NOTES ON MECHANICAL SHOCK AND VIBRATION WAVES
Revision B

By Tom Irvine
Email: tomirvine@aol.com
August 31, 2000

---

## INTRODUCTION

A wave is the phenomenon in which physical energy propagates through space relative to a medium. This definition is taken from Reference 1.

This tutorial has two goals.

The first is to discuss characteristics of many types of waves. Particular attention is given to the spectral representation of the waveforms. The spectral format results from both convention and the laws of physics.

The second goal is to explain why mechanical shock and vibration spectra are represented in terms of frequency rather than wavelength.

The tutorial takes this dual approach for the benefit of those readers approaching mechanical shock and vibration from some other discipline. The readers can thus compare and contrast the characteristics of shock and vibration to other types of waveforms with which they are more familiar.

## HARMONIC WAVES

Harmonic motion is periodic motion.

Some harmonic waveforms are represented by a governing equation of the form

$$\frac{\partial^2 E}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} \tag{1}$$

where E is some measure of amplitude energy,
x is a spatial coordinate,
t is the time coordinate,
c is a constant.

E is also referred to as a *field variable.*

Note the following characteristics of this equation:

1. It is given as one-dimensional for simplicity.

2. Both the time and space coordinates are second order.
3. It is a natural response equation
4. An attenuation or dissipation term is omitted for simplicity.
5. It assumes no dispersion.
6. It applies to certain longitudinal and transverse waveforms.

Dispersion and attenuation are defined later in this tutorial.

Equation (1) can be found in many references, including Reference 2.

A proposed solution to equation (1) is

$$E(x,t) = A\sin(kx - \omega t - \phi) \tag{2a}$$

where A is the amplitude
k is the wave number,
ω is the angular frequency,
ϕ is the phase angle.

Note that equation (2a) is a simple traveling-wave solution. A standing-wave solution would require a second sinusoidal term with an opposite polarity frequency. The proposed standing-wave solution would be

$$E(x,t) = A\sin(kx - \omega t - \phi) + A\sin(kx + \omega t - \phi) \tag{2b}$$

For simplicity, this tutorial deals mainly with traveling waves.

Taking derivatives of the proposed traveling-wave solution

$$\frac{\partial E}{\partial x} = kA\cos(kx - \omega t - \phi) \tag{3}$$

$$\frac{\partial^2 E}{\partial x^2} = -k^2 A\sin(kx - \omega t - \phi) \tag{4}$$

$$\frac{\partial E}{\partial t} = -\omega A\cos(kx - \omega t - \phi) \tag{5}$$

$$\frac{\partial^2 E}{\partial t^2} = -\omega^2 A\sin(kx - \omega t - \phi) \tag{6}$$

By substitution,

$$-Ak^2 \sin(kx - \omega t - \phi) = \left[\frac{1}{c^2}\right]\left[-\omega^2 A \sin(kx - \omega t - \phi)\right] \tag{7}$$

The following constraint results from the substitution

$$k^2 = \frac{\omega^2}{c^2} \tag{8}$$

$$k = \frac{\omega}{c} \tag{9}$$

The proposed solution shown as equation (2a) thus satisfied the governing equation (1). A complete solution requires knowledge of initial conditions and boundary conditions. Furthermore, a Fourier solution method is required because the general solution is a series of sinusoids. In other words, the natural response is the superposition of a series of sinusoids, each with its own amplitude, frequency, and phase angle.

Note that the wave number k is related to the wavelength $\lambda$ by

$$k = \frac{2\pi}{\lambda} \tag{10}$$

The angular frequency $\omega$ is related to the period T by

$$\omega = \frac{2\pi}{T} \tag{11}$$

The angular frequency $\omega$ is related to the frequency f by

$$\omega = 2\pi f \tag{12}$$

The frequency f is related to the period T by

$$f = \frac{1}{T} \tag{13}$$

The following relation is obtained by substitution

$$c = f\,\lambda \tag{14}$$

The wavelength is the spatial period, as shown in Figure 1.

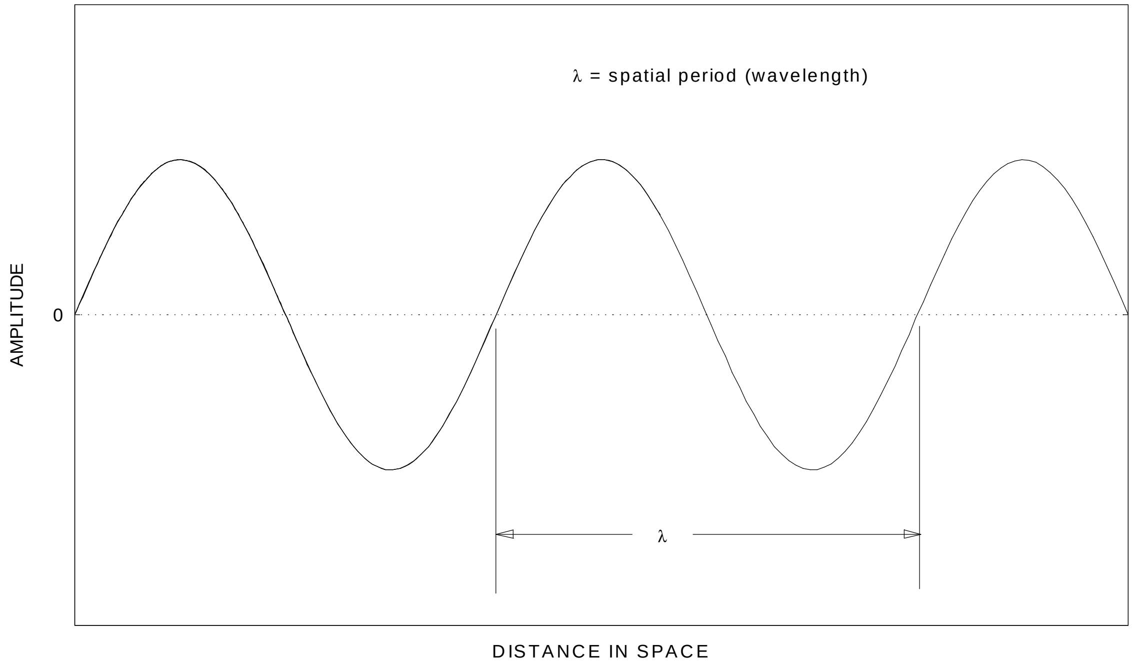



Figure 1.

The frequency is the inverse of the temporal period as shown in Figure 2.

