PENDULUM SEISMOMETER FOR MEASURING VERTICAL GROUND MOTION

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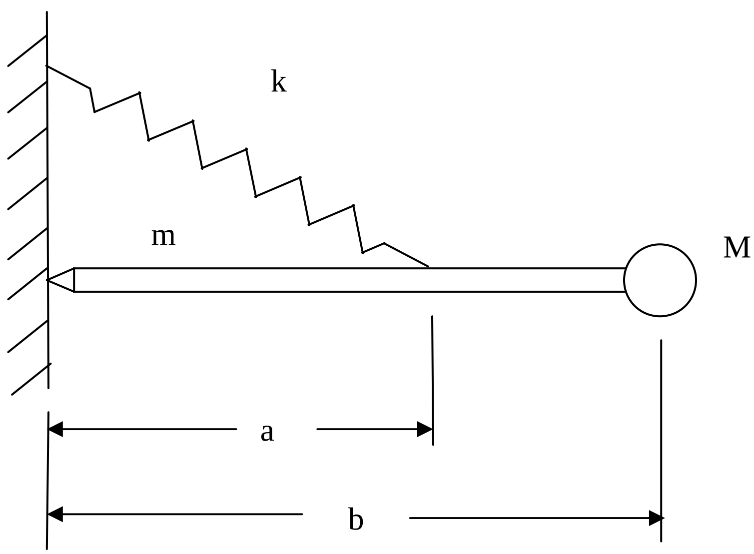

Figure 1.

The boom pivots against the wall. The system is constrained so that the only motion permitted is rotation about the axis perpendicular to the system's plane. The boom is suspended by a spring with stiffness k. The mass of the boom is m. The mass of the end mass is M.

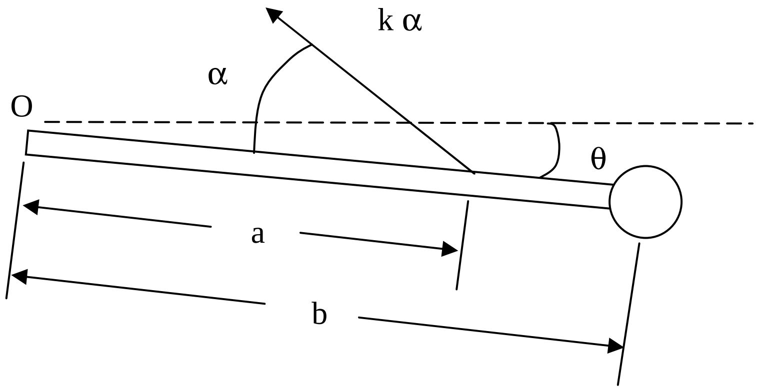

Figure 2. Free-body Diagram

Assume a very small angular displacement θ.

$$\sum M_O = J_O \ddot{\theta} \tag{1}$$

$$J_O \ddot{\theta} = -\left(ka^2 \sin\alpha\right)\theta \tag{2}$$

$$J_O \ddot{\theta} + \left(ka^2 \sin\alpha\right)\theta = 0 \tag{3}$$

$$\ddot{\theta} + \frac{\left(ka^2 \sin\alpha\right)}{J_O}\theta = 0 \tag{4}$$

$$\ddot{\theta} + \omega_n^2\,\theta = 0 \tag{5}$$

$$\omega_n^2 = \frac{\left(ka^2 \sin\alpha\right)}{J_o} \tag{6}$$

$$\omega_n = a\sqrt{\frac{k\sin\alpha}{J_o}} \tag{7}$$

The total mass moment of inertia is

$$J_o = \left[M + \frac{1}{3}m\right]b^2 \tag{8}$$

$$\omega_n = a\sqrt{\frac{k\sin\alpha}{\left[M + \frac{1}{3}m\right]b^2}} \tag{9}$$

The natural frequency is thus

$$\omega_n = \frac{a}{b}\sqrt{\frac{3k\sin\alpha}{3M + m}} \tag{10}$$