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SPACE VEHICLE
DESIGN CRITERIA
(STRUCTURES)

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STRUCTURAL VIBRATION PREDICTION

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FOREWORD

NASA experience has indicated a need for uniform criteria for the design of space vehicles. Accordingly, criteria are being developed in the following areas of technology:

Environment
Structures
Guidance and Control
Chemical Propulsion

Individual components of this work will be issued as separate monographs as soon as they are completed. A list of all previously issued monographs in this series can be found at the end of this document.

These monographs are to be regarded as guides to design and not as NASA requirements, except as may be specified in formal project specifications. It is expected, however, that the criteria sections of these documents, revised as experience may indicate to be desirable, eventually will become uniform design requirements for NASA space vehicles.

This monograph was prepared under the cognizance of the Langley Research Center. The Task Manager was G. W. Jones, Jr. The author was J. S. Archer of TRW Systems Group/TRW Inc. A number of other individuals assisted in developing the material and reviewing the drafts. In particular, the significant contributions made by R. E. Blake of Lockheed Missiles & Space Company; R. Chen and D. L. Keeton of McDonnell Douglas Corporation; S. A. Clevenson of NASA Langley Research Center; H. Himelblau of North American Rockwell Corporation; W. C. Hurty of the University of California, Los Angeles; G. Morosow of Martin Marietta Corporation; and A. G. Piersol of Measurement Analysis Corporation are hereby acknowledged.

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CONTENTS

STRUCTURAL VIBRATION PREDICTION

1. INTRODUCTION

A space vehicle is subjected to significant vibratory loads by the natural and induced environments encountered during its life. To consider properly the effect of vibration in the design of a space vehicle, it is necessary to predict the structural responses and internal loads resulting from the vibratory inputs to the structure.

This monograph is concerned with the determination of the space-vehicle structural vibration resulting from induced or natural environments, and with determining internal structural loads and stresses caused by such vibrations. The vibration sources are assumed to be described adequately; the content of the monograph is therefore an assessment of analytical and experimental methods of determining the resulting vibrations, and internal loads and stresses, and an enumeration of means of demonstrating the validity of these data.

Structural vibration is an oscillatory motion of a structural system which can be characterized by certain parameters that are invariant with respect to time, in contrast to a transient or shock response. After the initial shock, however, the structural response to some transients or shock loadings characteristically varies slowly enough with time so that this portion of the response may be treated as a vibration. Vibration may be periodic or random (or a combination of both).

Structural vibration causes internal loads and stresses in structural components and localized loads at attachment points where equipment is supported by the structure. The degree or magnitude of structural response may be expressed as an acceleration or displacement of various critical points as a function of time, as a weighted average acceleration (root mean square), or as spectra of acceleration or stresses as functions of frequency at discrete times. Severe vibration of space-vehicle structural elements usually results from rocket noise and from aerodynamic noise and buffet. Severe vibration can also result from other sources, such as thrust oscillation and rocket-engine resonances, wind gust and shear, transportation, tests, operation of internal equipment, and unstable dynamic coupling of the structure with the control system or with the propulsion system.

Table I lists the operational phases of a space-vehicle mission and the possible sources of vibration in each phase. Figure 1 illustrates a typical time history of the vibration of a launch vehicle, measured during the launch operation. It can be seen that structural vibration is significant during the launch phase of flight, and may also be significant during prelaunch, space, and entry operations.

TABLE I. – SOURCES OF VIBRATION
IN VARIOUS VEHICLE OPERATIONAL PHASES

Operation	Phase	Source
Prelaunch	Functional checkout	Vibration testing Static firing
	Transportation: Air	Air turbulence Propeller noise
	Ground	Rough highways
	Water	Rough water
	Launch readiness	Ground wind
Launch	Liftoff	Ignition Engine noise Tie-down release
	Ascent	Engine roughness Aerodynamic noise and buffet Pogo phenomena Control-system instability
	Staging	Separation Stage ignition
Space	On station	Control-system instability
Atmospheric	Entry	Aerodynamic noise and buffet Aerodynamic stability

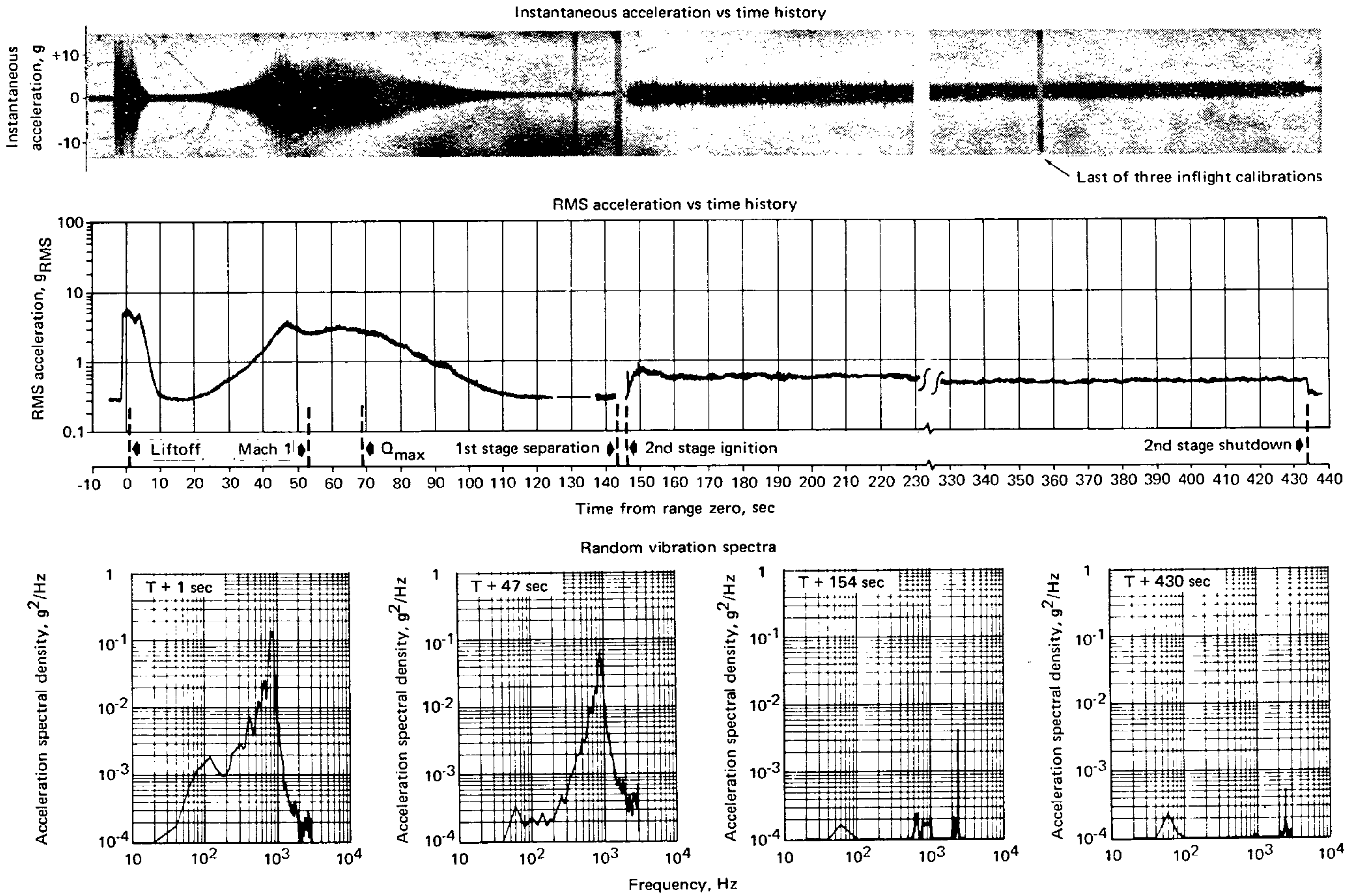

Figure 1. – Vibration measured on the S-IVB stage of the uprated Saturn I space vehicle during Apollo-Saturn flight SA-203.

Structural vibration may cause overstressing, fatigue from oscillation of the induced loads and stresses, malfunction of electrical and mechanical components (such as switch chatter and valve leakage), and excessive structural deflections which render equipment inoperative, or the vibration may exceed limits for pilot or crew. Structural vibration was of concern or actually resulted in failure in the following instances:

- Torsion vibration during staging of a major launch vehicle required close design attention to payload torsion characteristics to minimize loads and accelerations on the spacecraft structure.

- Control-system coupling with a launch-vehicle structure in the launch mode required engine shutdown to prevent failure from vibration of the structure while the vehicle was supported on the launch stand, thus requiring control-system redesign.

- Several launch vehicles experienced pogo-type longitudinal vibration (caused by unstable coupling of propulsion system with longitudinal structural vibration) that caused excessive loads and resulted in booster malfunction.

- Failure of structure in prototype space-vehicle testing frequently occurred from overstress caused by inadequate analysis during the development and qualification phases of the program.

Consideration of instabilities associated with coupling between structural and control or propulsion systems is closely related to the vibration response problem. Although stability considerations of this nature are dealt with in other monographs and regarded primarily as a control problem or a special structure-propulsion interaction problem (e.g., pogo), it is important that the structural engineer provide adequate support for accurate modeling of the structure for use in stability analysis. The structural model used for response analyses, at least in the lower frequencies, is often the appropriate model to be used for the stability analysis.

