Structural-Acoustics Tutorial Part 1 - Fundamentals

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Overview

- Instructor
- Structural Vibrations
  - Modes in beams and plates
  - Mobility
- Sound radiated by structural waves
  - Radiation efficiency
- Structural waves generated by impinging sound
  - Transmission loss
- Brief introduction to numerical methods
  - FE, BE, SEA

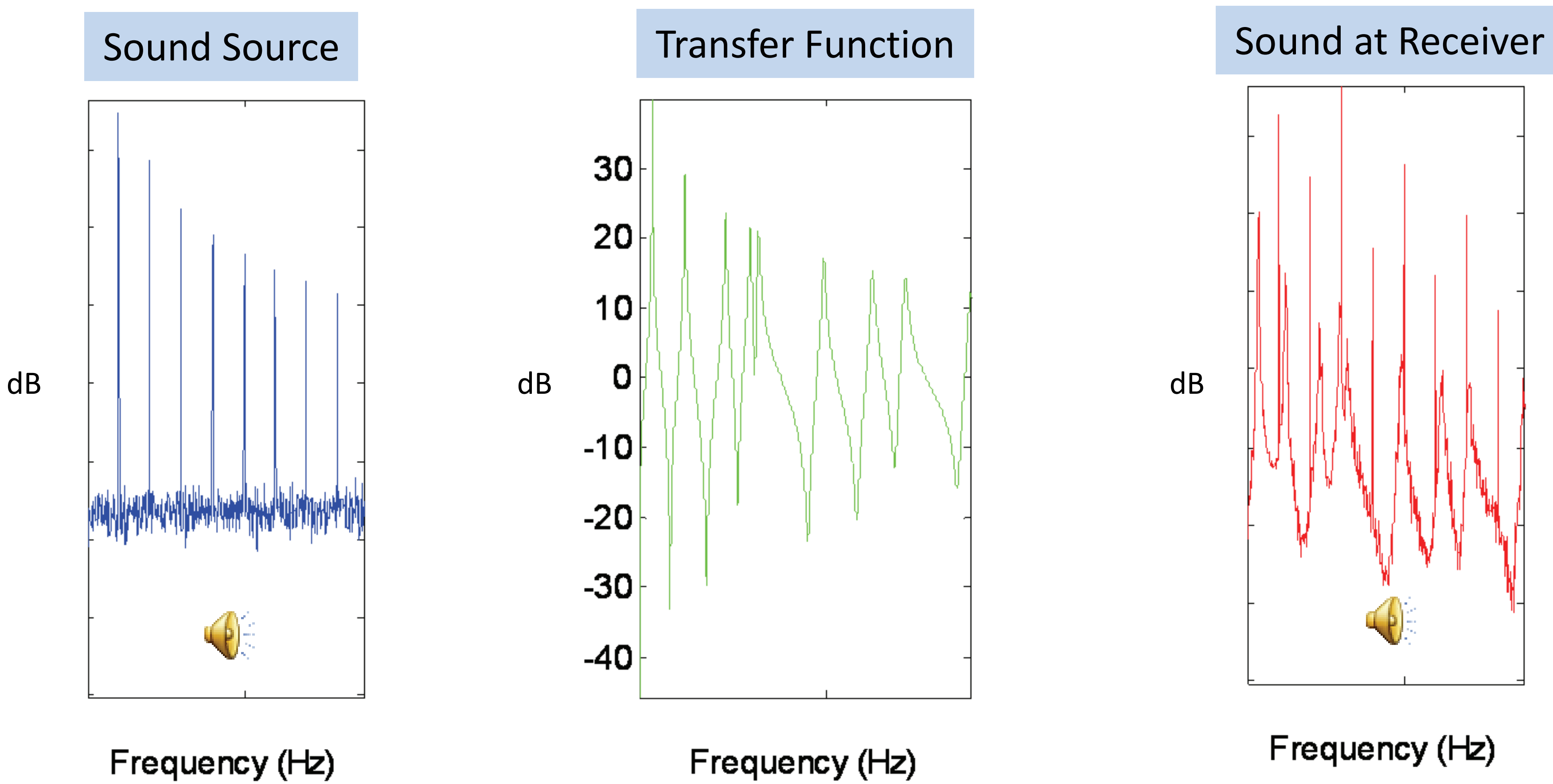
Instructor

- Dr. Stephen A. Hambric
  - Applied Research Lab, Penn State University
  - Also Professor in Penn States' Graduate Program in Acoustics
  - Associate Director, Penn State Center for Acoustics and Vibration (CAV)

- Accompanying materials:
  - Structural Acoustics Tutorials, Parts 1 and 2, from *Acoustics Today* magazine
  - Download at www.hambricacoustics.com

Motivation

- Structures can amplify (and attenuate) sound sources substantially



Structural Acoustic Transfer Functions

- Define transfer function:

$$\frac{P_{rad}}{\overline{F}^2}$$

P_{rad} ← Sound power

$\overline{F}^2$ ← Applied force²

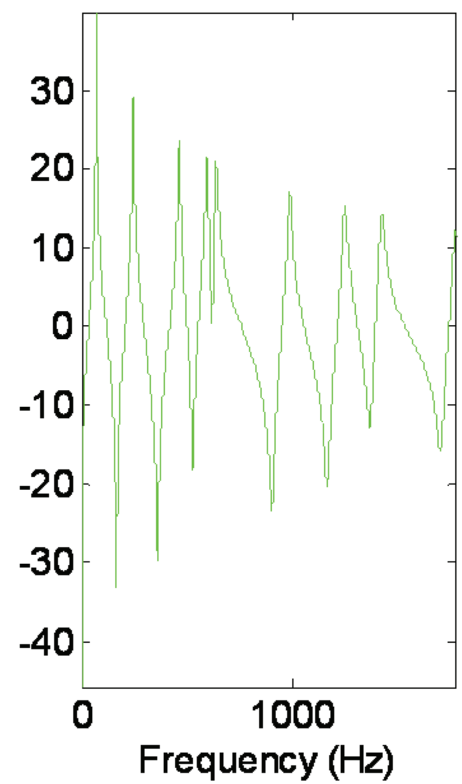
- How is it defined?

$$\frac{P_{rad}}{\overline{F}^2} = \frac{\left\langle \left|\overline{v}\right|^2 \right\rangle}{\overline{F}^2} \frac{P_{rad}}{\rho_o c_o A \left\langle \left|\overline{v}\right|^2 \right\rangle} \rho_o c_o A$$

$\frac{\left\langle \left|\overline{v}\right|^2 \right\rangle}{\overline{F}^2}$: Surface averaged mobility

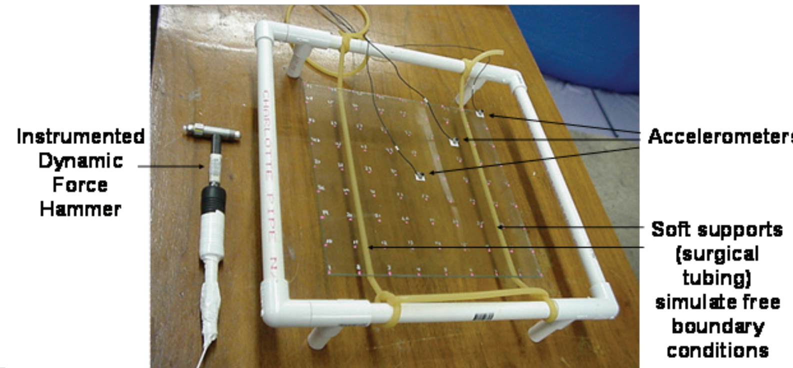
$\frac{P_{rad}}{\rho_o c_o A \left\langle \left|\overline{v}\right|^2 \right\rangle}$: Sound power radiation efficiency

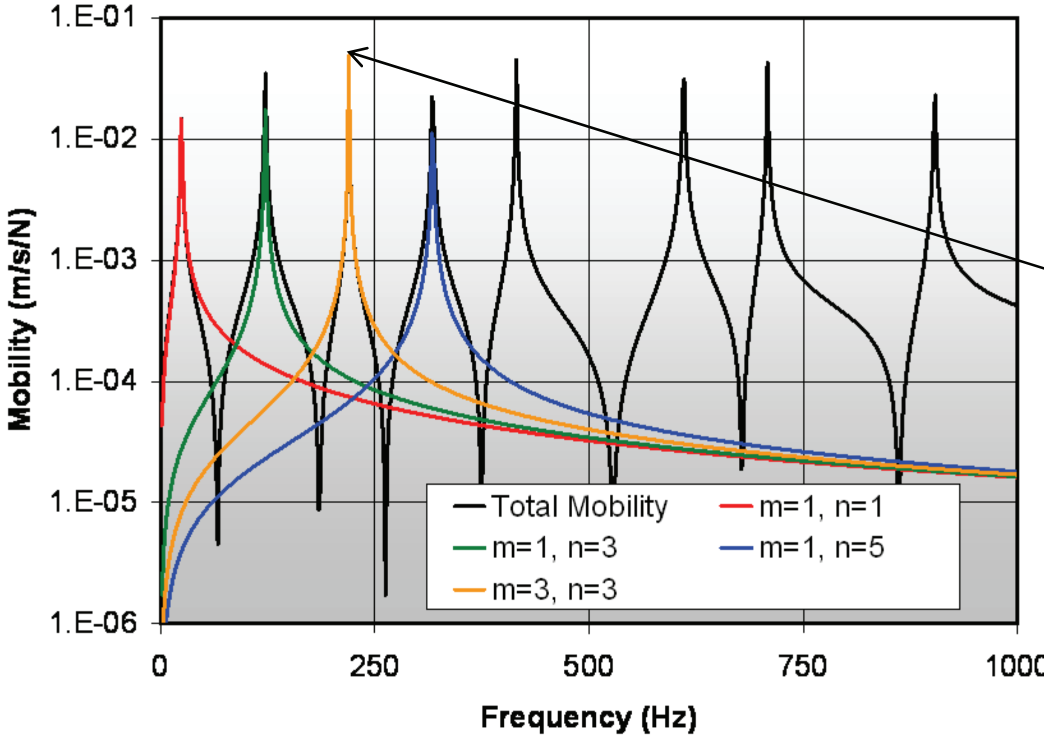
$\rho_o c_o A$: Fluid impedance



Surface Averaged Mobility $\frac{\langle |\bar{v}|^2 \rangle}{\bar{F}^2}$

- Averaged surface vibration amplitudes caused by known forces (mobilities)

