SHOCK ANALYSIS OF AVIONICS MOUNTING BOLTS

By Tom Irvine
Email: tomirvine@aol.com

May 20, 2005

Introduction

The shock analysis method is shown by the following example. The method is simple and somewhat conservative.

Consider an avionics box that is mounted to a bulkhead via six 10-32 high-strength cap screws. The bolts are in a 4 inch by 6 inch rectangular pattern as shown in Figure 1. The box mass is 4 lbm.

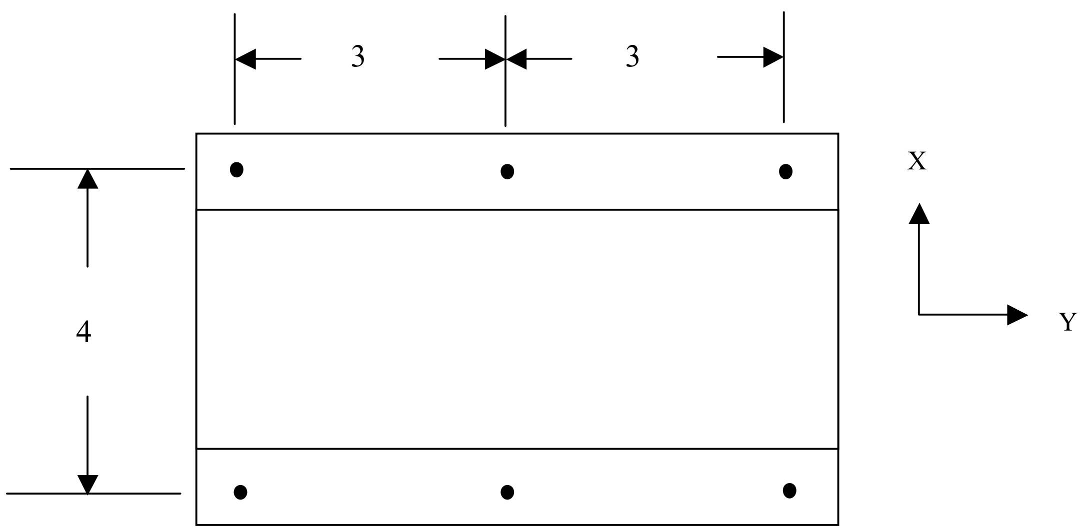


Figure 1.

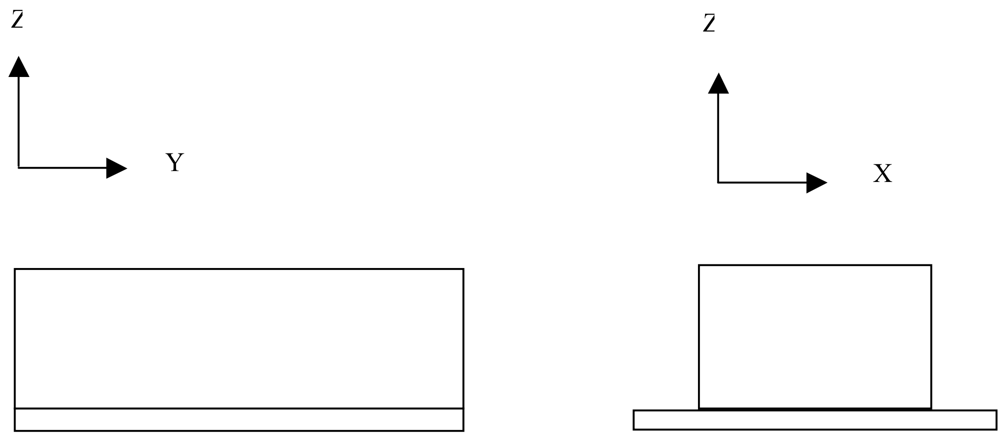


Figure 2. Front and Side Views

The avionics box is subjected to a base excitation shock load in each of its lateral axes.

Material Properties

The mechanical properties for 10-32 high-strength cap screws are:

Tensile yield strength	= 162 ksi
Ultimate tensile strength	= 180 ksi
Tensile stress area	= 0.020 in^2
Thread area for shear	= 0.0175 in^2