Important considerations in vibration analysis which affect the accurate determination of the vibratory displacements, loads, and stresses are the adequacy of the description of the applied loading, the selection and formulation of an adequate mathematical model, and the accurate assessment of the magnitude of the damping of the structural system.

Once the externally applied loading from natural or induced environments which cause structural vibration has been determined, the basic approach to determination of the vibrations and the resulting internal loads and stresses is to rely on empirical and theoretical analysis, verified by vibration tests under simulated environments.

Analytical techniques utilizing linear structural parameters for prediction of structural vibration in the lower frequency range are well established. However, heavy reliance is placed upon extrapolation of flight-vibration measurements on earlier vehicles to verify and supplement deterministic analysis, upon additional vibration data taken during development and qualification vibration tests, and on supplementary data taken during static-deflection tests to verify adequacy of the structural analysis.

This monograph is related to other planned or published NASA Design Criteria monographs which treat the inputs to and the responses of vehicle structure in various natural and induced environments. These related monographs cover natural vibration modal analysis, determination of environmental vibration loading, special vibration-instability problems, and other structural dynamic transients.

2. STATE OF THE ART

In general, space-vehicle structural design is based mainly on the assessment of the steady-state or quasi-static loads, with some factors added to allow for dynamic loads, including vibratory response loading. The design is then examined to see if it can withstand the various dynamic-load effects. The primary problems from vibration are fatigue and overstressing of the structure and the effects of the amplitude and frequency of the vibration as an input to equipment or nonstructural systems attached to the structure.

Analytical techniques for prediction are not highly refined for high-frequency responses. Thus, experimental techniques are often used to supplement analysis. A considerable amount of literature is available on both analytical and experimental techniques, providing a framework within which adequate guides may be obtained for design purposes.

2.1 Analytical Determination of Vibration Response

Various methods are available for determining the magnitude of structural vibrations and predicting equipment vibration loads. These methods may be divided into the following categories:

1. Extrapolation methods

2. Deterministic analysis

3. Statistical-energy analysis

Extrapolation methods use experimental data obtained on one vehicle to predict the vibration of a new vehicle. In *deterministic analysis,* the vehicle structure is represented by a mathematical model; the applied load is characterized in either the time or the frequency domain and, in addition, particularly for aeroacoustic loading, in the spatial domain; and the predicted vibration at various locations on the structure is directly calculated, using classical dynamic response techniques applied to the mathematical model. *Statistical-energy analysis* has been developed to estimate the vibration of complex structures subjected to random loading at the higher resonant frequencies.

2.1.1 Extrapolation Response Analysis

The extrapolation approach is a common technique for predicting vibration levels for a new vehicle, particularly in preliminary design. Extrapolation methods are customarily used to predict vibration motion, such as acceleration versus frequency for sinusoidal excitation or acceleration spectral density versus frequency (refs. 1 and 2)* for random excitation, rather than vibration stress. Since the assumptions made for the extrapolation of data from one vehicle to another are only partially valid, errors may result. Extrapolation methods may be divided into scaling methods and frequency response methods. A discussion of the merits and limitations of these methods may be found in references 3 to 5.

2.1.1.1 Scaling Methods

Scaling methods consist of the extrapolation of specific vibration data from a selected reference vehicle with similar structural and configuration characteristics to predict vibration levels on a new vehicle. Vibration levels may be predicted for any section of the structure by extrapolating vibration data measured on that section of the structure in the reference vehicle. Furthermore, predictions may be obtained for any time of flight by extrapolating the measurements from the reference vehicle for flight conditions that have been identified on the new vehicle (ref. 3).

Three specific scaling methods that have been reported on are the Condos and Butler Method (ref. 6), the Barrett Method (ref. 7), and the Winter Method No. 2 (ref. 3). Reference 6 describes the scaling of Titan I data to predict vibration levels caused by acoustic and aerodynamic noise on the structurally similar Titan II. Reference 7 similarly describes the scaling of Saturn I vibration measurements to predict acoustically induced vibration levels on uprated Saturn I and Saturn V vehicles.

*These references define acceleration spectral density as the mean squared acceleration per unit bandwidth.

Reference 7 describes a scaling method for mechanically transmitted vibration from liquid-fueled rocket engines through a structure which is not primarily excited by acoustic or aerodynamic noise. For example, engine components and structure near engines which do not have large exposed surface areas are excited primarily by mechanically transmitted vibration. The Winter Method No. 2 (ref. 3) is similar to the frequency response methods.

The major advantage of these scaling methods is that the vibratory motion of a new vehicle can be readily estimated by the designer even without his having knowledge of the detailed characteristics of its structure. This is highly desirable during early development of a vehicle, when the structure is incompletely defined and detailed calculations are not possible. The procedure can be applied to any type of structure in any flight vehicle for any flight condition, provided appropriate measurements are available from the reference vehicle. However, accuracy of the predictions is heavily dependent upon the quantity and quality of the measurements from the reference vehicle and upon the similarity of the reference vehicle to the new vehicle.

In addition to these techniques, the theory of mechanical impedance has recently been applied to the scaling of interface vibration environments based on measured flight data (refs. 8 and 9). This approach exploits the advantages of including the effects of the dynamic characteristics of the booster and payload in defining the vibration environment. If the payload and booster mechanical impedance are defined and the response at the interface is known, then by classical impedance methods the interface response with a new payload can be determined. Results obtained by this method have shown promise toward increasing accuracy of interface-vibration-level definition. The major difficulties encountered in the application of this concept are the lack of adequate flight data and the difficulty in generating an adequate definition of the mechanical impedance. The latter problem often requires the combined use of analytical and experimental results.

2.1.1.2 Frequency Response Methods

Frequency response methods extrapolate from pooled general vibration data measured on one or more general vehicles to predict vibration levels on a new vehicle. This approach includes procedures using empirical relationships developed from regression studies of past data to predict vibration levels in future vehicles. Seven specific procedures which have been reported on (ref. 3) are the Mahaffey and Smith Method (ref. 10); the Brust-Himelblau Method (ref. 11); the Eldred, Roberts, and White Method No. 1 (ref. 12); the Eldred, Roberts, and White Method No. 2 (ref. 12); the Curtis Method (ref. 13); the Franken Method (ref. 14); and the Winter Method No. 1 (ref. 3).

Frequency response methods determine the ratio of vibration to the magnitude of the fundamental source as a function of frequency or bandwidth. The fundamental source is generally acoustic or aerodynamic noise applied to the vehicle surface. Reference 10 describes the first use of this method, which was based on data obtained on the B-52 and B-58 aircraft; reference 11 extends the reference 10 approach to provide average acceleration spectral density rather than peak acceleration. Reference 12 describes use of the technique based on vibration and acoustic measurements obtained on the Snark missile; reference 13 correlates vibration measurements on high-speed aircraft to the aerodynamic pressure. Reference 14 describes a similar vibration and acoustic measurement study of Jupiter and Titan I external structure behavior during rocket-engine static firing; Winter has modified the results of reference 14 after adding Minuteman, Skybolt, and Genie data.

Frequency response methods are used to calculate the vibration response of a new vehicle without considering the detailed characteristics of its structure. In some cases, the overall accuracy is good, while in other cases considerable overprediction and underprediction have resulted (e.g., refs. 3, 6, and 11). Although these methods are not generally used for determining vibration stresses, they can be modified (refs. 15 to 17) to establish acceleration-versus-stress relationships which in turn can be used for extending frequency response methods. Stress extrapolation comparisons have been poor, however, even when acceleration extrapolation comparisons are good (ref. 14).

2.1.2 Deterministic Response Analysis

Deterministic analyses are most useful in predicting vibration stresses and motions in the lower frequency range. Methods used for these analyses are generally well known (refs. 2, 5, and 18 to 26). Typically, a rigorous analysis is restricted to that frequency range which encompasses the lower 10 to 50 modes of the structure. The lack of structural detail in the model or simplification of the loading description often reduces further the useful frequency range.

Deterministic analyses require modeling of the environmental source, as well as of the structure to be analyzed. These sources are discussed in other NASA Design Criteria monographs.

2.1.2.1 Mathematical Structural Modeling

Selection of the mathematical model for a vibration analysis is influenced by the desired accuracy and frequency range, the nature of the loading, the details to be employed in describing the loading, and the cost of computation and model formulation. The mathematical model is a prime factor in obtaining satisfactory vibration response analyses. The structural stiffness distribution, mass distribution, and boundary conditions are each given careful consideration in the synthesis of the mathematical model.

Generally, the two types of models available are continuous-parameter representations and lumped-parameter representations. Uniformly distributed continuous-parameter representations are used for fairly simple structures for which classical solutions are known, e.g., domes, conical and cylindrical shells, and plates and beams, all with simple boundary conditions (refs. 27 to 29). In addition, parametric studies frequently use this approach (ref. 30). Typical complex space-vehicle structures, however, usually consist of different types of substructure, which makes a distributed-parameter analysis difficult and often impossible to formulate. Thus, a lumped-parameter representation is usually preferred for modeling the structure.

