SPRING SURGE NATURAL FREQUENCIES

By Tom Irvine
Email: tomirvine@aol.com

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Assume that the spring is "massive."

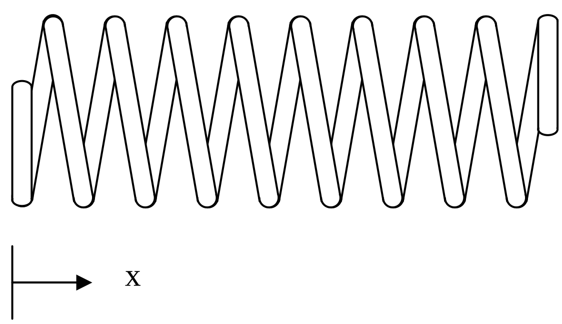
k = spring stiffness

L = undeformed spring length

m = spring's mass per unit length

M = mass mounted on the end of the spring

Free-Free

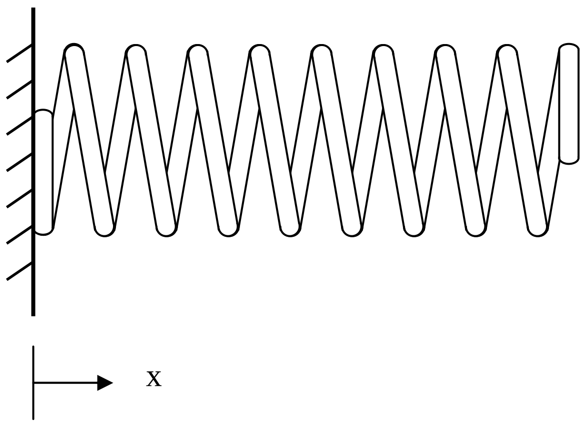


$$f_n = \frac{\lambda_n}{2\pi}\sqrt{\frac{k}{mL}} \quad , \quad \lambda_n = n\pi , \quad n = 1, 2, 3\ldots$$

Mode Shape

$$\cos\left(n\pi x / L\right) , \quad n = 1, 2, 3,\ldots$$

Fixed-Free

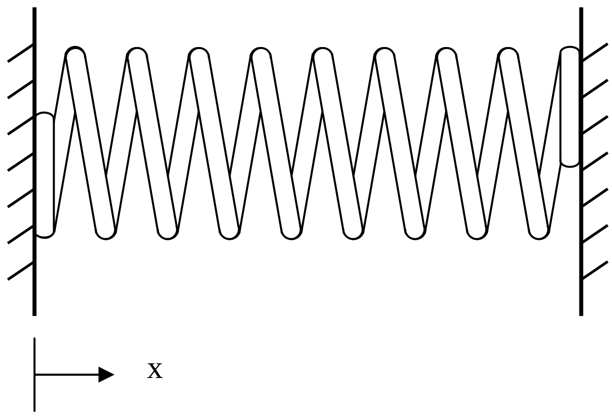


$$f_n = \frac{\lambda_n}{2\pi}\sqrt{\frac{k}{mL}} \quad , \quad \lambda_n = \left[\frac{2n-1}{2}\right]\pi, \quad n = 1, 2, 3\ldots$$

Mode Shape

$$\sin\left((2n-1)\frac{\pi x}{2L} \right) , \quad n = 1, 2, 3, \ldots$$

Fixed-Fixed



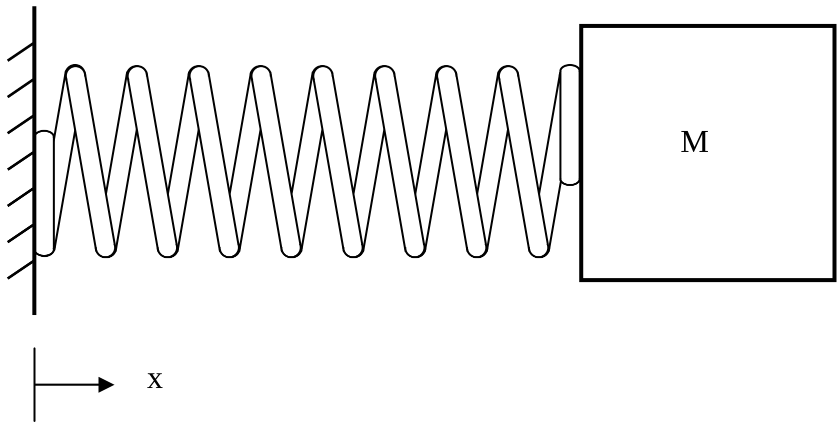
$$f_n = \frac{\lambda_n}{2\pi}\sqrt{\frac{k}{mL}} \quad , \quad \lambda_n = n\pi , \quad n = 1, 2, 3\ldots$$

Mode Shape

$$\sin\left(n\pi x / L\right) , \quad n = 1, 2, 3,\ldots$$

Note that the Fixed-Fixed and Free-Free cases have the same natural frequency equation.

Fixed-Mass

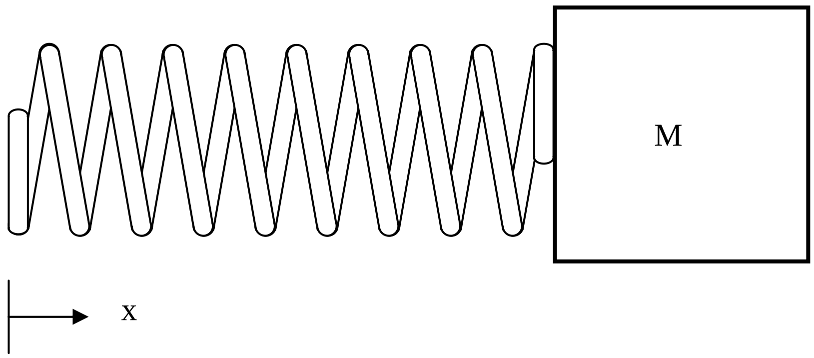


$$f_n = \frac{\lambda_n}{2\pi}\sqrt{\frac{k}{mL}} \quad , \quad \cot(\lambda_n) = \left[\frac{M}{mL}\right]\lambda_n \quad , \quad n = 1, 2, 3\ldots$$

Mode Shape

$$\sin(\lambda_n \, x / L) \; , \quad n = 1, 2, 3, \ldots$$

Free-Mass



$$f_n = \frac{\lambda_n}{2\pi}\sqrt{\frac{k}{mL}} \quad , \quad \tan(\lambda_n) = -\left[\frac{M}{mL}\right]\lambda_n \quad , \quad n = 1, 2, 3\ldots$$

Mode Shape

$$\sin(\lambda_n \, x / L) \ , \quad n = 1, 2, 3, \ldots$$

Reference

1. R. Blevins, Formulas for Natural Frequency and Mode Shapes, R. Krieger, Malabar, Florida, 1979.