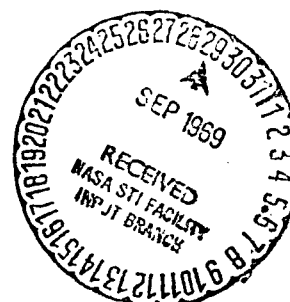


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SPACE VEHICLE
DESIGN CRITERIA
(GUIDANCE AND CONTROL)

NASA SP-8016

EFFECTS OF STRUCTURAL FLEXIBILITY ON SPACECRAFT CONTROL SYSTEMS



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FOREWORD

NASA experience has indicated a need for uniform design criteria for space vehicles. Accordingly, criteria are being developed in the following areas of technology:

Environment
Structures
Guidance and Control
Chemical Propulsion

Individual components of this work will be issued as separate monographs as soon as they are completed. This document, Effects of Structural Flexibility on Spacecraft Control Systems, is one such monograph. A list of all monographs in this series issued prior to this one can be found on the last page of this document.

These monographs are to be regarded as guides to design and not as NASA requirements, except as may be specified in formal project specifications. It is expected, however, that the criteria sections of these documents, revised as experience may indicate to be desirable, eventually will become uniformly applicable to the design of NASA space vehicles.

This monograph was prepared under the cognizance of the NASA Electronics Research Center. Principal contributors were R. B. Noll and J. Zvara of Kaman AviDyne and Dr. J. J. Deyst, Massachusetts Institute of Technology.

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Contributions in the form of design and development practices were also provided by many other engineers of NASA and the aerospace community.

Comments concerning the technical content of these monographs will be welcomed by the National Aeronautics and Space Administration, Office of Advanced Research and Technology (Code RVA), Washington, D.C. 20546.

April 1969

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

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EFFECTS OF STRUCTURAL FLEXIBILITY ON SPACECRAFT CONTROL SYSTEMS

1. INTRODUCTION

Elastic behavior of vehicle structure is usually considered during the design of a spacecraft control system and the possibility of severe control-system/structure interaction is usually foreseen. Less severe interaction may go undetected, however, but may still be of such magnitude as to result in a possible failure of the vehicle to complete all or part of its mission. Thus, *the control system designer must be aware of the numerous and subtle ways in which a control system and flexible structure may interact.*

Spacecraft control-system/structure interactions commonly take one of the following forms: (1) transient phenomena, such as the motion of a satellite resulting from sudden bending of a flexible boom caused by solar radiation; (2) unstable motions, such as may occur when the attitude control system of two docked spacecraft senses and responds in a destabilizing fashion to bending in the docking attachment; and (3) stable limit-cycle oscillations, such as when a spacecraft attitude control system is driven into nonlinear operation at or beyond the boundary of a region of locally unstable linear operation.

Design and flight experience has shown that the control-system parameters pertinent to control-system/structure interaction include:

- Time constants (or natural frequencies) and damping ratios of the elements of the control loop
- Nonlinear system parametric values (saturation limits, dead-zone widths, hysteresis characteristics, etc.)
- Sample intervals and quantization increments (digital systems)

Pertinent structural parameters are:

- Modal frequencies and damping ratios
- Mode shapes
- Inertial properties (masses, moments of inertia)
- Local flexibility characteristics

Other significant factors affecting control-system/structure interaction are:

- Solar radiation and pressure
- Magnetic torques
- Gravitational torques
- Docking and other maneuvers
- Dynamics of contained liquids
- Appendage deployment
- Crew motion
- Operation of propulsive devices

The present monograph discusses the selection, design, and evaluation of a spacecraft control system for operation in the presence of a flexible structure. Control-system design, structural design, and input disturbances to the control system from the natural environment will be treated in separate monographs. Launch vehicle interaction problems will be the subject of another monograph.

2. STATE OF THE ART

There is a long history of experience with control-system/structural-flexibility interaction problems (e.g., refs. 1, 2, and 3). Present technology and analysis techniques are usually adequate for predicting or alleviating interaction problems. Some designs, however, experience difficulties from unforeseen sources. The occurrence of such interaction problems is usually caused, not by a lack of available analysis and design procedures, but by lack of recognition of the numerous ways the interactions can occur and failure to analyze the spacecraft dynamic system in sufficient depth and detail.

Recognition of potential interaction problems in spacecraft is seldom straightforward. Most spacecraft are designed to accomplish a multitude of tasks with varying performance and accuracy requirements. Thus the control system and the structure of a spacecraft operate in a wide variety of configurations with an associated variety of potential interaction problems.

2.1 Review of Design and Flight Experience

Both the control system and the structure contribute to the interaction. Since the nature of the interaction is greatly dependent upon the shape of the vehicle, structural configuration can be used as a guide for identifying potential interaction problems and for categorizing spacecraft.

Four pertinent structural configuration categories have been identified from a review of spacecraft interaction problems.

- (1) Compact near-rigid body
- (2) Compact body with flexibly connected rigid appendages
- (3) Compact body with flexible appendages
- (4) Large flexible body

Category (1) applies to spacecraft which, while appearing to be rigid bodies, have sufficient structural flexibility to produce undesired motion through energy dissipation. Category (2) includes spacecraft that consist of flexibly connected rigid bodies, usually a large central body and one or more appendages. Category (3) is similar to category (2), differing in that the appendages are flexible. This category includes long, flexible, extendable booms (ref. 4) used as antennas or for providing the mass distribution required for gravity-gradient stabilization (refs. 5 and 6). Category (4) applies to all large vehicles, including docked spacecraft, which may possess significant elastic modes.

Table I presents a categorization of spacecraft by both structural configuration and the control mode for which an interaction was of concern. The spacecraft listed in the table are representative of vehicles for which interaction problems were important. The table is intended to provide a general awareness of known potential problem areas and should not be construed to mean that these are the only areas that should be investigated.

Cognizance of the variety of possible interactions is best afforded by examples of problems actually experienced. Specifically, the example interaction problems discussed herein are listed in table I.

Table I.—Spacecraft Control-System/Structure Categories^a

Spacecraft control mode	Spacecraft structural configuration			
	Compact near-rigid body	Compact body with rigid flexibly connected appendages	Compact body with flexible appendages	Large flexible body
Spin stabilization	Explorer I ATS D&E		Alouette I	
Gravity-gradient stabilization			1963-22A DODGE	
Active attitude control in thrust phase		Ranger		Gemini/Agena Apollo CSM/LM
Active attitude control in coast phase		Nimbus	OGO-III OGO-IV	

^aThe table lists spacecraft that are representative examples of each category and is not intended to be a complete listing.

2.1.1 Compact Near-Rigid Body, Spin Stabilized

The Explorer I Earth-orbiting spacecraft was a long cylindrical body with four flexible whip antennas extending laterally (fig. 1). The vehicle was to be passively spin stabilized about its principal axis of minimum moment of inertia. After only one orbit, however, the spacecraft exhibited precessional motion with a half angle of about 1 radian. Within a few days, the vehicle achieved an essentially stable motion, rotating about its axis of maximum moment of inertia. Instability of rotation about the minimum inertia axis was unsuspected before the flight.

Rotation about either the maximum or minimum moment of inertia axis of a completely rigid body is stable. Explorer I, however, was not a rigid body since bending of the whip antennas afforded a mechanism for energy dissipation through structural damping. As mechanical energy was lost, conservation of angular momentum forced the vehicle to precess about an axis inclined to the axis of rotational symmetry. Dynamic coupling of the precessional and bending motions continued the energy dissipation process until the minimum-energy dynamic state (rotation about the principal axis of maximum moment of inertia) was achieved. Detailed descriptions of the Explorer I problem can be found in references 7, 8, and 9.

The experience achieved with Explorer I resulted in a design practice of spin-stabilizing spacecraft only about the principal axis of maximum moment of inertia. In some cases, however, mission requirements may be best served by spin stabilization about the principal axis of minimum moment of inertia. This is the situation for the Application Technology Satellites (ATS) D and E which use spin stabilization about the minimum inertia axis during

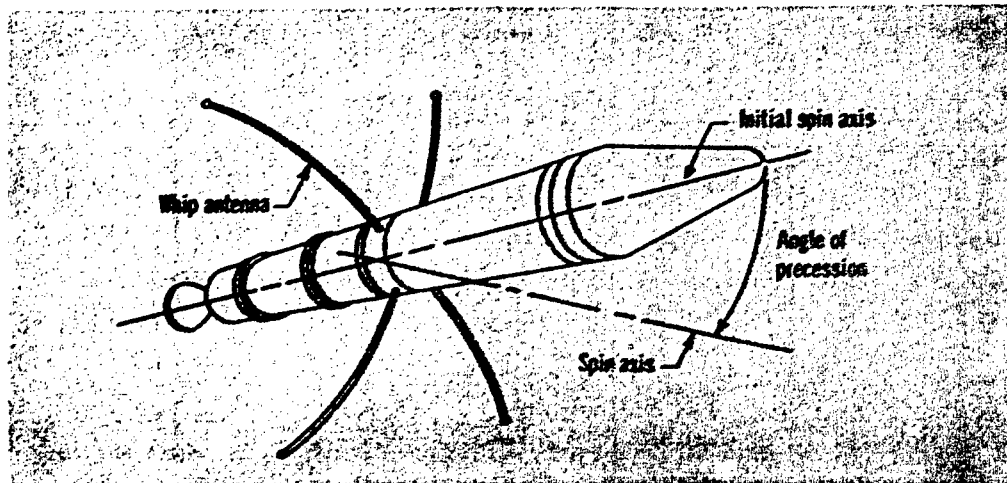


Figure 1.—Explorer I.

orbit transfer. The destabilizing effects of structural energy dissipation are offset by an active pulse-jet nutation damper system (ref. 10).