The mathematical modeling of a structure for a deterministic vibration analysis is essentially identical to that required for natural vibration modal analysis covered in the NASA monograph on that subject (ref. 25). The remainder of this section, presented for completeness, is largely an excerpt from Section 2.1.1 of reference 25.

Lumped- or discrete-parameter models of structure are generally formulated by a finite-element approach using matrix notation. These models are synthesized from independently modeled structural components (finite elements) joined at node points through common displacement and force coordinates. Finite-element models are available for beams (ref. 31), flat-plate elements (refs. 32 to 37), shell elements (refs. 38 to 40), and sandwich-plate elements (ref. 37). The model elements are used in conjunction with displacement and force coordinates that prescribe the interfacing geometry of the structural system. The finite-element model leads to the definition of a stiffness or flexibility matrix and a mass matrix for the discretized system (ref. 20).

Several methods are used to define the mass distribution. These include the *lumped-mass method,* the *consistent-mass method,* and a number of approaches that use various velocity-interpolation functions to define a mass matrix (ref. 22). The *lumped-mass method* distributes the element masses in concentrations located at the coordinate points in a manner that maintains the center of mass in the structure (refs. 22 and 41 to 43). This method of distribution is well suited for analysis of structures with preponderantly concentrated masses. Local rotary masses are frequently used with this method to represent the effect of significant transverse-mass distributions. The disadvantage of the lumped-mass method is the relatively large number of coordinate points required for accurate analysis of systems with primarily distributed masses.

The *consistent-mass method* represents the mass as it is actually distributed in the structure (refs. 31, 32, 38, 39, 44, and 45). Although only recently codified, this method is being widely adopted and incorporated in modern analytical computer programs. The consistent-mass-distribution method has been shown to yield more accurate results than the concentrated-mass technique for systems where the mass is largely distributed in the structure (refs. 41 and 44). A minor disadvantage is that the nondiagonal mass matrix tends to increase the complexity of the analysis.

The treatment of nonstructural mass, such as liquids in a fuel tank, requires special consideration. Mechanical coupling of the liquid mass with the structure is generally accomplished through an equivalent pendulum or spring-mass analogy that simulates the free-surface lateral-sloshing effect, as described in reference 46. Longitudinal mechanical coupling may be accomplished through an equivalent spring for the tank's end bulkhead that supports the liquid (ref. 47). Such analogies permit the solution of vibratory response problems in a direct fashion without recourse to treatments in fluid mechanics. A more complex liquid-structure coupling may be treated efficiently with finite-element models that satisfy the hydroelastic boundary conditions at a number of points along the liquid boundary. Several such techniques have been developed and applied (refs. 39 and 48 to 51).

Substructure methods (refs. 52 and 53) may be used in modeling large, complex structural configurations. These methods offer several advantages, particularly in such structures as large multistage vehicle systems where the several stages may be structurally dissimilar. Substructure methods can be used to extract a larger number of realistic and accurate high-frequency modes than can be obtained by using a single model of the entire structure having the same number of degrees of freedom.

The finite-element technique is the most generally applicable of the available analytical methods. This technique, which is readily adapted to digital-computer solution, takes maximum advantage of matrix notation in mathematical manipulations. The technique is being rapidly improved and has been adopted in all major computer programs for general structural analysis. Vibration response analyses can be performed for almost any configuration using the finite-element technique. The primary disadvantage of the finite-element technique is that, owing to the large-sized matrices involved, complex models use large amounts of computer time.

2.1.2.2 Response Formulation and Solution

Most real structures vibrate nonlinearly; hence, the ratio of the displacement and stresses to the vibration input is a function of loading magnitude. Unfortunately, there have been few nonlinear-vibration analyses of mathematical models which approach the complexity characteristic of nearly all space-vehicle structures (ref. 23). When sufficiently severe nonlinear behavior is expected, it is common practice to assume a linear model and perform the analysis using linear values of structural stiffness and damping which represent the expected effect of the nonlinearity on the response. Otherwise, using results of nonlinear-vibration studies performed on simple models, such as those reviewed in reference 23, the linear response is scaled to provide an approximate or conservative estimate of the nonlinear response.

Mathematical formulation of the linear-vibration response problem generally leads to a set of second-order, linear, simultaneous differential equations, in terms of structural-coordinate displacements and forces. These are known as equations of motion. When arranged using matrix notation, the elastic characteristics of the system are defined by the displacement-vector coefficient, i.e., the stiffness matrix; the damping characteristics of the system are defined by the velocity-vector coefficient, i.e., the damping matrix; and the mass inertial characteristics of the system are defined by the acceleration-vector coefficient, i.e., the mass matrix. When properly formulated, the equations of motion yield the natural vibration mode shapes and frequencies or the structural responses to the inputs. The structural responses may be described as accelerations and displacements, or as internal loads (the force responses within the structure) and stresses.

The response solution is frequently obtained by uncoupling the equations of motion by use of a transformation of coordinates in which the transformation is derived from the natural modes of the undamped structure (ref. 20). Uncoupled equations may be solved by several techniques, some of which are the mode-displacement method, the mode-acceleration method, and the mechanical-admittance or -impedance methods (refs. 5 and 22). The mechanical-admittance or -impedance methods may be used either with uncoupled equations or with the coupled system. The choice of the method of solution is based primarily on the physical nature of the forcing functions. When the environment can be described in terms of discrete analytical functions in time, displacement, or velocity, the mode-displacement method has its advantages, since response for each mode is readily computed and the total response is obtained by superposition of all modes.

Solution of the homogeneous form of the equations of motion without damping provides the resonant frequencies and mode shapes which characterize the mathematical model, and thus, the structure (ref. 25). The modal data are used principally to determine the vibration response. The resonant frequencies and mode shapes are also useful in selecting mounting locations for equipment and control sensors. This is important when known critical frequencies will have detrimental effects on the equipment or structure. The response of a structure at various frequencies, or at desired intervals throughout the frequency range of interest, is usually calculated by summing the response in each of the orthogonal modes which characterize the structural vibration.

When the applied loading is random in nature, the response is also random. In this case, the solution for the vibration response at a particular location on the structure is expressed as a function of frequency and referred to as spectral density. References 1, 2, 20, and 23 provide equations for obtaining the displacement spectral-density response due to spatially distributed applied random loading. References 2, 23, 26, and 54 describe several distributions of cross-correlation functions that might be used in defining the random characteristics of various aeroacoustic-noise loadings. The acceleration spectral density, usually preferred in specifying design and test requirements for equipment, is readily obtained from the displacement spectral density. Internal loads from which the stresses throughout the structure can be calculated are readily determined from root-mean-square displacement derivations. Response data related to fatigue analyses, such as the number of response cycles and the probability of peak levels, may also be predicted (refs. 1 and 23).

A major problem confronting the analyst in predicting an accurate vibration response is how to determine the proper amount and type of damping which characterizes the various vibrating modes. Viscous damping is often assumed in the equations of motion because it is compatible with the assumption of linear vibration and is therefore easy to solve for dynamic response. The damping of real aerospace structures is not viscous, but usually occurs as a combination of material damping, friction damping, and acoustic radiation (i.e., air damping). An apparent damping effect may also be present because of nonlinear structural stiffness, although no actual energy dissipation occurs.

It has been shown experimentally that material damping depends on the stress distribution (refs. 55 and 56): if the maximum stress exceeds the endurance limit, the damping may increase significantly for most materials.

Friction damping is the result of energy dissipation caused by slipping or sliding between mated surfaces. For aerospace structures made of parts which are bolted or riveted together, friction is the dominant source of damping. The friction force is approximately constant and controlled by the force normal to the mated surfaces (refs. 55, 57, and 58). It is difficult to estimate friction damping and its distribution in the various modes because of its nonlinearity, the distribution of the normal pressure over the mated surfaces between the fasteners, the variations in the force exerted by each fastener that result from manufacturing, and the spatial variations in the static and kinetic coefficients of friction.

Another form of damping, acoustic radiation, occurs when acoustic waves, generated by the vibration of the structure, are propagated to other structures and to surrounding space. Radiation damping can be appreciable for panels and other structures with low surface densities, unless the structure vibrates at high altitude (where the radiation is reduced) or unless the surrounding space is reverberant. For simple cases, such as acoustic radiation from a rectangular panel into a free field or a reverberant space with known surface characteristics, radiation damping in the various modes can be estimated (refs. 59 to 64). For more complex cases, the estimating can be quite involved. Other forms of damping may include surface and boundary damping and air pumping (refs. 65 and 66). It is difficult to estimate properly the damping in the various modes, even for the simplest structural configurations. Thus, to predict damping, heavy reliance is usually placed on data from previous tests on similar structures.

2.1.3 Statistical-Energy Response Analysis

Deterministic methods are frequently ineffective in providing vibration data in the high-order modes of a structural system. Statistical-energy analysis (refs. 67 to 72) is an alternate approach for estimating the vibration response of complex structures subjected to random loading at the higher resonance frequencies. This approach permits the use of gross structural properties and employs a statistical description of a structure as a vibrating system; i.e., motion of the system is assumed to be dominated by resonant response rather than by forced nonresonant response. The response is predicted on the basis of the average vibration energy contained within a band of frequencies. The approach is relatively new and has not been widely used as a practical tool for space-vehicle-vibration predictions.

