PRELOADED SPRING-MASS SYSTEM SUBJECTED TO BASE EXICITATION

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Derivation

Consider a single-degree-of-freedom system as shown in Figure 1. This is a piecewise-linear system.

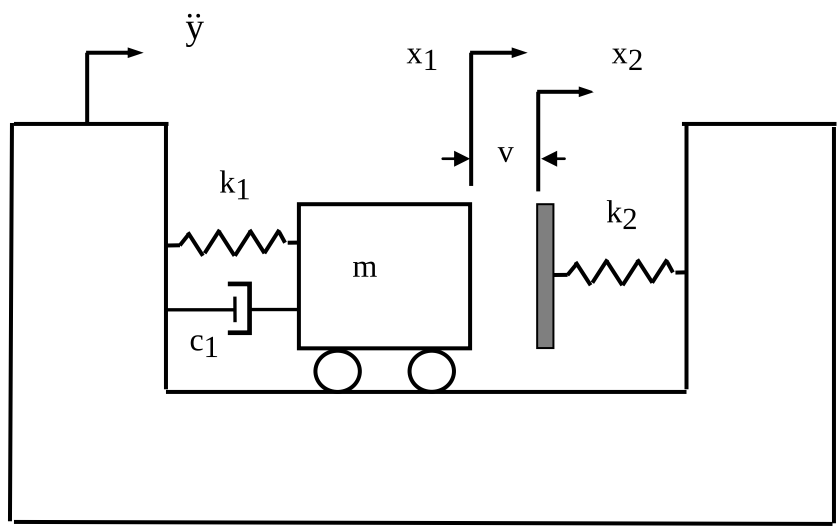

Figure 1.

The variables are

m	=	mass
c_1	=	damping coefficient
k_1, k_2	=	stiffness
$\ddot{y}$	=	base acceleration

The mass and bumper are initially in contact with a preload P.

The displacements x_1 and x_2 are with respect to the initial preloaded rest condition

The relative displacements are

$$z_1 = x_1 - y \tag{1}$$

$$z_2 = x_2 - y \tag{2}$$

Initially,

$$z_1(0) = z_2(0) \tag{3}$$

The undeflected bumper spring k_2 has a relative displacement

$$u = -P / k_2 \tag{4}$$

Spring-Mass System, Gap Condition

The equation of motion for the spring-mass system subjected to base excitation is

$$m\ddot{x}_1 + c_1\dot{x}_1 + k_1 x_1 = c_1\dot{y} + k_1 y \ , \quad \text{for} \quad z_1 < u \tag{5}$$

$$m\left(\ddot{z}_1 + \ddot{y}\right) + c_1\left(\dot{z}_1 + \dot{y}\right) + k_1\left(z_1 + y\right) = c_1\ddot{y} + k_1 y \tag{6}$$

$$m\ddot{z}_1 + c_1\dot{z}_1 + k_1 z_1 = -m\ddot{y} \tag{7}$$

$$\ddot{z}_1 + \left(\frac{c_1}{m}\right)\dot{z}_1 + \left(\frac{k_1}{m}\right)z_1 = -\ddot{y} \tag{8}$$

Spring-Mass-Bumper System, Contact Condition

The following formula is for the case then the mass and bumper are in contact.

$$\ddot{z}_1 + \left(\frac{c_1}{m}\right)\dot{z}_1 + \left(\frac{k_1 + k_2}{m}\right)z_1 = -\ddot{y} \quad , \quad \text{for} \quad z_1 \geq u \tag{9}$$

In this case,

$$z_2(t) = z_1(t) \tag{10}$$

Solution Method

The ordinary differential equations can then be solved in the time domain for an arbitrary base excitation using the Runge-Kutta fourth order method, or some other numerical method.

Reference

1. T. Irvine, Free Vibration of a Single-degree-of-freedom System, Revision B, Vibrationdata, 2005.

APPENDIX A

Example

The system in Figure 1 has the following parameters.

m	=	0.167 kg
c_1	=	1.18 N s/m
k_1	=	840 N/m
k_2	=	50000 N/m
P	=	5 N

Note that the spring-mass system has 5% damping in the gap condition.

The system is subjected to a 40 G, 11 msec terminal sawtooth pulse.

The results are calculated using the formulas in the main text using Matlab script: spring_mass_preloaded_base.m.

The displacement and velocity are shown in Figures A-1 and A-2, respectively.

