Equation (1) is given in Reference 1. The derivation of equation (1) is given in Appendix A.

The n$\sigma$ VRS can readily be calculated by applying equation (1) to the method in Reference 3. The formula is given in Appendix B.

As an example, consider that a component has been tested to the level in Table 1. Calculate the corresponding n$\sigma$ VRS.

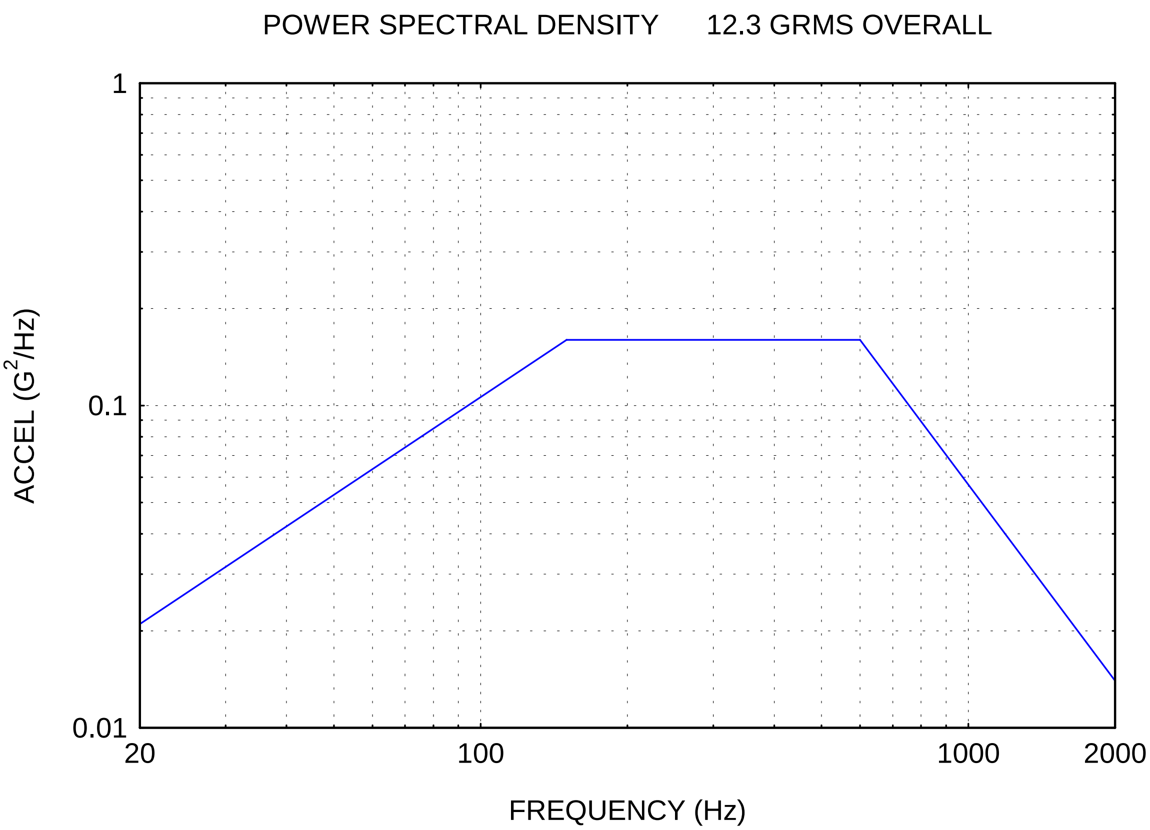


Figure 1.

| Table 1. PSD, Test Level, 12.3 GRMS, 180 sec/axis | |
|---|---|
| Frequency (Hz) | Accel (G^2/Hz) |
| 20 | 0.021 |
| 150 | 0.160 |
| 600 | 0.160 |
| 2000 | 0.014 |