Statistical-energy analysis techniques have been applied to large, complex vehicle configurations. Sections of the Saturn V launch vehicle were analyzed to predict vibration for acoustic noise at liftoff, subsonic and supersonic boundary-layer turbulence, and shock-induced separation and disturbed flow at various times during the ascent phase of the mission. The vibration and strain were calculated for each loading condition (ref. 72). Acoustical and mechanical vibration were investigated by this technique for a simplified physical model of the OGO spacecraft (refs. 69 and 73). Although still in an early developmental phase, statistical-energy analysis may be a valuable tool for interpretation and rough estimates of vehicle vibration above the lower resonant frequencies.

2.2 Tests

Various experimental tests are used to predict structural vibration response or to refine earlier predictions during various phases of vehicle development. Tests may be conducted in the laboratory, in the field, or in flight. Laboratory tests may be static tests, vibration tests, or acoustic tests.

2.2.1 Static Tests

Static load-displacement tests are basic to the design-development phase of a space-vehicle structure and perform an important function in supporting vibration analyses. These tests may provide some of the data needed to generate the mathematical model defining the elastic characteristics of the structure, as well as to verify the mathematical model used in the deterministic analysis. Frequently, static tests are performed to determine the influence coefficients that define boundary conditions at an interstage connection with a supporting stage structure. These data are then used to develop the analytical model. For some large full-scale structures, these experimental data are approximated by use of detailed subscale models, also known as replica models (ref. 74). In some cases, the required load-displacement data are obtained during proof testing of the prototype vehicle.

2.2.2 Vibration Tests

Various laboratory vibration tests are performed to determine the vibration response. Often these tests are made in conjunction with other tests and analyses. Laboratory vibration tests include *design-development, qualification,* and *acceptance tests.*

Design-development vibration tests are performed to provide information on the modal characteristics of the structure (ref. 25), to determine the magnitude of the vibration response, to evaluate the adequacy of the design, and to identify the vibration failure mechanisms. These vibration tests may consist of sine-sweep, random, or modal tests. The test-to-failure test is often used during design development to establish margins for flight and qualification test conditions. These tests are designed and instrumented to provide vibration response data for specific dynamic-forcing functions.

Qualification vibration tests are performed on flight-quality hardware to demonstrate the adequacy of the design and fabrication methods for flight. Instrumentation of these tests is primarily for diagnostic purposes, but it is also useful in providing vibration response data.

Acceptance vibration tests are performed on articles intended for use in flight to demonstrate that adequate workmanship has been achieved during fabrication and assembly. These tests are normally not instrumented and provide no significant vibration response data.

Other tests include reliability tests (to demonstrate the variation of the failure mechanism between items of hardware under prescribed types of loading); fragility tests (to map the failure-threshold perimeter under a variety of loadings or frequencies); and interim tests between development and qualification.

Vibration tests are generally performed on equipment, on spacecraft structures, and on major launch-vehicle structures. For example, reference 75 describes the electrodynamic-shaker excitation of the Ranger spacecraft to envelop the vibration applied by the launch vehicle; reference 76 describes a similar test on the Gemini spacecraft; reference 77 describes excitation of the Surveyor spacecraft simulating vibration during the lunar-descent phase of the mission; reference 78 describes the vibration tests of several Saturn V sections.

A wide variety of test equipment is available for performing laboratory vibration tests. Electrodynamic shakers are the most popular because they are easiest to control (in contrast to hydraulic shakers) with regard to frequency, force output, acceleration output, and waveform. They are used mostly in the 5- to 2000-Hz frequency range, although the useful upper-frequency limit usually depends upon the shaker's force capacity and size. Electrodynamic shakers with force-generating capacities up to 30 000 lb (133 kN) are commerically available. New-generation electrodynamic systems provide solid-state amplifiers with direct coupling to the shaker armature, and can permit operation, within shaker-stroke constraints, down to 0 Hz. Improvements in shaker construction to give relatively large strokes permit the use of such overall systems, including multishaker applications, for testing of transients (ref. 79).

Many investigators favor hydraulic shakers principally because of their relatively large stroke and because of the very-low-frequency performance which is better than can be obtained with electrodynamic-shaker systems having transformer coupling between amplifiers and shakers. Hydraulic shakers are used primarily in the 0- to 500-Hz frequency range, although the useful upper-frequency limit depends on the design features of the shakers, which vary widely, as well as on their force capacity and size. Hydraulic shakers with force-generating capacities up to 250 000 lb (1.1 MN) are commercially available.

In vibration tests, the shaker applies the vibration force in only one direction; the flight excitation is applied in all directions simultaneously. Usually the assumption is made that vibration applied sequentially in each of three orthogonal directions for a specified duration per direction has the same response magnitude as vibration applied in all three directions simultaneously for the same duration. This assumption is usually valid only for resonant behavior in one direction; it is valid only by coincidence when multidirectional resonances are excited.

Vibration test inputs may vary. Some tests use sine-sweep inputs, some random inputs, and others a combination of both. Random inputs are generally representative of stationary random environments only because of vibration-system-control limitations. Test levels usually envelop the expected environments for the various prelaunch and mission events which exhibit high vibration. Often, the time duration selected for the vibration test is the sum of the effective durations of these prelaunch and mission events.

2.2.3 Acoustic Tests

Since two of the major causes of structural vibration are acoustic noise at liftoff and aerodynamic noise during the transonic and maximum dynamic-pressure regimes, it is natural to consider acoustic noise as a laboratory source of space-vehicle vibration. The Titan program first utilized acoustic testing for the development of vehicle structure subjected to acoustic noise (ref. 80). Acoustic testing was used on the Apollo program for qualification of equipment, for qualification of structure, and for revision of vibration design and test requirements (refs. 81 to 84).

Many different (some odd) facilities have been used for acoustic tests. For example, the Mercury (ref. 85) and OGO spacecraft were tested on the flatbeds of trucks located in an open field near the discharge nozzle of a large blowdown wind tunnel (ref. 85). A similar arrangement may be used with a rocket engine as the noise source. Large sections of the Saturn V launch vehicle were tested in a large reverberant test facility (ref. 86). The facility used for the acoustic tests of the Apollo lunar module and the command and service modules is described in references 82, 87, and 88.

Air modulators, such as those used in the Saturn and Apollo acoustic tests, work on the principle of exhausting high-pressure air through an orifice whose cross-sectional area is modulated by an electromagnet controlled from an external electrical signal, usually a random-noise generator. Air modulators generally have limited spectrum range and control (usually from 50 Hz to 1 kHz), with a spectral maximum in the vicinity of 100 Hz. Noise generation above 1 kHz is usually determined by the "unmodulated" flow noise of the air through the orifice. Air modulators are commercially available in acoustic power capacities up to 200 kW.

Acoustic testing has several advantages over vibration-shaker testing. In acoustic testing, a large prototype specimen can be tested; thus, the modal characteristics of the test specimen are similar to those of the flight configuration. The load is distributed over the external surface, rather than applied as point loading at one or more structural interfaces. Finally, a reasonable spectrum is often obtained during the acoustic test, compared with the difficulty of providing a test spectrum by means of a vibration shaker. The major disadvantage of acoustic testing is the high initial cost of the facilities to perform these tests properly (ref. 5).

2.2.4 Field Tests

Various field tests, such as rocket-engine and stage- or payload-static firings, are run to demonstrate the operational performance of various space-vehicle subsystems prior to flight. These tests are frequently used to provide data for predicting structural vibration or revising analytically determined data.

Solid- and liquid-propelled rocket engines are usually designed and tested quite early in a vehicle's development program, sometimes even before the prime contractors are selected. Because of this early scheduling, tests can be invaluable in providing the acoustic- and engine-vibration data that are used for initial vibration predictions. Data obtained from these tests are especially important when a rocket engine incorporates new design features which can affect acoustic noise and vibration generation, but have not been previously tested or instrumented. However, in making use of the rocket-engine acoustic-noise data, the designer must take into consideration any differences in the test-stand and launch-pad configurations that may affect the noise generation or transmission, such as the flame deflector. Also, many of the components attached to the engine during the early firings are usually not of flight-weight configuration, and may invalidate the vibration data.

2.2.5 Flight Tests

The flight test is almost always used as the final demonstration of the adequacy of the vibration-load analysis for both vehicles and stages. Nevertheless, in many cases, there is insufficient instrumentation because, in planning, too little weight and too few telemetry channels are allocated for instrumentation.

A large number of vibration and acoustic measurements are usually needed during flight test to demonstrate the adequacy of the vibration and acoustic analysis, to help in determining the probable cause of a flight failure, and to provide additional data for the design of future space vehicles. The measurement requirements usually far exceed the capacity of the telemetry subsystem. Most vibration and aeroacoustic measurements require a wide bandwidth (ref. 89). Techniques occasionally employed to reduce the demand on the telemetry subsystem include onboard frequency analysis, onboard tape recording, and time-division multiplexing.

3. CRITERIA

In the design of a space vehicle, account shall be taken of the effects of the structural vibration resulting from the environments to which the vehicle may be subjected. The vibration data needed for structural design shall be determined and verified by a suitable combination of analysis and tests.