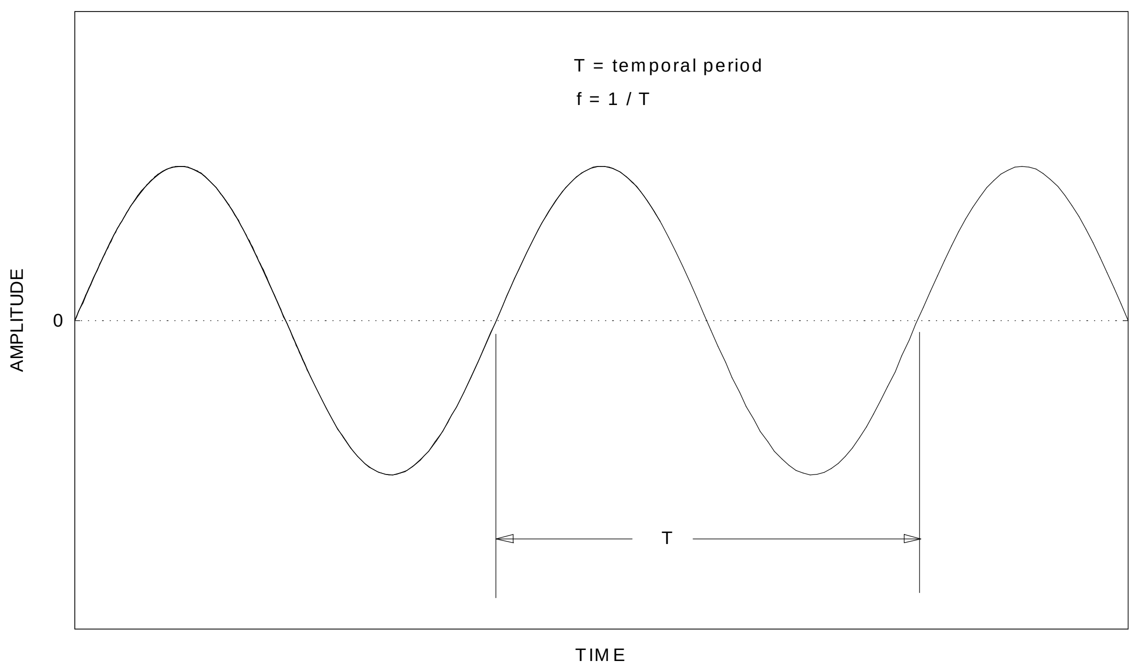



Figure 2.

## DISPERSION

The classical harmonic wave equation assumes that the wave speed is constant regardless of frequency and wavelength. Specifically, equation (14) requires the product of frequency and wavelength to be a constant. This constant is the wave speed.

The classical harmonic wave equation is an approximation of physical reality. Waves travel through media. Some media tend to be dispersive. The speed of wave in a dispersive medium varies with frequency, or with wavelength.

## ATTENUATION

Attenuation is a loss of energy due to air resistance, viscosity, friction, or some other dissipation mechanism. Attenuation causes the amplitude to decrease. Attenuation is not considered in this tutorial.

## ELECTROMAGNETIC WAVES

### Basic Description

Electromagnetic waves propagating in a vacuum are governed by an equation of the form of equation (1).

Electromagnetic waves include X-rays, visible light, radio waves, and other types.

Electromagnetic waves are transverse waves because the electric and magnetic fields are perpendicular to the direction of propagation.

Note that electromagnetic waves do not require a physical medium for conduction. They can travel through the vacuum of interplanetary and interstellar space.

Electromagnetic waves propagate at the speed of light. The speed of light in a vacuum is denoted by c. There is a simple relationship between the frequency f and wavelength $\lambda$ of an electromagnetic wave propagating in a vacuum.

$$c = f\,\lambda \tag{15a}$$

Note that equation (15a) is the same as equation (14).

### Light Dispersion

Note that the speed of light changes as the light passes from one medium into another. This refraction causes light waves to bend. Consider a light wave traveling from a vacuum into a dense medium, such as glass or water. The velocity in the dense medium drops immediately by a factor of one over the refractive index n of the medium.

White light is composed of a spectrum of color components. White light entering a prism at a particular angle is dispersed into a rainbow of colors. Each spectral color propagates at a different wave speed.

Equation (15a) must be modified for a dispersive medium with refractive index n. The speed of light u in a dispersive medium is

$$u = c / n \tag{15b}$$

The wavelength $\hat{\lambda}$ in a dispersive medium relative to the wavelength in a vacuum $\lambda$ is given by

$$\hat{\lambda} = \lambda / n \tag{15c}$$

Thus

$$u = f \ \hat{\lambda} \tag{15d}$$

And

$$c / n = f \ (\lambda / n) \tag{15e}$$

The parameter which remains constant, regardless of medium, is the frequency f.

Consider the two monochromatic light waves shown in Table 1. The parameters are taken from Reference 3.

| Table 1. Light Wave Example | | |
|---|---|---|
| Color | Violet | Red |
| Wavelength in vacuum (Angstroms) | 4100 | 6600 |
| Wave speed in vacuum (km/sec) | 300,000 | 300,000 |
| Refractive index in crown glass | 1.5380 | 1.5200 |
| Wave speed in crown glass (km/sec) | 195,059 | 197,368 |
| Wavelength in crown glass (Angstroms) | 2666 | 4342 |

Red light thus has a higher wave speed than violet light in crown glass. Also note the wavelength inside crown glass is shifted downward.

Spectral Representation

Note that the electromagnetic spectrum is typically represented in terms of wavelength, as shown in Figure 3.

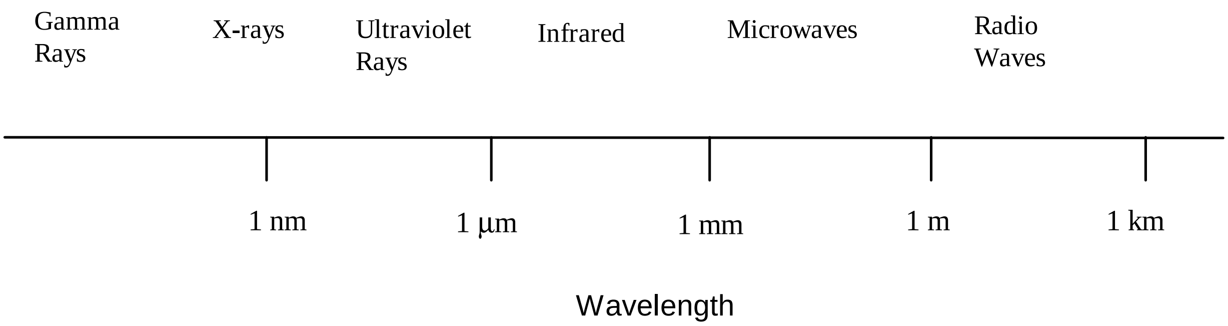



Figure 3. Electromagnetic Spectrum

Note that visible light has a narrow spectral domain from 0.38 µm to 0.76 µm.

As an alternative, the horizontal axis in Figure 3 could be represented in terms of frequency. Note that radio receiver dials are represented in terms of a frequency scale by convention. The choice of horizontal axis dimension is somewhat arbitrary because frequency and wavelength are related in a simple manner by equation (15).

Note that figure 41-1 in Reference 2 uses a double X-axis approach to represent the spectrum in terms of both frequency and wavelength.

Also note that the spectral plot in Figure 3 does not have a vertical axis. The plot is simply meant to show the wavelengths of certain radiation types.

Nevertheless, the amplitude of the electrical field can be measured in terms of potential difference with units of volts. An alternative amplitude measurement is power in units of watts.

## ACOUSTIC WAVES

### Fundamental Characteristics

Acoustic waves are similar to electromagnetic waves in the sense that both are governed by an equation of the form of equation (1).

Acoustic waves differ from electromagnetic waves because acoustic waves require a physical medium through which to propagate. Sound cannot travel in a vacuum. On the other hand, sound can travel through the air, water, Earth, metal, wood, and other physical objects.

Sound waves are longitudinal waves which alternately push and pull the material through which it propagates. Furthermore, the amplitude disturbance is parallel to the direction of propagation.

The frequency and wavelength associated with and an acoustic wave are related by the familiar formula

$$c = f\,\lambda \tag{16}$$

where the constant c is the speed of sound.

Note that sound waves in air are nearly non-dispersive even in the ultrasonic frequency range.

The speed of sound c is given by

$$c = \sqrt{\frac{B}{\rho_o}} \tag{17a}$$

where B is the bulk modulus,
$\rho_o$ is the equilibrium density.

The bulk modulus B is defined in terms of the pressure P and volume V as

$$B = \frac{\Delta P}{-\Delta V / V} \tag{17b}$$

Equations (17a) and (17b) are taken from Reference 2.