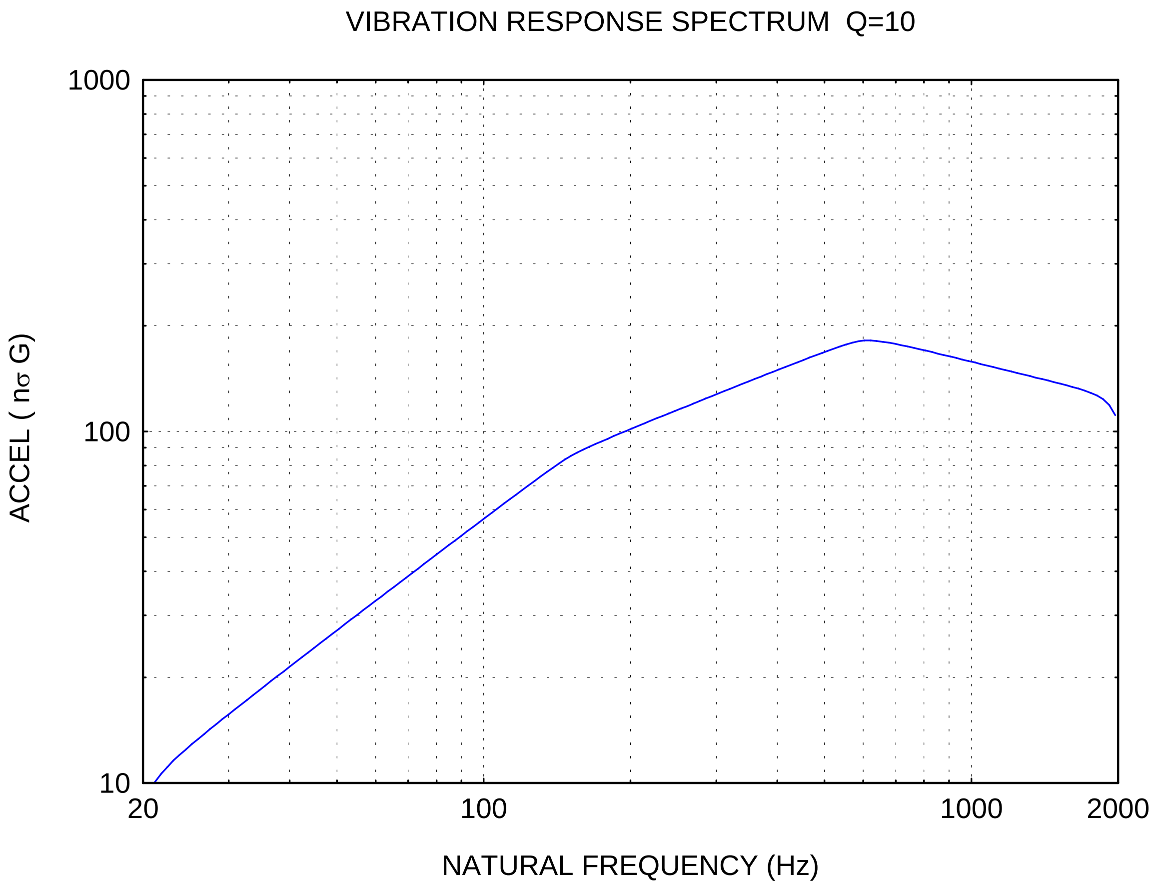


Figure 2.

The equivalent SRS is assumed to be equal to the nσ VRS in Figure 2.

Method 2, Time Domain Synthesis

A time history can be synthesized to satisfy the PSD using the method in Reference 4. Then, an SRS can be calculated from the time history. This method is shown through the following example using the same PSD from the first method.