2.1.2 Compact Body With Flexibly Connected Rigid Appendages

2.1.2.1 Active Control During Thrust Maneuvers

There are a number of examples of successful control systems designed for compact bodies with flexibly connected appendages. One of the earliest was Ranger, which had a shallow parabolic antenna and two solar arrays (fig. 2). The most critical flight operation, involving the possibility of interaction, was the spacecraft autopilot mode. This mode utilized rate integrating gyros and jet vanes to control spacecraft attitude during the midcourse thrust maneuver. Control-system/structure interaction was considered early in the design phase and

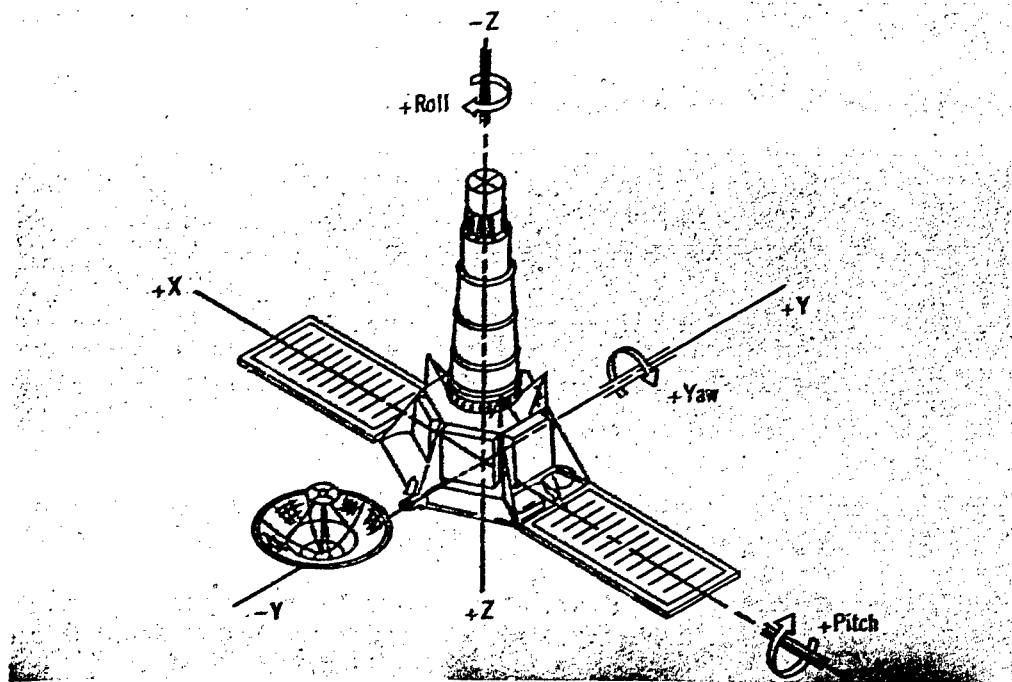


Figure 2.—Ranger.

a mathematical model of the structure was devised. The model consisted of four masses (main body, antenna, and two solar arrays) connected by hinges with linear torsional spring restraints (fig. 3). No structural damping was assumed. The rate gyros, utilizing compensation networks in their torquer loops, provided sufficient lead compensation to stabilize the spacecraft. Although the vehicle bending modes were lightly damped by the attitude-control loops, the final pointing error was not appreciably affected by the transient response. Accurate simulation of the control loops and the spacecraft dynamics justified the design for the mid-course maneuver. The Ranger design experience is described in reference 11.

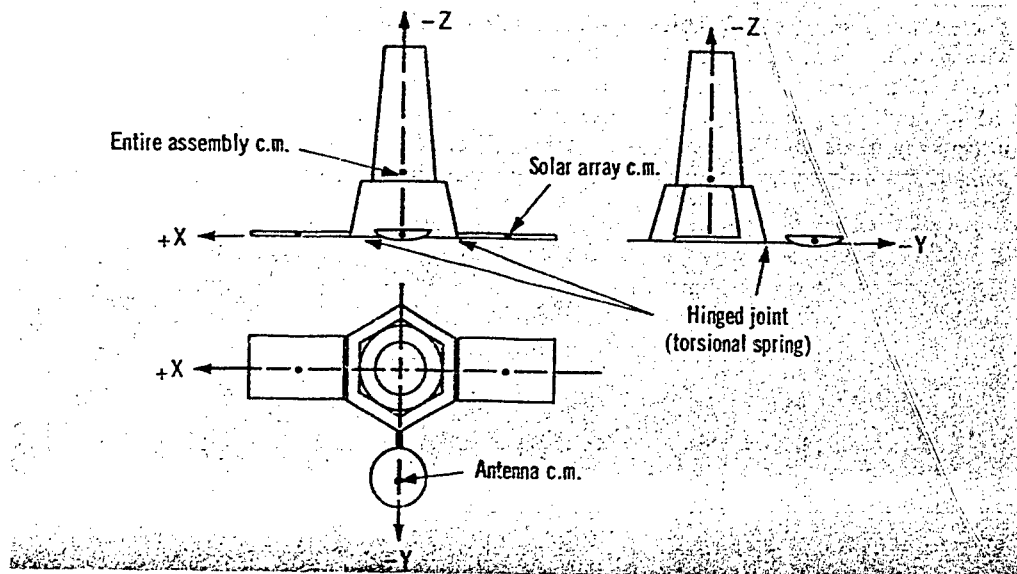


Figure 3.—Ranger simplified structural model.

2.1.2.2 Active Control During Coast Phase

Flexibly attached appendages are generally subjected to lower magnitude forces during coasting flight than during thrust maneuvers. Past experience for spacecraft with flexibly attached appendages, particularly solar arrays, has not revealed any significant problems. For example, the Nimbus satellites were designed to have sufficient structural rigidity so that interaction was prevented by frequency separation between the attitude control system and the structure (ref. 12).

2.1.3 Compact Body With Flexible Appendages

2.1.3.1 Spin-Stabilized Spacecraft With Booms—Environmental Excitation

Alouette I (ref. 13) had a compact central body with four antennas, two 22.9 meters (75 feet) long and two 11.4 meters (37.5 feet) long. The antennas were flexible, extendable booms, stored during launch and deployed after orbital insertion. The satellite was spin stabilized about its axis of maximum moment of inertia, and after successful deployment of the booms, it achieved its desired spin rate of 1.5 rpm. An excessive rate of spin decay, as illustrated in figure 4, was soon detected, however, and after about 3 years of operation the vehicle had essentially stopped spinning.

Analysis revealed the despin mechanism to be the interaction of boom flexibility with both solar radiation and pressure (ref. 14). Briefly, the explanation is as follows: solar radiation causes thermal boom bending. As the satellite spins, timelags present in the thermal distortion produce asymmetrical bending of the booms (fig. 5). Asymmetry of the spacecraft geometry, in the presence of solar pressure, produces a net torque on the body. Depending upon the particular geometry of the vehicle, the net torque may either help or oppose the spin. In the case of Alouette I, the spin rate was opposed.

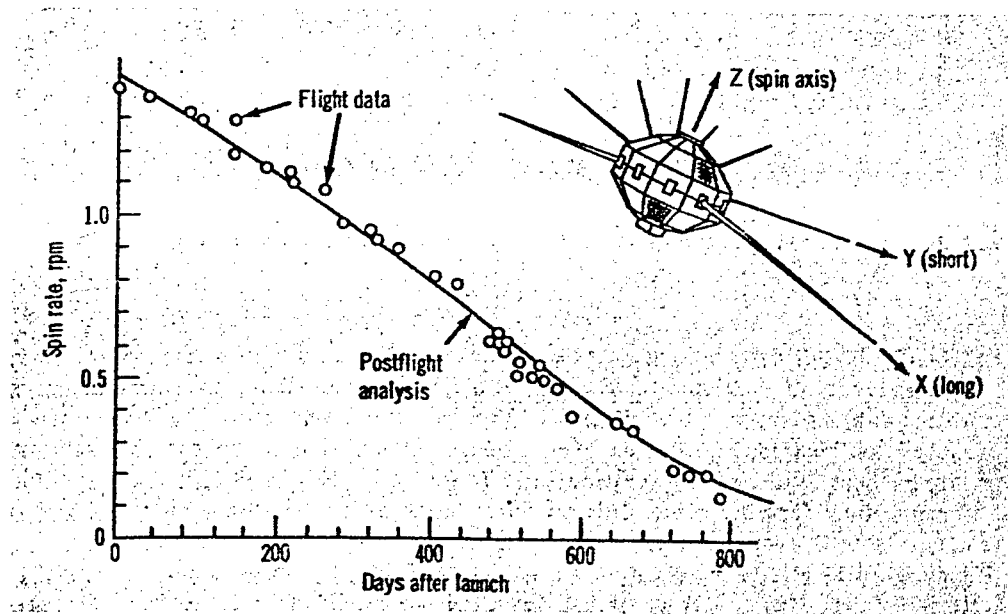


Figure 4.—Alouette I spin decay (ref. 14).

The despin problem was resolved successfully for the flight of Alouette II by mounting small metallic reflector plates on the ends of the booms. The plates provided a compensating torque of sufficient magnitude to essentially cancel the despin rate caused by asymmetric thermal bending.

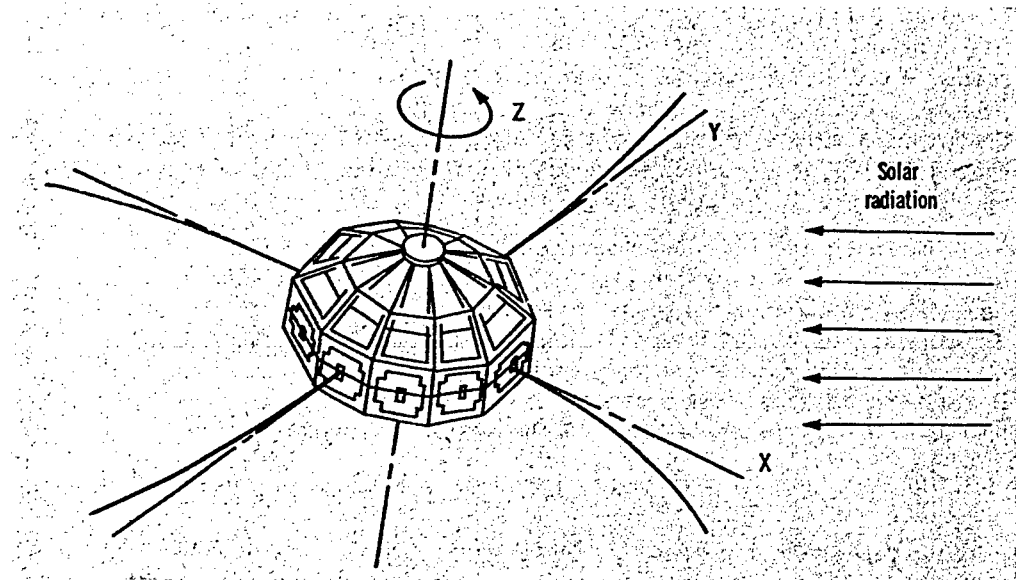


Figure 5.—Alouette I thermal distortion.