Music Example

As an example, a certain piano key is an "A" note with a fundamental frequency of 440 Hz.[1] The speed of sound in air is about 340 m/sec at sea level and at a temperature of 59 degrees F, per Reference 4. The corresponding wavelength of the fundamental frequency of the "A" note is 0.774 m.

The fundamental frequency of a piano key is determined by the piano wire length, tension, and material density. These parameters are independent of air temperature and air density.

[1] Note that this same piano key would produce harmonic tones at integer multiples of 440 Hz.

Now assume that the piano is moved to a mountain settlement at an altitude of 10,000 ft.

The speed of sound decreases to 329 m/sec at the new altitude per Reference 4. The "A" note still has a fundamental frequency of 440 Hz, however. The corresponding wavelength in air decreases by 3.5% to 0.747 m.

There is no reason to re-tune the piano on the basis of the changing wavelength in air.[2] The human ear is sensitive to the frequency rather than the wavelength. In summary, music notes are characterized by frequency rather than wavelength.

## Acoustic Power Spectra

Sound waves in air cause pressure fluctuations of the air molecules. The amplitude of sound waves in air or water is typically measured in terms of pressure units. Typically, the root-mean-square (rms) pressure is measured over some time interval. The rms pressure value is converted into a decibel form.

By convention, acoustic power spectra are represented in terms of frequency rather than wavelength. Reference 5 is an example of this convention. A figure from this reference is given in Appendix A. More precisely, the acoustic power spectra are almost always represented in terms of a proportional bandwidth, such as 1/3 octave bands. Note that these bands are *frequency bands.*

# SEISMIC WAVES

## Seismic Waveforms

This section is taken from Reference 6.

There are four types of seismic waves: primary, secondary, Rayleigh, and Love. A diagram of the waveforms is shown in Appendix B.

The primary and secondary waves are both body waves which travel through the body of the Earth. The Rayleigh and Love waves are both surface waves which can travel along the Earth's surface.

[2] The piano would likely need re-tuning because the new environment would affect the tension in the wires. This tension change occurs because the piano is composed of materials which react differently to temperature and humidity changes.

Body Waves

*Primary Wave*

The primary wave, or P-wave, is a sound wave. It thus has longitudinal motion. Note that the primary wave is the fastest of the four waveforms.

The wave speed c for P-waves is given by

$$c = \sqrt{\frac{B + \left(\frac{4}{3}\right)G}{\rho}} \tag{18}$$

where B is the bulk modulus,
G is the shear modulus,
ρ is the mass per unit volume.

*Secondary Wave*

The secondary wave, or S-wave, is a shear wave. This wave produces an amplitude disturbance which is at right angles to the direction of propagation.

The wave speed c for S-waves is given by

$$c = \sqrt{\frac{G}{\rho}} \tag{19}$$

where G is the shear modulus,
ρ is the mass per unit volume.

Note that water cannot withstand a shear force. S-waves thus do not propagate in water.

Surface Waves

*Common Characteristics*

Surface waves are dispersive. The velocity varies with frequency.

Specifically, low-frequency surface waves propagate faster than high-frequency surface waves. This is also true of ocean waves.

*Love Waves*

Love waves are shearing horizontal waves. The motion of a Love wave is similar to the motion of a secondary wave except that Love wave only travel along the surface of the Earth.

Love waves do not propagate in water.

*Rayleigh Waves*

Rayleigh waves produce retrograde elliptical motion. The ground motion is thus both horizontal and vertical. The motion of Rayleigh waves is similar to the motion of ocean waves except that ocean waves are prograde.

<u>Seismologists' Perspective</u>

Seismologists are largely content to analyze their data in the time domain since seismic waves are transient. They convert the displacement value into a single logarithmic magnitude value so that the energy released can be compared to that of other seismic events.

On the other hand, Seismologists must determine whether a given event is natural or man-made. A particular concern is clandestine underground nuclear explosions.

Explosions have a different spectral content than natural earthquakes as shown in Appendix C. Note the spectral plot in Appendix C is represented in terms of frequency rather than wavelength.

<u>Civil Engineering Perspective</u>

Surface waves cause more property damage because they cause larger ground displacements, velocities, and accelerations. They also travel more slowly, and may collect wave forms from the entire fault rupture; thus the duration of strong shaking may last for several minutes.

Civil engineers are interested in frequency content since they must design buildings to withstand ground-shaking. A particular concern is equipment which must withstand ground or floor shaking. Thus, Reference 7 gives seismic design and testing guidelines in terms of a frequency spectrum.

OCEAN WAVES

Ordinary

Ocean waves are driven by the wind. The wind is often generated by storms at sea.

The wave energy travels along the ocean surface, but the water molecules do not propagate forward with the waves. Moving water is called a current. Waves are not current.

The wave will drag along the bottom by friction if the water depth is shallow enough.

The wavebase is the depth at which waves "feel the bottom." The wavebase of all waves is half the wavelength. Thus, waves with wavelengths of about 100 m feel the bottom at depths of about 50 m.

As a wave approaches the shoreline, three effects occur:

1. Frictional dragging with the sea floor causes it velocity to decrease.
2. The wavelength decreases as the waves begin to bunch from incoming waves.
3. Wave height increases, typically to 3 m.

Ordinary waves are characterized by velocity, amplitude, and wavelength in common literature. A diagram of the wave motion is shown in Figure 4.

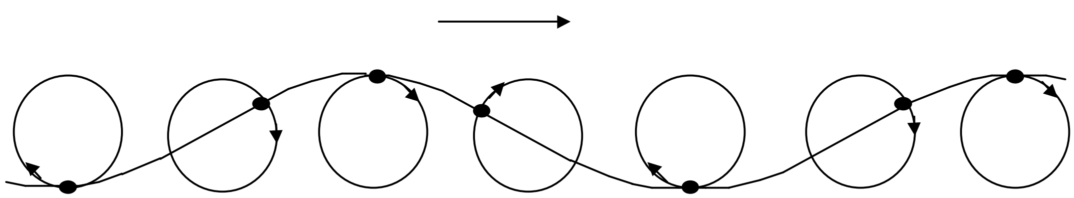


Figure 4. Ocean Waveform

Tsunami

This section is based on Reference 6.

Tsunami waves are generated by earthquakes, volcanoes, and underwater landslides. An earthquake with strong vertical motion is more likely to generate a tsunami than an earthquake with strong horizontal motion.

Tsunami waves characterized in common references in terms of amplitude, wavelength, and velocity. A frequency characterization is also possible; but this format may be limited to esoteric research journals, if the format is presented at all.

A tsunami has a typical wavelength of 200 km. The speed is proportional to the square root of ocean depth. It is 800 km/hr at a depth of 5 km. Specifically, the wave velocity v is

$$v = \sqrt{g\,d} \tag{20}$$

where d is the depth and g is the gravitational acceleration.

The amplitude of a tsunami in the open sea is typically less than 1 m. A ship at sea would thus be unable to distinguish a tsunami from an ordinary wave.

Nevertheless, a tsunami has tremendous kinetic energy. The amplitude of the tsunami increases as it approaches the shore. It can increase to a height of 30 m.

## WIND WAVES

This section is based on Reference 8.

Consider an object subjected to wind loading. Air pressure oscillation occurs both in oncoming wind turbulence and in the wake. A particular wake effect of interest is vortex shedding.

Note that the air molecules flow with the wind. The whole medium is moving. In this sense, wind does not meet the classical criteria for wave motion. Wind is perhaps best treated simply as a forcing function upon an object.

Nevertheless, the effects of wind are often described in terms of wave parameters in common literature. Specifically, the "spectral content" of wind oscillation is represented both in terms of frequency and wavelength.

On coming turbulence has a fairly random character. Its peak amplitude corresponds with wavelengths of about 500 m.

Wake turbulence tends to have a more sinusoidal character. Most of its energy is centered at a wavelength corresponding to the diameter of the object.