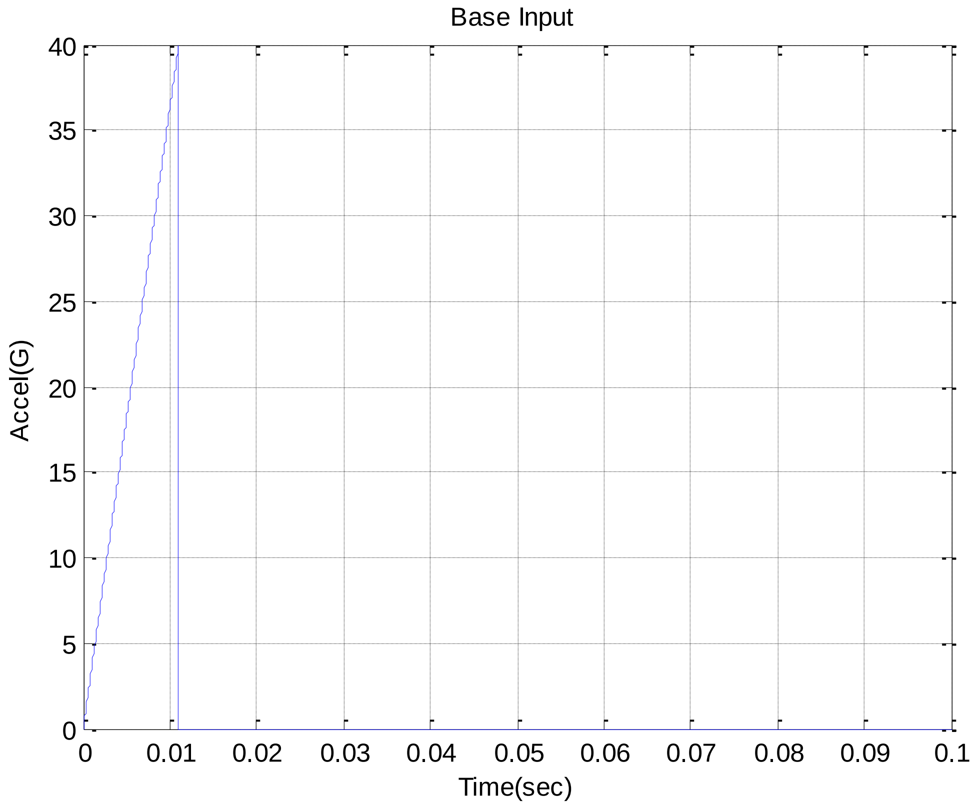
Base Input
Accel(G)
0
5
10
15
20
25
30
35
40
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.1
Time(sec)

Figure A-1.

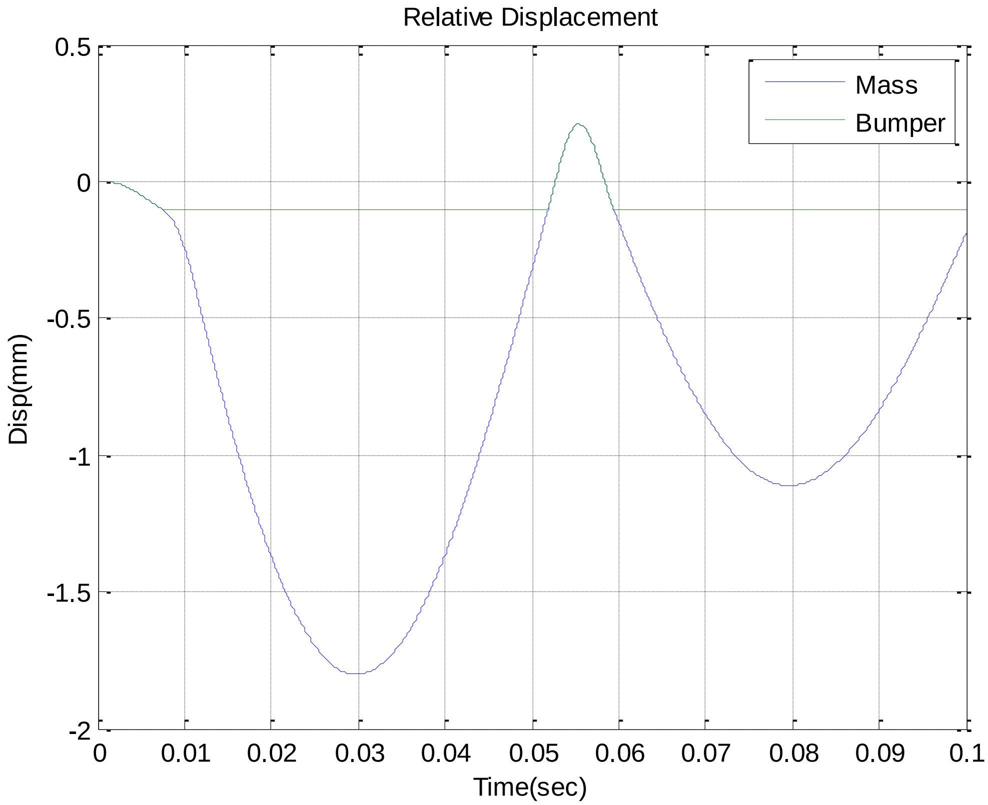

Figure A-2.

The initial deflection of spring 2 = - 0.1 mm.

The mass and bumper are in contact when the curves coincide.