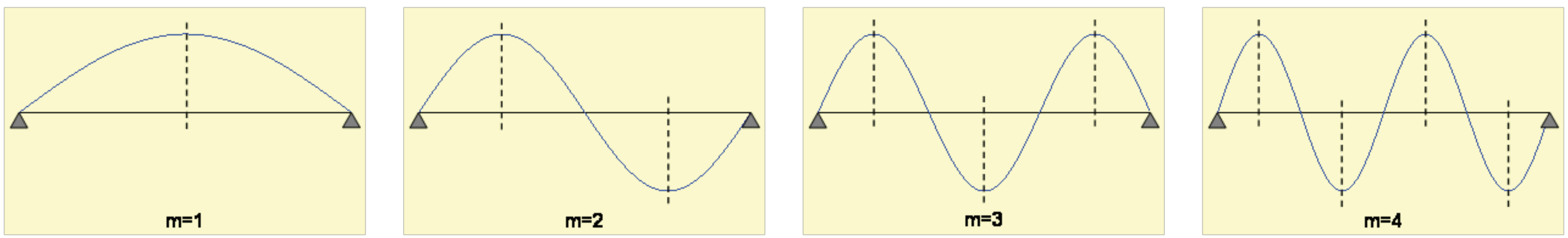




Peaks in the mobility function are caused by <u>modes of resonance</u>

Modes of Resonance

- Superposition of forward and backward traveling waves
- Example for flexure of simply supported beam:



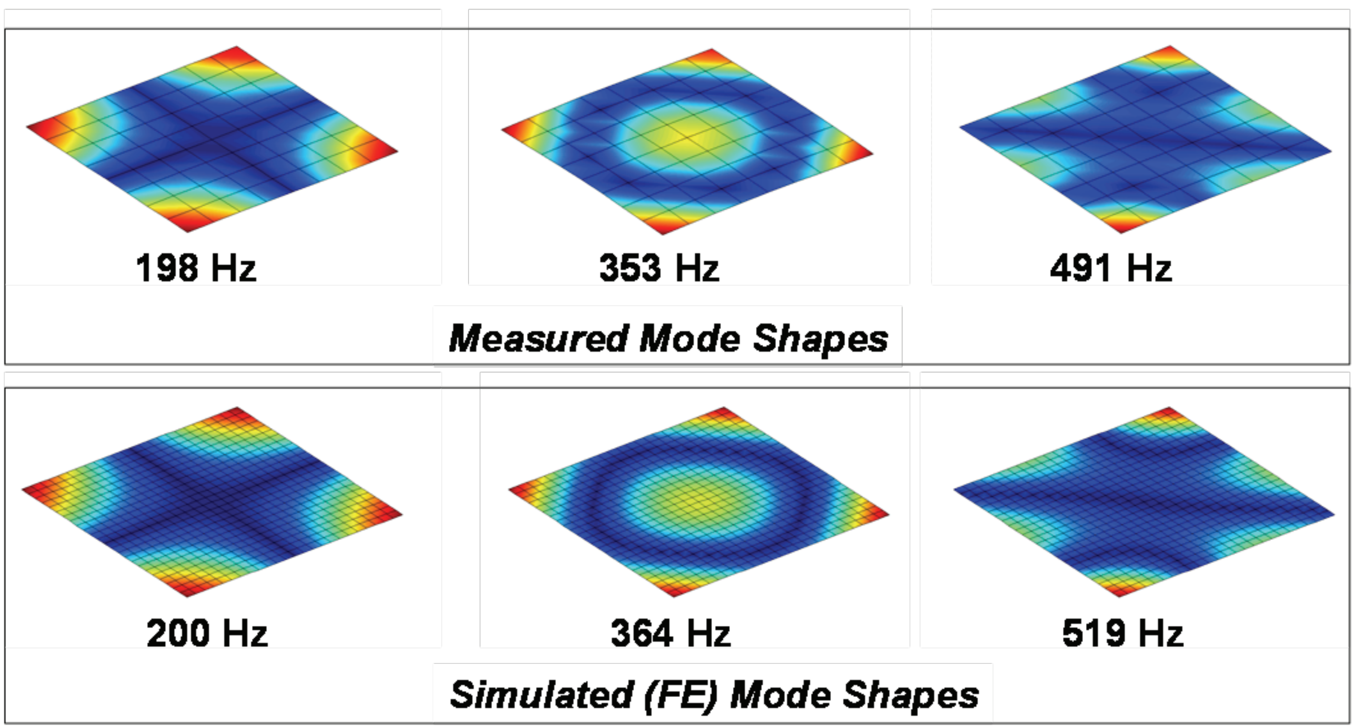
$$\omega_m = k_m^2 \sqrt{\frac{EI}{\rho A}} = \frac{m^2 \pi^2}{a^2} \sqrt{\frac{EI}{\rho A}}$$

- For other boundary conditions, wavenumbers change, e.g., for free boundaries:

$$\omega_m \cong \frac{\pi^2 (2m-1)^2}{4a^2} \sqrt{\frac{EI}{\rho A}}$$

Modes of Resonance

- Can also be computed with Finite Element (FE) models or measured
  - Measured and simulated resonance frequencies seldom match exactly



Glass plates with free boundary conditions

Mobility as a Summation of Modes

- A structure's mobility function is a simple series summation of modal responses to a driving force
  - Example: simply supported rectangular plate

$$\frac{v(x,y)}{F(x_o,y_o)} = \frac{i\omega}{\left(\frac{\rho h a b}{4}\right)} \times \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{1}{\omega_{mn}^2 - \omega^2}\left[\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}\right]\times\left[\sin\frac{m\pi x_0}{a}\sin\frac{n\pi y_0}{b}\right]\right)$$

Modal mass (fraction of static mass)

Difference between drive and resonance frequencies

Mode shape at <u>response</u> location

Mode shape at <u>drive</u> location

Modal Response Peaks

- Why isn't modal response infinite at resonance frequencies?

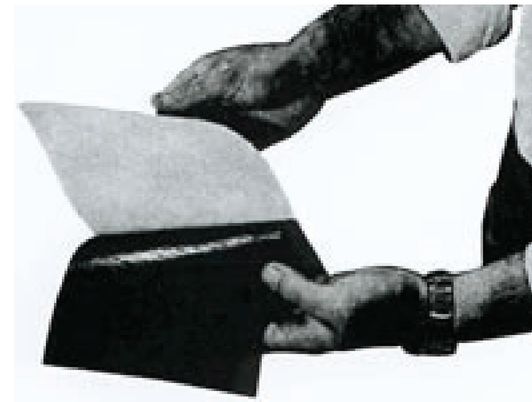
$$\frac{v(x,y)}{F(x_o,y_o)} \propto \frac{1}{\omega_{mn}^2 - \omega^2}$$