## AERODYNAMIC SHOCK WAVES

This section is based on Reference 9.

Aerodynamic shock waves represent a change in entropy. They result in a discontinuity of flow parameters, such as pressure.

Consider an aircraft flying at a supersonic speed. The aircraft actually has two shock waves: a bow wave and a tail wave. The waves have nearly the shape of a cone. Hence, they are called mach cones. The pressure between the two cones is a region of overpressure, relative to atmospheric pressure. Consider an observer on the ground. The bow wave sweeps over the observer causing a brief duration of high pressure. The air pressure then drops below the ambient value. Next, the tail wave sweeps over the observer, restoring the air pressure to its ambient value. This explains the double crack of a sonic boom. People viewing the landing of a space shuttle hear this effect, for example. The time history has an *N signature* as shown in Figure 5.

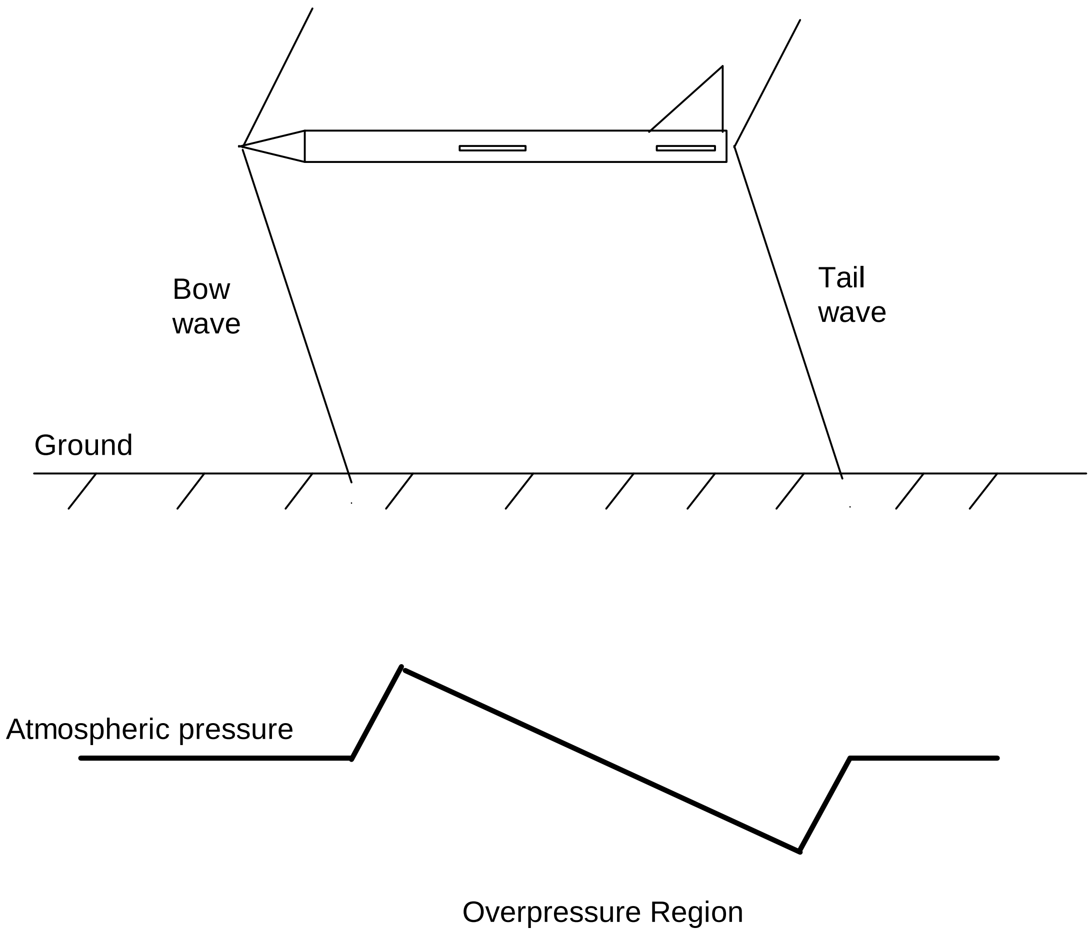



Figure 5.

Aerodynamic shock waves are characterized by time histories in common literature. They do not seem to have any individual spectral representation, either in terms of frequency or wavelength.

On the other hand, the pressure fields surrounding aircraft are sometimes characterized by frequency. Wind tunnel testing is performed to determine these pressure fields. Note that the pressure fields represent the combined effects of shock waves, turbulent boundary layers, flow separation, flow recirculation, etc.

## FUNDAMENTAL MECHANICAL SHOCK AND VIBRATION WAVES

### Mechanical Waveform Types

This section is based on Reference 10.

There are many types of mechanical waves. The three common types are: longitudinal, transverse, and bending. These type definitions are largely established by convention. Real-world mechanical waveforms do not necessarily fit purely into any one category.

### Mechanical Longitudinal Waves

Longitudinal mechanical waves may be called structure-borne sound. They are similar to seismic P-waves. Each of these sound waves is governed by an equation of the form of equation (1).

The wave speed c for a mechanical longitudinal wave is given by

$$c = \sqrt{\frac{E}{\rho}} \tag{21}$$

where E is the elastic modulus,
$\rho$ is the mass per unit volume.

Note that equation (21) is equivalent to equation (17) with the elastic modulus replacing the bulk modulus.

The frequency and wavelength are related by the familiar formula

$$c = f\,\lambda \tag{22}$$

The fundamental longitudinal frequency fn of a free-free beam with length L is given by

$$fn = \frac{c}{2L} \tag{23}$$

Equation (23) has been used for designing beams to meet shock response spectrum tests. Each beam was sized so that its natural frequency matched the knee frequency of the shock response spectrum.

### Mechanical Transverse Waves

Transverse mechanical waves are characterized by shear deformation. They are similar to the seismic S-wave.

There are two subtypes of transverse waves: transverse plane waves and torsional waves. Each is governed by an equation of the form of equation (1).

The transverse wave speed C for both types is given by

$$c = \sqrt{\frac{G}{\rho}} \tag{24}$$

where G is the shear modulus,
$\rho$ is the mass per unit volume.

The frequency and wavelength are related by the familiar formula

$$c = f\,\lambda \tag{25}$$

Mechanical Bending Waves

Bending waves are also called flexural waves. They are fundamentally different from all other waveforms.

Again, the classic harmonic wave equation is a second-order equation with respect to both the spatial and time coordinates.

The bending wave equation remains second-order in time, but it is fourth-order in space. The one-dimensional form for the natural response is

$$-B\frac{\partial^4 E}{\partial x^4} = m\frac{\partial^2 E}{\partial t^2} \tag{26}$$

where E is a field variable,
B is the flexural stiffness,
m is the mass per unit length.

The field variable E can represent translation amplitude, rotational amplitude, bending moment, or shear force.

Note that equation (26) assumes that the bending wavelength is large compared to the dimensions of the thickness of the beam or plate. Otherwise, correction terms are needed.

A proposed traveling-wave solution is again

$$E(x,t) = A\sin(kx - \omega t - \phi) \tag{27}$$

The solution assumes that the bending-wave is propagating along an infinite length. For practical purposes, this assumption is important only for low-frequency bending-modes.

Substitution into equation (26) yields the following constraint

$$\frac{B}{m}k^4 = \omega^2 \tag{28}$$

The bending wave phase velocity $C_B$ is related to the wave number k and the angular frequency $\omega$ by the formula.

$$k = \frac{\omega}{C_B} \tag{29}$$

Substitution of equation (29) into (28) yields the following phase velocity relationship

$$C_B = [\omega]^{1/2} \left[\frac{B}{M}\right]^{1/4} \tag{30}$$

The consequence of equation (30) is that the phase velocity is no longer a constant. Rather it is proportional to the square root of the frequency.

