NATURAL FREQUENCIES OF FLUID SYSTEMS

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Cylinder

Consider a solid cylinder with a specific gravity less than that of water. The cylinder is depressed slightly and then released. Calculate the natural frequency of oscillation of the cylinder if it remains upright.

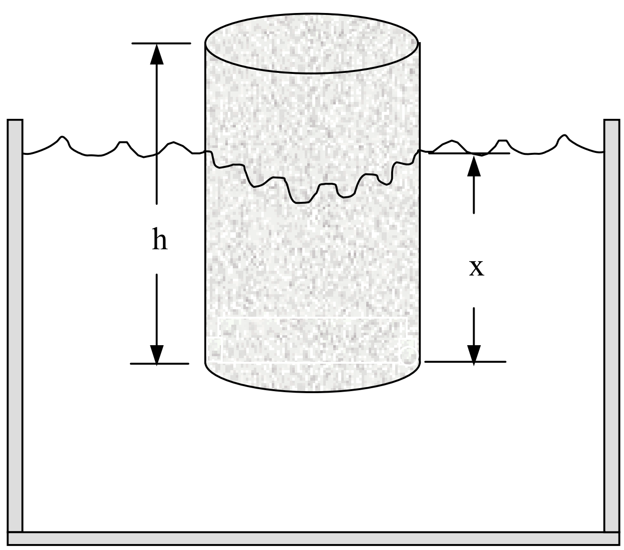

Figure 1-1.

The weight of the displaced water W is

$$W = A \rho g x \tag{1-1}$$

where

- A is the cross sectional area of the cylinder parallel to the water surface
- ρ is the mass density of water
- g is the acceleration of gravity
- x is the initial displacement, relative to the equilibrium position

The weight of the displaced water is the restoring force.

Let m be the mass of the cylinder. Apply Newton's law.

$$\sum F = m\ddot{x} \tag{1-2}$$

$$m\ddot{x} = -W \tag{1-3}$$

$$m\ddot{x} + W = 0 \tag{1-4}$$

$$m\ddot{x} + A\rho g x = 0 \tag{1-5}$$

$$\ddot{x} + \left[\frac{A\rho g}{m}\right] x = 0 \tag{1-6}$$

Let ω_n be the natural frequency.

$$\ddot{x} + \omega_n^2 x = 0 \tag{1-7}$$

$$\omega_n^2 = \left[\frac{A\rho g}{m}\right] \tag{1-8}$$

$$\omega_n = \sqrt{\frac{A\rho g}{m}} \tag{1-9}$$

U-tube Manometer

Determine the natural frequency of the fluid in the manometer.

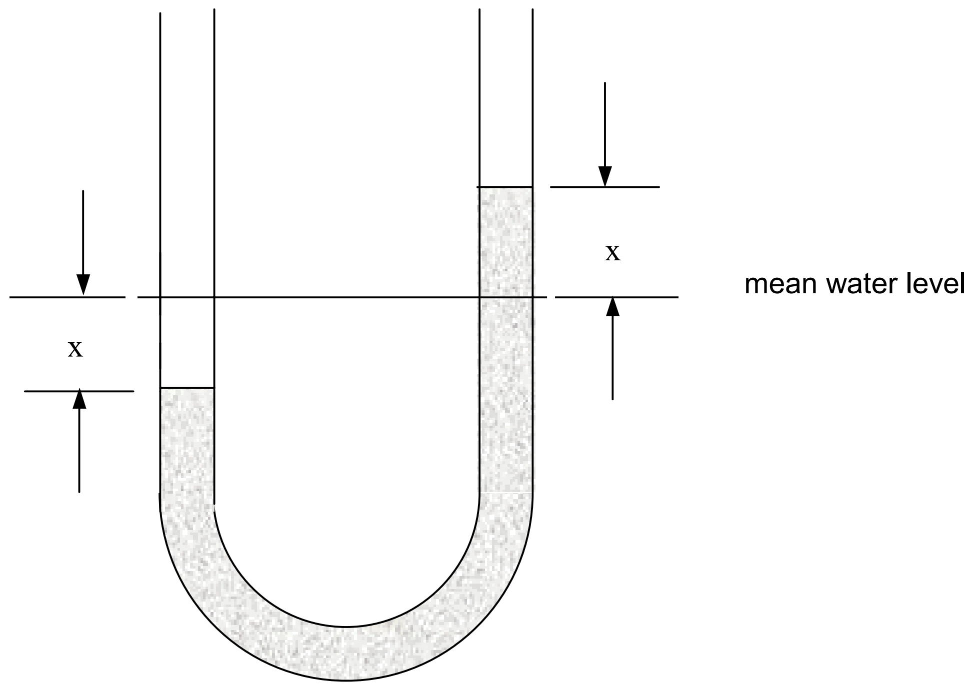

Figure 2-1.

Let L be the total length of the fluid column.
Let A be the tube cross-section area.

The potential energy PE relative to the mean water level is

$$PE = A \rho g x^2 \tag{2-1}$$

The kinetic energy KE is

$$KE = \frac{1}{2} A L \rho \dot{x}^2 \tag{2-2}$$

Apply the energy method.

$$\frac{d}{dt}\left(KE + PE\right) = 0 \tag{2-3}$$

$$\frac{d}{dt}\left[\frac{1}{2} A L \rho \dot{x}^2 + A \rho g x^2\right] = 0 \tag{2-4}$$

$$A L \rho \dot{x} \ddot{x} + 2 A \rho g x \dot{x} = 0 \tag{2-4}$$

$$L \dot{x} \ddot{x} + 2 g x \dot{x} = 0 \tag{2-6}$$

$$L \ddot{x} + 2 g x = 0 \tag{2-7}$$

$$\ddot{x} + \left[\frac{2g}{L}\right] x = 0 \tag{2-8}$$

Let ω_n be the natural frequency.

$$\omega_n^2 = \left[\frac{2g}{L}\right] \tag{2-9}$$

$$\omega_n = \sqrt{\frac{2g}{L}} \tag{2-10}$$

Density Values

The density values for water are shown in Table 1.

Table 1. Water		
Liquid	Density (kg/m^3)	Density (lbm/ft^3)
Water (fresh) at 20 °C	998	62.3
Water (sea) at 13 °C	1026	64.1

Reference

1. W. Seto, Mechanical Vibrations, McGraw-Hill, New York, 1964.