- Because ω_{mn} is complex, due to ***damping***
  – For lightly damped systems:

$$\omega_{mn} \cong \tilde{\omega}_{mn}\left(1 + i\frac{\eta}{2}\right)$$

where η is the ***loss factor***

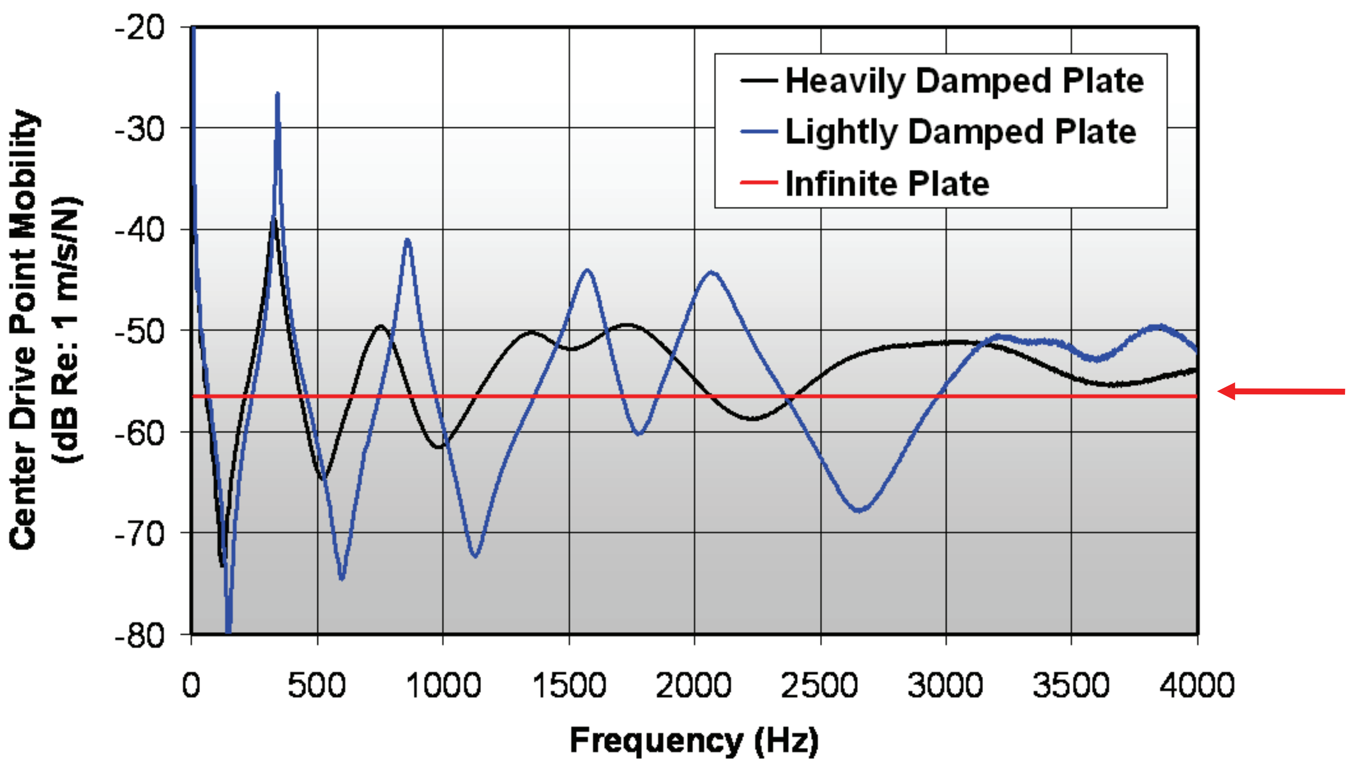
Damping Mechanisms



- Thin sheets of rubber adhered to the surface, or sandwiched between structures
- Adjoining structures
  - Coupling losses
  - Joint losses (friction)
- Sound radiation

Effects of Damping on Modal Response Peaks

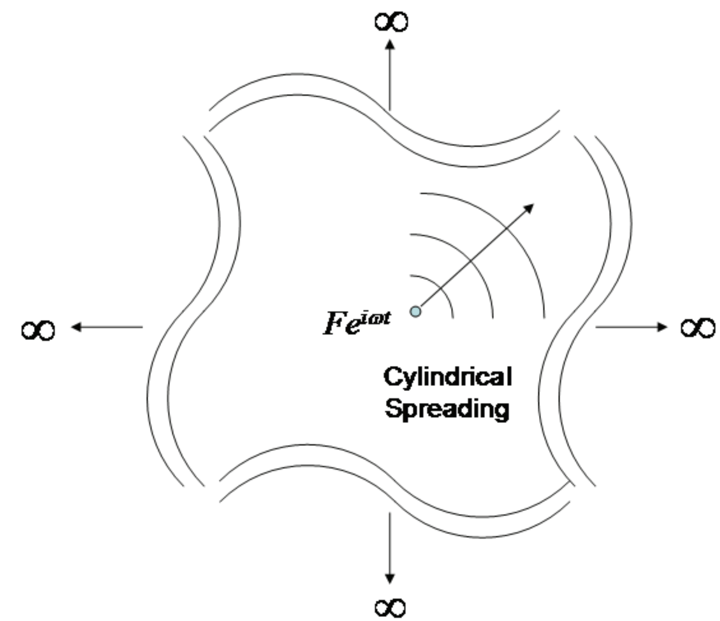
- Peak (and ‘anti-peak’) amplitudes decrease with increasing damping
  - Mobility approaches the <u>mean levels</u> of an infinite plate



How can mobility of an infinite plate be computed?

Infinite Structure Mobilities

- A structure is effectively infinite when waves reflected from boundaries are very weak with respect to the original waves emanating from a source

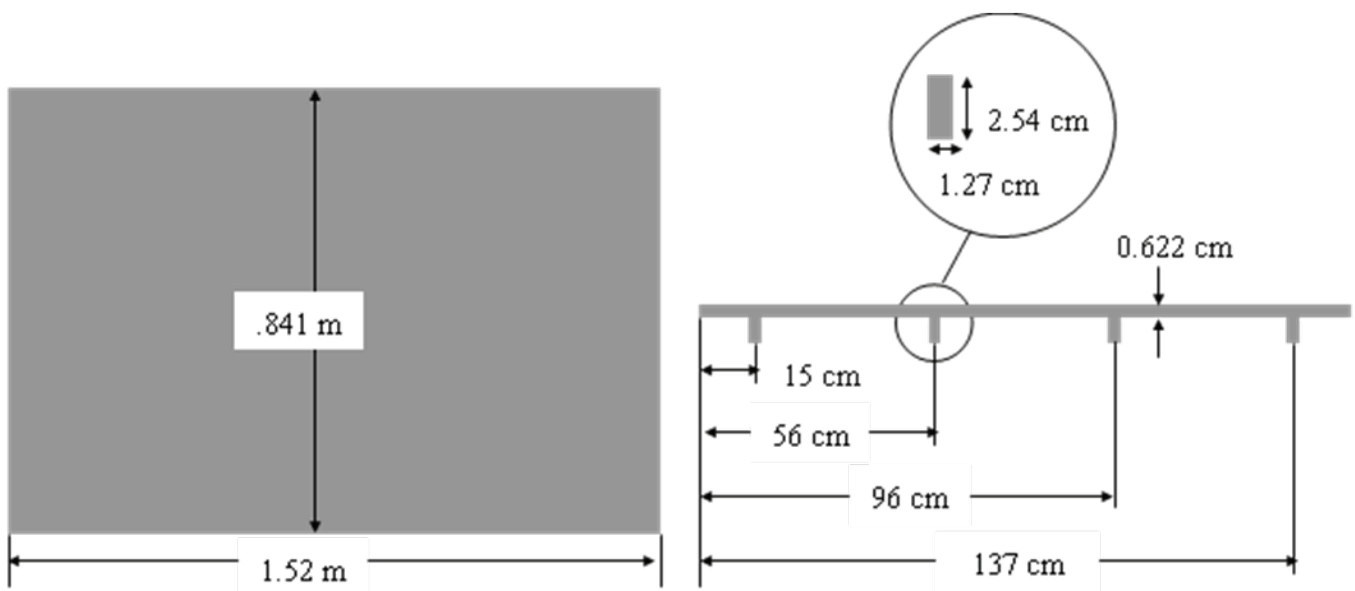


- For plates: $Y_{\text{inf}} = \left(v / F\right)_{\text{inf}} = \dfrac{1}{8\sqrt{D\rho h}}$

- Useful for:
  - scaling mobilities
  - performing engineering estimates of proposed material changes
  - Checking mobility measurement accuracy

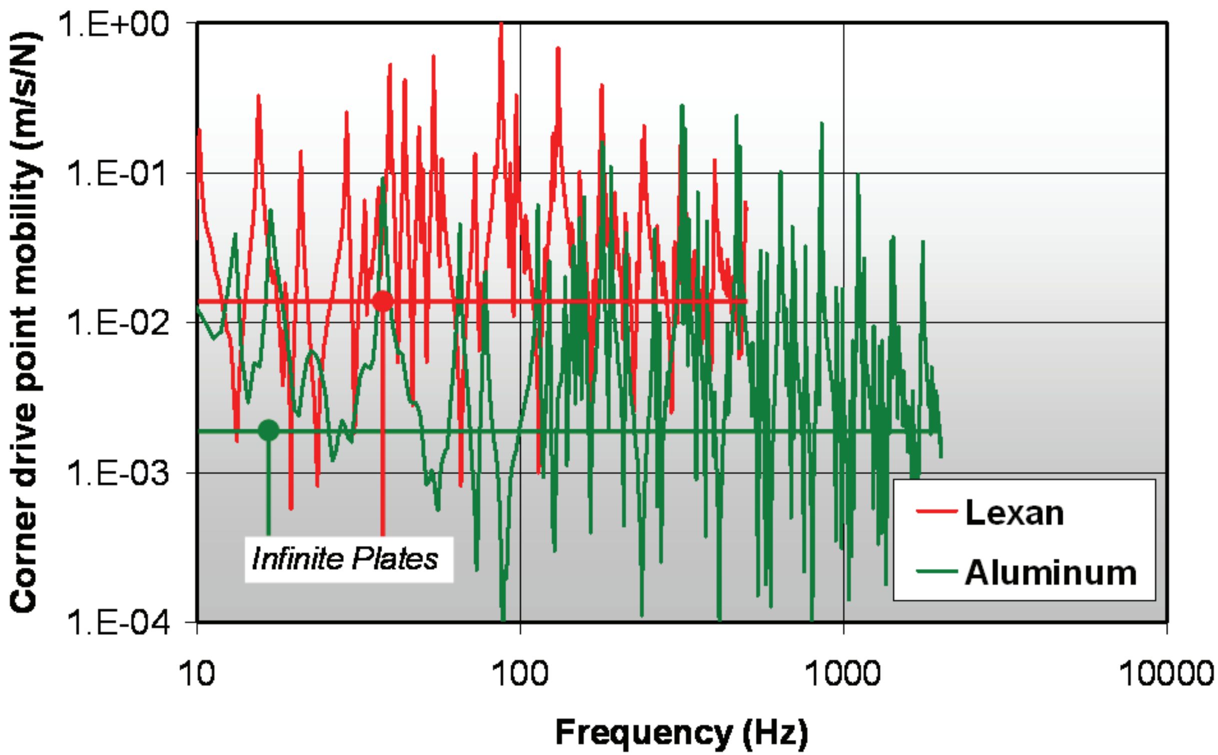
Infinite Structure Mobility Scaling Example