Shock Specification

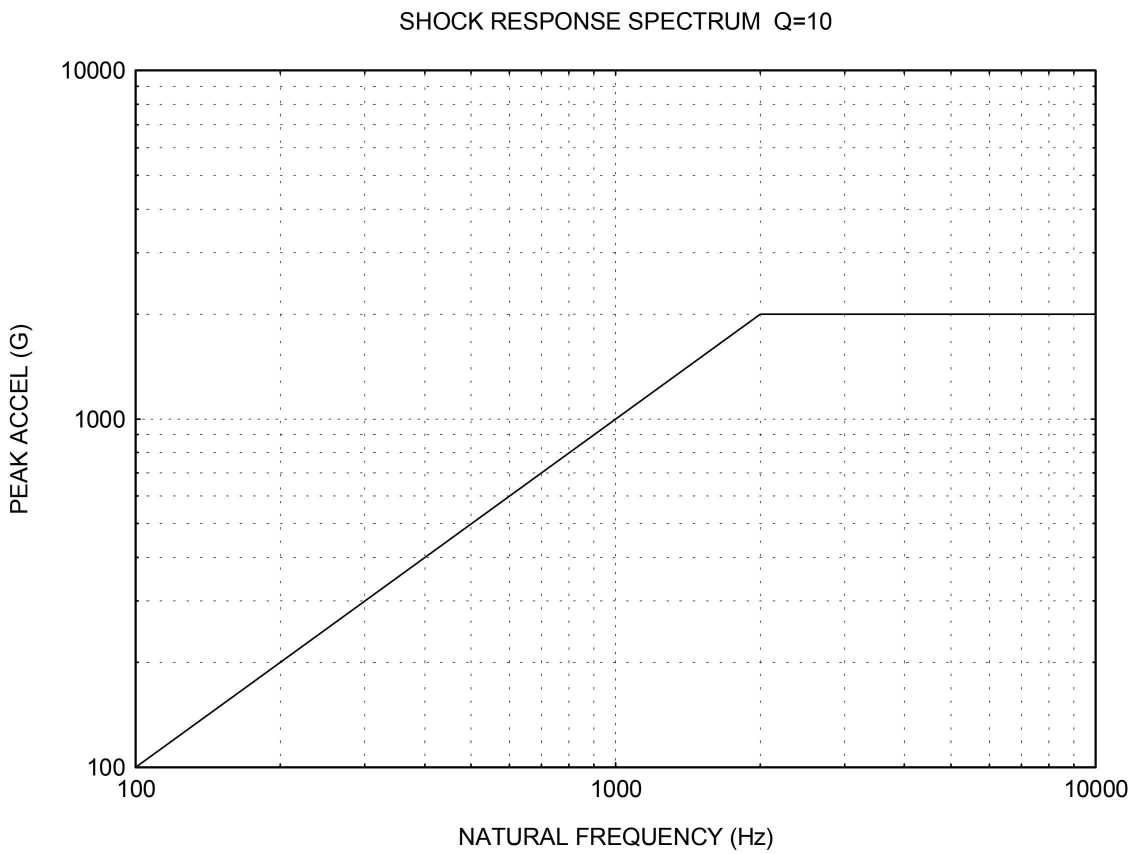


Figure 3.

The box must withstand the shock response spectrum shown in Figure 3. The natural frequency of the box is unknown, however. Thus, assume that response is 2000 G.

Shock in the X-direction (Overturning about the Y-axis)

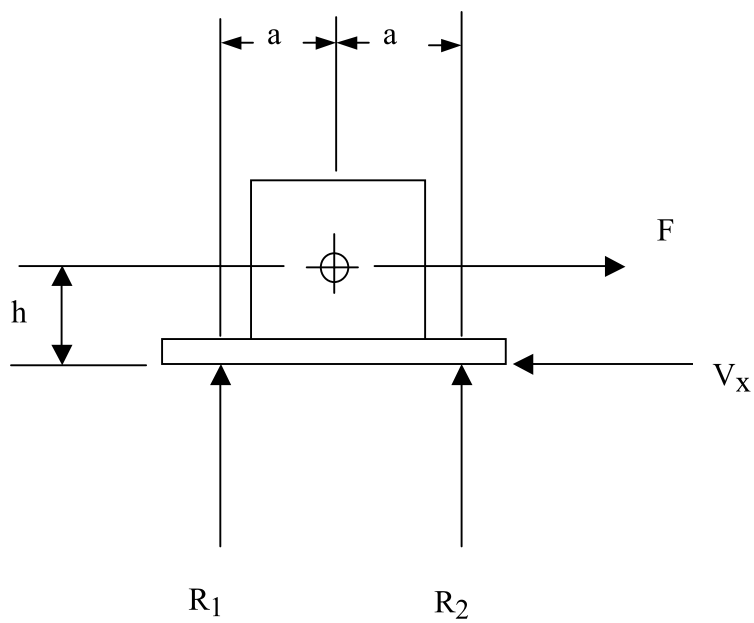


Figure 4. Free-body Diagram

The distance from the base to the center of gravity is h=2 inch. The lateral distance from the CG to each bolt is a=2 inch. The reaction loads are represented by R_1 and R_2. Each reaction load represents the effect of three bolts, since there are six bolts total.