2.1.3.2 Gravity-Gradient Stabilized Spacecraft With Booms—Environmental Excitation

Extendable booms have been used extensively for obtaining gravity-gradient stabilization. While magnetic, gravitational, and aerodynamic efforts contribute to errors in gravity-gradient stabilization, a more significant problem has been caused by boom bending produced by solar radiation. This phenomenon was experienced by the first successful gravity-gradient stabilization experiment, flown on the 1963-22A satellite in a 500-nautical-mile circular orbit. This satellite utilized a bare beryllium-copper boom, 100 feet long with a lossy spring damper mounted at the tip (fig. 6). Although attitude errors displayed by 1963-22A were within allowable tolerances, the magnitudes of these errors were larger than expected. It was found that thermal boom bending caused by solar thermal heating was responsible (ref. 15). Both static and dynamic effects were noted. The static effect was caused by a steady-state temperature gradient across the diameter of the boom as a result of the mean solar radiation heating. This effect produced a steady-state error in

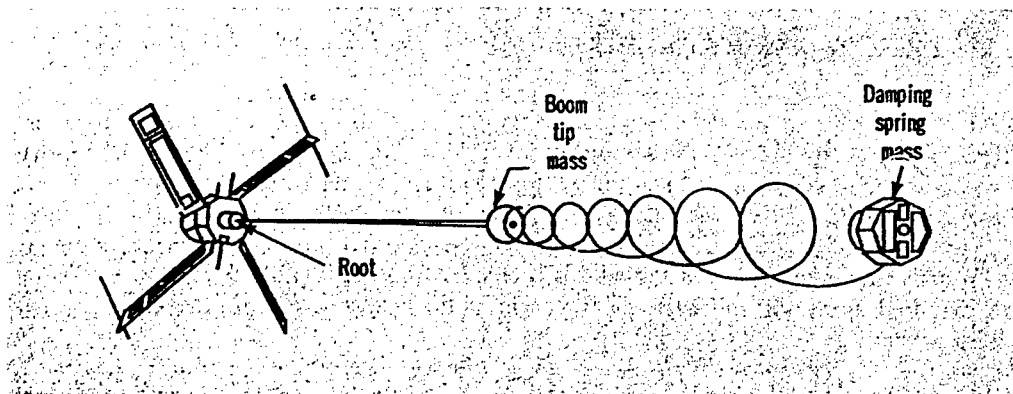


Figure 6.—The 1963-22A satellite with a lossy spring damper (not drawn to scale).

excess of 5° in the orientation of the satellite, relative to the local vertical. Dynamic boom bending was observed when the satellite passed from the Earth's shadow into sunlight. When ensuing motion of the spacecraft exhibited peak excursions as high as 5° . The net result of both effects was a maximum error in attitude of about 10° from local vertical. Although well within the specification value of 20° for 1963-22A, such errors are intolerable for many scientific satellite experiments (ref. 16).

The lossy spring damper was effective in reducing the dynamic boom-bending deflections to less than 5° . A subsequent flight (1964-83D), with an identical boom but without the damper, exhibited boom oscillations occasionally in excess of 12° (ref. 17).

The DODGE (Department of Defense Gravity Experiment) satellite was an experiment in obtaining gravity-gradient stabilization at near synchronous altitude. Two boom end masses on this satellite contained silicone fluid splash dampers to reduce dynamic boom bending. No detectable dynamic bending was evident in the flight data, indicating that this technique may successfully solve or at least significantly reduce the problem.

Boom bending caused by solar effects can also be alleviated by the use of high reflectivity surfaces, such as silver plate. The effectiveness of silver plating was demonstrated by satellite 1963-49B, which experienced maximum static and dynamic boom bending of less than 2.0° and 0.2° , respectively (ref. 17). A lossy spring damper was also used on the boom.

Extendable booms used for gravity-gradient stabilization are also susceptible to a self-excited oscillation driven by solar radiation. The anomalous behavior of GGSE-V, GGSE-VI, and OV1-10 spacecraft have been ascribed to a phenomenon termed thermal flutter. (See refs. 18 and 19 for details.)

2.1.3.3 Active Control of Rigid Body With Booms — Self-Excitation

The OGO-III (Orbiting Geophysical Observatory) satellite also experienced boom-bending difficulties (ref. 20). This vehicle, shown in figure 7, had two 6.10-meter (20-foot) experiment booms (designated EP-5 and EP-6) mounted parallel to the pitch axis and one 9.14-meter (30-foot) boom parallel to the roll axis. An active feedback system controlled roll attitude, utilizing infrared Earth-horizon scanners and inertia reaction wheels, with gas jets to unload momentum when the wheels became saturated. In addition, two solar arrays were mounted along the roll axis and the angle of the arrays was controlled to maximize solar energy input.

As the satellite approached perigee at the end of its first orbit, roll-axis oscillations with a frequency of 0.41 hertz were detected from telemetry data. These motions increased in amplitude until the reaction wheels, gas jets, and solar-array drive were all excited. A 10-second delay in the gas-jet system was activated by ground control and the limit-cycle amplitude decreased until the solar-array drive stopped oscillating. The reaction wheels, however, sustained the oscillation until the vehicle approached apogee of its second orbit. On most succeeding orbits, as the spacecraft neared perigee, the reaction-wheel and solar-array oscillations were excited and the oscillations were sustained for large fractions of the succeeding orbit.

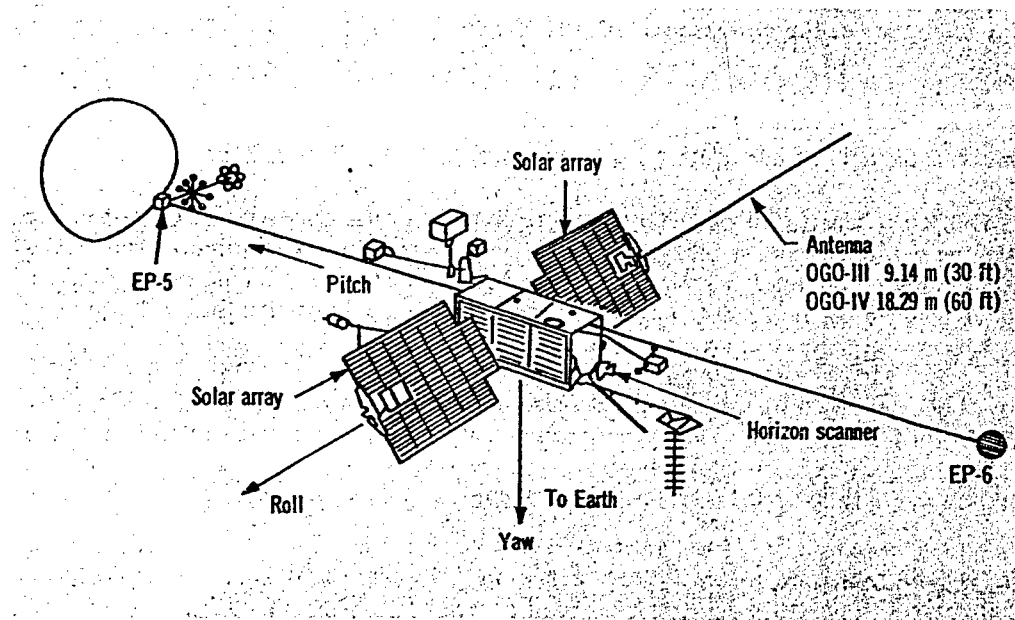


Figure 7.—OGO-III and OGO-IV.

The OGO-III design analysis had been conducted with the flexible booms modeled by mass-spring-damper systems as shown in figure 8. Roll-axis bending mode frequencies of 0.24 and 0.42 hertz (fig. 9) were calculated using the dynamic model. Structural damping ratios between 1 and 2 percent were assumed for the booms. Although a preflight closed-loop stability analysis had not predicted it, control-system/boom-flexibility interaction was suspected as the cause of the sustained oscillations, because of the proximity of the second bending mode frequency to the in-flight oscillation frequency. Reevaluation of the spacecraft dynamics, subsequent to the flight problem, revealed that a sustained oscillation at a frequency of 0.42 hertz could be obtained if boom damping ratios of 0.3 percent were used rather than the preflight values. Excellent correlation of analytic results with flight data indicated that an interaction had occurred and failure to predict it had been because of the discrepancy in the amount of structural damping assumed. In addition, the in-flight oscillation was excited on each orbit, near perigee, when the solar-array drive was most active. Hence, it is suspected that the solar-array drive motion excited the reaction-wheel loop into its limit-cycle oscillations.

In order to eliminate the limit-cycle oscillations, a reaction-wheel delay logic similar to the gas-jet delay logic was devised for OGO-IV. This logic prevents alternating torques from being applied. For example, when positive torque is demanded, the system is inhibited from applying a negative torque for 5 seconds. Hence, if the error signal alternates sign more frequently than every 5 seconds, no torque is applied. Low frequency inputs are not affected by the delay logic so the system operates efficiently to control roll attitude while inhibiting the roll limit cycle noted on OGO-III.

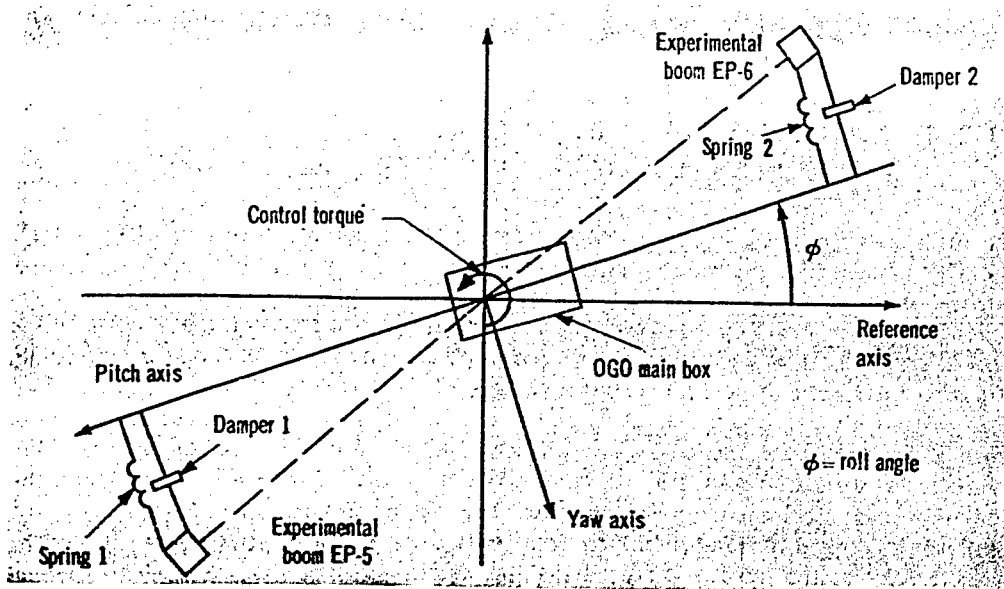


Figure 8.—OGO simplified structural model.