3.1 Data Required

The input sources of vibration and the type, amount, and accuracy of the structural vibration data needed for design shall be defined.

3.2 Analytical Determination of Vibration Response

When measured vibration data are available from a similar vehicle or for a similar structure and similar environments, a suitable extrapolation method of analysis shall be used to predict the structural vibration for preliminary design requirements.

A deterministic analysis shall be made to obtain the vibration data needed for design, except for minor redesign where it can be demonstrated that the results of the extrapolation analysis are acceptable. The deterministic analysis shall use a mathematical model which adequately represents the mass distribution, stiffness distribution, damping, and boundary conditions of the flight vehicle. It shall be demonstrated, by tests if necessary, that vibration data from the deterministic analysis are of the type, amount, and accuracy required for structural design.

3.3 Tests

The significant characteristics of load displacement, vibration, and damping of the space-vehicle structure, if not otherwise adequately established, shall be determined or verified by a combination of static, vibration, and aeroacoustic testing, as applicable, on a realistic structure or on an actual vehicle. If full-scale tests are not feasible, replica-model dynamic tests (for lower order modes) shall be performed.

4. RECOMMENDED PRACTICES

The total plan for investigation of structural vibration resulting from anticipated environmental sources should be defined and initiated early in the space-vehicle development program. Vibration and acoustic development tests to confirm the calculated response and to demonstrate structural adequacy should be planned as an integral part of the total plan for the structural vibration analysis.

The following steps should be taken in the investigation of structural vibration:

1. Determine the input sources and the type, amount, and accuracy of the structural vibration data required for structural design of the vehicle.

2. Collect and extrapolate measured input and response data on similar vehicles.

3. Construct a mathematical model (or models) adequate for the vibration analyses, and select or devise a computational method.

4. Analyze the response of the model to the anticipated vibration environments.

5. Conduct vibration and static tests on segments of the structure or on the complete structure to confirm the calculated response.

4.1 Data Required

All sources of disturbance for all phases of vehicle operation should be considered during the determination of data required for structural vibration analysis. Vibration data available from ground and flight tests on similar vehicles should be examined.

Inputs for the vibration analysis should include at least the following:

- Vibration testing during prelaunch functional checkout
- Static firing during prelaunch functional checkout
- Winds during launch readiness
- Ignition and engine noise during launch liftoff
- Engine roughness during launch ascent
- Aerodynamic noise and buffet during launch ascent
- Engine cutoff and stage ignition during launch staging
- Aerodynamic noise and buffet during atmospheric entry

Reference should be made to other NASA Design Criteria monographs for guidance in obtaining these inputs to the vibration analysis.

No single type of analytical model or degree of model complexity can be recommended to analyze all configurations of a space vehicle. Since the accuracy of the response analysis varies widely with the complexity of the structure and the order of the responding modes, the following accuracy numbers should be regarded as guides only, and should not be taken as requirements without a study to determine their suitability for the individual problem. In general, calculation of maximum internal loads in the low-order modes should be accurate to about ±25 percent. Calculation of maximum internal loads at high-order modes, with respect to attached equipment, should be accurate to about ±50 percent.

4.2 Analytical Determination of Vibration Response

4.2.1 Extrapolation Response Analysis

During the early development phase of a new vehicle, when the structure is incompletely defined and detailed calculations are not possible, the initial estimate of structural vibration should be obtained by extrapolation of input-versus-response data measured on similar vehicles. When data are available on an existing vehicle having structural and configuration characteristics similar to those of the proposed vehicle, a scaling technique should be used to obtain the expected vibration response (refs. 3, 5, and 8). This approach is recommended when the design of an existing launch vehicle is modified, such as for thrust augmentation or for adding an extra stage, and when a new spacecraft is designed to fly on a proven launch vehicle for which data are available from previous flights of similarly proportioned spacecraft. However, if data are not available from a similar vehicle, a frequency response technique should be used to estimate the expected vibration response (refs. 3 and 5).

4.2.2 Deterministic Response Analysis

An evaluation of the structural vibration response, using a deterministic-analysis approach, should be undertaken as soon as the preliminary vehicle configuration is selected and an initial determination is made of the structural characteristics. Since many or most of the structural details are lacking during the preliminary design phase, the structure should be represented for analytical purposes by relatively simple mathematical models which are improved as the structural design is defined in more detail.

The structural characteristics should be represented for analysis by an equivalent linear mathematical model. It should account for all stress-strain effects that influence the structural distortions, such as beam shear, torsion, and axial extension, as well as plate shear and twist, unless their effect on the vibration response has been proven negligible (refs. 90 and 91). The effects of internal forces which modify the effective-stiffness characteristics should be included in the mathematical model. These internal forces may result from dead loads, quasi-static accelerations during boost, built-in preloads, and other static or quasi-static loads applied to the structure.

In evaluating the stiffness of the model, the effect of local structure, such as joints between interstage adaptors and vehicle stages, trusses on which payload or engines are mounted, or play in joints (such as engine gimbal blocks when the engine is not under thrust) should be evaluated. Depending upon the characteristics of the joint, the combination of axial and bending loads could lead to variations in stiffness during different periods of operation, both in flight and on the ground. The variation of joint stiffness under these conditions is difficult to determine by analysis and should be ascertained by test.

Finite-element techniques are recommended for modeling complex structures mathematically. In this approach, the mathematical model should be defined in terms of assumed displacement functions in component parts leading to direct construction of a structural-stiffness matrix. The relative proportion of adjacent individual components in the model must be chosen to minimize extreme variations in stiffness or flexibility which result in loss of accuracy in the structure-matrix coefficients (ref. 92). Unless the equivalent of IBM 360 double-precision arithmetic is used, the ratio of numerical values between direct-coupled diagonal elements in the elastic matrix should not usually be allowed to exceed 1:1000. Substructure methods of modeling the structure should be used for the analysis of large multistage vehicle systems where the several stages may be structurally dissimilar (ref. 52).

The mathematical model should closely represent the distribution of mass throughout the structure. The use of distributed-mass models such as the consistent-mass-matrix technique (ref. 44) is recommended. If a lumped-mass technique is used, it should be demonstrated by a parameter-variation study that a sufficient number of discrete mass points are used to represent the modal characteristics adequately (refs. 93 and 94). For accurate modal-data determination of a lumped-mass, unsupported, one-dimensional system, the number of discrete masses should be about ten times the order of the highest mode to be determined. For a more complex structure, such as multidimensional frames, the relationship between the number of discrete points and the accuracy of the computed frequency is not established, and reliance must be placed on the experience of the analyst.

Mathematical models of tanks containing liquids should simulate at least the first-mode lateral sloshing effect. A mass-spring model, based on the pendulum analogy, is recommended to simulate this phenomenon (ref. 95). The equivalent hydrostatic loads obtained from the computed slosh-mass acceleration response should be introduced into the structure for internal-load analysis. Longitudinal mechanical coupling of the liquid with the structure must also be provided for in the longitudinal-mode analysis (ref. 47). Care should be exercised to ensure that only the effective portion of the liquid mass is represented in the model; that is, a linear model of a smooth cylindrical tank rotating about its geometric axis of revolution does not cause rotation of any contained liquid.

The mode-displacement or mode-acceleration response technique should be used to calculate response of the low-order modes caused by various disturbances. The total response is obtained by summing the response in each of the orthogonal modes which represent the structural vibration (refs. 20 and 22). The resonant frequencies and mode shapes obtained to perform the response analysis should be used as guides to select mounting locations for equipment. When the applied loading can be characterized as random, the modal response technique should be used to obtain the structural response (refs. 2, 20, and 23).

The magnitude of damping used in the vibration response analysis should be chosen with care. To estimate realistic damping levels for the various modes of vibration, test data on similar structure should be used when available. The recommended damping factors provided in table II should be used only as guides in lieu of more pertinent test data which may be available.

4.2.3 Statistical-Energy Response Analysis

Because this type of analysis is relatively new and has not been widely used, no recommended practices are provided. However, the lack of a recommendation is not intended to discourage this type of analysis.

4.3 Tests

Static tests should be conducted to verify the analytically derived load-displacement characteristics and boundary conditions of the analytical model of the vehicle structure. So far as is feasible, the test boundary conditions should simulate flight conditions. The analytically derived vibration characteristics of the vehicle structure should be confirmed by vibration tests when not adequately established by prior analytical and experimental correlations. Furthermore, if vibration response data obtained by analysis cannot otherwise be demonstrated to be adequate, the analytical data should be replaced or confirmed by the results of vibration tests conducted during development testing. The scope of the test program should depend on the confidence placed in the analytical results.