- Two panels with identical geometries and different materials
  - Lexan (plastic) and Aluminum



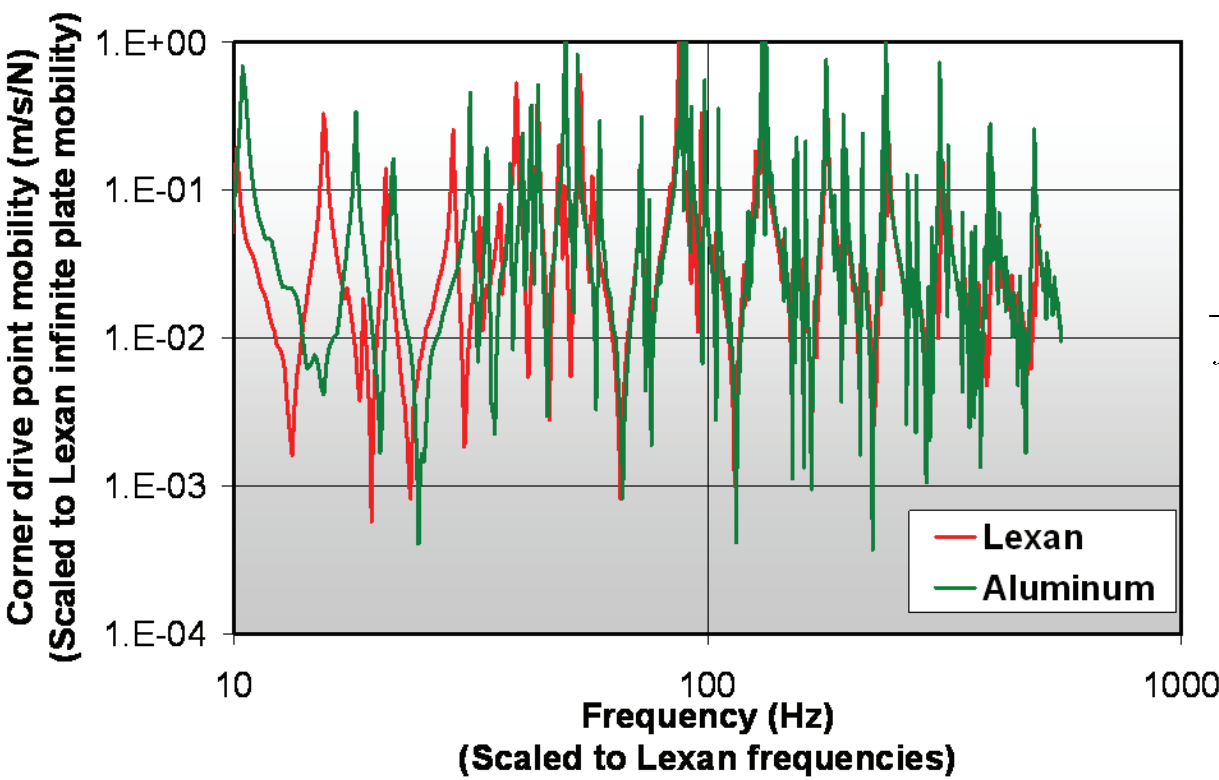
Infinite Structure Mobility Scaling Example

- Measured mobilities of both panels



Infinite Structure Mobility Scaling Example

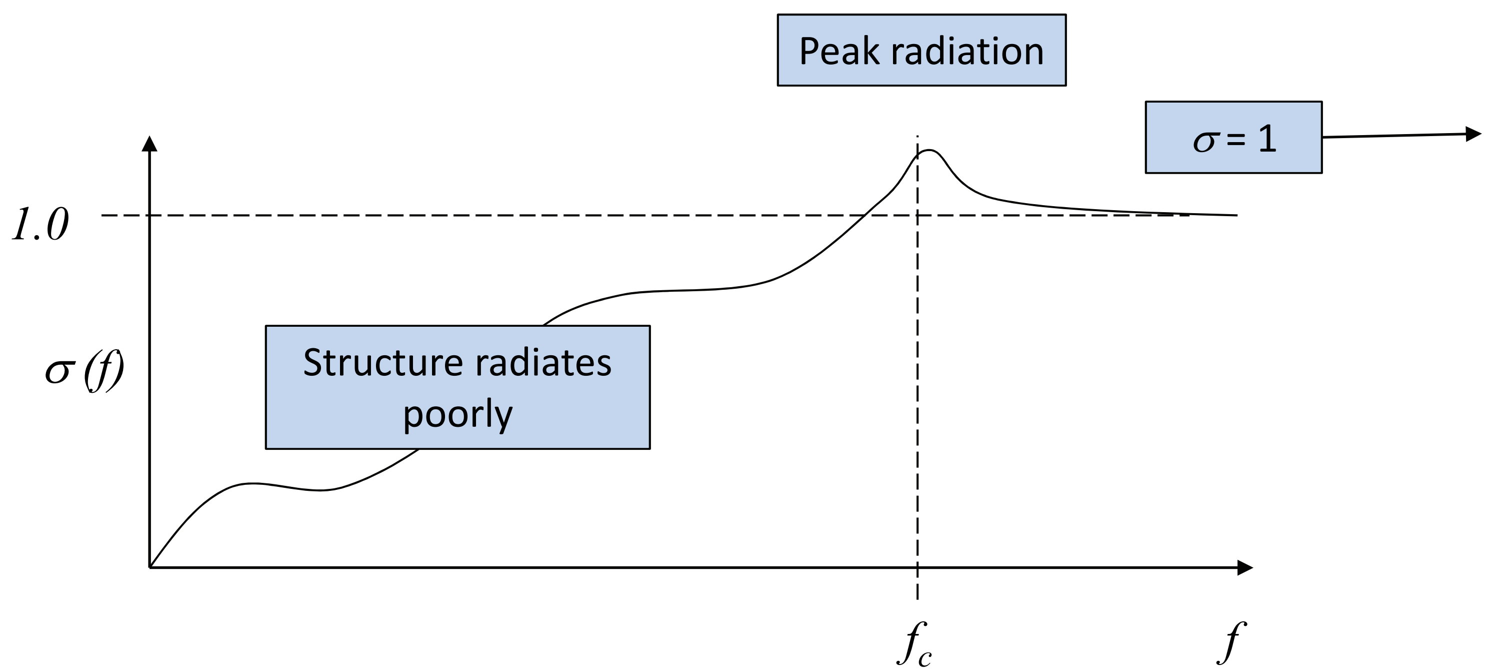
- Aluminum panel mobility scaled to that of Lexan



$$\frac{f_{Al}}{f_{Lexan}} = \frac{c_{Al}}{c_{Lexan}} = \frac{\sqrt{E_{Al} / \rho_{Al}}}{\sqrt{E_{Lexan} / \rho_{Lexan}}}$$