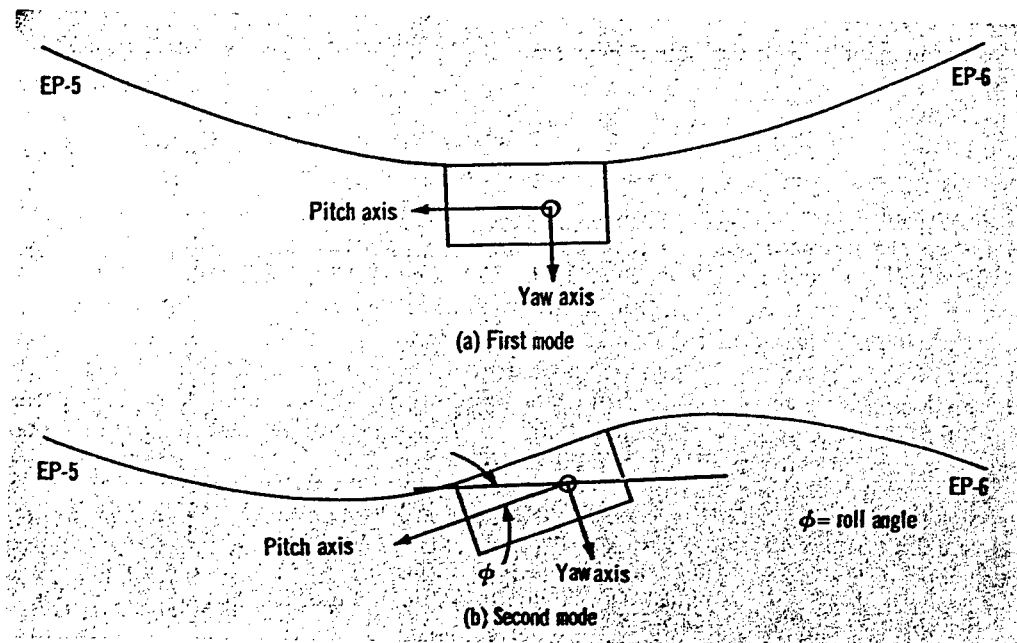


Figure 9.—OGO roll-axis bending mode displacements.

2.1.3.4 Active Control of Rigid Body With Booms—Environmental Excitation

The reaction-wheel delay logic was incorporated into the control system of OGO-IV (ref. 21). This modification prevented oscillations of the two 6.10-meter (20-foot) booms (EP-5 and EP-6 of fig. 7) from interacting with the roll control system. However, a reaction-wheel oscillation with a period of approximately 35 seconds was observed in the pitch axis during the 42d revolution (rev 042). By rev 065, the pitch oscillation had increased in amplitude, saturating the reaction wheel and exciting the gas jets. The oscillation was attributed to an 18.29-meter (60-foot) boom extending from the solar panel along the roll axis, as shown in figure 7. The boom was excited by solar radiation as evidenced by decay of the oscillation whenever the spacecraft passed into eclipse. While the pitch response was predominant, the oscillation was also evident in roll and yaw. This led to speculation that the boom was coning in a coupled torsion-bending motion similar to that described in reference 22. The characteristics of the motion indicated that the boom oscillation was caused by thermal flutter which is discussed in references 18 and 19 (see sec. 2.1.3.2).

2.1.4 Large Flexible Body, Active Control During Thrust Maneuvers

2.1.4.1 Gemini/Agena

The in-flight connection of two spacecraft was accomplished when the Gemini VIII spacecraft docked with an Agena Target Vehicle as illustrated in figure 10. The Gemini X flight plan included docking with the additional requirement for firing the Agena engine to perform maneuvers in the docked configuration. Stability studies were made using an estimated first body-bending modal frequency of 5 hertz in order to determine the performance capability of the Agena thrust vector control system with the Gemini spacecraft as payload. The studies revealed that inadequate gain margins existed. The Agena control system was modified by adding a 5-hertz attenuation filter to the compensation networks of the autopilot to gain stabilize the bending mode. However, the first bending modal frequency was subsequently reestimated closer to 3 hertz which, again, impaired the Agena performance. The Agena control system was then modified by altering an autopilot lead-lag compensation network to give a 16- to 18-decibel gain margin for the 3-hertz frequency. The effectiveness of the modification is shown in figure 11.

A ground vibration test was conducted with the Gemini spacecraft attached to the Agena target docking adapter. The docking adapter was bolted to a fixture that was cantilevered from the laboratory floor. Test data were then adjusted analytically to correspond to the in-flight free-free configuration. The adjusted test results, shown in table II, yielded estimates of the first body-bending modal frequency and damping ratio, and the maximum cross-axis coupling present in the system. These results were used to support the stability analysis. The final phase of the control-system/structural-flexibility interaction study included an in-flight test of the docked Gemini X/Agena vehicle. The pilot fired a pair of spacecraft pitch-plane attitude thrusters for 3 seconds followed by a 3-second firing of the

*Table II.—Comparison of Gemini/Agena In-Flight Data
With Ground-Test Data (Ref. 21)*

Test	Frequency, Hz	Damping ratio, percent	Maximum cross-axis coupling, percent
Ground	3.6	3 (Ambient temperature)	3 to 6
In-flight	4.0	4.5 to 6.5 (Temperature unknown)	3 to 6

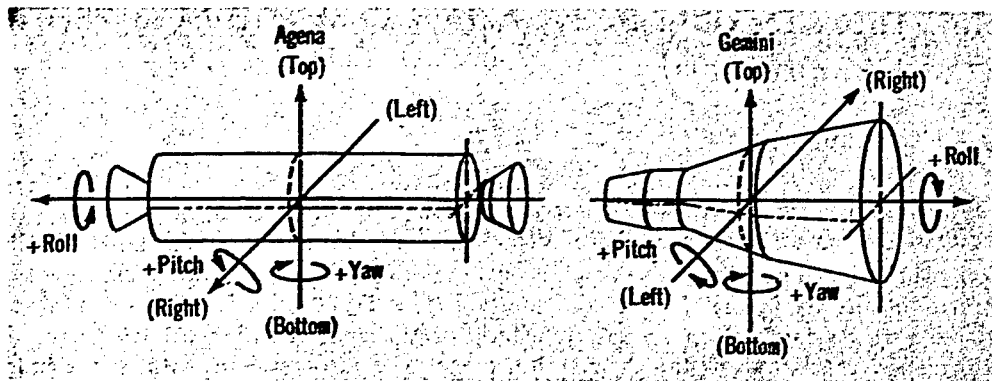


Figure 10.—Gemini/Agena.

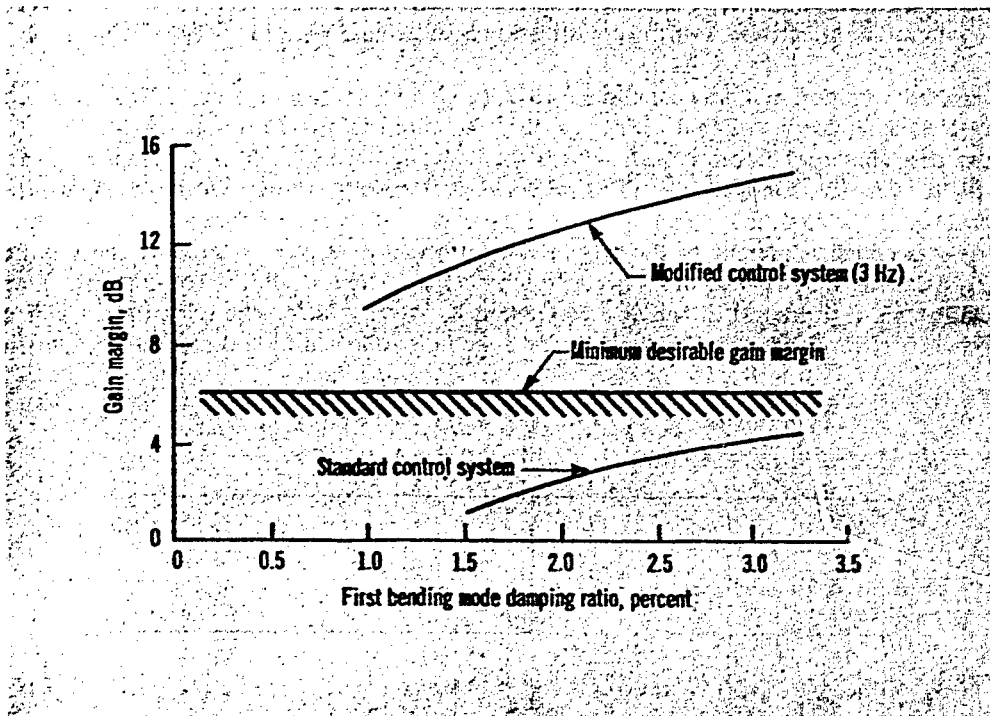


Figure 11.—Stability study of maneuverable Gemini/Agena.

opposing pair of pitch-plane thrusters. The maneuver was then repeated in the yaw plane. Vibration data, sensed by accelerometers located in the Gemini adapter section, were transmitted to the ground station and evaluated prior to firing of the Agena primary propulsion system. The first body-bending frequency was found to be about 12 percent higher than that obtained in the ground tests, damping was higher than in ground test, and the cross-axis coupling was approximately the same as determined previously (table II). After the satisfactory evaluation of the flight data was completed, the Agena engine was ignited and no interaction problems were encountered. For more details on this aspect of the Gemini program, see references 23 and 24.

2.1.4.2 Apollo Command and Service Module Lunar Module

The Apollo system utilizes a docked configuration on its translunar trajectory. The Apollo Command and Service Module (CSM) docked with the Lunar Module (LM) (fig. 12) is a very

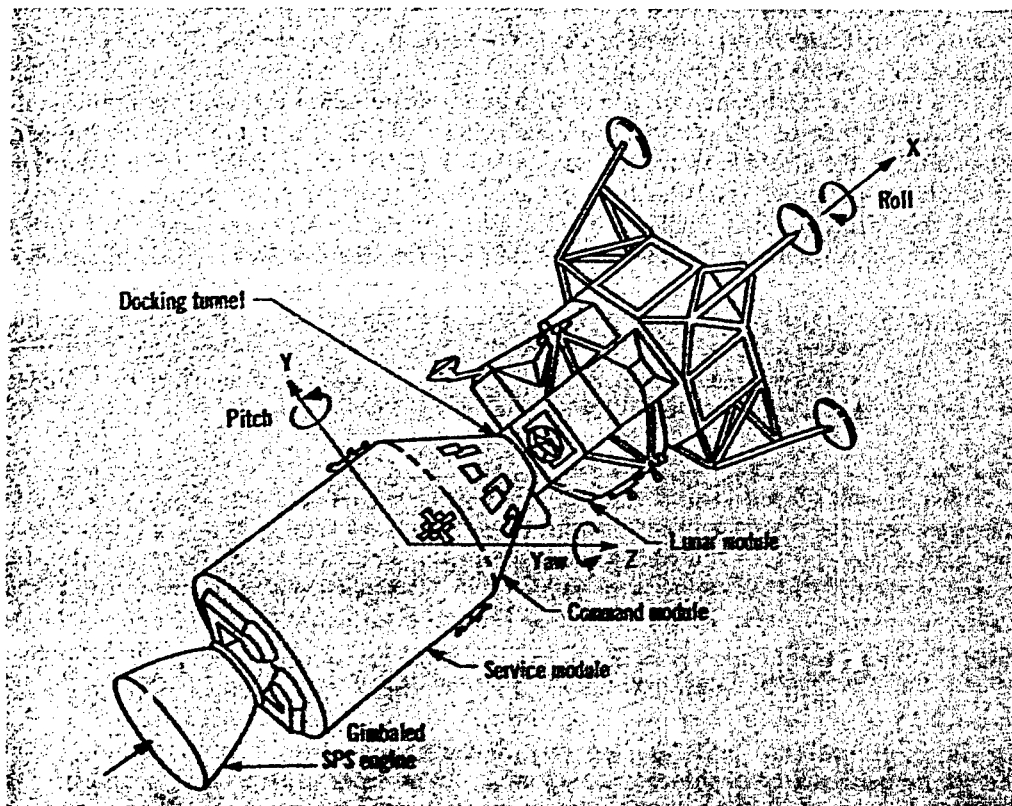


Figure 12.—Apollo Command and Service Module and Lunar Module.

flexible vehicle because of bending in the coupling structure. During thrusting maneuvers, spacecraft pitch and yaw attitude control is achieved by driving the service module propulsion system (SPS) engine gimbal servos. The resultant direction of the thrust vector, relative to the vehicle center of mass, produces control torques about the pitch and yaw axes. Vehicle attitude (pitch, yaw) is available as feedback information from the inertial measurement unit located in the command module. The thrust vector control digital autopilot (TVC DAP) utilizes this information to generate appropriate gimbal servo commands, so that the vehicle will respond to steering commands from the guidance and navigation system. Simultaneously, the TVC DAP must stabilize bending at the docking tunnel (coupling structure) and fuel-slosh motion. A high computer sampling rate of 12.5 samples per second was selected so that a digital filter could be designed to provide substantial attenuation for the higher frequency bending modes.