Extensive testing is recommended where a radically new space-vehicle configuration is involved. On the other hand, simple changes of payload on a standard launch vehicle may require only an analytical determination of the new response data. Changes in

TABLE II. – RECOMMENDED DAMPING FACTORS

Type of structure	Percent of critical viscous damping	Remarks
Homogeneous-element configurations (machined brackets, solid beams, welded construction)	1 to 2	Damping factor depends on stress levels induced during vibration (refs. 56 and 96)
Riveted or bolted structures	3 to 10	Damping caused by friction at joints significantly reduces the vibration amplification
Laminated plastics	4 to 10	Phenolic laminate (refs. 56 and 96)
Honeycomb-core panels	3 to 6	Damping factor depends on method of fabrication (brazed versus adhesive bond) and on acoustic radiation
Vibration-isolated components	10 to 20	Damping factor depends on the isolator design
Nonviscous fluids	0.5 1.0 1.5	Frequency < 5 Hz 5 Hz $\leqslant$ frequency $\leqslant$ 15 Hz Frequency $>$ 15 Hz

mass usually can be adequately handled by changes in the mathematical model without additional tests, although significant changes in stiffness usually require test verification. To validate the analyses of structural vibration response, vibration and acoustic tests should be performed on subsystems, large structural sections and complete vehicle systems when feasible.

If the actual design incorporates seriously nonlinear features, such as looseness in joints and backlash in gears, then the structural behavior cannot be adequately predicted by linear analysis. If such features exist in the design, either by intent or by virtue of uncontrollable factors, then the analyst, designer, and experimenter must all be careful in applying linear analytic approximations to nonlinear real-life test results.

4.3.1 Static Tests

Tests of static-load-displacement and boundary-condition-influence coefficients should be performed on full-scale engineering models that have static characteristics of the primary structure identical to those of the prototype and flight structure. If full-scale tests are not feasible, data from replica models such as the one-fifth-scale Saturn replica model (ref. 74) may be sufficient. The load conditions under which the displacement data are obtained should simulate the quasi-static conditions expected for the time of flight for which the vibration response data will apply.

Load-displacement data should be obtained to determine the elastic characteristics for the primary load-carrying element in the structure, with loads applied at the location of the primary masses or at major attachment points. For a simple spacecraft structure, the load-displacement characteristics may be determined for only a single major load point, as at the cantilevered end of the major structural element, at the location of the major equipment platform cantilevered from a central cylinder, or at the support points for an injection rocket attachment. For a launch vehicle or a large spacecraft structure, load-displacement measurements should be determined for the interstage structure between attach points, at major transverse bulkheads, for engine-support trusses, and for payload-support points.

The load-displacement characteristics and boundary-condition data obtained should be compared with the mathematical model. These data should correlate within about 20 percent for displacement under a given load. If necessary, the mathematical model should be changed to agree with the static load-displacement test data. These load-displacement tests (or influence-coefficient measurements) should not be confused with static structural-qualification tests.

4.3.2 Vibration Tests

To provide a basis for evaluating the quality of analytically derived response data, vibration tests should preferably be performed on a full-scale engineering model, on a prototype, or on flight-type structures which have dynamic characteristics closely approximating those of the flight structure. Scaled replica models should be considered for determining low-order vibration-mode characteristics where full-scale tests are not feasible (ref. 74).

Sine-sweep tests on spacecraft are recommended for verifying analytical modal frequencies and low-order mode shapes, for determining modal damping, and for extrapolating vibration magnitudes. If analytical data are not available, or if more precise experimental data on large spacecraft and launch vehicles are desired than are available from sine-sweep tests, then modal-vibration survey tests should be performed to obtain more precise modal data. Where modal density is high, survey techniques giving coincident and quadrature response (ref. 97), or the methods of reference 98 should be used for identifying and determining modal characteristics.

Qualification tests should be instrumented to the fullest extent possible to provide vibration response data for design-load verification. Because of the unrealistic test-boundary conditions in most shaker tests, the vibration-test levels at frequencies in the vicinity of the structure's fundamental resonance may cause loads far in excess of flight loads, and may require careful control to limit the response in certain critical low-order modes to within the expected flight response.

4.3.3 Acoustic Tests

Acoustic tests should be used to supplement or supersede vibration testing of vehicle structural systems and equipment when the source of the environmental disturbances is acoustic or aerodynamic noise. The acoustic tests should be performed on a sufficiently large section of the vehicle so that the modal characteristics of the test specimen are similar to those of the flight configuration at the low side of the frequency bands of interest. A meaningful acoustic test should be designed to simulate, so far as is feasible, the exact environment and the dynamic characteristics of the boundaries of the specimen (refs. 5 and 81 to 84).

4.3.4 Field Tests

No specific recommendations are made for performing field tests. However, if the scope of a program includes static-firing tests of rocket engines, stages, or payloads, such tests should be evaluated as a source of data for structural vibration prediction.

4.3.5 Flight Tests

Flight tests should be used to provide data to confirm vibration analysis and to provide environmental data to support design improvements. Instrumentation should be installed to identify unusual load situations and permit diagnosis of causes of partial or total failure due to vibration loads.

REFERENCES

1. Crandall, S. H. and Mark, W. D.: Random Vibration in Mechanical Systems. Academic Press, 1963, p. 111.

2. Robson, J. D.: Introduction to Random Vibration. Elsevier Publishing Co., 1964.

3. Barnoski, R. L.; Piersol, A. G.; Van Der Laan, W. F.; White, P. H.; and Winter, E. F.: Summary of Random Vibration Prediction Procedures. NASA CR-1302, 1969.

4. Piersol, A. G.: The Development of Vibration Test Specifications for Flight Vehicle Components. J. Sound and Vibration, vol. 4, no. 1, July 1966, pp. 88-115. (Also NASA CR-234, 1965.)

5. Himelblau, H.; Fuller, C. M.; and Scharton, T. D.: Assessment of Space Vehicle Aeroacoustic-Vibration Prediction, Design, and Testing. NASA CR-1596, 1970.

6. Condos, F. M.; and Butler, W.: A Critical Analysis of Vibration Prediction Techniques. Proc. Inst. Envir. Sci., 1963, pp. 321-326.

7. Barrett, R. E.: Techniques for Predicting Localized Vibratory Environments of Rocket Vehicles. NASA TN D-1836, 1963.

8. On, F.: A Verification of the Practicality of Predicting Interface Dynamical Environments by Use of the Impedance Concept. The Shock and Vibration Bull. No. 38, Part 2, Aug. 1968, pp. 249-260.

9. Jones, G.; and On, F.: Prediction of Interface Random and Transient Vibratory Environments Through the Use of Mechanical Impedance Concepts. Presented to the Fortieth Shock and Vibration Symposium (Hampton, Virginia), Oct. 1969.

10. Mahaffey, P. T.; and Smith, K. W.: Method for Predicting Environmental Vibration Levels in Jet Powered Vehicles. Noise Control, vol. 6, no. 4, July 1960, pp. 20-26. (Also The Shock and Vibration Bull. No. 28, Part 4, Aug. 1960, pp. 1-14. Available from DDC as AD 244857.)

11. Brust, J. M.; and Himelblau, H.: Comparison of Predicted and Measured Vibration Environments for Skybolt Guidance Equipment. Presented to Symposium on Shock, Vibration and Associated Environments (Washington, D.C.), Dec. 1963.

12. Eldred, K. M.; Roberts, W. H.; and White, R. W.: Structural Vibrations in Space Vehicles. WADD TR 61-62, Dec. 1961. (Available from DDC as AD 273334.)

13. Curtis, A. J.: A Statistical Approach to Prediction of the Aircraft Flight Vibration Environment. The Shock and Vibration Bull. No. 33, Part 1, Feb. 1964, pp. 1-13. (Available from DDC as AD 348503.)

14. Franken, P. A.: Sound-Induced Vibrations of Cylindrical Vehicles. J. Acoust. Soc. Am., vol. 34, no. 4, Apr. 1962, pp. 453-454.

15. Hunt, F. V.: Stress and Strain Limits on the Attainable Velocity in Mechanical Vibration. J. Acoust. Soc. Am., vol. 32, no. 9, Sept. 1960, pp. 1123-1128.

16. Ungar, E. E.: Maximum Stresses in Beams and Plates Vibrating at Resonance. J. Eng. for Industry (Trans. ASME), vol. 82B, no. 1, Feb. 1962, pp. 149-155.

17. Crandall, S. H.: Relation Between Strain and Velocity in Resonant Vibration. J. Acoust. Soc. Am., vol. 34, no. 12, Dec. 1962, pp. 1960-1961.

18. Harris, C. M.; and Crede, C. E., eds.: Shock and Vibration Handbook. McGraw-Hill Book Co., Inc., 1961.

19. Crandall, S. H., ed.: Random Vibration Technology. Press and Wiley (New York), 1958.

20. Hurty, W. C.; and Rubinstein, M. F.: Dynamics of Structures. Prentice-Hall, Inc., 1964.

21. Pestel, E. C.; and Leckie, F. A.: Matrix Methods in Elasto-mechanics. McGraw-Hill Book Co., Inc., 1963.

22. Bisplinghoff, R. L.; Ashley, H.; and Halfman, R. L.: Aeroelasticity. Addison-Wesley Publ. Co., 1955.

23. Lin, Y. K.: Probabilistic Theory of Structural Dynamics. McGraw-Hill Book Co., Inc., 1967.

24. Thomson, W. T.: Laplace Transformation. Prentice-Hall, Inc., 1960.

25. Anon.: Natural Vibration Modal Analysis. NASA Space Vehicle Design Criteria (Structures), NASA SP-8012, 1968.

26. Schweiker, J. W.; and Davis, R. E.: Response of Complex Shell Structures to Aerodynamic Noise. NASA CR-450, 1966.

27. Timoshenko, S. P.: Vibration Problems in Engineering. 3rd Ed., D. Van Nostrand Co., Inc., 1955.

28. Nowacki, W.: Dynamics of Elastic Systems. John Wiley & Sons, Inc., 1963.

29. Kalnins, A.: Dynamic Problems of Elastic Shells. Appl. Mech. Rev., vol. 18, no. 11, Nov. 1965, pp. 867-872.

30. Forsberg, K.: A Review of Analytical Methods Used to Determine the Modal Characteristics of Cylindrical Shells. NASA CR-613, 1966.