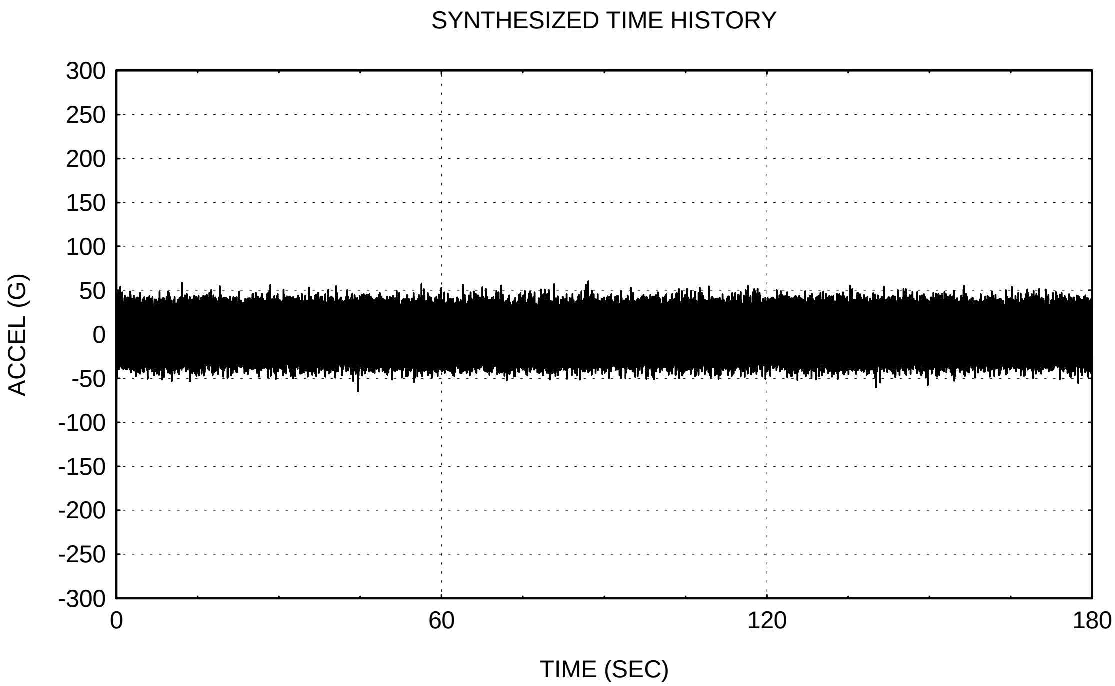


Figure 3.