The lowest frequency mode, whose shape is dominated by bending at the docking tunnel, had a predicted natural frequency of 2.31 hertz for the one-half full propellant condition. Because of the large uncertainty in predictions of structural parameters, the autopilot was designed to be stable for a comparatively wide range of first bending mode frequencies from 1 to 3.8 hertz. A combination of gain and phase lag stabilization was used to stabilize this mode. As a result of the conservative approach to the design of the digital autopilot, bandwidth of the closed-loop system was quite low (0.05 hertz). Hence, initial condition transients are prolonged and velocity errors at SPS engine cutoff could be large. In order to reduce the effects of initial condition transients, a second autopilot operational mode with a bandwidth of 0.1 hertz was implemented for use during the first few seconds of the SPS burn. Although this system quickly reduces initial errors, it also destabilizes the first propellant-slosh mode, occurring at 0.4 hertz. Before the slosh oscillations become significant, however, control is switched to the low bandwidth system previously described, which stabilizes the slosh mode. Both autopilot modes stabilize all spacecraft bending modes so the structure is not subjected to excessive dynamic oscillations.

Extensive simulation was used to verify the Apollo TVC DAP. Analog, digital, and hybrid computer techniques were applied. Much of the spacecraft hardware, including the guidance and navigation digital computer, was included in the simulations. In addition, extensive testing of the CSM/LM structure was conducted to determine vibration characteristics. The results of a ground modal survey compared favorably with analytical predictions.¹ Mode shapes were essentially as predicted and frequencies were within 15 percent of the computed values. The first modal frequency for the one-half full propellant condition was found to be 2.65 hertz. An in-flight excitation through the SPS engine was conducted on the Apollo 9 mission. This "stroking" test verified the ground-test results and demonstrated the proper performance of the control system in the presence of structural flexibility. For more detail on the docked Apollo interaction, see references 25, 26, 27, and 28.

¹ Unpublished data.

2.1.5 Summary

The foregoing resume of the status of control-system/structure interaction effects in spacecraft has considered the most significant problems to date as well as areas illustrating successful design for flexible spacecraft. The compact, near-rigid satellites have been relatively free of difficulty.

Spacecraft with solar paddles attached to compact rigid central bodies have been successfully designed to include structural-flexibility effects in the design of the control system. Flexible booms deployed from compact rigid central-body spacecraft have been the source of most in-flight problems. Initially, difficulties existed because of insufficient analysis of the spacecraft dynamics. More recent occurrences of boom-flexibility interactions have stemmed from the difficulty in analyzing the booms, insufficient or inaccurate structural data for the booms, and insufficient knowledge of the effects of environmental phenomena on the booms. The docked configurations of manned spacecraft are, to date, the best examples of the flexible-body spacecraft. The possibility of an interaction problem in Gemini/Agena was explored analytically. Design changes were supported by simulation and ground test, and final verification of the absence of an interaction problem was obtained from flight test.

2.2 Potential Problems

As spacecraft designers gain in experience and the payload potential of launch vehicles increases, it is likely that spacecraft will grow in size, complexity, and sophistication. Control-system/structure interaction considerations will continue to be a significant design consideration of these future spacecraft. Several potential problem areas are discussed in the following examples.

2.2.1 Dual-Spin Stabilized Spacecraft With Flexibility

Spacecraft which have two sections that rotate relative to each other about a common axis are called dual-spin spacecraft (ref. 29). These spacecraft, like single-body spin-stabilized spacecraft, are susceptible to instabilities caused by energy dissipation through structural flexibility (refs. 30, 31, and 32). The dual-spin concept has been applied successfully to the Orbiting Solar Observatory satellites and to ATS C.

2.2.2 Spin-Stabilized Spacecraft — Spin Resonance

Spin resonance is an interaction in which a coincidence of spin rate and the natural frequencies of transverse bending modes of slender bodies result in excessive structural deformation similar to the critical speeds of a shaft. Although it has not been experienced by spacecraft, it has occurred in the flight of a spin-stabilized launch vehicle.

2.2.3 Spacecraft With Flexibility in the Control Loop

This class of interaction problem differs from any previously discussed in that the elastic structural elements serve as couplings between sensor and actuator. This interaction is a basic consideration in launch vehicle design and will become more important for large, flexible spacecraft. For example, ATS F and G will utilize a self-erecting 9.14-meter (30-foot) diameter, flexible antenna connected by a truss structure to an Earth-viewing equipment body (ref. 33). The attitude control system is located at the antenna, and the sensors are mounted on the Earth-viewing equipment body. Structural flexibility between the sensors and actuators is a primary design consideration.

2.2.4 Spacecraft With Multiple Controllers

Multiple controllers may be employed in future designs in order to maintain independent control of bodies flexibly attached to a central body, to provide load control of large flexible members, and to introduce known deflections and artificial damping to flexible members. Multiple controllers are also present when several spacecraft are docked as will be the case for the numerous configurations of the Apollo Applications program (ref. 34). The interaction problems illustrated in section 2.1 are compounded in the presence of multiple controllers.

2.2.5 Larger, More Flexible Vehicles

The interaction potential of all spacecraft increases with vehicle size. The Apollo Applications program previously discussed presents more difficulties than the Apollo CSM/LM. Other examples of magnified problems are: a radio-telescope satellite, 1500 meters (4921 feet) in diameter, composed of flexible aluminum ribbons (ref. 35); and the Radio Astronomy Explorer with four extendable booms, each 228 meters (750 feet) long (ref. 36).

3. CRITERIA

It shall be demonstrated to a high, quantitative level of confidence that there exists no instability or other behavior, involving interaction of the stabilization and control system with structural deformations of the spacecraft, which could impair flightworthiness or compromise crew safety. It shall also be demonstrated that system performance requirements, including pointing accuracy, fuel consumption, and vibratory accelerations, relating to mission success are satisfactorily met when account is taken of the effects of static or dynamic interactions. These demonstrations should be made by a suitable combination of analytical studies, simulations, component tests, system tests, and, when necessary, flight tests. All anticipated flight configurations and modes of operation should be considered. The stabilization and control system should be designed so that it is relatively

insensitive to changes in the characteristics of the structure and/or control-system hardware, and so that it has sufficient inherent versatility to handle limited changes in guidance and control requirements.

3.1 Analytical Studies

Analytical studies shall be performed in sufficient detail to determine structural-flexibility effects on the control system, and to demonstrate system efficiency and compliance with system requirements and specifications.

The analysis shall consider:

- The effects of environmental as well as vehicle originated excitations of the structure and control system
- Vehicle and control system configuration changes
- Changes in spacecraft center of mass
- Changes in spacecraft mass and inertial characteristics
- Effects of structural parameters including structural damping
- Effects of control system parameter changes

3.2 Simulation Studies

Simulation studies shall be conducted whenever possible to demonstrate system efficiency and compliance with system requirements and specifications, and for use as a design tool. In order to achieve realistic system response, as much flight hardware as is feasible shall be included in the simulation. If the spacecraft is to be manually controlled, pilot-in-the-loop simulation studies shall be included.

3.3 Tests

Tests shall be performed on spacecraft which exhibit control-system/structure interaction effects. These tests shall demonstrate that interaction effects do not impair operation of the dynamic system, and that system criteria and requirements are satisfied. Preliminary tests shall be conducted during the development program on a timely basis so that maximum utilization can be made of the test results. Ground tests should include component testing, deformation and vibrational testing of the structure, environmental tests, control system operation tests, and overall system tests including both structure and control system. If the spacecraft is manned, flight tests shall be conducted to demonstrate compliance with the crew safety criteria.

4. RECOMMENDED PRACTICES

Coordination must be established between control system and structural design groups, so that both may actively participate in selecting the best overall design. Interchange of information and intelligent compromise on all parameters affecting interaction should take place during the spacecraft development phase. All participating design groups should be made aware of configuration and hardware changes so that the effects of the changes may be evaluated from the viewpoint of each group's particular area of responsibility.

Existing analysis and design techniques, available to the control system designer are usually sufficient for modeling interaction phenomena and aiding proper design. It is necessary, however, that the range of structural parameter values be available to the control designer for use in modeling the system. From experience gained in the past, it is clear that unforeseen interaction problems usually result from a lack of knowledge about the mechanism causing the interaction. Special emphasis should, therefore, be placed on investigating possible interaction problems and accurately determining the possible ranges of structural parameters.

4.1 Analytical Studies

The static and dynamic structural characteristics of spacecraft, particularly of flexible or flexibly connected appendages such as solar arrays and extendable booms, must be carefully analyzed and related to the control system dynamics to determine possible interaction effects. Estimates of the accuracy of structural data should be made available to the control system designer as early as possible. For determination of basic structural response characteristics, see NASA SP-8012 "Natural Vibration Modal Analysis."

4.1.1 Structural Analysis

To study the effects of flexibility, an accurate structural dynamic model must be developed. Using structure represented as discrete parameter or distributed parameter (continuous) systems, the structural dynamic model is used to obtain the dynamic equations which (in conjunction with rigid body motion) describe flexible spacecraft motion. Because of the wide variation in spacecraft structural configuration, several methods of modeling the structural dynamic system have evolved. Reference 37 describes the four major methods used in attitude control analysis: energy sink method, discrete parameter method, modal coordinate method, and hybrid coordinate method.

Each method is briefly described below. It should be noted that only qualitative boundaries of applicability can be defined.

Energy Sink Method.—The energy sink method is applicable to the compact, near-rigid body class of vehicles (e.g., ref. 7). A typical use is analyzing spin axis nutation for spin-stabilized spacecraft. The method assumes that the motion of the flexible spacecraft is closely approximated in terms of short period dynamics as a rigid body. However, dissipation of energy through structural damping is accounted for by postulating a small negative rate of change of kinetic energy with no corresponding change in angular momentum. Thus, a hypothetical "energy sink" is assumed which, in fact, violates Newton's laws, but provides a useful description of the spacecraft motion. See appendix A for more detail.