The phase velocity represents the propagation velocity of only one infinite sinusoidal wave. Waveforms are typically composed of a number of sinusoids, however. A distortion or dispersion effect occurs because the higher frequency waveforms propagate with a higher phase velocity than the lower-frequency components.[3]

The bending wave phase velocity $C_B$ is related to the frequency f and wavelength $\lambda$ by the familiar formula

$$C_B = f\,\lambda \tag{31}$$

Again, phase velocity $C_B$ is no longer a constant.

Consider a plate with thickness h and a longitudinal wave velocity of $C_L$. The longitudinal velocity is a constant for a given material. The phase velocity is

$$C_B \approx \sqrt{1.8\ C_L h f} \tag{32}$$

[3] An interesting analysis would be to revisit pyrotechnic shock data taken at various stations of rocket vehicle during both flight and ground tests. The data could be analyzed for dispersion effects. An evaluation could be made to determine if there is any relationship between dispersion and attenuation.

By substitution,

$$f\,\lambda \approx \sqrt{1.8\ C_L h\, f} \tag{33}$$

Dividing both sides by $\sqrt{f}$,

$$\lambda\sqrt{f} \approx \sqrt{1.8\ C_L h} \tag{34}$$

Contrast

Equation (32) shows that bending-wave phase velocity increases with frequency. The opposite is true for seismic and ocean surface waves!

Bending Wavelet Example

Consider a steel plate with 20 mm thickness and "very large" surface area. A hypothetical excitation source is at the center of the plate.

At a certain time, the source generates a 100 Hz traveling wavelet with unit amplitude. A wavelet pulse is defined as 1.5 cycles for this example. The wavelength is 136 cm per equation (34). The phase velocity is 13,600 cm/sec.

A certain time later, the source generates a 2000 Hz traveling wavelet with half the amplitude as the first pulse. The wavelength of the second pulse is 30.5 cm per equation (34). The phase velocity is 61,000 cm/sec.

The high-frequency pulse eventually overtakes the low-frequency pulse as shown in Figures 5a and 5b.