Radiation Efficiency

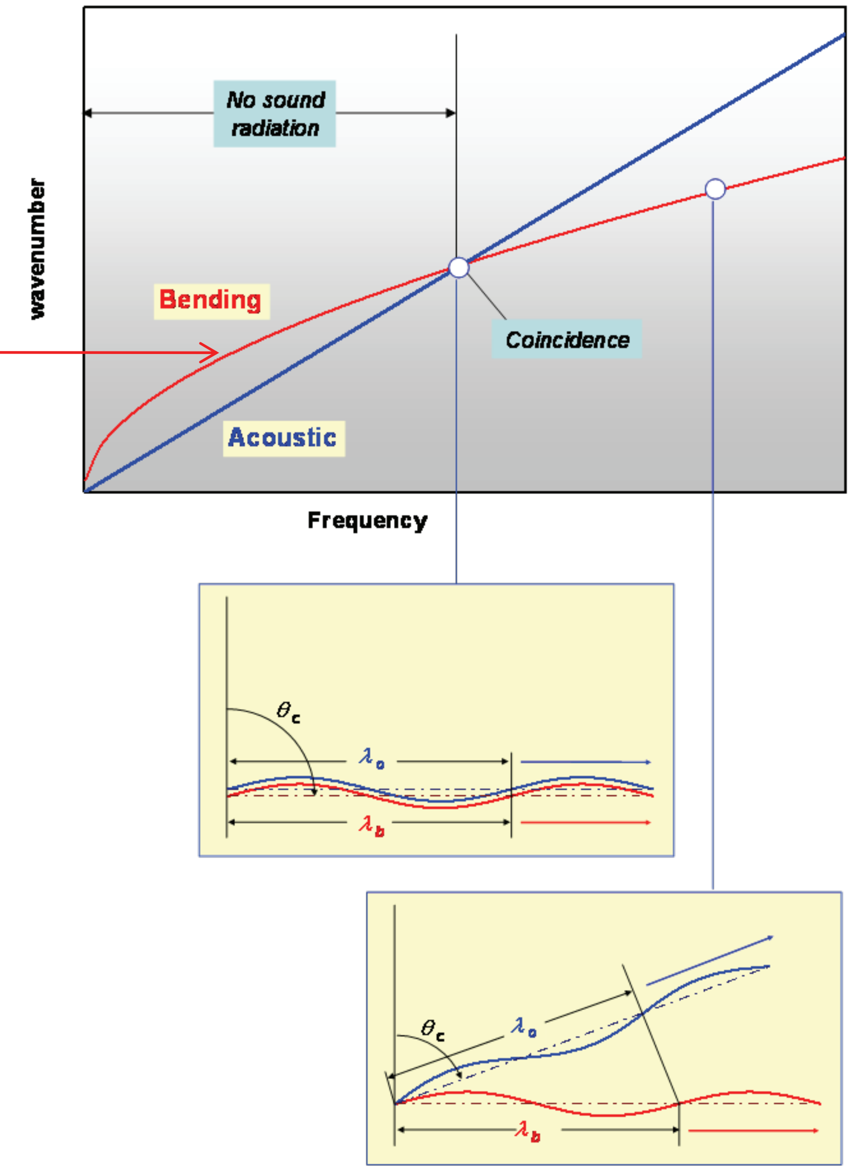
$$\frac{P_{rad}}{\rho_o c_o A \left\langle \left| \overline{v} \right|^2 \right\rangle}$$



Critical Frequency, where bending and acoustic wavespeeds match

Coincidence

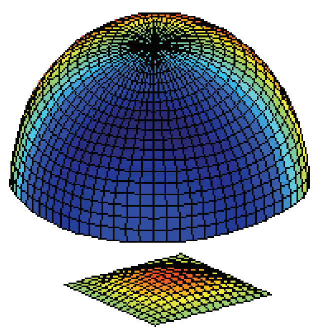
Bending waves are <u>dispersive</u>, slower than acoustic waves at low frequencies (subsonic), and faster at high frequencies (supersonic)



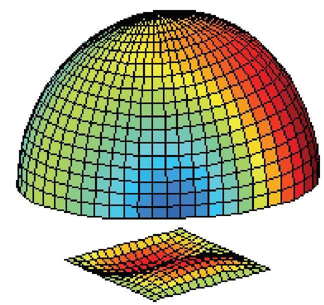
Radiating Flat Baffled Panel Below Coincidence

Pressures

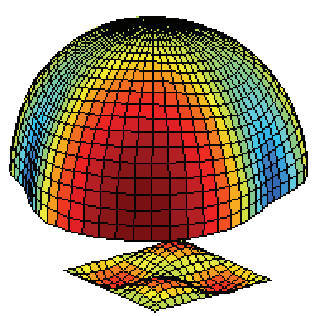
$m = 1, n = 1, k_o/k_{mn} = 0.638$

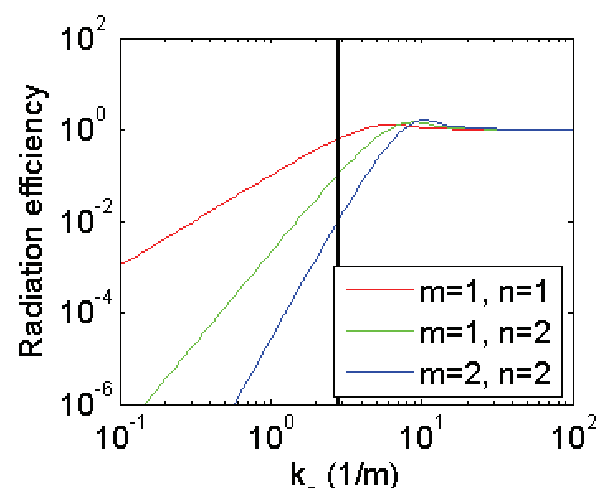


$m = 1, n = 2, k_o/k_{mn} = 0.404$



$m = 2, n = 2, k_o/k_{mn} = 0.319$



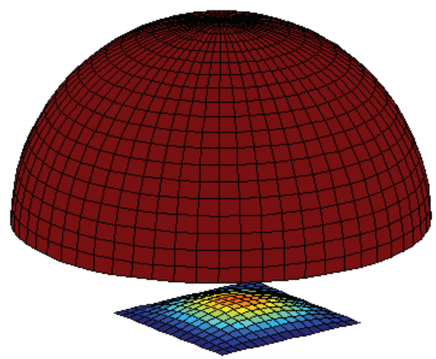


Odd/Odd >
Odd/Even >
Even/Even

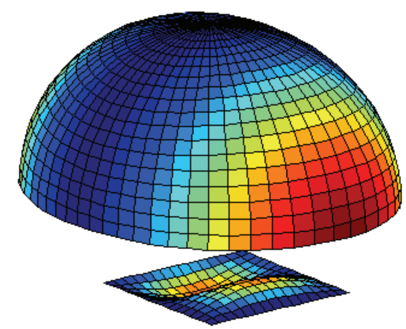
Radiating Flat Baffled Panel – Increasing Frequency

Intensities (proportional to square of pressure)