Discrete Parameter Method.—The discrete parameter method is best suited to spacecraft characterized as compact bodies with flexibly connected appendages, although it can be applied to the flexible body classes as well (e.g., refs. 11 and 38). Typically, the method assumes a model composed of discrete concentrations of mass, connected by massless elastic springs and possibly massless dampers. Equations of motion are derived by developing the equilibrium equations for each rigid subbody. When used to model flexible structure, the degree of subdivision of the structure into mass units is dependent upon the complexity of the structure, computer capacity, and the experience and ingenuity of the analyst. Here, one aspect of the analysis that often causes difficulty is the estimation of equivalent spring constants and damping ratios to produce an accurate model. The discrete parameter method yields a set of coupled ordinary differential equations, usually too large to be solved analytically. However, the equations are well suited to numerical solution on a computer.

Modal Coordinate Method.—The modal coordinate method may be used to model large, flexible spacecraft and others for which the previous methods are not readily applicable (e.g., refs. 39 and 40). This method is restricted to flexible structures with small internal damping which may perform large-angle rigid-body rotations but whose deformations are everywhere small compared with body dimensions, that is, deformation behavior can be described by linear equations of motion (e.g., refs. 41 and 42). Therefore, the spacecraft elastic motions can be construed to be linear in deformation coordinates and, hence, can be described by the superposition of normal modes of undamped free vibration. The use of a series of normal modes permits a practical limitation of the complexity of the analysis by the exclusion of all modes above a selected frequency on the basis that their contribution to the interaction is not significant. The undamped modes may be obtained either analytically or experimentally. The use of normal modes yields equations of elastic motion in which elastic and inertia forces are uncoupled. See appendix B for more detail on this method. The scope of the equations may be increased by the addition of a damping ratio term for each mode, with the restriction that internal damping is small and uncoupled. The differential equations of elastic motion must be solved simultaneously with the rigid-body equations of motion, the latter usually being nonlinear because of large angular rotations. The dynamic system is usually so complex that a computerized solution is necessary.

Hybrid Coordinate Method.—The hybrid coordinate method of analysis combines the discrete parameter method and the modal coordinate method. The discrete parameter method has

the advantage of being able to incorporate large deflections and nonlinearities, while the modal coordinate method is efficient by virtue of the ability to truncate higher modes, thereby eliminating unnecessary high frequency response from the analysis. Since most spacecraft have structural systems that are a combination of one or more near-rigid bodies with one or more flexible bodies, the hybrid system provides the most efficient method for analyzing the structure. To the present, applications of the hybrid system have been few, but there is increasing interest in this method (refs. 43 and 44).

4.1.2 Control-System Analysis

If the spacecraft control system does not contain active control elements (viz, a passive system), it may be modeled as an integral part of the structure (e.g., gravity-gradient booms), with control dynamics included in the structural model. Active control systems, however, require additional modeling to describe the dynamic character of the controller.

The first step in analysis of a control system usually involves modeling the controller as a linear system. In many cases, however, "hard" nonlinearities (i.e., dead bands, etc.) are purposely designed into the controller and there is no region of linear system operation. In that event, quasi-linearization (describing function) or phase-plane analysis is performed in order to handle the nonlinear elements (refs. 45 and 46). If the controller contains sampled-data elements (i.e., a digital system), it is often helpful to utilize z-transform techniques to study the system (ref. 47). In some cases, when the sampling frequency is high compared to other modes of the system, the sampled data subsystem may be analyzed as an equivalent continuous system. Caution should be used, however, because the continuous analysis will not display instability and poor transient response which may be produced by the effects of a finite sampling rate. A z-transform analysis is usually necessary, if the sample frequency is less than five times higher than the highest significant frequency component of the input to the sampled-data system, or if the sample-data output frequency can excite elements of the control loop.

4.1.3 Control-System/Structure Dynamic System

A complete set of coupled equations of motion for the control-system/structure dynamic system is essential to an accurate assessment of interaction potential (ref. 48). The general form of the closed-loop control system, including the dynamics of the flexible structure is illustrated in figure 13. Both control forces and external disturbances excite the rigid and flexible body modes. These modes sum to produce sensor motions. Sensor outputs become the feedback signals used by the controller to command actuators which in turn produce the control forces.

The control forces are functions of the particular actuator dynamics of the control system. These forces are inherently included in the spacecraft dynamic equations used in the energy sink method as illustrated in appendix A. In the discrete parameter method, control forces

can be applied directly as a forcing function or as an external source of energy. For the modal coordinate method, the control forces contribute to the generalized forces, as illustrated in appendix B.

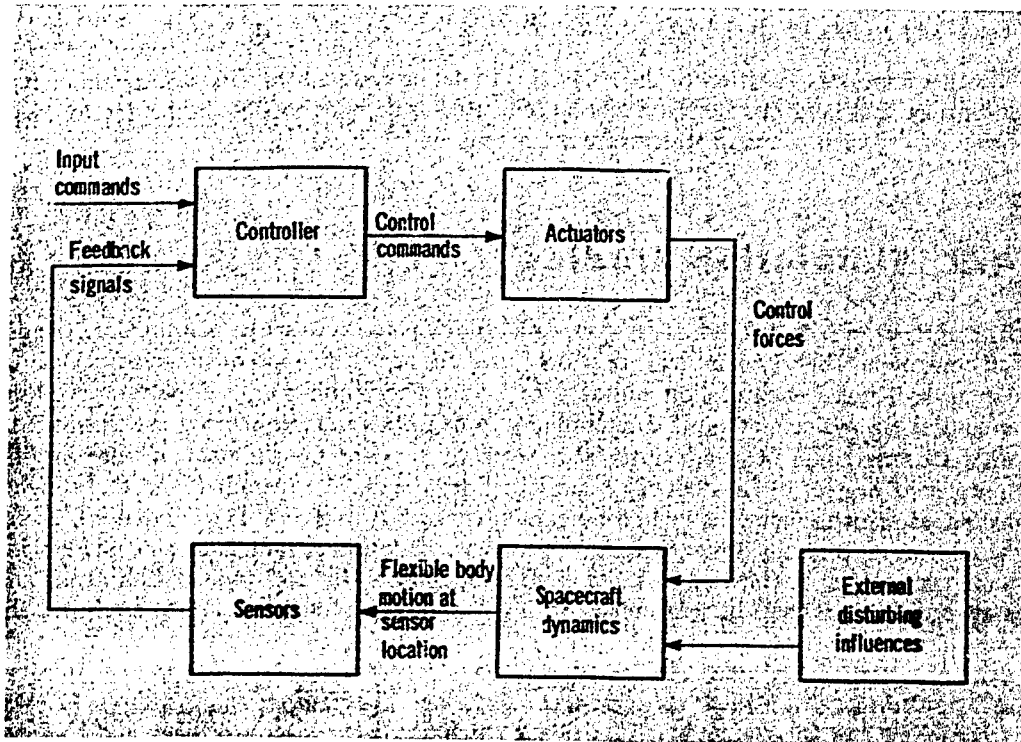


Figure 13. - Block diagram of control loop with flexible body dynamics.

After structure and control-system models have been constructed, system stability is studied. Laplace transform methods utilizing Nyquist diagrams, Bode plots, Nichols charts and/or root locus diagrams are useful (e.g., refs. 49 to 52). The Nyquist diagram and Bode plot indicate stability and determine stability margins. A root locus provides insight into the effects of gain changes on system stability and can provide information about system transient response. Closed-loop frequency response is determined from the Nichols chart which is especially useful for lightly damped structural systems where gain and phase characteristics change rapidly with frequency. These techniques may also be applied to nonlinear systems using quasi-linear describing function techniques. Stability analyses should be supported by detailed computer simulations.

Careful consideration of the response of the mathematical model to typical input signals and disturbances can give useful indications of both design and modeling inadequacies. Special attention should be given to the mechanics by which control-system/structure interaction takes place in order to determine possible modifications to the dynamic model. Sensor and actuator motions caused by localized and overall structural flexibility can produce significant effects which should be considered.

Another important function of the analysis is to define system hardware requirements. It is important that hardware requirements be generated as early as possible in the development program, especially if major structural components are involved.

4.2 Simulation Studies

The analysis of complex control-system/structure dynamic systems is facilitated by the use of computers. Digital computers are widely used for structure and control-system analysis where high speed computation and accuracy are prime requirements. Structural analysis, including eigenvalue methods for determining mode shapes and frequencies, is well suited to digital computation. The computer also helps to alleviate the problem of selecting the number of degrees of freedom since additional degrees may be included, and their effects examined. This technique is limited by computer capacity and cost. The computer is useful as an aid to control-system analysis and design. Bode plots, root loci, and phase-plane analyses can all be facilitated by digital computation. In addition, optimization and statistical analysis may be performed. Analog computers are used most effectively for the solution of arrays of differential equations. Simulation of control-system and structural dynamics, particularly when nonlinearities are involved and a number of degrees of freedom are desired, is well suited to analog computation. The Ranger analysis (ref. 11) was accomplished by analog computation. Where high accuracy, extensive logic, and complex dynamic simulation are required, hybrid computers provide the capabilities of both digital and analog computers. Hybrid simulation aided in the development of the Apollo CSM/LM TVC DAP (ref. 53).

Computer simulation is an indispensable tool for control-system design analysis and verification. Effects which are difficult to analyze mathematically are most suitably studied by simulation (ref. 48). Input disturbance effects such as gravitational perturbations, solar radiation (heat and pressure), magnetic disturbance torques, engine vibration, and meteoroid impacts are often examined by this method. Various spacecraft operational modes (staging, docking, deployment of appendages, fuel venting, maneuvering, etc.) may be simulated and control-system/structure interaction investigated under each condition. The simulation may also include spacecraft internal dynamics involving center-of-mass shifts, crew motion, and fuel slosh. Combinations of disturbance effects can also be studied. Real-time simulation, using actual flight hardware, is strongly recommended. The extent of the simulation should be chosen considering other program factors, but should include control-system hardware

and computer simulation of structural dynamics. Man-in-the-loop simulation is mandatory if the crew plays an active role in controlling the spacecraft. The simulation experience obtained in Project Gemini (refs. 21, 54, and 55) vividly illustrates the value of these techniques.

