Rotating unbalance in machinery is an example of an applied force. Model a machine with rotating unbalance as a single-degree-of-freedom system.

The variables are:

- \( M \) is the total mass
- \( m \) is the eccentric mass representing the unbalance
- \( c \) is the viscous damping coefficient
- \( k \) is the stiffness
- \( x \) is the absolute displacement of the non-rotating mass
- \( e \) is the eccentricity
- \( \omega \) is the rotational frequency in radians/sec
The free-body diagram is

![Free-body diagram with forces](image)

Figure 2.

The rotating unbalance force is

\[ f(t) = -m \frac{d^2}{dt^2} [e \sin \omega t] \]  

\[ f(t) = m \omega^2 e \sin \omega t \] (1) (2)

Sum the forces using Newton’s law. The upward direction is positive.

\[ \sum F = m \ddot{x} \] (3)

\[ M \ddot{x} = f(t) - kx - c \dot{x} \] (4)

\[ M \ddot{x} + c \dot{x} + kx = f(t) \] (5)

\[ M \ddot{x} + c \dot{x} + kx = m \omega^2 e \sin \omega t \] (6)

\[ \ddot{x} + (c/M) \dot{x} + (k/M)x = (m/M) \omega^2 e \sin \omega t \] (7)

\[ c/M = 2\xi \omega_n \] (8)
\[
\omega_n = \sqrt{\frac{k}{m}}
\]  

(9)

\[
\ddot{x} + 2\xi\omega_n\dot{x} + \omega_n^2 x = (m/M)\omega^2 e^{\sin \omega t}
\]  

(10)

Equation (10) can be solved via Reference 1.

The steady-state displacement magnitude \(X\) is

\[
X = \left\{\frac{m\omega^2 e}{k}\right\} \frac{1}{\sqrt{\left(1-\rho^2\right)^2 + (2\xi\rho)^2}}
\]  

(11)

where

\[
\rho = \frac{\omega}{\omega_n}
\]

The steady-state displacement magnitude can be scaled as

\[
\frac{M}{me} X = \left\{\frac{M\omega^2}{k}\right\} \frac{1}{\sqrt{\left(1-\rho^2\right)^2 + (2\xi\rho)^2}}
\]  

(12)

\[
\frac{M}{me} X = \left\{\frac{\omega^2}{\omega_n^2}\right\} \frac{1}{\sqrt{\left(1-\rho^2\right)^2 + (2\xi\rho)^2}}
\]  

(13)

\[
\frac{M}{me} X = \frac{\rho^2}{\sqrt{\left(1-\rho^2\right)^2 + (2\xi\rho)^2}}
\]  

(14)
The corresponding phase angle is

\[ \theta = \arctan \left( \frac{2 \xi \rho}{1 - \rho^2} \right) \]  \hspace{1cm} (15)

Reference