AN INTRODUCTION TO FLUID SLOSH
Revision A

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Variables

a	=	Length
b	=	Width
d	=	Diameter
f_{ij}	=	Natural frequency
g	=	Acceleration of Gravity
h	=	Height of fluid surface
D_{ij}	=	Term in the rectangular basin calculation
R	=	Radius
J_i	=	Bessel function of order i
λ_{ij}	=	A root of the first derivative of the Bessel function

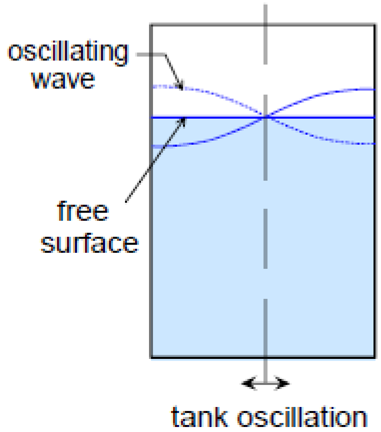

Figure 1. Slosh Wave, Fundamental Mode, Reference 5.

Introduction

Sloshing is the free-surface oscillation of a fluid in a basin or partially filled tank. The oscillations are typically excited by base excitation, where the excitation may be lateral, longitudinal, or rotational. The image in Figure 1 shows an example of lateral excitation, which has a higher potential of provoking slosh waves than axial excitation.

The main purpose of this tutorial is to give formulas for calculating the slosh natural frequencies for common basin and tank geometries. Some consideration is also given to damping.

Assumptions

1. The liquid is homogeneous, inviscid, irrotational, & incompressible.
2. The boundaries of the basin are rigid.
3. The wave amplitudes are sufficiently small in comparison with the wavelengths and depths so that nonlinear effects are negligible.
4. The influence of the surrounding atmosphere is negligible.
5. The influence of surface tension is negligible.
6. The basin or tank has no baffles or partitions.

Note that if the amplitude becomes too large then the slosh modes will break up into turbulent splashing.

Upright Cylindrical Basin

The slosh natural frequency from Reference 1 is

$$f_{ij} = \frac{1}{2\pi}\sqrt{\frac{\lambda_{ij}\, g}{R}\tanh\left(\frac{\lambda_{ij}\, h}{R}\right)} \quad , \quad i=0,\ 1,\ 2,\ \ldots, \quad j=0,\ 1,\ 2,\ \ldots. \tag{1}$$

where the roots are found from the derivative of the Bessel function

$$\frac{d}{dx} J_i\left(\lambda_{ij}\right) = 0 \tag{2}$$

Or equivalently,

$$J_{i-1}\left(\lambda_{ij}\right) - \frac{i}{\lambda_{ij}} J_i\left(\lambda_{ij}\right) = 0 \tag{3}$$

Note that

i = number of nodal diameters, equal to the Bessel function order
j = number of nodal circles

The roots of equation (3) can be found via numerical analysis. An example is given in Appendix A.

Upright Annular Cylinder

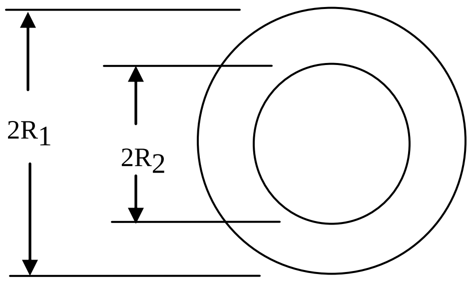

Figure 2.

The slosh natural frequency from Reference 1 is

$$f_{ij} = \frac{1}{2\pi}\sqrt{\frac{\lambda_{ij}\, g}{R_1}\tanh\left(\frac{\lambda_{ij}\, h}{R_1}\right)} \quad , \quad i=0, 1, 2, \ldots, \quad j=0, 1, 2, \ldots. \tag{4}$$

where the roots are found from the following Bessel function equation from Reference 6 & 7,

$$\frac{d}{dx} J_i\left(\lambda_{ij}\right) \frac{d}{dx} Y_i\left(\lambda_{ij}\frac{R_2}{R_1}\right) - \frac{d}{dx} J_i\left(\lambda_{ij}\frac{R_2}{R_1}\right) \frac{d}{dx} Y_i\left(\lambda_{ij}\right) = 0 \tag{5}$$

Note that

i = number of nodal diameters, equal to the Bessel function order
j = number of nodal circles

The roots of equation (5) can be found via numerical analysis. An example is given in Appendix B.

