Introduction

Steinberg gives a method for estimating the transmissibility $Q$ for a sinusoidal input in Reference 1.

The two main conclusions from empirical data are:

1. Increasing the natural frequency will increase the transmissibility.
2. Increasing the input acceleration $G$ level will decrease the transmissibility.

Steinberg’s equation for approximating $Q$ for a system is:

$$Q = A \left( \frac{f_n}{G_{in}^{0.6}} \right)^{0.76} \quad (14.21)$$

where

- $A = 1.0$ for beam-type structures
- $A = 0.5$ for plug-in PCBs or perimeter supported PCBs
- $A = 0.25$ for small electronic chassis or electronic boxes
- $f_n$ = natural frequency (Hz)
- $G_{in}$ = sinusoidal vibration input acceleration $G$ level

Definitions of structures:

- **Beam structures** = several electronic components with some interconnecting wires or cables
- **PCB** = well populated with an assortment of electronic components
Small electronic chassis = 8-30 inches in its longest dimension, with a bolted cover to provide access to various types of electronic components such as PCBs, harnesses, cables, and connectors.

Random Vibration Derivation

Steinberg’s methods can be adapted for a random vibration input.

The system is assumed to be a single-degree-of-freedom system.

Again, Steinberg’s equation for sine vibration is

\[
Q = A \left( \frac{f_n}{(G_{in})^{0.6}} \right)^{0.76}
\]  \hspace{1cm} (1)

The sine and random equivalence formula is taken from Reference 2.

\[
(G_{in}) = \frac{1.95 \left( G_{1\sigma} \right)}{Q}
\]  \hspace{1cm} (2)

where \( G_{1\sigma} \) is the 1-sigma response to random vibration.

Substitute equation (2) into (1).

\[
Q = A \left( \frac{f_n Q^{0.6}}{\left( 1.95 \left( G_{1\sigma} \right) \right)^{0.6}} \right)^{0.76}
\]  \hspace{1cm} (3)

\[
Q = 0.6 A f_n^{0.76} Q^{0.46} G_{1\sigma}^{-0.46}
\]  \hspace{1cm} (4)

\[
\frac{Q}{Q^{0.46}} = 0.6 A f_n^{0.76} G_{1\sigma}^{-0.46}
\]  \hspace{1cm} (5)
\[ Q^{0.64} = 0.6 A f_n^{0.76} G_{1\sigma}^{-0.46} \]  

(6)

\[ Q = \left[ 0.6 A f_n^{0.76} G_{1\sigma}^{-0.46} \right]^{1/0.64} \]  

(7)

\[ Q = \left[ 0.6 A f_n^{0.76} G_{1\sigma}^{-0.46} \right]^{1.56} \]  

(8)

\[ Q = 0.45 A^{1.56} f_n^{1.2} G_{1\sigma}^{-0.72} \]  

(9)

Miles equation for the 1-sigma response \( G_{1\sigma} \) to random vibration is

\[ G_{1\sigma} = \left( \frac{\pi}{2} f_n Q \hat{Y}_{\text{PSD}}(f_n) \right)^{0.5} \]  

(10)

where

\( \hat{Y}_{\text{PSD}}(f_n) \) is the power spectral density \((G^2/Hz)\) at the natural frequency \( f_n \).

Substitute Miles equation (10) into equation (9).

\[ Q = 0.45 A^{1.56} f_n^{1.2} \left[ \left( \frac{\pi}{2} f_n Q \hat{Y}_{\text{PSD}}(f_n) \right)^{0.5} \right]^{-0.72} \]  

(11)

\[ Q = 0.45 A^{1.56} f_n^{1.2} f_n^{-0.36} Q^{-0.36} \left( \frac{\pi}{2} \right)^{-0.36} \left[ \hat{Y}_{\text{PSD}}(f_n) \right]^{-0.36} \]  

(12)
Note that the transmissibility $Q$ is equivalent to $\left( \frac{G_{\text{out}}}{G_{\text{in}}} \right)$ at the system’s natural frequency.

The power transmissibility is $Q^2$.

References