31. Archer, J. S.: Consistent Matrix Formulations for Structural Analysis Using Influence Coefficient Techniques. AIAA Paper no. 64-488, 1964.

32. Argyris, J. H.: Continua and Discontinua. AFFDL TR-66-80, Proc. Conference on Matrix Methods in Structural Mechanics (Wright-Patterson AFB, Ohio), Dec. 1965, pp. 11-189. (Available from DDC as AD 646300.)

33. Gallagher, R. H.; Rattinger, I.; and Archer, J. S.: A Correlation Study of Methods of Matrix Structural Analysis. Report to the Fourteenth Meeting, Structures and Materials Panel, AGARD, NATO, Pergamon Press Ltd., 1964.

34. Turner, M. J.; Clough, R. W.; Martin, H. C.; and Topp, L. J.: Stiffness and Deflection Analysis of Complex Structures. J. Aeron. Sci., vol. 23, no. 9, Sept. 1956, pp. 805-823.

35. Gallagher, R. H.; and Huft, R. D.: Derivation of the Force-Displacement Properties of Triangular and Quadrilateral Orthotropic Plates in Plane Stress and Bending. Rept. D2114-950005, Bell Aerosystems Co., Jan. 1964.

36. Melosh, R. J.: Basis for Derivation of Matrices for the Direct Stiffness Method. AIAA J., vol. 1, no. 7, July 1963, pp. 1631-1637.

37. Melosh, R. J.; and Christiansen, H. N.: Structural Analysis and Matrix Interpretive System (SAMIS) Program: Technical Report. TM-33-311, Jet Propulsion Lab., Nov. 1, 1966.

38. Percy, J. H.; Pian, T. H. H.; Navaratna, D. R.; and Klein, S.: Application of Matrix Displacement Method to Linear Elastic Analysis of Shells of Revolution. AIAA J., vol. 3, no. 11, Nov. 1965, pp. 2138-2145.

39. Archer, J. S.; and Rubin, C. P.: Improved Analytic Longitudinal Response Analysis for Axisymmetric Launch Vehicles. NASA CR-345, 1965.

40. Jones, R. E.; and Strome, D. R.: Direct Stiffness Method Analyses of Shells of Revolution Utilizing Curved Elements. AIAA J., vol. 4, no. 9, Sept. 1966, pp. 1519-1525.

41. Fowler, J. R.: Accuracy of Lumped Systems Approximations for Lateral Vibrations of a Free-Free Uniform Beam. Rept. EM 9-12, Engineering Mechanics Laboratory, Space Technology Laboratories, Inc. (Los Angeles), July 1959.

42. Norris, C. H.; Hansen, R. J.; Holley, M. J., Jr.; Biggs, J. M.; Namyet, S.; and Minami, J. K.: Structural Design for Dynamic Loads. McGraw-Hill Book Co., Inc., 1959.

43. Archer, J. S.: A Stiffness Matrix Method of Natural Mode Analysis. Proc. IAS National Specialists Meeting on Dynamics and Aeroelasticity (Dallas, Tex.), Nov. 6-7, 1958.

44. Archer, J. S.: Consistent Mass Matrix for Distributed Mass Systems. Proc. ASCE Structural Division, Third Conference on Electronic Computation (Boulder, Colo.), vol. 89, no. ST4, June 19-21, 1963, pp. 161-178.

45. Melosh, R. J.; and Lang, T. E.: Modified Potential Energy Mass Representations for Frequency Prediction. AFFDL TR-66-80, Proc. Conference on Matrix Methods in Structural Mechanics (Wright-Patterson AFB, Ohio), Dec. 1965, pp. 445-455.

46. Anon.: Propellant Slosh Loads. NASA Space Vehicle Design Criteria (Structures), NASA SP-8009, 1968.

47. Pinson, L. D.: Longitudinal Spring Constants for Liquid-Propellant Tanks with Ellipsoidal Ends. NASA TN D-2220, 1964.

48. Guyan, R. J.; Ujihara, B. H.; and Welch, P. W.: Hydrostatic Analysis of Axisymmetric Systems by Finite Element Method. Proc. 1968 Conference on Matrix Methods (Wright-Patterson AFB, Ohio), 1969.

49. Chiu-Hung-Luk: Finite Element Analysis for Liquid Sloshing Problems. Master of Science Thesis, Mass. Inst. Technol., June 1969. Available from DDC as AD 693619.

50. Zienkiewicz, O. C.; and Cheung, T.: The Finite Element Method in Structural and Continuum Mechanics. McGraw-Hill Publishing Co., Inc., 1967.

51. Fujino, T.: Analysis of Hydrodynamic Problems by Finite Element Method. Presented to 1969 Conference on Matrix Methods in Structural Analysis Design (Tokyo, Japan), Aug. 25-30, 1969.

52. Hurty, W. C.: Dynamic Analysis of Structural Systems Using Component Modes. AIAA J., vol. 3, no. 4, Apr. 1965, pp. 678-685.

53. Przemieniecki, J. S.: Theory of Matrix Structural Analysis. McGraw-Hill Book Co., Inc., 1968.

54. White, R. W.: Theoretical Study of Acoustic Simulation of In-Flight Environments. The Shock and Vibration Bull. No. 37, Part 5, Jan. 1968, pp. 55-75.

55. Lazan, B. J.; and Goodman, L. E.: Material and Interface Damping. Vol. II of Shock and Vibration Handbook, ch. 36, C. M. Harris and C. E. Crede, eds., McGraw-Hill Book Co., Inc., 1961.

56. Lazan, B. J.: Damping of Materials and Members in Structural Mechanics. Pergamon Press Ltd., 1968.

57. Metherill, A. F.; and Diller, S. V.: Instantaneous Energy Dissipation Rate in a Lap Joint – Uniform Clamping Pressure. J. Appl. Mech., vol. 35E, no. 1, Mar. 1968, pp. 123-128.

58. Goodman, L. E.; and Klumpf, J. H.: Analysis of Slip Damping with Reference to Turbine Blade Vibration. J. Appl. Mech., vol. 23, no. 3, Sept. 1956, pp. 421-429.

59. Smith, P. W.; and Lyon, R. H.: Sound and Structural Vibration. NASA CR-160, 1965.

60. Lyon, R. H.; and Maidanik, G.: Statistical Methods in Vibration Analysis. AIAA J., vol. 2, no. 6, June 1964, pp. 1015-1024.

61. Smith, P. W.: Response and Radiation of Structural Modes Excited by Sound. J. Acoust. Soc. Am., vol. 34, no. 5, May 1962, pp. 640-647.

62. Manning, J. E.; and Maidanik, G.: Radiation Properties of Cylindrical Shells. J. Acoust. Soc. Am., vol. 36, no. 9, Sept. 1964, pp. 1691-1698.

63. Chertock, G.: Sound Radiation from Vibrating Surfaces. J. Acoust. Soc. Am., vol. 36, no. 7, July 1964, pp. 1305-1313.

64. Chertock, G.: A Fortran Program for Calculating the Sound Radiation From a Vibrating Surface of Revolution. Rept. DTMB 2083, David Taylor Model Basin, Dec. 1965. (Available from DDC as AD 627745.)

65. Ross, D.; Ungar, E. E.; and Kerwin, E. M.: Damping of Plate Flexural Vibrations by Means of Viscoelastic Laminae. Structural Damping, J. E. Ruzicka, ed., Booklet published by ASME, 1959, pp. 49-87.

66. Maidanik, G.: Energy Dissipation Associated with Gas-Pumping in Structural Joints. J. Acoust. Soc. Am., vol. 40, no. 5, Nov. 1966, pp. 1064-1072.

67. Lyon, R. H.: Random Noise and Vibration in Space Vehicles. Shock and Vibration Mono. 1, Shock and Vibration Information Center, DOD, 1967.

68. Ungar, E. E.: Fundamentals of Statistical Energy Analysis of Vibration of Vibrating Systems. AFFDL TR-66-52, Apr. 1966. (Available from DDC as AD 637304.)

69. Manning, J. E.; Lyon, R. H.; and Scharten, T. D.: The Transmission of Sound and Vibration to a Shroud Enclosed Spacecraft. NASA CR-81688, 1966. (Also Rept. 1431, Bolt Beranek and Newman, Oct. 1966.)

70. Maidanik, G.: Response of Ribbed Panels to Reverberant Acoustic Fields. J. Acoust. Soc. Am., vol. 34, no. 6, June 1962, pp. 809-826.

71. Scharton, T. D.; and Yang, T. M.: Statistical Energy Analysis of Vibration Transmission Into an Instrument Package. SAE Paper no. 670876, Oct. 1967.