Rectangular Basin

Define term D as

$$D_{ij} = \sqrt{\frac{i^2}{a^2} + \frac{j^2}{b^2}} \quad , \quad i=0, 1, 2, \ldots, \quad j=0, 1, 2, \ldots. \tag{5}$$

where a is the length and b is the width.

The slosh natural frequency from Reference 1 is

$$f_{ij} = \frac{1}{2\sqrt{\pi}} \sqrt{g D_{ij} \tanh(\pi h D_{ij})} \tag{6}$$

where

i=number of nodal lines along length
j=number of nodal lines along width

An example is given in Appendix C.

Cone

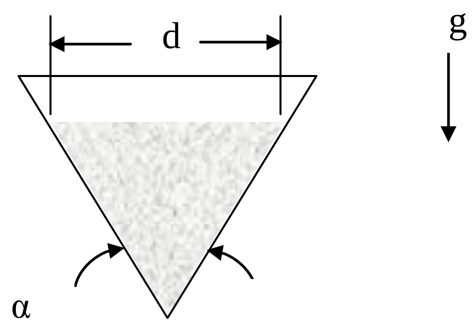

Figure 3.

The fundamental frequency of a cone from Reference 2 is

$$f_n = \frac{\beta}{2\pi}\sqrt{g/d} \tag{7}$$

where

$$\beta \approx 6.65e\text{-}9\,\alpha^4 - 3.03e\text{-}6\,\alpha^3 + 5.90e\text{-}4\,\alpha^2 - 0.0680\,\alpha + 4.12 \tag{8}$$

with α in units of degrees.

Note that the exact values of β could be found via the appropriate Bessel functions, which will be included in a future revision.

As a special case,

$$f_n = \frac{1}{\pi}\sqrt{g/d} \quad \text{for} \quad \alpha = 45° \tag{9}$$

An example is given in Appendix D.

Partially Filled Spherical Tank

The natural frequency from Reference 3 is

$$f_i = \frac{\beta_i}{2\pi}\sqrt{g/r} \tag{10}$$

Note that r is the radius and d is the diameter.

Define a nondimensional parameter: $x=h/d$

The β parameters for the first three modes are

$$\beta_1 \approx 12.1x^5 - 24.2x^4 + 18.7x^3 - 6.22x^2 + 1.27x + 0.975 \tag{11}$$

$$\beta_2 \approx -0.200x^5 + 14.11x^4 - 26.19x^3 + 19.87x^2 - 7.04x + 3.28 \tag{12}$$

$$\beta_3 \approx 19.3x^4 - 37.7x^3 + 29.5x^2 - 10.8x + 4.50 \tag{13}$$

Note that the exact values of β could be found via the appropriate Bessel functions, which will be included in a future revision.

An example is given in Appendix E.

Damping

The main source of damping in a tank without baffles is the wiping action of the liquid against the interior walls. Baffles may be installed to provide additional damping.

Other Design Considerations

The tank must be designed to withstand the pressure from the sloshing liquid.
The natural frequency can be increased by subdividing the tank.

References

1. R. Blevins, Formulas for Natural Frequency and Mode Shapes, R, Krieger, Malabar, Florida, 1979.

2. NASA SP-8009, Propellant Slosh Loads, 1968.

3. NASA TN-D1281, Analytical and Experimental Investigations of Forces and Frequencies Resulting from Liquid Sloshing in a Spherical Tank, 1962.

4. NASA TN D-5412, Natural Frequency of Liquids in Annular Cylinders under Low Gravitational Conditions, 1969.

5. F. Dodge, The New "Dynamic Behavior of Liquids in Moving Containers," Southwest Research Institute.

6. NASA SP-106, The Dynamic Behavior of Liquids in Moving Containers, with Applications to Space Vehicle Technology, 1966.

7. MTP-AERO-61-16, The Effect of Propellant Sloshing on the Stability of an Accelerometer Controlled Rigid Space Vehicle, George C. Marshall Space Flight Center, 1961.

APPENDIX A

<u>Cylindrical Basin Example, via a Matlab Script</u>

```
>> slosh_cylinder

 slosh_cylinder.m   ver 1.2   June 6, 2013
 by Tom Irvine

 This program calculates the slosh frequencies in
 a cylindrical basin.

 Assume
 1. The liquid is homogeneous, inviscid, irrotational, &
incompressible
 2. The boundaries of the basin are rigid
 3. Small wave amplitudes, linear behavior
 4. The influence of the surrounding atmosphere is negligible
 5. The influence of surface tension is negligible

 Enter acceleration of gravity (in/sec^2)
 386
 Enter diameter (inch)
 48
 Enter height (inch)
 36

 i=number of node diameters
 j=number of nodal circles

 Freq(Hz)      lambda       i     j
   0.8626      1.84118      1     0
    1.115      3.05424      2     0
    1.249      3.83171      0     1
    1.308      4.20119      3     0
    1.472      5.31755      4     0
    1.474      5.33144      1     1
    1.653      6.70613      2     1
    1.691      7.01559      0     2
    1.807      8.01524      3     1
    1.865      8.53632      1     2
    1.945       9.2824      4     1
    2.015      9.96947      2     2
```