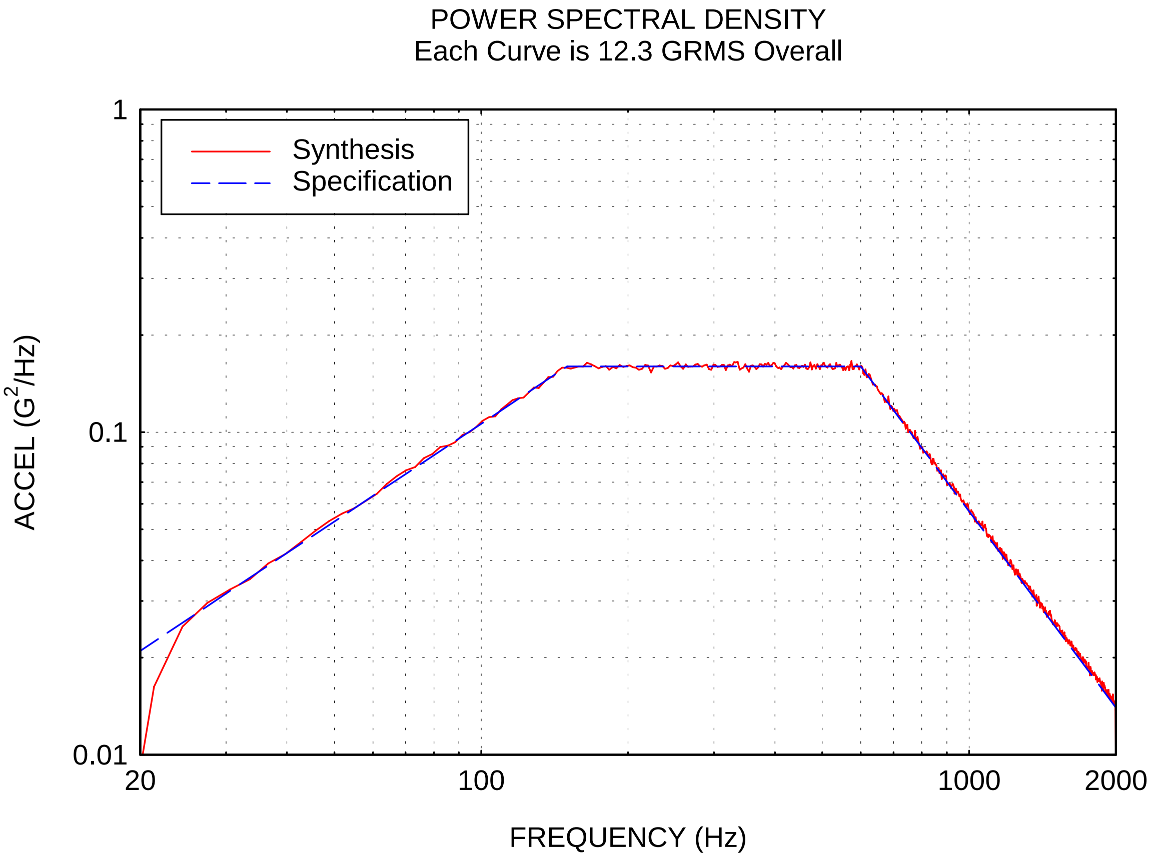


Figure 4.

The PSD of the synthesized time history agrees very well with the specification.

The difference at 20 Hz is not important for this exercise.

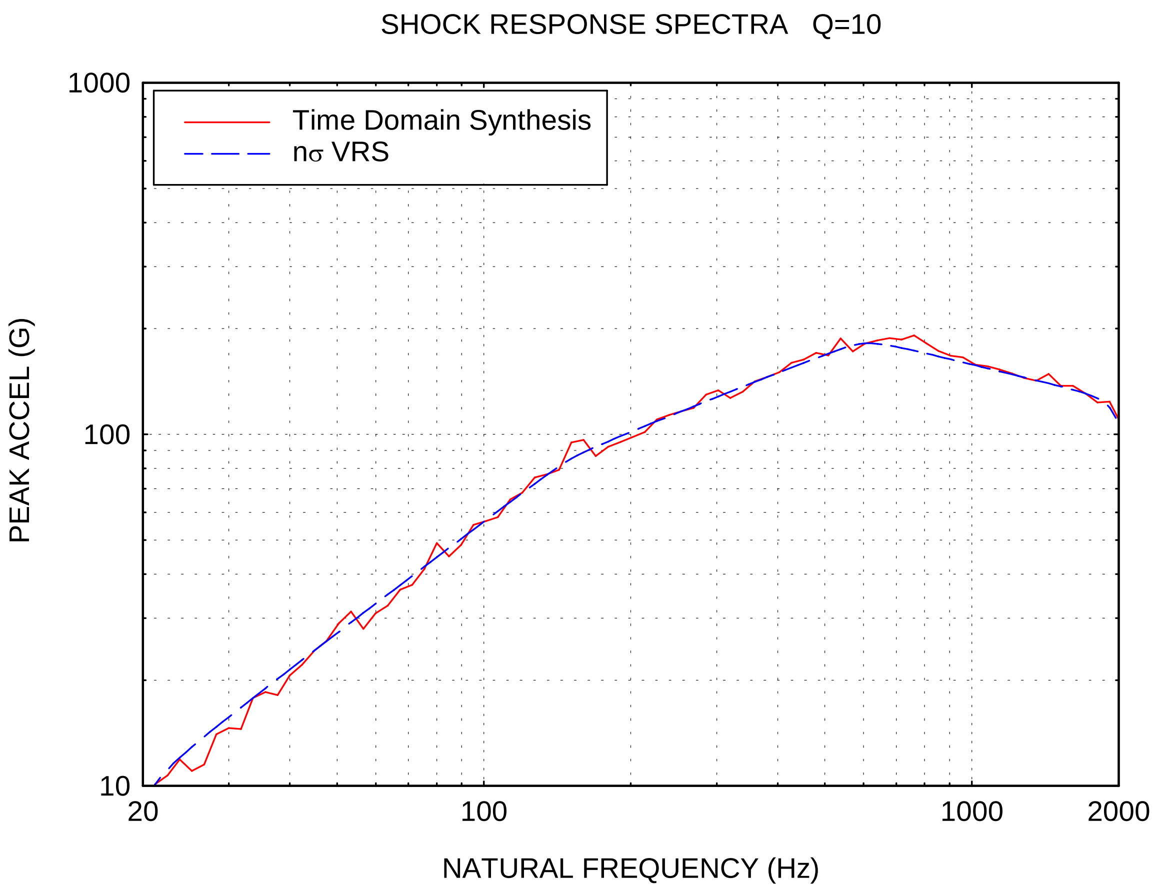


Figure 5.

The two SRS curves agree very well.