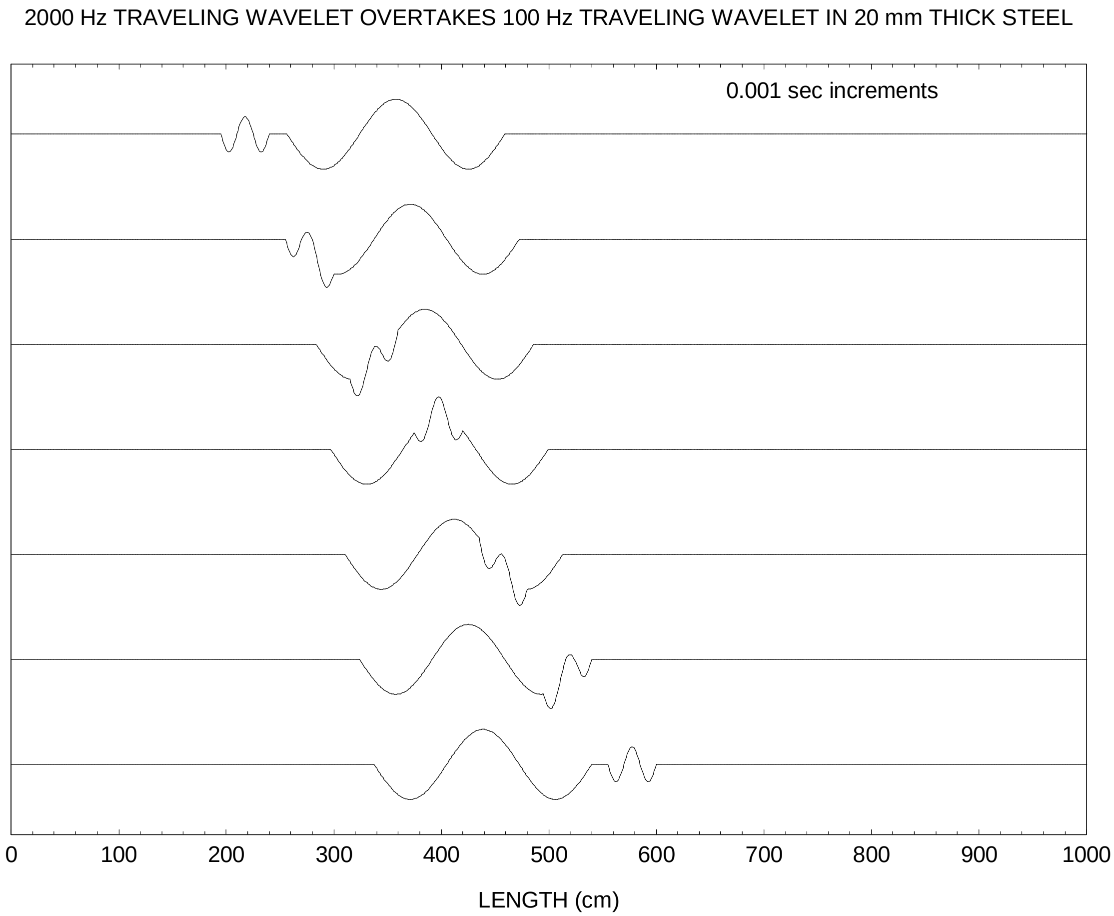



Figure 5a. Wavelet Example, Amplitude versus Length

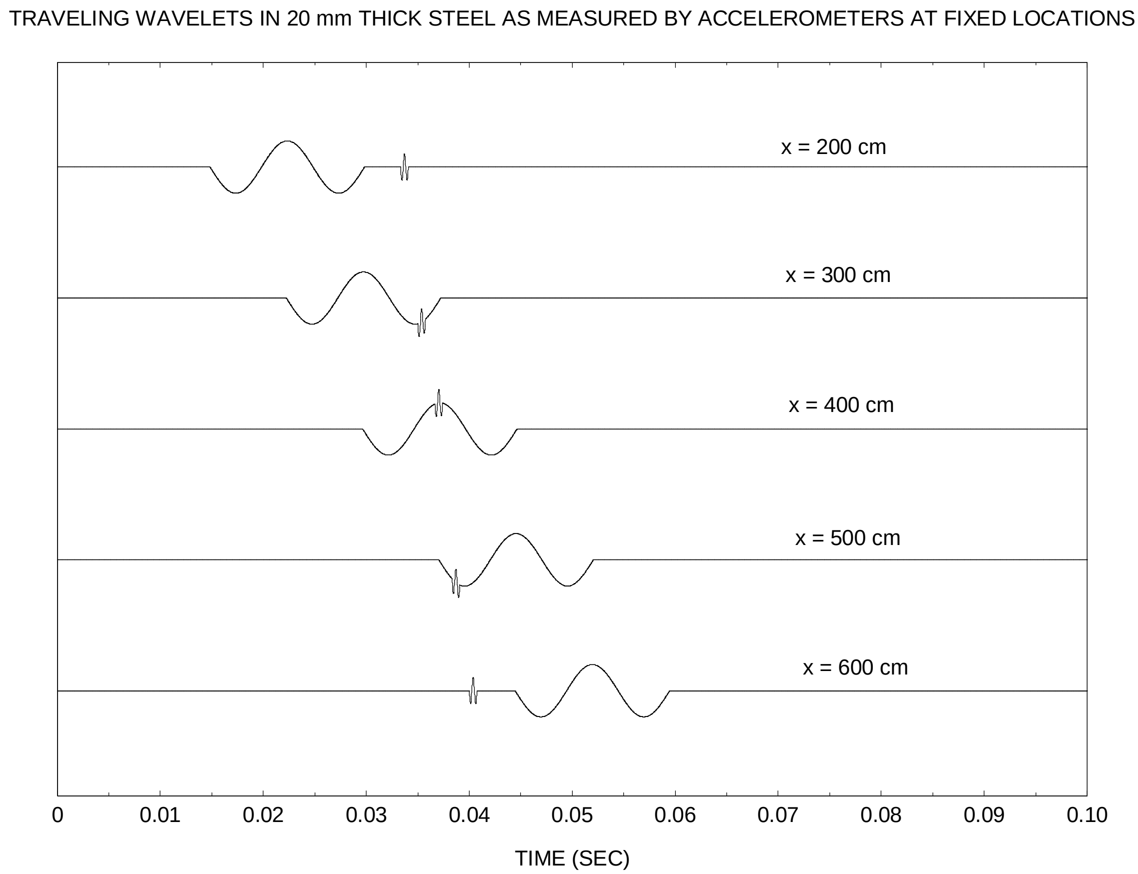



Figure 5b. Wavelet Example, Amplitude versus Time

Test Considerations

Consider a hypothetical component acceptance vibration test. A mechanical engineer specifies the test format as a sine-dwell test at 10,000 Hz at some amplitude. Somehow this test represents the flight environment to which the component will be exposed.

Furthermore, the specification calls for the component to be mounted on the end of a "long" steel rod. The rod is considered to behave as an elastic-body rather than a rigid-body for this test. A control accelerometer is mounted at the end of the rod adjacent to the component. A shaker is used to excite the opposite end.

The test engineer readily implements the mechanical engineer's specification.

The component has a circuit board which acts as single-degree-of-freedom system with a 700 Hz natural frequency. It readily passes the vibration test since it filters out the 10,000 Hz excitation.

Sometime later, a second mechanical engineer notes that the longitudinal wave speed in steel is 517,000 cm/sec. He correctly calculates that the wavelength at 10,000 Hz is 51.7 cm.

Feeling clever, this second mechanical engineer decides to rewrite the specification in terms of wavelength rather than frequency. He specifies the test at the same amplitude but at a wavelength of 51.7 cm. For brevity, he omits the requirement that the test be performed using the rod.

The test engineer receives the revised specification. He realizes that the specification no longer requires a rod. Also feeling clever, he decides to substitute a plate instead. The plate is a 20 mm thick steel plate. The length and width are both "large."

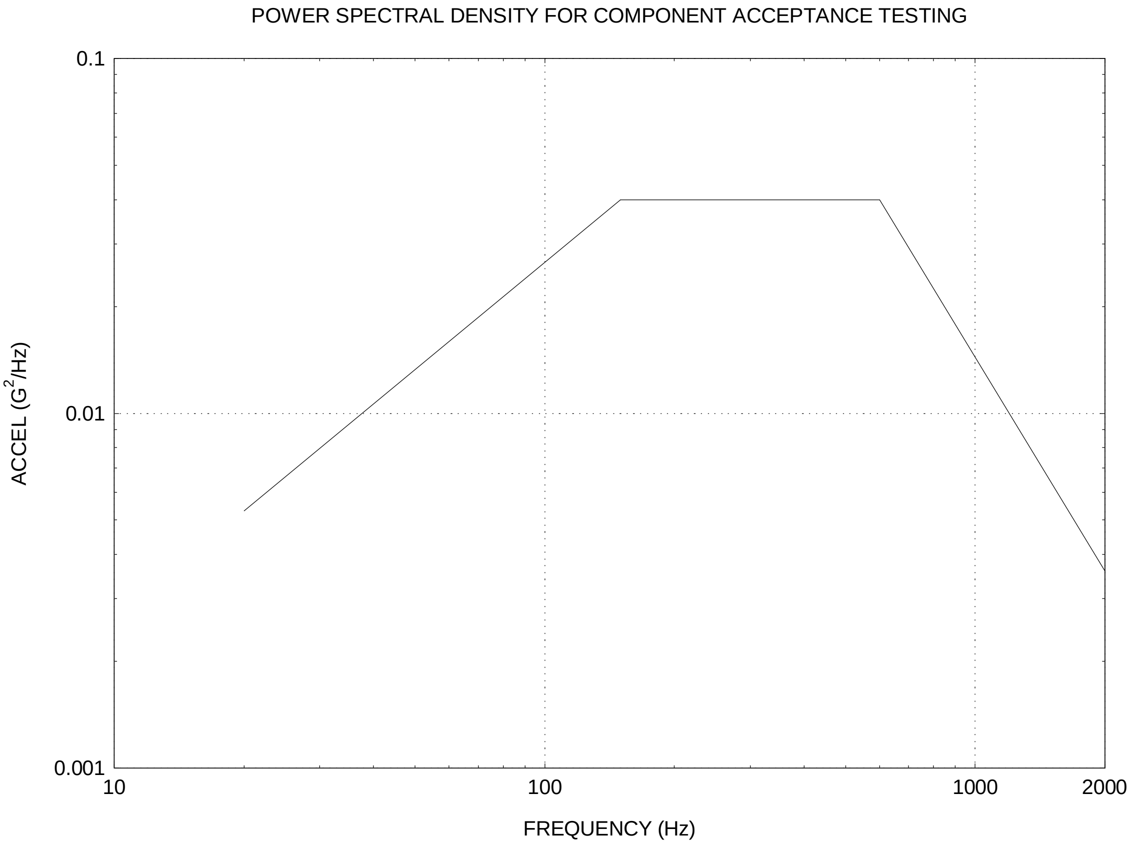
The test engineer uses formula (34) to determine that the excitation frequency should be 696 Hz to achieve a 51.7 cm bending wavelength in this plate.

As a result, the component is now tested at 696 Hz instead of 10,000 Hz. The component's 700 Hz natural frequency is excited, and the component fails.

This example is far-fetched, but it demonstrates another reason that shock and vibration tests are specified in terms of frequency bands rather than wavebands.

Also note that shaker base input shock and vibration tests should be specified in terms of frequency as discussed in Appendix D.

A sample test specification taken from Reference 5 is shown in Figure 6. Note that the spectral dimension is frequency.




| Breakpoints | |
|---|---|
| Freq (Hz) | PSD (G^2/Hz) |
| 20 | 0.0053 |
| 150 | 0.04 |
| 600 | 0.04 |
| 2000 | 0.0036 |

Figure 6. MIL-STD-1540C Component Acceptance Test Level

The overall level is 6.1 GRMS.

## MECHANICAL PLATE WAVES

### Description

Consider the case of wave propagation in a plate where the thickness is greater than the wavelength.  These types of waves are called “plate waves.”

This is a very simple description.  A scholarly description is given in Reference 10.

There are several types of plate waves.  One is the Rayleigh wave which was discussed in the seismology section.  Another is the Lamb wave.

### Lamb Wave

The following description is taken from Reference 11.

A Lamb wave is type of ultrasonic wave propagation in which the wave is guided between two parallel surfaces of the test object.  For an object sufficiently thin to allow penetration to the opposite surface, e.g. a plate having a thickness of the order of a wavelength or so, Rayleigh waves degenerate to Lamb waves, which can propagate in a number of modes, either symmetrical or antisymmetrical.  The velocity is dependent on the product of frequency and material thickness.

Lamb waves are named for Horace Lamb, in honor of his fundamental contributions to this subject. Investigation on Lamb and leaky Lamb waves have been carried out continuously since their discovery.   Researchers have done theoretical and experimental work for different purposes, ranging from seismology and the ship construction industry to acoustic microscopy, non-destructive testing and acoustic sensors.