The applied lateral acceleration load is represented by V. The inertial load against the CG is F.

The force is equal to mass m times acceleration a.

$$F = m\,a \tag{1}$$

$$F = 4 \text{ lbm x } 2000 \text{ G} = 8000 \text{ lbf} \tag{2}$$

For simplicity, solve for the reaction forces assuming static equilibrium.

The sum of the vertical forces equals zero. Neglect the weight of the box.

$$R_1 + R_2 = 0 \tag{3}$$

$$R_1 = - R_2 \tag{4}$$

The sum of the moments about the CG equals zero. Clockwise is positive.

$$R_1\, a - R_2\, a + V_x\, h = 0 \tag{5}$$

$$2\, R_1\, a + V_x\, h = 0 \tag{6}$$

$$2\, R_1\, a = - V_x\, h \tag{7}$$

$$R_1 = - V_x\, h / (2\, a) \tag{8}$$

Furthermore,

$$R_1 = - V_x\, h / (2\, a) \tag{9}$$

$$V_x = F \tag{10}$$

$$R_1 = - F\, h / (2\, a) \tag{11}$$

$$R_1 = -\frac{(8000\,\mathrm{lbf})(2\,\mathrm{in})}{2\,(2\,\mathrm{in})} \tag{12}$$

The negative sign indicates tension. It is omitted in equation (13) for brevity.

$$R_1 = 4000\ \mathrm{lbf} \tag{13}$$

Three bolts act in tension. The tension in each of the three bolts is

$$\frac{R_1}{3} = \frac{4000}{3} \text{ lbf} \tag{14}$$

$$\frac{R_1}{3} = 1333 \text{ lbf} \tag{15}$$

The tensile stress is

$$\sigma = \frac{R_1}{A} \tag{16}$$

where A is the tensile stress area.

$$\sigma = \frac{1333 \text{ lbf}}{0.020 \text{ in}^2} \tag{17}$$

$$\sigma = 66.7 \text{ ksi} \tag{18}$$

The shear is taken by all six bolts.

$$\frac{V}{6} = \frac{8000 \text{ lbf}}{6} \tag{19}$$

$$\frac{V}{6} = 1333 \text{ lbf} \tag{20}$$

The shear stress in each of the six bolts is

$$\tau = \frac{1333 \text{ lbf}}{0.0175 \text{ in}^2} \tag{21}$$

$$\tau = 76.2 \text{ ksi} \tag{22}$$

An elliptical interaction curve from MIL-HDBK 5 and NASA 1228 represents the behavior of high strength bolts under combined tension and shear. The equation is as follows:

$$\left(\frac{\tau}{\tau_{ult}}\right)^3 + \left(\frac{\sigma}{\sigma_{ult}}\right)^2 \leq 1 \tag{23}$$

A safety factor SF is inserted into the equation as

$$\left((SF)\frac{\tau}{\tau_{ult}}\right)^3 + \left((SF)\frac{\sigma}{\sigma_{ult}}\right)^2 \leq 1 \tag{24}$$

Assume that SF =1.25 with respect to the ultimate stress limit.

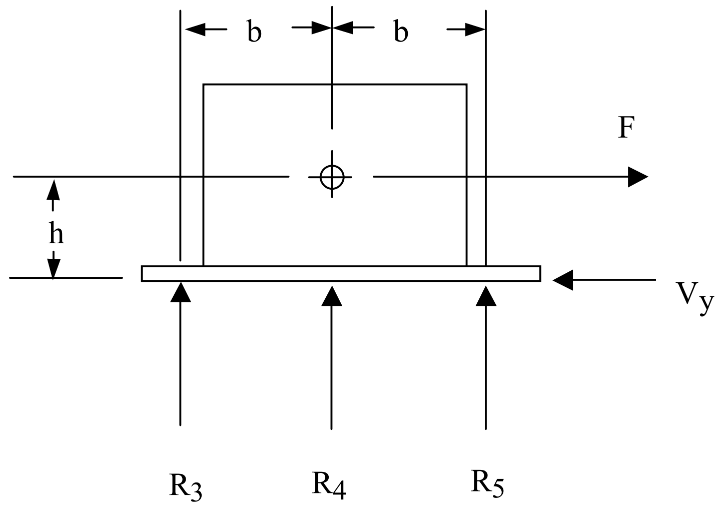
Furthermore, the ultimate stress limit is multiplied by 0.6 for shear, per <u>Fastener Standards</u>, Sixth Edition. The limit is thus reduced from 180 ksi to 108 ksi for shear.