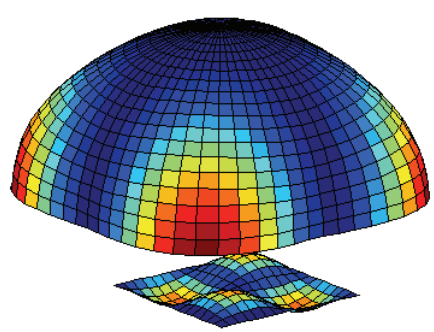
$m = 1, n = 1, k_o/k_{mn} = 0.090$

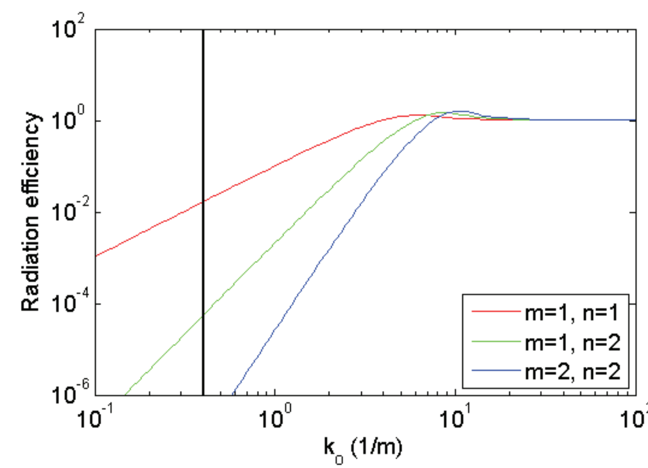


$m = 1, n = 2, k_o/k_{mn} = 0.057$

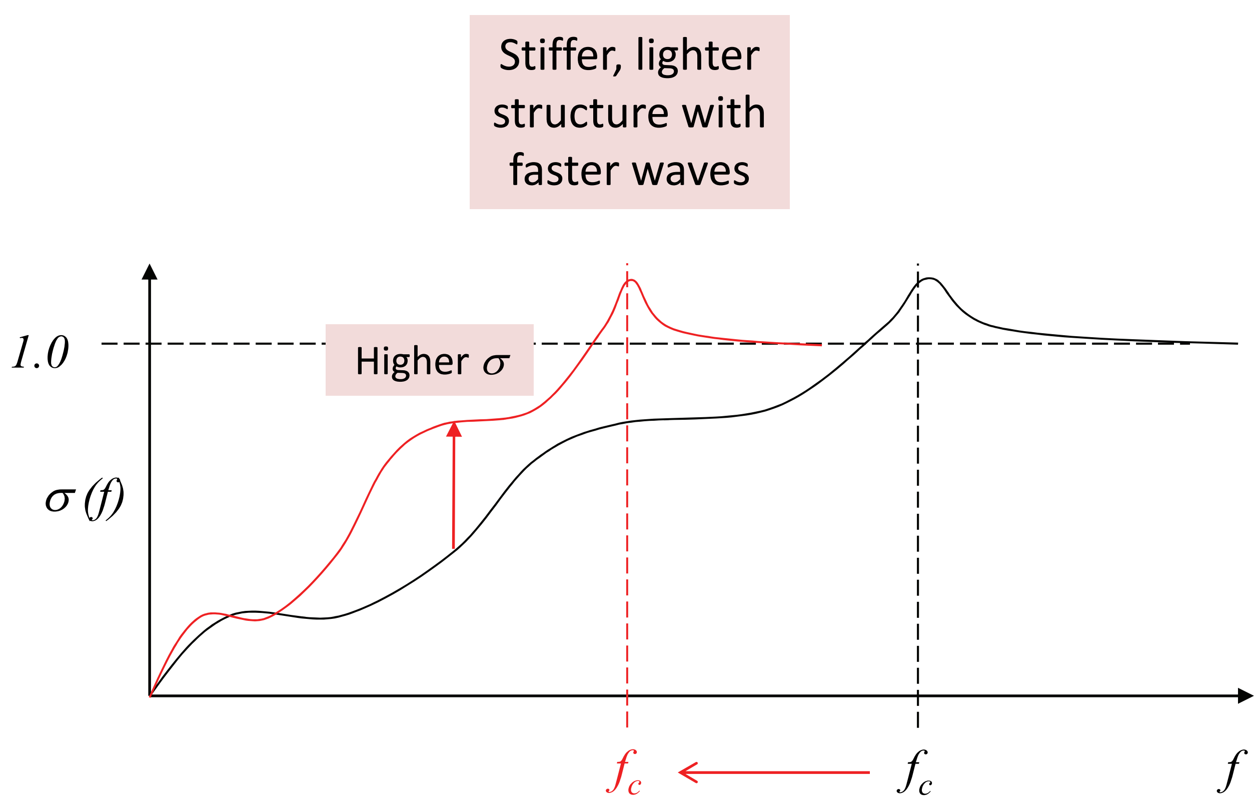


$m = 2, n = 2, k_o/k_{mn} = 0.045$

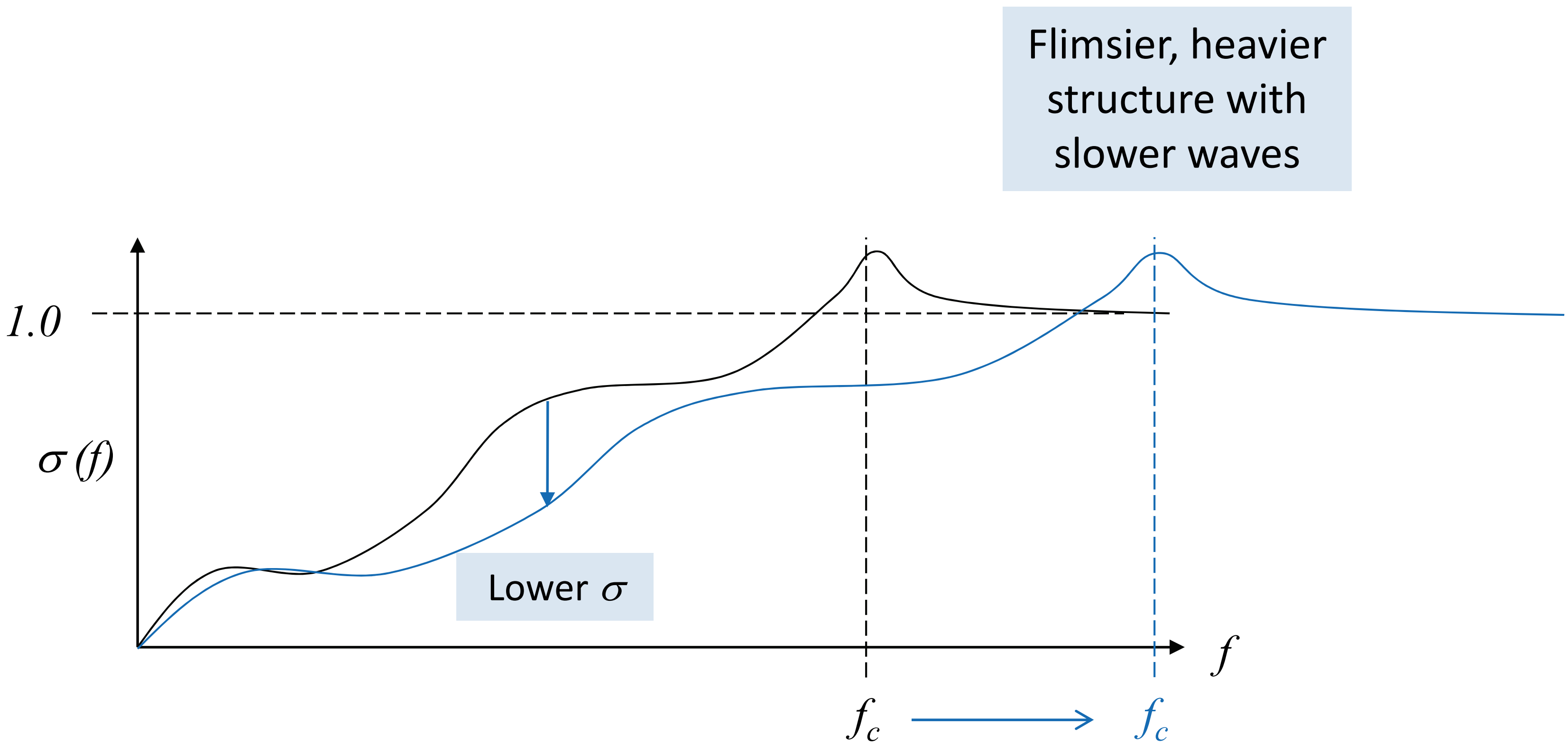




Radiation Efficiency – Effects of Coincidence Frequency Shifts

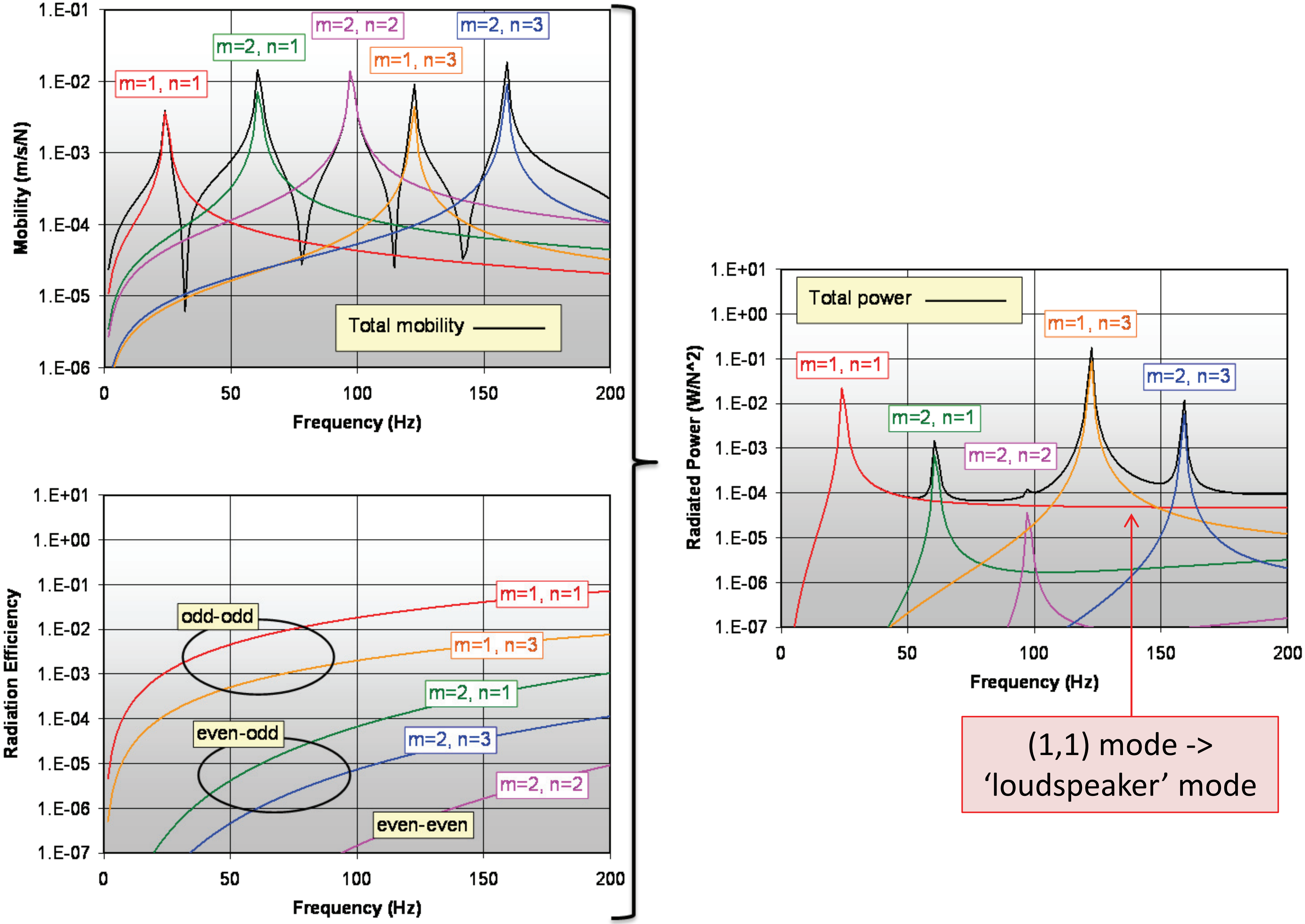


Radiation Efficiency – Effects of Coincidence Frequency Shifts

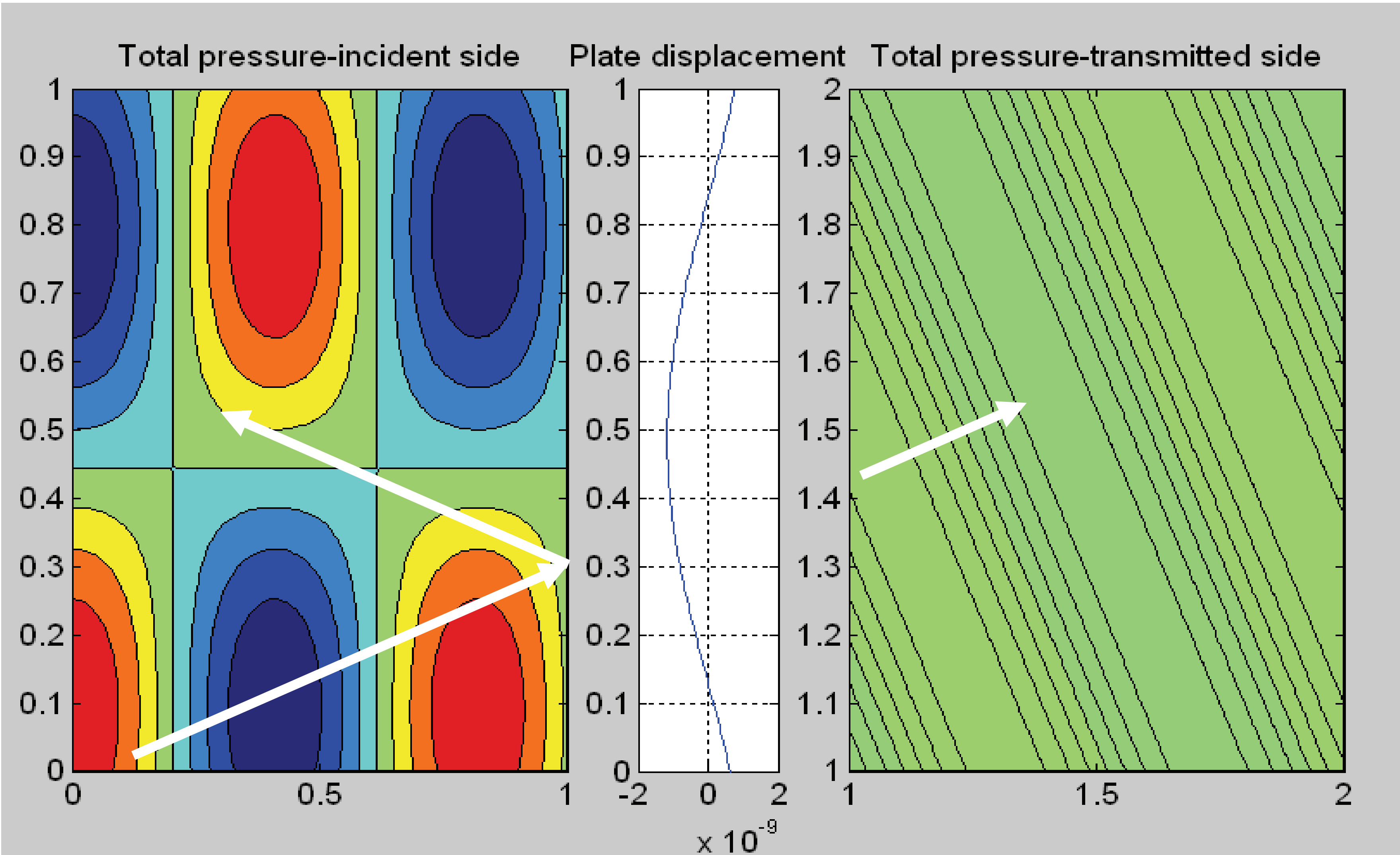


Sound Power Transfer Functions

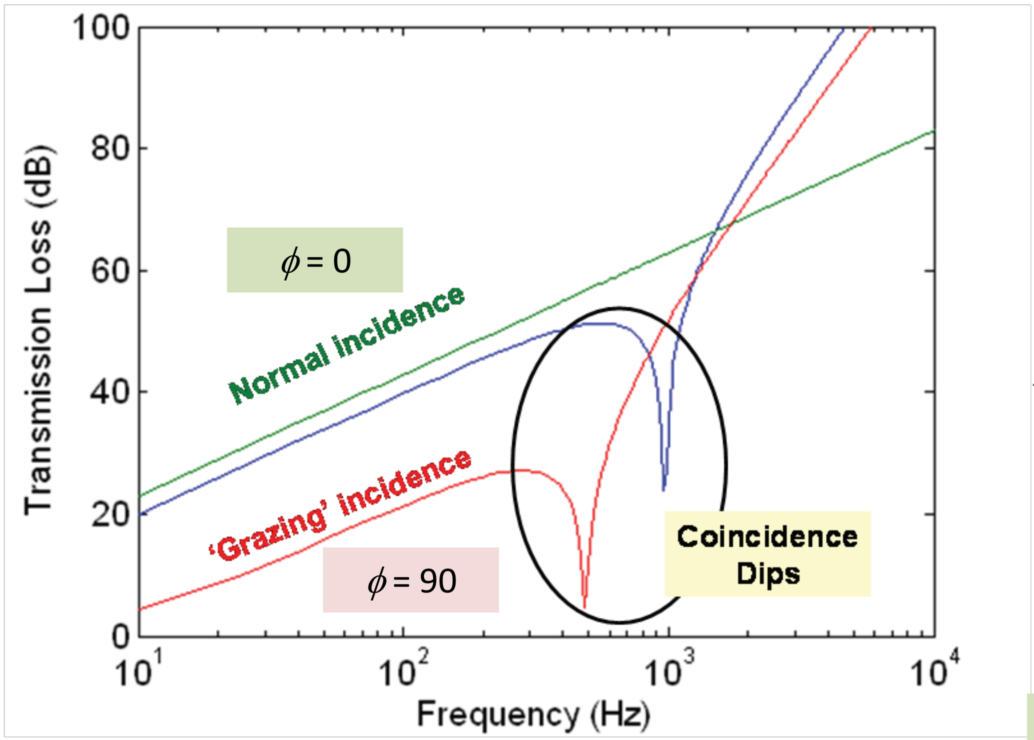
- Combining surface averaged mobility with radiation efficiency tells us how well a structure radiates sound when driven by a known force


Mobility (m/s/N)
1.E-01
1.E-02
1.E-03
1.E-04
1.E-05
1.E-06
0
50
100
150
200
Frequency (Hz)
m=1, n=1
m=2, n=1
m=2, n=2
m=1, n=3
m=2, n=3
Total mobility
Radiation Efficiency
1.E+01
1.E+00
1.E-01
1.E-02
1.E-03
1.E-04
1.E-05
1.E-06
1.E-07
0
50
100
150
200
Frequency (Hz)
odd-odd
even-odd
even-even
m=1, n=1
m=1, n=3
m=2, n=1
m=2, n=3
m=2, n=2
Radiated Power (W/N^2)
1.E+01
1.E+00
1.E-01
1.E-02
1.E-03
1.E-04
1.E-05
1.E-06
1.E-07
0
50
100
150
200
Frequency (Hz)
Total power
m=1, n=1
m=2, n=1
m=2, n=2
m=1, n=3
m=2, n=3
(1,1) mode ->
'loudspeaker' mode