## SUMMARY

The convention for representing the spectral distribution of  waveforms is given in Table 2.  Lamb waves are not included.

| Table 2. Conventional Spectral Representation of Waveforms | |
|---|---|
| Wave Type | Spectral Notes |
| Electromagnetic | Can be represented in terms of either frequency or wavelength. |
| Acoustic | Represented in terms of frequency. |
| Seismic | Frequency representation. |
| Ocean | Represented in terms of wavelength. |
| Wind | No wave propagation in classical sense. Nevertheless, represented in terms of either frequency or wavelength. |
| Aerodynamic Shock | Represented with other forcing functions in terms of frequency. |
| Shaker with Rigid Fixture | No wave motion. |
| Mechanical Shock and Vibration | Represented in terms of frequency. |

Again, the information in Table 1 is taken from common convention. Possible exceptions may be found in esoteric journals.

REFERENCES


1. Frank Fahy, Sound and Structural Vibration, Academic Press, London, 1985.
2. Halliday and Resnik, Physics Parts I & II, Wiley, New York, 1978.
3. Jenkins and White, Fundamentals of Optics, McGraw-Hill, New York, 1976.
4. I. Shames, Mechanics of Fluids, McGraw-Hill, New York, 1982.
5. MIL-STD-1540C, Military Standard Test Requirements for Launch, Upper-stage, and Space Vehicles, U.S. Department of Defense, 1994.
6. Bruce Bolt, Earthquakes, Freeman, New York, 1988.
7. C. Harris, editor; Shock and Vibration Handbook, 4th edition; W.J. Hall, "Chapter 24 Part I: Vibration of Structures Induced by Ground Motion," McGraw-Hill, New York, 1995.
8. C. Harris, editor; Shock and Vibration Handbook, 4th edition; W.J. Hall, "Chapter 29 Part II: Vibration of Structures Induced by Wind," McGraw-Hill, New York, 1995.
9. P. Tipler, Physics, Worth, New York, 1976.
10. L. Cremer and M. Heckl, Structure-Borne Sound, Springer-Verlag, New York , 1988.
11. Ultrasonic Testing Encyclopedia, http://www.ndt.net/index.html

# APPENDIX A

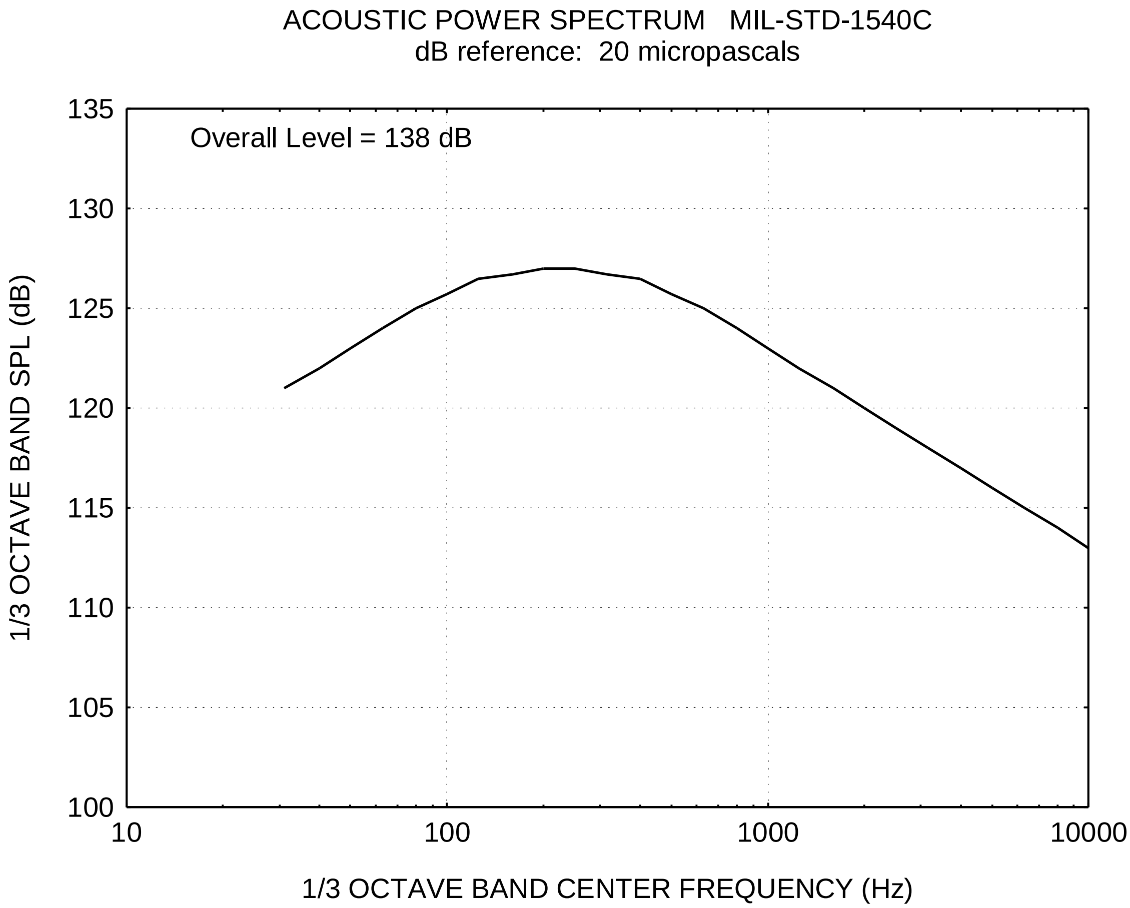



Figure A-1.

Table A-1. Coordinates for Acoustic Power Spectrum

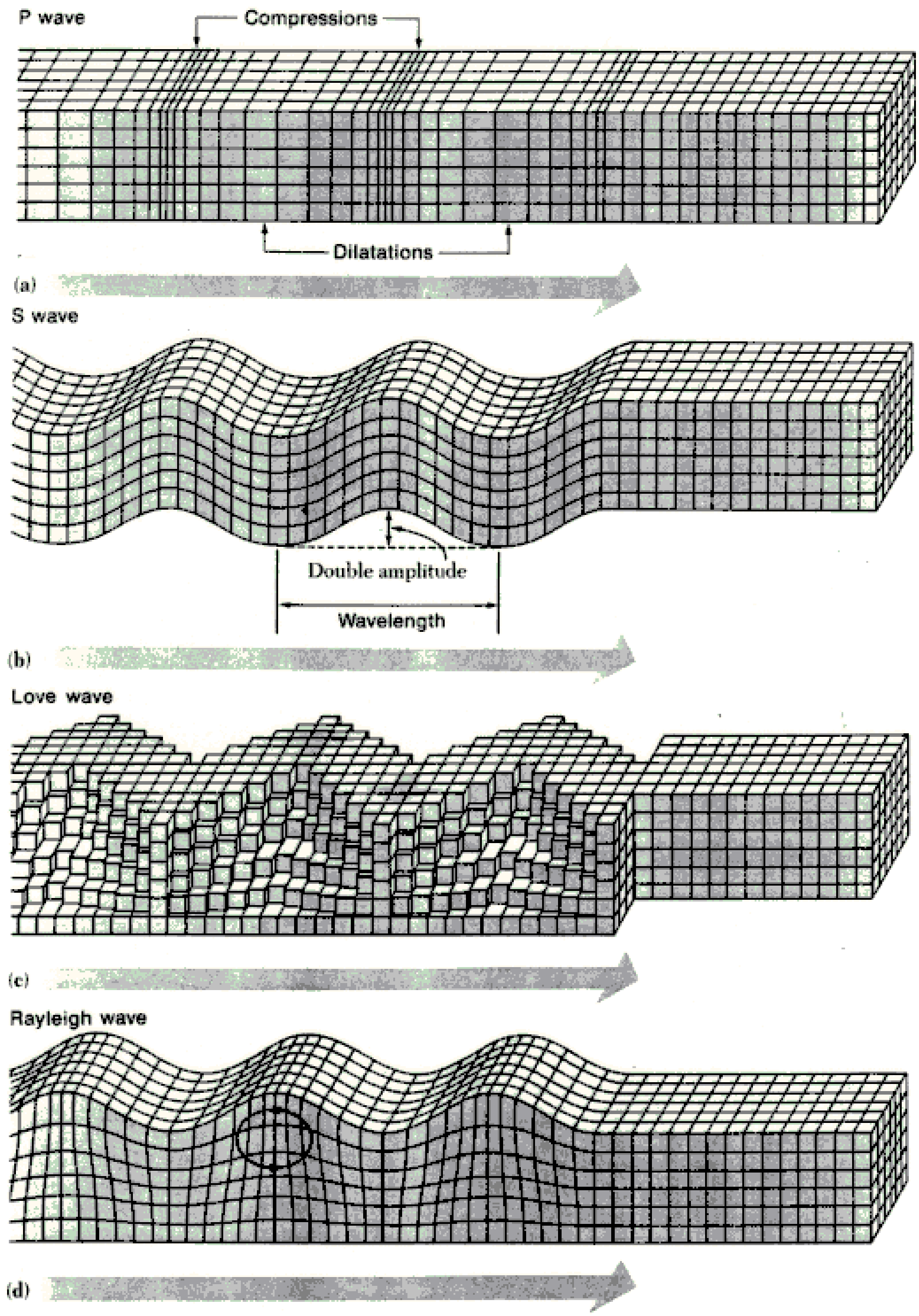
| 1/3 Octave Center Frequency (Hz) | Sound Pressure Level (dB) |
|---|---|
| 31 | 121 |
| 40 | 122 |
| 50 | 123 |
| 63 | 124 |
| 80 | 125 |
| 100 | 125.7 |
| 125 | 126.5 |
| 160 | 126.7 |
| 200 | 127 |
| 250 | 127 |
| 315 | 126.7 |
| 400 | 126.5 |
| 500 | 125.7 |