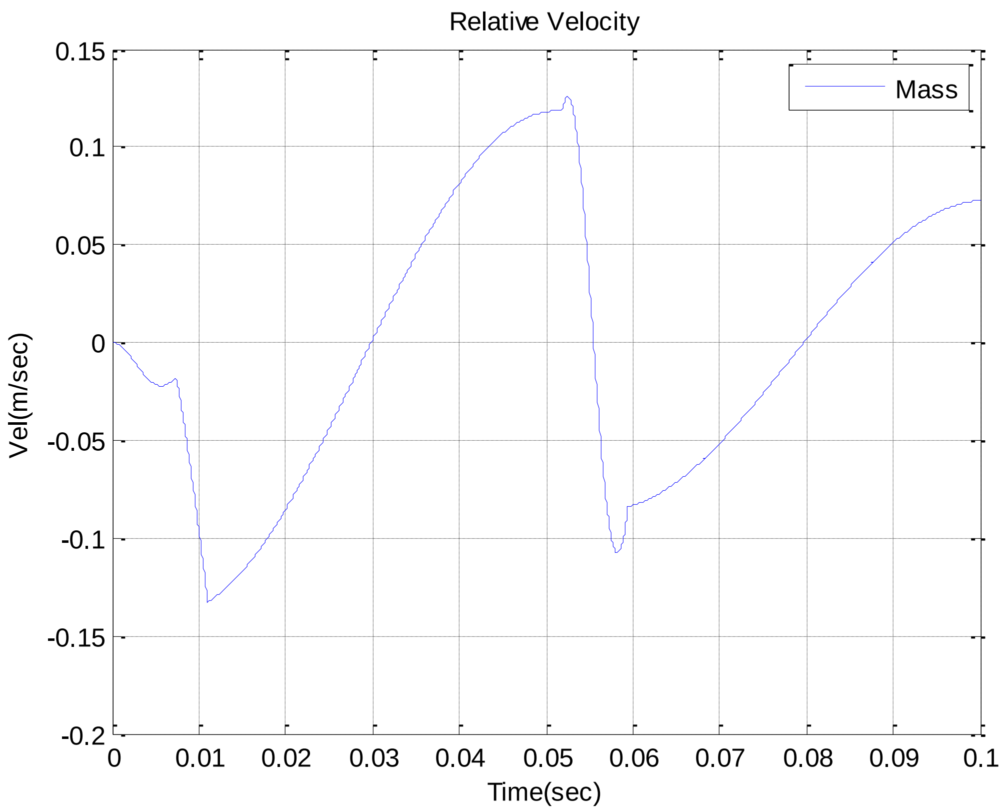
Relative Velocity
Mass
Vel(m/sec)
Time(sec)
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.1

Figure A-3.

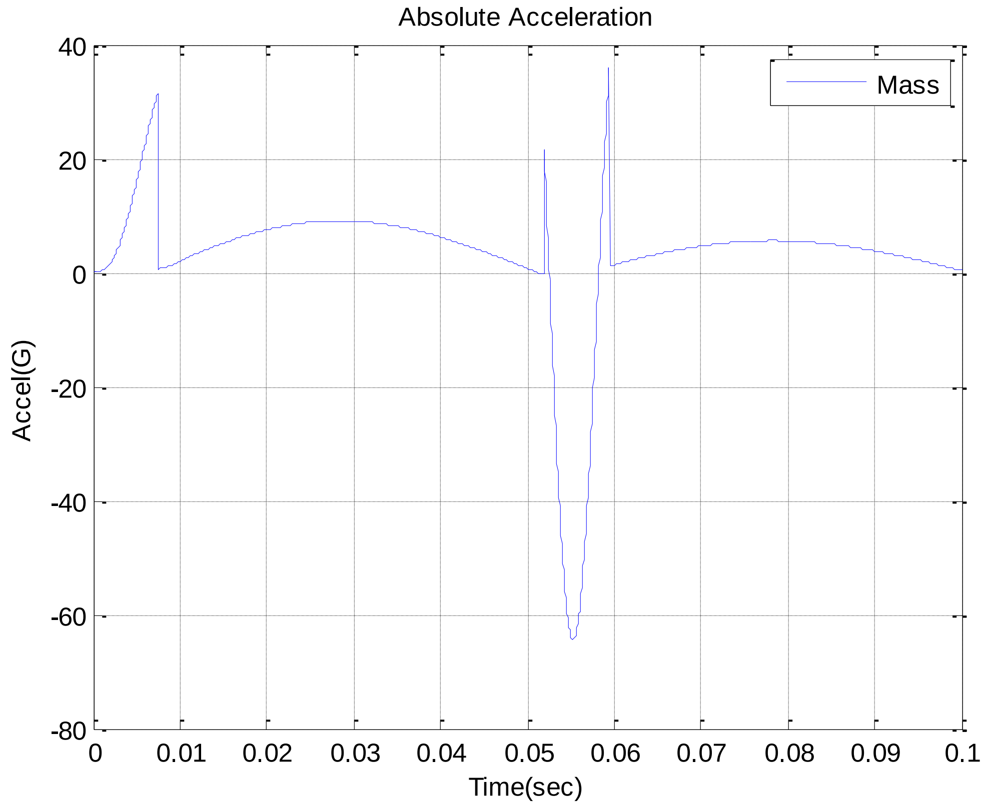

Figure A-4.

The re-contact shock pulse occurs between 0.05 and 0.06 seconds.