4.3 Tests

Vibration testing is of great value in investigating possible control-system/structure interaction. It is recommended that ground vibration tests of the in-flight configuration of the spacecraft be conducted to provide adequate structural response data, if analysis and simulation indicate that a control-system/structural-flexibility interaction problem may exist. These tests are particularly significant for manned spacecraft, and omission of the tests should be permitted only if the tests are technically unfeasible or if adequate technical information is available (such as flight-test data) which provides evidence that the interaction is within acceptable limits. Sinusoidal inputs should be applied to the spacecraft to determine the response characteristics, including damping, of the total system. Most spacecraft vibration tests are implemented by vibration tables (e.g., refs. 13, 56, and 57). Attempts should be made to obtain a realistic simulation to determine the free-free modes of the system. One method is to suspend the spacecraft by springs taking care to eliminate the static load, and to use several electromagnetic shakers to excite the spacecraft as was done for Surveyor (ref. 58), Gemini (ref. 59), and Apollo (ref. 60). Local response as well as overall response should be monitored (ref. 61). If feasible, vibration tests should be made on the control-system/structure combination (i.e., closed loop) to determine if instabilities exist. The practicality of this test may be limited by difficulty in simulating in-flight conditions such as providing the driving force of a high thrust engine. Any instability should be investigated, considering differences in design and test environmental and operating conditions. For manned vehicles, the crew should be in the spacecraft with precautions taken for their safety (ref. 62).

In order to investigate the effect of structural flexibility on the control system, testing with appendages deployed is recommended. This can be done with solar paddles which, to date, have had sufficient stiffness to be considered rigid. However, the highly flexible extendable booms, as used on Alouette I and OGO-III, are not amenable to test. Flexible appendages, such as these, should be considered as experiment packages until sufficient flight-test data is available on their response characteristics. Retraction or jettison of the appendage should be considered if serious interaction problems are anticipated.

If, after all analyses, simulation, and ground tests have been completed, the flightworthiness of the spacecraft is in question, then a flight-test program is recommended. Provision should be made to facilitate postlaunch evaluation of the spacecraft and to allow in-flight adjustments of the control system to negate any interaction effects.

4.4 Recommended Practices for Specific Categories of Spacecraft

4.4.1 Compact Near-Rigid Body, Spin Stabilized

The following practices are recommended for spin-stabilized compact spacecraft:

1. Whenever possible, spin-stabilize about the axis of maximum moment of inertia.
2. If the spacecraft is expected to maintain its spin about an axis of minimum moment of inertia for an extended period of time, then all possible sources of energy dissipation are to be avoided. The following are specific recommended practices:
 - a. The spacecraft should be as rigid as possible.
 - b. If there is flexibility in the spacecraft body or appendages, care should be taken that the resulting motion does not give rise to excessive mechanical energy dissipation.
 - c. Fluids that could dissipate energy in a sloshing mode should be avoided.
 - d. Passive nutation dampers should be caged during this time.
3. If the spacecraft is expected to maintain its spin about an axis of minimum moment of inertia for an extended period and energy dissipation cannot be prevented, then an appropriate source of energy should be incorporated. The active nutation dampers discussed in reference 10 are representative of the devices to be utilized.
4. If the spacecraft is expected to maintain its spin about an axis of maximum moment of inertia, then it should be designed so that the interaction of the flexible appendages with the space environment decreases the ratio of angular momentum to angular kinetic energy.

4.4.2 Compact Body With Flexibly Connected Rigid Appendages

4.4.2.1 Active Control During Thrust Maneuvers

In order to control the effects of one or more flexible appendages the following are recommended:

1. If possible, design the bandpass of the control system to be lower than the lowest flexible appendage natural frequency and separated from it by as large a margin as possible.
2. Make the damping ratio associated with each flexible appendage as large as possible. Artificial stiffening or artificial damping by means of separate control loops should be used as applicable.

3. Keep the mass and moment arm of the flexible appendages as small as possible in relation to the spacecraft's rigid-body inertia.
4. Keep the control gain as high as possible consistent with stability limitations to minimize attitude error in the presence of environmental or motion-induced disturbance torques.
5. If expedient, design the control system to suppress structural flexibility effects actively.
6. Consider the time-varying nature of the flexible appendage and control dynamics, and select control parameters which will ensure stability over the anticipated range of structural parameter changes.
7. Large amplitude stability of a nonlinear controller should be demonstrated by simulation. For example, bending coordinates should be initialized at the structural limit and the resulting simulation should show that the closed loop is stable and that structural limits are not exceeded.
8. Perform a simulation with fuel-slosh coordinates initialized at maximum possible amplitude and check for violation of a performance requirement for attitude control, for excessive wear of thrust vector servos, and excessive fuel expenditure rate in the reaction-jet system. If the results of the worst amplitude study indicate slosh is a problem, then the stability of the slosh modes should be examined.
9. Harmonic resonance of engine vibration and structure should be avoided to prevent feedback to the control system. Engine misalignment should be minimized; the torque caused by engine misalignment must be included in the simulation.

4.4.2.2 Active Control During Coast Phase

The recommended practices of section 4.4.2.1 are applicable with the exception of 9.

4.4.3 Compact Body With Flexible Appendages

Long, flexible, extendable booms have been involved in interaction problems. The following recommendations are made for all booms:

1. Reduce the boom thermal bending caused by solar radiation and pressure by:
 - a. Using modified booms which include the following: closed cross sections which afford higher torsional rigidity; perforations which admit solar radiation to the interior so that temperature gradient across the boom is significantly reduced; wire-mesh construction both to achieve rigidity and to alleviate the temperature distribution problem; and black interiors to increase the radiation absorption properties, again, to provide uniform temperature distribution (refs. 63 to 66).
 - b. Using high reflectivity surfaces such as silver plate polished to a high luster (refs. 67 and 68).

- c. Using a boom motion damper either at the boom root, along its length, or at the boom end mass
- d. Making the boom as short as possible
- 2. Check the structural dynamic response of booms and other lightly damped structures for zero damping as well as for nominal damping.

Choice of the boom depends on the amount of bending which can be tolerated. The following discussion indicates the amount of accuracy which can be achieved with 100-foot booms.

Extendable booms designed to minimize thermal bending (viz, made from screening or with perforations to heat the side opposite from the Sun) can be expected to attain angular tip deflections on the order of 1° . If the boom is continuous, but has an outside silver-plated surface polished to a high luster, then peak angular tip deflections would be between 2° and 5° . If the boom exterior surface has a comparatively high solar absorptivity, then the satellite designer can readily expect peak boom tip angular deflections in excess of 10° .

4.4.3.1 Spin-Stabilized Spacecraft With Booms—Environmental Excitation

If it is desired to retain the spin rate of spinning spacecraft with long extendable booms, then it is recommended that:

- 1. The booms be designed for minimum thermal bending as discussed above in this section.
- 2. A means for maintaining spacecraft spin rate, by reaction against the Earth's magnetic field or by the use of spinup rockets or gas jets, be included; or specially designed plates be installed at the ends of booms to compensate for the combined effect of thermal boom bending and solar radiation pressure.

4.4.3.2 Gravity-Gradient Stabilized Spacecraft With Booms—Environmental Excitation

This class of spacecraft is particularly susceptible to static and dynamic thermal bending of the required long extendable booms. Specific recommendations to reduce solar-induced attitude disturbances in these flexible structures are as follows:

- 1. Use booms designed as recommended in section 4.4.3.
 - a. A polished silver-plated exterior is a minimum requirement.
 - b. Use a boom motion damper as part of the end mass.
 - c. Consider gravitational and aerodynamic effects on boom bending.

2. Obtain the required moments of inertia for the spacecraft by using extendable booms that are as short as possible with as heavy an end mass as can be used within the spacecraft's weight budget.

4.4.3.3 Active Control of Rigid Body With Booms— Self-Excitation and Environmental Excitation

When a spacecraft with long, flexible, extendable booms has an active, closed-loop control system, the following practices are recommended in addition to those of section 4.4.2.1:

1. Use booms designed as recommended in section 4.4.3.
2. If possible have the natural frequency of the extendable boom separated as far as possible from the control-system bandpass.

4.4.4 Large Flexible Body, Active Control During Thrust Maneuvers

The recommended practices of section 4.4.2.1 are applicable.

4.4.5 Dual-Spin Stabilized Spacecraft With Flexibility

1. The energy dissipation of the spacecraft should be managed as follows:
 - a. Design the member with the highest spin rate to be structurally stiff.
 - b. Limit the slosh of onboard fluids to insensitive frequencies. In particular, no spinning-member sensitive frequency should occur at or near the spin minus the precessional rate.
 - c. Incorporate a dissipative damper of proper design on the more slowly spinning member to achieve stability; or mount an active or passive damping mechanism on the spinning or despun members as indicated in reference 32.
2. The center of mass of the rotating member should lie as closely as possible to the bearing axis.
3. The bearing axis should, to a high accuracy, be a principal axis of the rotating member. The nutational amplitude when one body is despun is proportional to the rotating member's moment of inertia cross products and center of mass offsets from the bearing axis.
4. If the transverse moments of inertia of the spinning member differ significantly, conventional linear analysis regarding stability and system response is not applicable. A Floquet analysis or computer simulation is then required to establish adequate performance of a proposed design.
5. Simulation of the system equations is recommended for the general case in which the despun body may have significant moment of inertia cross products, may have its center of mass located off the bearing axis, and may have time-varying moments of inertia caused by moving appendages.

4.4.6 Spin-Stabilized Spacecraft—Spin Resonance

1. Spin rate should not exceed 70 percent of the lowest natural frequency of the transverse bending mode. This margin reflects uncertainties in the values of the natural frequency and of the spin rate which may be achieved.
2. Spin rates above the natural frequencies of the lower modes are not recommended. If, however, a spin rate above the lower natural frequencies is required,
 - a. The rate should be nearly midway between adjacent natural frequencies if practical.
 - b. The rate should be separated from the nearest natural frequency by a margin equal to at least 30 percent of the lowest natural frequency.
 - c. The spin acceleration should be high enough that the bending deformation developed during passage of the spin rate past the natural frequencies will not be excessive.
3. Bending deformation should be minimized by an arrangement of vehicle frame and internal components that will result in local centers of mass being on the centerline of the undeformed vehicle along the entire length.

4.4.7 Spacecraft With Flexibility in the Control Loop

The recommended practices given in section 4.4.2.1 are applicable except for recommendation 3 which is replaced by:

3. Choose the control bandpass so that it excludes the structural resonant frequencies. This may be achieved through natural stiffening of the structure, by artificially stiffening the structure through use of special inner control loops, through utilization of a low pass filter within the control amplifier, by artificially lowering the bandwidth of the control through use of a special actuator lag which inhibits sign reversal of the control at a rate higher than that needed to follow control commands, or through use of notch filters.

4.4.8 Spacecraft With Multiple Controllers

Specific recommendations are dependent on the spacecraft configuration and controller. In general, the following is recommended: simulation of the spacecraft dynamic system is essential to determine stability and optimum control system performance.

4.4.9 Larger, More Flexible Vehicles

Specific recommendations are the same as given in section 4.4.8. In addition, structures, such as solar arrays, which have been analyzed as rigid flexibly connected appendages, should be considered as flexible structures as they increase in size and as new concepts in construction are introduced.