| 1/3 Octave Center Frequency (Hz) | Sound Pressure Level (dB) |
|---|---|
| 630 | 125 |
| 800 | 124 |
| 1000 | 123 |
| 1250 | 122 |
| 1600 | 121 |
| 2000 | 120 |
| 2500 | 119 |
| 3150 | 118 |
| 4000 | 117 |
| 5000 | 116 |
| 6300 | 115 |
| 8000 | 114 |
| 10000 | 113 |

APPENDIX B

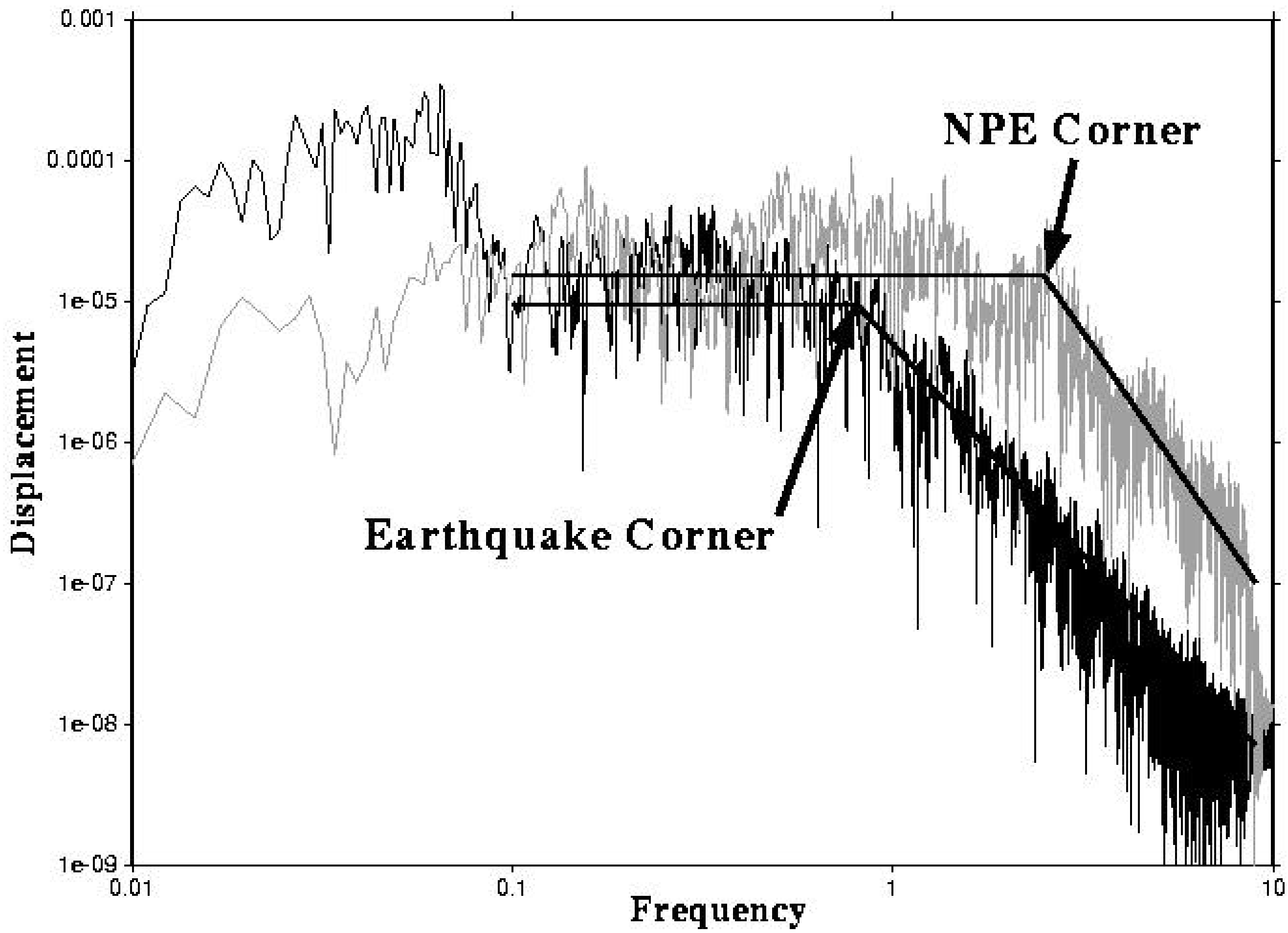
## SEISMIC WAVEFORMS

This diagram is taken from Reference 6.



APPENDIX C

SPECTRAL DISTRIBUTION OF SEISMIC ENERGY IN TERMS OF FREQUENCY




This plot is courtesy of Mark Tinker of the University of Arizona. The NPE curve is the result of an explosion.

## APPENDIX D

## SHAKER TABLE SHOCK AND VIBRATION

### ELECTROMAGNETIC SHAKER

Nearly all component vibration tests are performed by mounting the component on a fixture which is in turn mounted on a shaker. Most fixtures can be considered as rigid masses. The input to the component is controlled by an accelerometer mounted on the fixture. The acceleration amplitude Y(t) is described by the equation

$$Y(t) = A \sin(\omega t - \phi) \tag{D-1}$$

Equation (D-1) holds for sinusoidal inputs. Random inputs can be represented by a Fourier series of sinusoids.

Note that there is no spatial coordinate x in equation (D-1). In other words, the shaker and fixture do not have true wave motion. On the other hand, there is a time coordinate with an angular frequency multiplier. This is one of the reasons that base input vibration tests are specified in terms of frequency rather than wavelength.

Nevertheless, the component mounted on the fixture may display wave response to the base input. For example, the component may have a rectangular circuit board which has a bending wave response to the base input.