AN INTRODUCTION TO ROTATING UNBALANCE

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Rotating unbalance in machinery is an example of an applied force. Model a machine with rotating unbalance as a single-degree-of-freedom system.

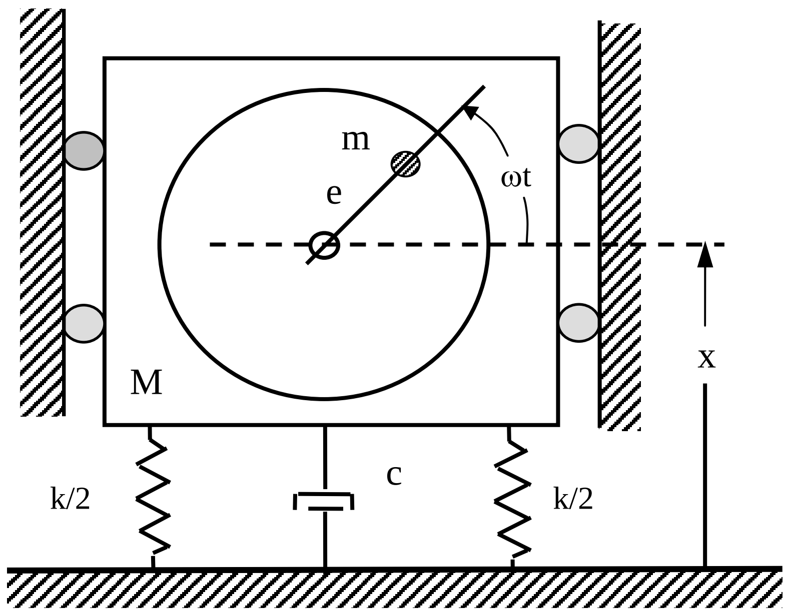

Figure 1.

The variables are:

M	is the total mass
m	is the eccentric mass representing the unbalance
c	is the viscous damping coefficient
k	is the stiffness
x	is the absolute displacement of the non-rotating mass
e	is the eccentricity
ω	is the rotational frequency in radians/sec

The free-body diagram is

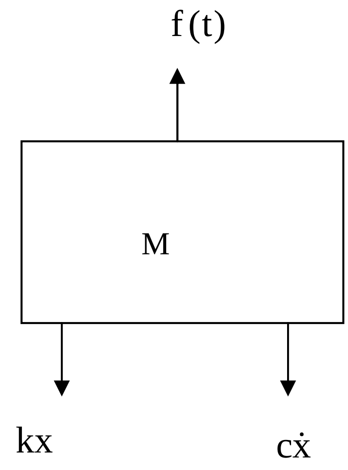

Figure 2.

The rotating unbalance force is

$$f(t) = -m \frac{d^2}{dt^2}\left[e \sin \omega t\right] \tag{1}$$

$$f(t) = m\,\omega^2\, e\, \sin \omega t \tag{2}$$

Sum the forces using Newton's law. The upward direction is positive.

$$\sum F = m\ddot{x} \tag{3}$$

$$M\ddot{x} = f(t) - kx - c\dot{x} \tag{4}$$

$$M\ddot{x} + c\dot{x} + kx = f(t) \tag{5}$$

$$M\ddot{x} + c\dot{x} + kx = m\,\omega^2\, e\, \sin \omega t \tag{6}$$

$$\ddot{x} + (c/M)\dot{x} + (k/M)x = (m/M)\,\omega^2\, e\, \sin \omega t \tag{7}$$

$$c/M = 2\xi\,\omega_n \tag{8}$$

$$\omega_n = \sqrt{\frac{k}{m}} \tag{9}$$

$$\ddot{x} + 2\xi\,\omega_n \dot{x} + \omega_n^2\, x = (m/M)\,\omega^2\, e\, \sin \omega t \tag{10}$$

Equation (10) can be solved via Reference 1.

The steady-state displacement magnitude X is

$$X = \left\{ \frac{m\,\omega^2\, e}{k} \right\} \frac{1}{\sqrt{\left(1-\rho^2\right)^2 + \left(2\xi\rho\right)^2}} \tag{11}$$

where

$$\rho = \frac{\omega}{\omega_n}$$

The steady-state displacement magnitude can be scaled as

$$\frac{M}{me} X = \left\{ \frac{M\omega^2}{k} \right\} \frac{1}{\sqrt{\left(1-\rho^2\right)^2 + \left(2\xi\rho\right)^2}} \tag{12}$$

$$\frac{M}{me} X = \left\{ \frac{\omega^2}{\omega_n^2} \right\} \frac{1}{\sqrt{\left(1-\rho^2\right)^2 + \left(2\xi\rho\right)^2}} \tag{13}$$

$$\frac{M}{me} X = \frac{\rho^2}{\sqrt{\left(1-\rho^2\right)^2 + \left(2\xi\rho\right)^2}} \tag{14}$$

The corresponding phase angle is

$$\theta = \arctan\left[\frac{2\xi\rho}{1-\rho^2}\right] \tag{15}$$

<u>Reference</u>

1. T. Irvine, The Time-Domain Response of a Single-Degree-of-Freedom System Subjected to a Sinusoidal Force, Vibrationdata, 1999.