72. Chandiramani, K. L.: Structural Response to Inflight Acoustic and Aerodynamic Environments. NASA CR-83211, 1966. (Also Rept. 1417, Bolt Beranek and Newman, June 1966.)

73. Manning, J. E.; and Koronaios, N.: Experimental Study of Sound and Vibration Transmission to a Shroud-Enclosed Spacecraft. NASA CR-96144, 1968. (Also Rept. 1592, Bolt Beranek and Newman, Aug. 1968.)

74. Mixson, John S.; and Catherine, John J.: Comparison of Experimental Vibration Characteristics Obtained From a 1/5-Scale Model and From a Full-Scale Saturn SA-1. NASA TN D-2215, 1964.

75. Wiksten, D. B.: Dynamic Environment of the Ranger Spacecraft: I through IX (Final Report). TR 32-909, Jet Propulsion Lab., May 1966.

76. Daiber, J. R.; and Noonan, V. S.: The Vibration Design Approval and Acceptance Test Program for the Gemini Spacecraft – Component, Module and Whole Vehicle Testing. The Shock and Vibration Bull. No. 35, Part 2, Jan. 1966, pp. 139-146. (Available from DDC as AD 628599.)

77. Curtis, A. J.; Abstein, H. T.; and Varga, R. J.: On the Use of Multiple (Multi-Point) Random Excitation with Application to Surveyor Spacecraft Tests. The Shock and Vibration Bull. No. 35, Part 2, Jan. 1966, pp. 49-74. (Available from DDC as 628599.)

78. Schock, R. W.; Everitt, J. M.; and Seat, J. R.: Saturn S-II, S-IVB, and Instrument Unit Subassembly and Assembly Vibration and Acoustic Evaluation Programs. The Shock and Vibration Bull. No. 37, Part 5, Jan. 1968, pp. 117-137.

79. Trubert, M. R.: An Analog Technique for the Equalization of Multiple Electromagnetic Shakers for Vibration Testing. J. Spacecraft Rockets, vol. 5, no. 12, Dec. 1968, pp. 1438-1443.

80. McGregor, H. W.; Dinicola, D.; Williamson, H.; Otera, J. M.; Pitsker, J. R.; and Condos, F. M.: Acoustic Problems Associated with the Underground Launching of a Large Missile. The Shock and Vibration Bull. No. 29, Part 4, June 1961, pp. 317-335. (Available from DDC as AD 259522.)

81. Peverley, R. W.: Vibroacoustic Test Methods for Vibration Qualification of Apollo Flight Hardware. The Shock and Vibration Bull. No. 37, Part 5, Jan. 1968, pp. 153-166.

82. Newbrough, D. E.; Bernstein, M.; and Baird, E. F.: Development and Verification of the Apollo Lunar Module Vibration Test Requirements. The Shock and Vibration Bull. No. 37, Part 5, Jan. 1968, pp. 105-115.

83. Stevens, R. A.; Allen, H. C.; and Pratt, H. K.: Vibration Qualification of Flyaway Umbilical by Acoustic Test Methods. Proc. Inst. Envir. Sci., 1967, pp. 437-442.

84. Chirly, A. E.; Stevens, P. A.; and Wood, W. R.: Apollo CSM Dynamic Test Program. The Shock and Vibration Bull. No. 39, Part 2, Feb. 1969, pp. 105-121.

85. Clevenson, S. A.; Hilton, D. A.; and Lauten, W. T.: Vibration and Noise Environmental Studies for Project Mercury. Proc. Inst. Envir. Sci., 1961, pp. 541-554.

86. Murray, F. M.: Operational Characteristics of a 100,000-Cubic-Foot Acoustic Reverberant Chamber. The Shock and Vibration Bull. No. 37, Part 5, Jan. 1968, pp. 13-24.

87. Wren, R. J.; Dorland, W. D.; and Eldred, K. M.: Concept, Design and Performance of the Spacecraft Acoustic Laboratory. The Shock and Vibration Bull. No. 37, Part 5, Jan. 1968, pp. 25-54.

88. Dorland, W. D.; Wren, R. J.; and Eldred, K. M.: Development of Acoustic Test Conditions for Apollo Lunar Module Flight Certification. The Shock and Vibration Bull. No. 37, Part 5, Jan. 1968, pp. 139-152.

89. Subcommittee G-5.9 on Telemetry Requirements, SAE Committee G-5 on Aerospace Shock and Vibration: Desired Telemetry System Characteristics for Shock, Vibration and Acoustic Measurements. Vol. II of Proc. International Telemetering Conf., 1966, pp. 232-253.

90. Alley, Vernon L.; and Guillottee, Robert J.: A Method of Determining Modal Data of a Nonuniform Beam with Effects of Shear Deformation and Rotary Inertia. NASA TN D-2930, 1965.

91. Argyris, J. H.; and Kelsey, S.: Energy Theorems and Structural Analysis. Butterworths (New York), 1960.

92. Rosanoff, R. A.; and Ginsburg, T. A.: Matrix Error Analysis for Engineers. AFFDL TR-66-80, Proc. Conference on Matrix Methods in Structural Mechanics (Wright-Patterson AFB, Ohio), Dec. 1965, pp. 887-910.

93. Alley, Vernon L.; and Gerringer, A. Harper: A Matrix Method for the Determination of the Natural Vibrations of Free-Free Unsymmetrical Beams with Application to Launch Vehicles. NASA TN D-1247, 1962.

94. Wingate, R. T.: Matrix Analysis of Longitudinal and Torsional Vibrations in Nonuniform Multibranch Beams. NASA TN D-3844, 1967.

95. Abramson, H. N., ed.: The Dynamic Behavior of Liquids in Moving Containers. NASA SP-106, 1966.

96. Lee, L. T.: A Graphical Compilation of Damping Properties of Both Metallic and Non-Metallic Materials. AFML TR-66-169, May 1966.

97. Stahle, C. V., Jr.: Phase Separation Technique for Ground Vibration Testing. Aerospace Engineering, vol. 21, no. 7, July 1962, pp. 56-57, 91-96.

98. Bishop, R. E. D.; and Gladwell, G. M. L.: An Investigation into the Theory of Resonance Testing. Philosophical Transactions of the Royal Society, vol. 255, no. A1055, Jan. 17, 1963, pp. 241-279.

NASA SPACE VEHICLE DESIGN CRITERIA MONOGRAPHS ISSUED TO DATE

SP-8001	(Structures)	Buffeting During Atmospheric Ascent, May 1964 – Revised November 1970
SP-8002	(Structures)	Flight-Loads Measurements During Launch and Exit, December 1964
SP-8003	(Structures)	Flutter, Buzz, and Divergence, July 1964
SP-8004	(Structures)	Panel Flutter, May 1965
SP-8005	(Environment)	Solar Electromagnetic Radiation, June 1965
SP-8006	(Structures)	Local Steady Aerodynamic Loads During Launch and Exit, May 1965
SP-8007	(Structures)	Buckling of Thin-Walled Circular Cylinders, September 1965 – Revised August 1968
SP-8008	(Structures)	Prelaunch Ground Wind Loads, November 1965
SP-8009	(Structures)	Propellant Slosh Loads, August 1968
SP-8010	(Environment)	Models of Mars Atmosphere (1967), May 1968
SP-8011	(Environment)	Models of Venus Atmosphere (1968), December 1968
SP-8012	(Structures)	Natural Vibration Modal Analysis, September 1968
SP-8013	(Environment)	Meteoroid Environment Model – 1969 [Near Earth to Lunar Surface], March 1969
SP-8014	(Structures)	Entry Thermal Protection, August 1968
SP-8015	(Guidance and Control)	Guidance and Navigation for Entry Vehicles, November 1968
SP-8016	(Guidance and Control)	Effects of Structural Flexibility on Spacecraft Control Systems, April 1969
SP-8017	(Environment)	Magnetic Fields – Earth and Extraterrestrial, March 1969
SP-8018	(Guidance and Control)	Spacecraft Magnetic Torques, March 1969
SP-8019	(Structures)	Buckling of Thin-Walled Truncated Cones, September 1968
SP-8020	(Environment)	Mars Surface Models (1968), May 1969
SP-8021	(Environment)	Models of Earth's Atmosphere (120 to 1000 km), May 1969
SP-8023	(Environment)	Lunar Surface Models, May 1969
SP-8024	(Guidance and Control)	Spacecraft Gravitational Torques, May 1969
SP-8028	(Guidance and Control)	Entry Vehicle Control, November 1969
SP-8029	(Structures)	Aerodynamic and Rocket-Exhaust Heating During Launch and Ascent, May 1969
SP-8031	(Structures)	Slosh Suppression, May 1969
SP-8032	(Structures)	Buckling of Thin-Walled Doubly Curved Shells, August 1969
SP-8035	(Structures)	Wind Loads During Ascent, June 1970
SP-8036	(Guidance and Control)	Effects of Structural Flexibility on Launch Vehicle Control Systems, February 1970
SP-8037	(Environment)	Assessment and Control of Spacecraft Magnetic Fields, September 1970
SP-8040	(Structures)	Fracture Control of Metallic Pressure Vessels, May 1970
SP-8046	(Structures)	Landing Impact Attenuation for Nonsurface-Planing Landers, April 1970

NASA-Langley, 1971 —— 32