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Appendix A

APPLICATION OF ENERGY SINK METHOD TO SPIN-STABILIZED SPACECRAFT

The torque-free precession of a rigid spin-stabilized spacecraft is illustrated in figure A-1. The angular momentum H is resolved into components aligned to the body axes using the precession angle θ . The magnitude of total angular momentum is given by

$$|H| = (I_x^2 \omega_x^2 + I_y^2 \omega_y^2 + I_z^2 \omega_z^2)^{1/2} \quad (\text{A-1})$$

where I and ω are the moment of inertia and rotational rate, respectively, about the body axes x , y , and z . The total kinetic energy is given by

$$T = \frac{1}{2}(I_x \omega_x^2 + I_y \omega_y^2 + I_z \omega_z^2) \quad (\text{A-2})$$

Referring to figure A-1, the precession angle is related to the angular momentum by

$$\sin \theta = \frac{(I_x^2 \omega_x^2 + I_y^2 \omega_y^2)^{1/2}}{H} \quad (\text{A-3})$$

Spin-stabilized spacecraft generally have cylindrical symmetry about the spin axis z . Applying the symmetry condition of $I_x = I_y$ to equations (A-1) and (A-2) yields

$$\omega_x^2 + \omega_y^2 = \frac{2T I_z - H^2}{I_x (I_z - I_x)} \quad (\text{A-4})$$

which, when substituted into the square of equation (A-3), gives

$$\sin^2 \theta = \left[\frac{2I_x I_z}{I_z - I_x} \right] \frac{T - \frac{I_x}{I_z - I_x} H^2}{H^2} \quad (\text{A-5})$$

The result given in equation (A-5) is for a rigid, torque-free body with constant moments of inertia and, hence, the angular momentum H is constant. However, if a small dissipation of kinetic energy is postulated, such as through structural flexibility, then

$$\frac{dT}{dt} \neq 0 \quad (\text{A-6})$$

Differentiating equation (A-5) yields

$$\frac{d\theta}{dt} = \frac{2I_x I_z}{(I_z - I_x) H^2 \sin 2\theta} \frac{dT}{dt} \quad (\text{A-7})$$

Because dT/dt is negative, the precession angle will decrease if $I_z > I_x$ and increase if $I_z < I_x$, indicating stable motion about the principal axis of maximum moment of inertia I_z .

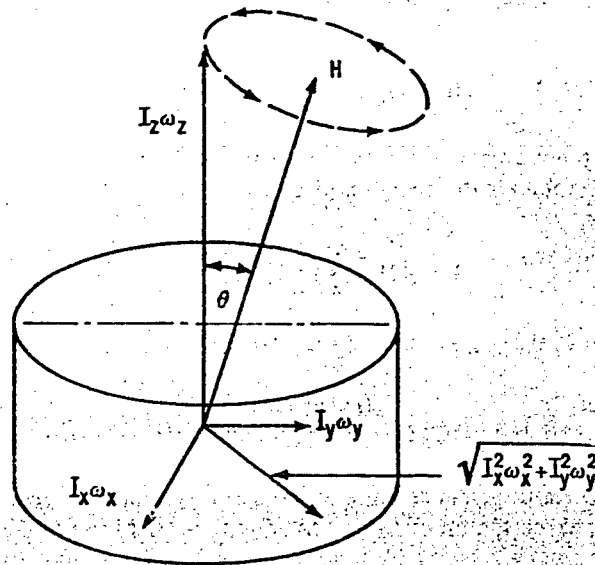


Figure A-1.—Torque-free precession of a spinning spacecraft.

Appendix B

DEVELOPMENT OF CONTROL-SYSTEM/STRUCTURE DYNAMICS IN MODAL COORDINATES

The general configuration of a linear elastic system is illustrated in figure B-1. As discussed in reference 69, body axes are associated with the instantaneously deformed structure with origin at the center of mass (c.m.) and directions so chosen that the instantaneous angular momentum of the deformation velocity $\dot{\mathbf{q}}(t)$ vanishes. $\mathbf{r} = \tilde{\mathbf{r}} + \mathbf{q}$ is the position vector of any mass element dm . In the absence of external forces and any mechanism for energy

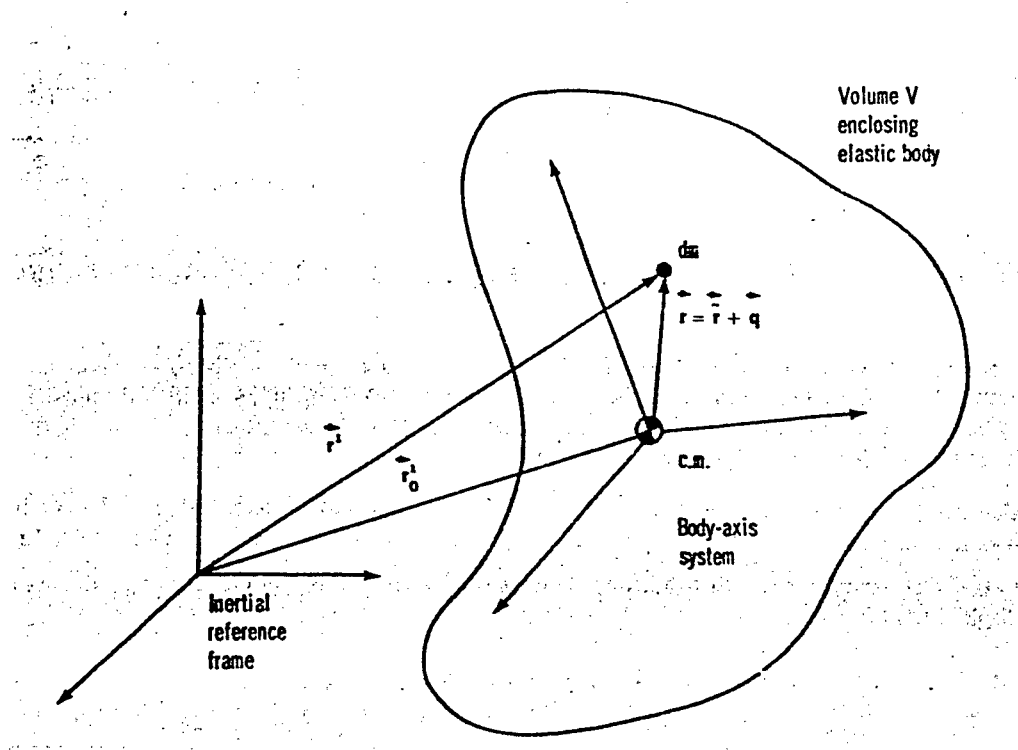


Figure B-1.—Coordinates used in describing a linear elastic system.

dissipation, the passive vehicle is characterized by a countable infinity of natural vibration frequencies ω_{o_i} and associated normal mode shapes $\phi_i(\tilde{r})$. In practical applications all these frequencies are distinct, and the modes then have the orthogonality property

$$\int_V \phi_i \cdot \phi_j dm = \begin{cases} M_i, & i = j \\ 0, & i \neq j \end{cases} \quad (B-1)$$

where M is the generalized mass. When control forces, disturbances, and damping are present, it is convenient to express the displacement

$$q = \sum_{i=1} q_i(\tilde{r}, t) = \sum_{i=1} \phi_i(\tilde{r}) \xi_i(t) \quad (B-2)$$

where ξ is the generalized displacement. The equations of motion for the elastic degrees of freedom become

$$M_i \left(\ddot{\xi}_i + \omega_{o_i}^2 \xi_i \right) = \Xi_i(t) \quad (B-3)$$

where Ξ is the generalized force, which is related to the sum f of all applied forces, per unit mass of the vehicle, by

$$\Xi_i(t) = \int_V \phi_i \cdot f dm \quad (B-4)$$

Equation (B-3) has the advantage of uncoupling the elastic and inertia forces associated with the various vehicle modes. In many cases the internal damping is small and tends also to be uncoupled so that it can be approximated by adding a term $-2\zeta_i \omega_{o_i} \dot{\xi}_i$ on the right of (B-3) where ζ is the damping ratio.

Control forces appearing on the right of (B-4) are generally concentrated and may be included in f using Dirac delta functions. If F_{c_j} is the j th concentrated control force and f_d is the disturbance force distribution, then (B-4) becomes (for point forces¹ and sensor displacements)

$$\Xi_i(t) = \int_V \phi_i \cdot f_d dm + \sum_{j=1} \phi_i(r_j) \cdot F_{c_j} \quad (B-5)$$

¹Control moments may be included by constructing force couples.

where r_j is the point of application of the j th control force. Including the damping terms on the left of (B-3) and substituting (B-5), one obtains

$$M_i(\ddot{\xi}_i + 2\zeta_i\omega_{o_i}\dot{\xi}_i + \omega_{o_i}^2\xi_i) = \int_V \phi_i \cdot f_d \, dm + \sum_{j=1} \phi_i(r_j) \cdot F_{c_j} \quad (B-6)$$

The transfer function relating the contribution of the i th mode to the motion of sensors located at point r_G is then

$$q_i(\tilde{r}_G, s) = \phi_i(\tilde{r}_G) \frac{\left(\int_V \phi_i \cdot f_d \, dm + \sum_{j=1} \phi_i(r_j) \cdot F_{c_j} \right)}{M_i(s^2 + 2\zeta_i\omega_{o_i}s + \omega_{o_i}^2)} \quad (B-7)$$

where s is the Laplace transform variable.

It should be noted that, for some systems, the application of control forces introduces terms in $\Xi_j(t)$ that depend on the generalized coordinates $\xi_j(t)$. If these forces are conservative, it is often convenient to define a new set of orthogonal modes for the active system and to use them as generalized coordinates in place of the passive modes in (B-2). The principal effect of this modification is to alter the frequencies ω_i and damping ratios ζ_i to be used in the sets of equations (B-6) and (B-7).

NASA SPACE VEHICLE DESIGN CRITERIA DOCUMENTS ISSUED TO DATE

SP-8001 (Structures)	Buffeting During Launch and Exit, May 1964
SP-8002 (Structures)	Flight-Loads Measurements During Launch and Exit, December 1964
SP-8003 (Structures)	Flutter, Buzz, and Divergence, July 1964
SP-8004 (Structures)	Panel Flutter, May 1965
SP-8005 (Environment)	Solar Electromagnetic Radiation, June 1965
SP-8006 (Structures)	Local Steady Aerodynamic Loads During Launch and Exit, May 1965
SP-8007 (Structures)	Buckling of Thin-Walled Circular Cylinders, revised August 1968
SP-8008 (Structures)	Prelaunch Ground Wind Loads, November 1965
SP-8009 (Structures)	Propellant Slosh Loads, August 1968
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SP-8012 (Structures)	Natural Vibration Modal Analysis, September 1968
SP-8014 (Structures)	Entry Thermal Protection, August 1968
SP-8015 (Guidance and Control)	Guidance and Navigation for Entry Vehicles, November 1968
SP-8018 (Guidance and Control)	Spacecraft Magnetic Torques, March 1969
SP-8019 (Structures)	Buckling of Thin-Walled Truncated Cones, September 1968