Acoustic Waves Impinging on Structures



Sound Transmission Loss of an Infinite Panel



Dips occur when mass and stiffness terms cancel

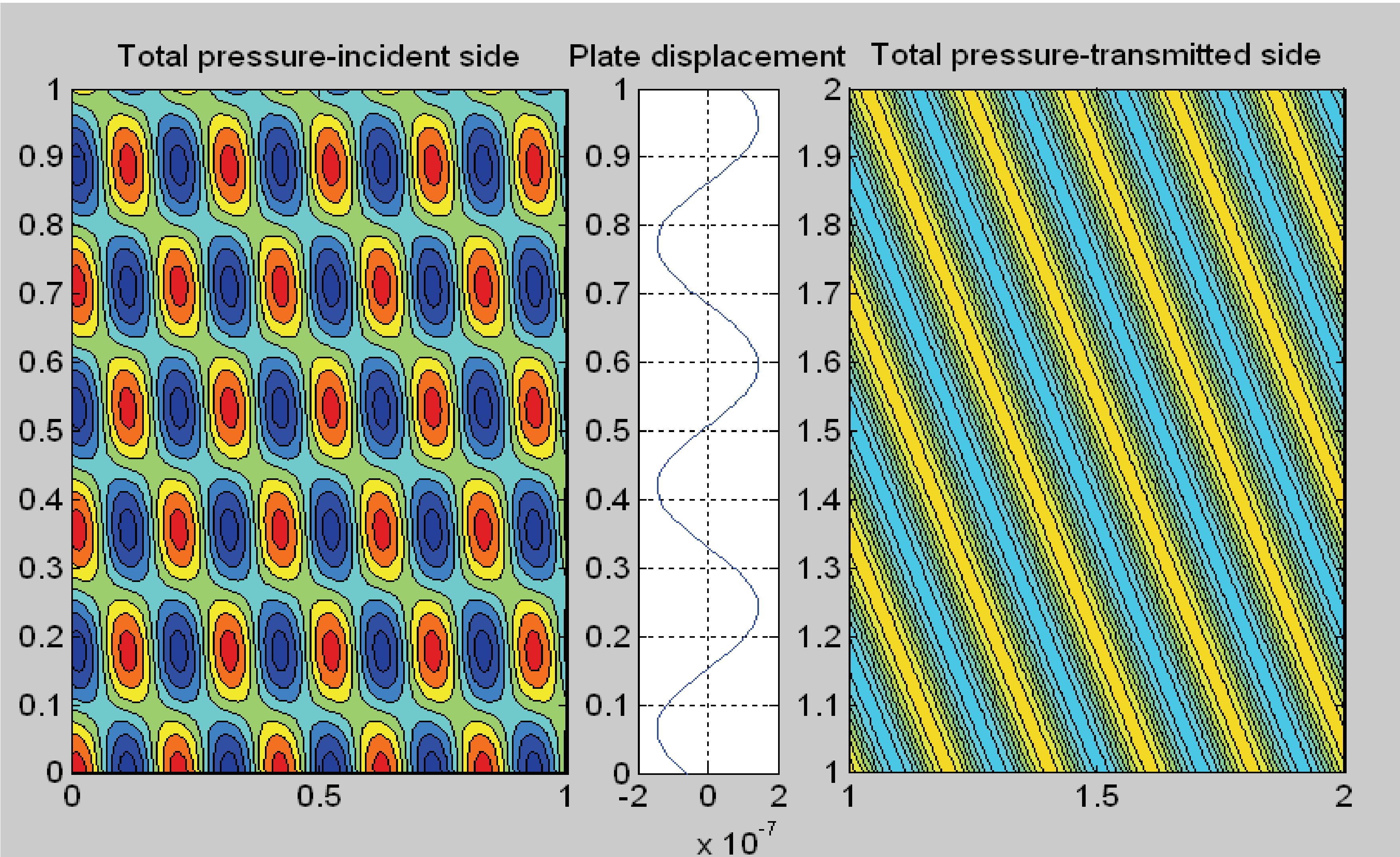
Damping important at coincidence

$$\tau = \frac{\left[2\rho_0 c_0/\sin\phi\right]^2}{\left[2\rho_0 c_0/\sin\phi + (D/\omega)\eta(k_0 \sin\phi)^4\right]^2 + \left[\omega\rho h - (D/\omega)(k_0\sin\phi)^4\right]^2}$$

Mass important below coincidence (mass law)

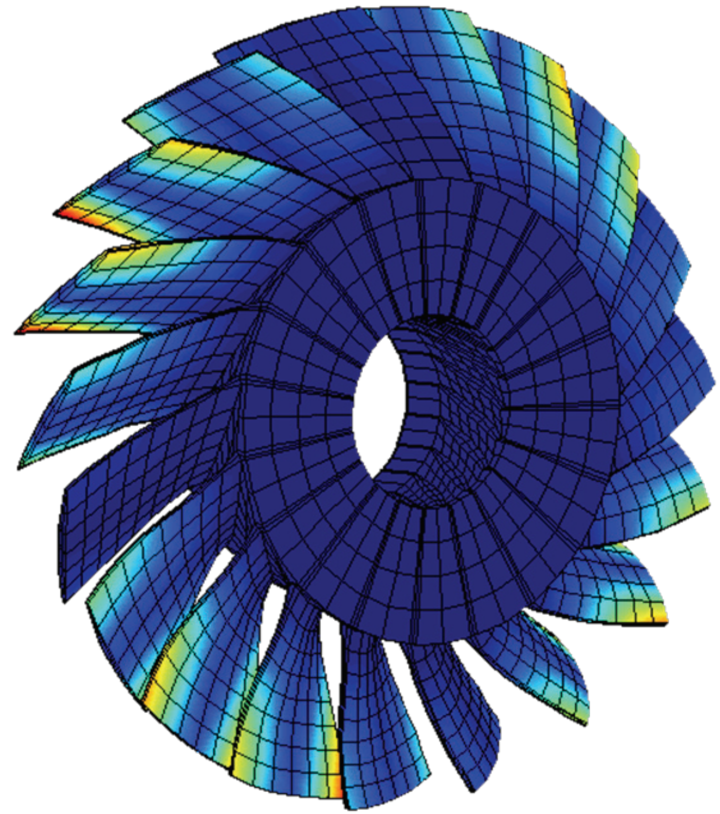
Stiffness important above coincidence

Sound Transmission near Coincidence



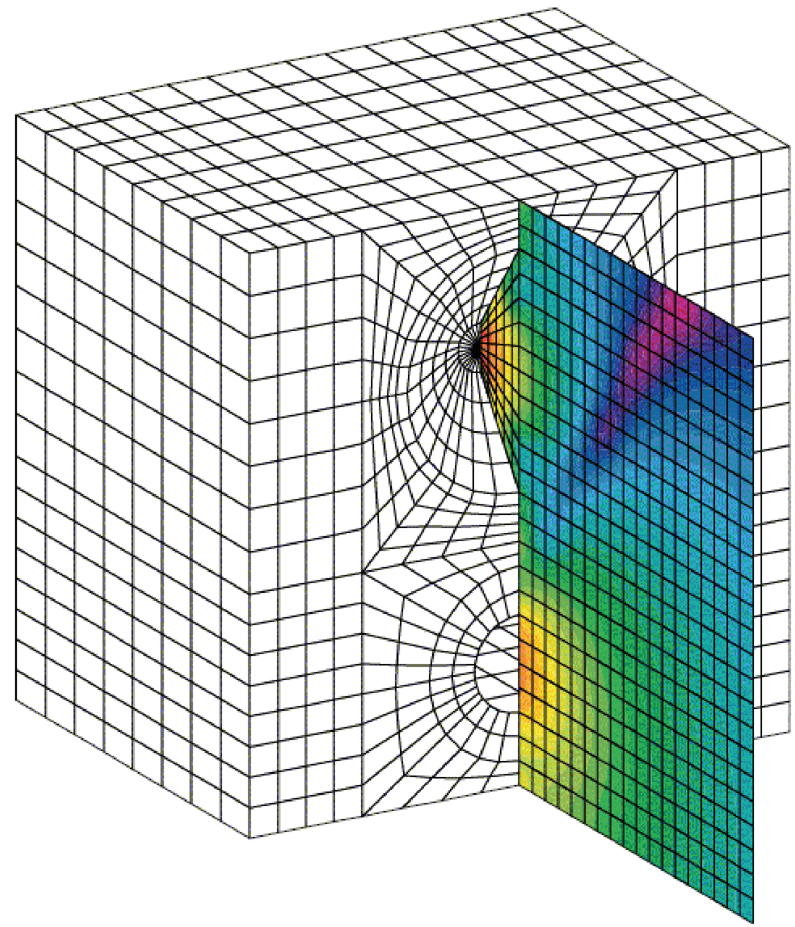
Finite Element (FE) Analysis

- Used generally to model structures
  - Plates, beams, and solids
- Also sometimes used to model acoustic regions, usually inside an object
- Many commercial software packages available



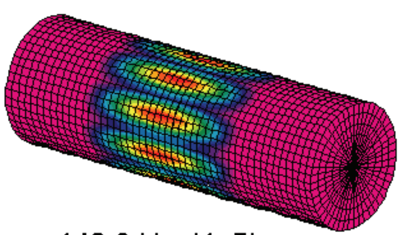
Boundary Element (BE) Analysis

- Used to model the *boundaries* of acoustic regions
  - Inside or outside a vibrating surface
- Once surface pressures and velocities are known, the acoustic field anywhere may be computed
- Some commercial software packages are available (not as many as FE packages)

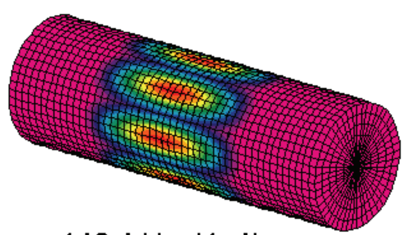


Coupled FE/BE Analyses

- Impedance matrices of structures (from FE) and acoustic regions (from BE) may be coupled
  - Enforce continuity of normal fluctuating velocity along boundary



143.8 Hz (1, 5)



146.4 Hz (1, 4)

		$f_{fluid} / f_{in\ vacuo}$						
Analysis Method	**NA**	**(1,5)**	**(1,4)**	**(1,6)**	**(1,3)**	**(1,7)**	**(2,6)**	**(1,2)**
Analytical	-	0.489	0.456	0.520	0.421	0.547	0.527	0.391
FE/BE	320	0.605	0.548	0.653	0.486	0.684	0.663	0.427
	720	0.545	0.498	0.589	0.448	0.629	0.601	0.401
	1440	0.491	0.455	0.523	0.418	0.553	0.538	0.382

Statistical Energy Analysis (SEA)

- SEA models the energy exchange between large groups of resonances in interconnected structures and acoustic regions
  - Usually used at high frequencies, where mode counts are very high
  - Exchange of energy is modeled statistically
    - Get calculations averaged over large regions, and over wide frequency (usually one-third octave) bands
  - Fast computations, independent of increasing frequency
  - Va-One software