Substitute the loads and limits into equation (24).

$$\left((1.25)\frac{76.2 \text{ ksi}}{108 \text{ ksi}}\right)^3 + \left((1.25)\frac{66.7 \text{ ksi}}{180 \text{ ksi}}\right)^2 = 0.90 \leq 1 \tag{25}$$

The criterion is thus satisfied with a safety factor of 1.25 with respect to ultimate.

Shock in the Y-direction (Overturning about the X-axis)



The sum of the vertical forces equals zero.

$$R_3 + R_4 + R_5 = 0 \tag{26}$$

The sum of the moments about the CG equals zero. Clockwise is positive.

$$F\,h + R_3\,b - R_5\,b = 0 \tag{27}$$

Assume

$$R_5 = -R_3 \tag{28}$$

$$F\,h + 2\,R_3\,b = 0 \tag{29}$$

$$2\,R_3\,b = -F\,h \tag{30}$$

$$R_3 = -F\,h\,/\,(2\,b) \tag{31}$$

$$R_5 = F\,h\,/\,(2\,b) \tag{32}$$

$$R_4 = 0 \tag{33}$$

$$R_3 = -\frac{(8000\,\text{lbf})(2\text{ in})}{2(3\,\text{in})} = -2667\text{ lbf} \tag{34}$$

Similarly,

$$R_5 = 2667\text{ lbf} \tag{35}$$

There are two bolts in tension. Thus

$$\frac{R_3}{2} = -1333\text{ lbf} \tag{34}$$

The tensile stress is

$$\sigma = \frac{1333\text{ lbf}}{0.020\text{ in}^2} = 66.7\text{ ksi} \tag{35}$$

The shear is taken by all six bolts.

$$V_y = F \tag{36}$$

$$\frac{V_y}{6} = \frac{8000\,\text{lbf}}{6} = 1333\text{ lbf} \tag{37}$$

$$\tau = \frac{1333\text{ lbf}}{0.0175\text{ in}^2} = 76.2\text{ ksi} \tag{38}$$

The stress result for the Y-axis load is the same as that for the X-axis.

$$\left((1.25)\frac{76.2 \text{ ksi}}{108 \text{ ksi}} \right)^3 + \left((1.25)\frac{66.7 \text{ ksi}}{180 \text{ ksi}} \right)^2 = 0.90 \leq 1 \tag{39}$$

The criterion is thus satisfied with a safety factor of 1.25 with respect to ultimate.

This approach has assumed that the loading in each of the two lateral axes occurs at separate times, so that the stress results can be considered independently. Furthermore, no consideration was given to the loading in the Z-axis, which would produce tension in the bolts but not shear.

Loading for an actual pyrotechnic shock event would occur in all three axes, but it is unlikely that the peak response in each of three axes would occur simultaneously.

Note that a more detailed stress equation is given in Appendix A. An alternate equation is given in Appendix B.

APPENDIX A

Detailed Stress Criterion

The elliptical interaction equation for a joint that remains clamped under loading with a safety factor is:

$$\left[(SF)\frac{P_s}{F_{us}}\right]^3 + \left[\frac{(SF)\,P_b + F_i}{F_{ut}}\right]^2 \le 1 \qquad \text{(A-1)}$$

$$P_b = \frac{K_b\,P_t}{K_b + K_m} \qquad \text{(A-2)}$$

where

SF	=	bolt safety factor to ultimate
P_b	=	portion of the tensile load taken by the bolt
P_s	=	external shear load on the bolt
P_t	=	external tensile load on the bolt
F_{us}	=	ultimate shear capability of the bolt
F_{ut}	=	ultimate tension capability of the bolt
F_i	=	bolt preload
K_b	=	spring constant of the bolt
K_m	=	spring constant of the joint members

APPENDIX B

Alternate Stress Criterion

Some references give the combined stress formula for a bolt as

$$\left(\frac{\tau}{\tau_{ult}}\right)^2 + \left(\frac{\sigma}{\sigma_{ult}}\right)^2 \le 1 \qquad \text{(B-1)}$$

The equation for a safety factor of 1.25 on ultimate stress is

$$\left(\frac{\tau}{\tau_{ult}}\right)^2 + \left(\frac{\sigma}{\sigma_{ult}}\right)^2 \le 0.80 \qquad \text{(B-2)}$$

Equation (B-1) is given in McGuire, Steel Structures, equation 6.13.

It is also given in the AISC Steel Construction Manual, Seventh Edition, Figure C1.6.3.1.

Furthermore, the shear ultimate limit should be multiplied by 0.6 for conservatism, per Fastener Standards, Sixth Edition.