Thus, the conversion can be performed via either method.

References

1. DiMaggio, S. J., Sako, B. H., and Rubin, S., Analysis of Nonstationary Vibroacoustic Flight Data Using a Damage-Potential Basis, AIAA Dynamic Specialists Conference, 2003 (also, Aerospace Report No. TOR-2002(1413)-1838, 1 August 2002).

2. W. Thomson, Theory of Vibration with Applications, Second Edition, Prentice- Hall, New Jersey, 1981.

3. T. Irvine, An Introduction to the Vibration Response Spectrum, Revision C, Vibrationdata, 2004.

4. T. Irvine, A Method for Power Spectral Density Synthesis, Revision B, Vibrationdata, 2000.

5. T. Irvine, An Introduction to the Shock Response Spectrum, Revision P, Vibrationdata, 2002.

APPENDIX A

Rayleigh Distribution

The following section is based on Reference 2.

Consider the response of single-degree-of-freedom distribution to a broadband time history. The response is approximately a constant frequency oscillation with a slowly varying amplitude and phase.

The probability distribution of the instantaneous acceleration is the same as that for the broadband random function.

The absolute values of the response peaks, however, will have a Rayleigh distribution, as shown in Table A-3.

| Table A-1. Rayleigh Distribution Probability | |
|---|---|
| $\lambda$ | Prob [ A > $\lambda\sigma$ ] |
| 0.5 | 88.25 % |
| 1.0 | 60.65 % |
| 1.5 | 32.47 % |
| 2.0 | 13.53 % |
| 2.5 | 4.39 % |
| 3.0 | 1.11 % |
| 3.5 | 0.22 % |
| 4.0 | 0.034 % |
| 4.5 | 4.0e-03 % |
| 5.0 | 3.7e-04 % |
| 5.5 | 2.7e-05 % |
| 6.0 | 1.5e-06 % |

The values in Table A-3 are calculated from

$$P\left(\lambda\sigma < A < \infty\right) = \exp\left\{-\frac{1}{2}\lambda^2\right\} \tag{A-1}$$

Determine the $\lambda$ value for which exactly one peak is expected to occur for a natural frequency fn and a duration T.

$$\exp\left\{-\frac{1}{2}\lambda^2\right\} \text{fn T} = 1 \tag{A-2}$$

$$\exp\left\{-\frac{1}{2}\lambda^2\right\} = \frac{1}{\text{fn T}} \tag{A-3}$$

$$-\frac{1}{2}\lambda^2 = \ln\left[\frac{1}{\text{fn T}}\right] \tag{A-4}$$

$$\lambda^2 = -2\ln\left[\frac{1}{\text{fn T}}\right] \tag{A-5}$$

$$\lambda^2 = 2\ln\left[\text{fn T}\right] \tag{A-6}$$

$$\lambda = \sqrt{2\ln(\text{fn T})} \tag{A-7}$$

Let

$$n = \lambda = \sqrt{2\ln(\text{fn T})} \tag{A-8}$$

APPENDIX B

VRS Equation

The following equation is taken from Reference 3. The acceleration response $\ddot{x}_{GRMS}$ of a single-degree-of-freedom system to base excitation is

$$\ddot{x}_{GRMS}\left(f_n,\xi\right)=\sqrt{\sum_{i=1}^{N}\frac{\left\{1+(2\xi\rho_i)^2\right\}}{\left\{\left[1-\rho_i^{\,2}\right]^2+\left[2\xi\rho_i\right]^2\right\}}\hat{Y}_{A\,PSD}(f_i)\Delta f_i}\,,\quad \rho_i=f_i\,/\,f_n \qquad \text{(B-1)}$$

where

| | | |
|---|---|---|
| $\hat{Y}_{A\,PSD}(f)$ | = | base input PSD |
| $\xi$ | = | damping ratio |
| $f_n$ | = | natural frequency |
| $f_i$ | = | base excitation frequency |

Assume a zero mean. The n$\sigma$ response is

$$\ddot{x}_{n\sigma}\left(f_n,\xi\right)=\sqrt{2\ln(fn\,T)}\left\{\ddot{x}_{GRMS}\left(f_n,\xi\right)\right\} \qquad \text{(B-2)}$$