2.036	10.1735	0	3
2.150	11.3459	3	2
2.184	11.706	1	3
2.273	12.6819	4	2

APPENDIX B

Annular Cylindrical Basin Example, via a Matlab Script

```
>> slosh_annular

 slosh_annular.m   ver 1.1   July 16, 2010
 by Tom Irvine

 This program calculates the slosh frequency in
 a annular cylinder basin.

 Assume
 1. The liquid is homogeneous, inviscid, irrotational, & incompressible
 2. The boundaries of the basin are rigid
 3. Small wave amplitudes, linear behavior
 4. The influence of the surrounding atmosphere is negligible
 5. The influence of surface tension is negligible

 Enter acceleration of gravity (in/sec^2)
 386
 Enter the liquid surface height (inch)
 36
 Enter the outer diameter (inch)
 50
 Enter the inner diameter (inch)
 15

 i=number of nodal diameters
 j=number of nodal circles

 Freq(Hz)      lambda         i     j
   0.7784       1.58206       1     0
    1.077        2.9685       2     0
    1.279       4.18011       3     0
    1.357       4.70578       0     1
    1.417       5.13739       1     1
    1.441       5.31297       4     0
    1.566       6.27367       2     1
    1.584       6.41468       5     0
    1.738       7.72126       3     1
    1.887       9.10423       0     2
    1.892       9.15259       4     1
    1.908       9.30827       1     2
    1.969         9.918       2     2
    2.024        10.475       5     1
```

APPENDIX C

Rectangular Basin Example, via a Matlab Script

```
>> slosh_rectangular

 slosh_rectangular.m   ver 1.2   June 6, 2013
 by Tom Irvine

 This program calculates the slosh frequencies in
 a rectangular basin.

 Assume
 1. The liquid is homogeneous, inviscid, irrotational, &
incompressible
 2. The boundaries of the basin are rigid
 3. Small wave amplitudes, linear behavior
 4. The influence of the surrounding atmosphere is negligible
 5. The influence of surface tension is negligible

 Enter acceleration of gravity (in/sec^2)
 386
 Enter length (inch)
 72
 Enter width (inch)
 48
 Enter height (inch)
 36

 i=number of nodal lines along length
 j=number of nodal lines along width

 Freq(Hz)      i    j
   0.6255      1    0
   0.7928      0    1
   0.8739      1    1
    0.922      2    0
    1.032      2    1
    1.131      0    2
    1.131      3    0
    1.161      1    2
    1.196      3    1
     1.24      2    2
```

APPENDIX D

<u>Conical Basin Example, via a Matlab Script</u>

```
>> slosh_cone

 slosh_cone.m  ver 1.2   June 7, 2013
 by Tom Irvine

 This program calculates the slosh frequency in
 a conical basin.

 Assume
 1. The liquid is homogeneous, inviscid, irrotational, &
incompressible
 2. The boundaries of the basin are rigid
 3. Small wave amplitudes, linear behavior
 4. The influence of the surrounding atmosphere is negligible
 5. The influence of surface tension is negligible

 Enter acceleration of gravity (in/sec^2)
 386
 Enter diameter of fluid surface (inch)
 48
 Enter cone apex angle (deg)
 45

  fn=  0.9053 Hz    beta=       2
```

APPENDIX E

Spherical Example, via a Matlab Script

```
>> slosh_spherical

 slosh_cone.m   ver 1.0   July 14, 2010
 by Tom Irvine

 This program calculates the slosh frequency in
 a partially filled spherical container.

 Assume
 1. The liquid is homogeneous, inviscid, irrotational, &
incompressible
 2. The boundaries of the basin are rigid
 3. Small wave amplitudes, linear behavior
 4. The influence of the surrounding atmosphere is negligible
 5. The influence of surface tension is negligible

 Enter acceleration of gravity (in/sec^2)
 386
 Enter diameter of sphere (inch)
 48
 Enter fluid height (inch)
 24

 f1 =    0.803 Hz  beta=   1.258
 f2 =    1.483 Hz  beta=   2.323
 f3 =    1.895 Hz  beta=   2.969
```