# An Abbreviated Time Domain Analysis Method for a PSD Base Input

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## Variables

There is a need in certain structural dynamics finite element analysis problems to perform a response analysis in the time domain. This would be the case, for example, if the stiffness was non-linear. Loss-of-clearance or contact between two adjacent vibrating parts is another analysis example.

Consider the case of base excitation in the form of a PSD. A time history can be readily synthesized to satisfy the PSD. The corresponding number of time history samples may overwhelm the finite element program's allowable array sizes or cause excessive processing time, however.

Certain military and aerospace PSD specifications have an upper frequency of 2000 Hz. A synthesized time history should thus have a sample rate of 20 KHz, which is ten times the highest PSD frequency. The total number of time history points would be 1.2 million if the specified duration was 60 seconds.

Now consider that the peak response must be calculated and that fatigue is unimportant. This would be the case for evaluating the resulting response with respect to yielding, ultimate stress limit, or relative displacement.

A shorter duration time history can be synthesized to satisfy this need as presented in this paper.

The abbreviated time history is derived using response spectra. Its response spectrum must envelop that of the full-duration PSD as determined from Rayleigh distribution of the PSD response.

The derivation method is demonstrated by example.

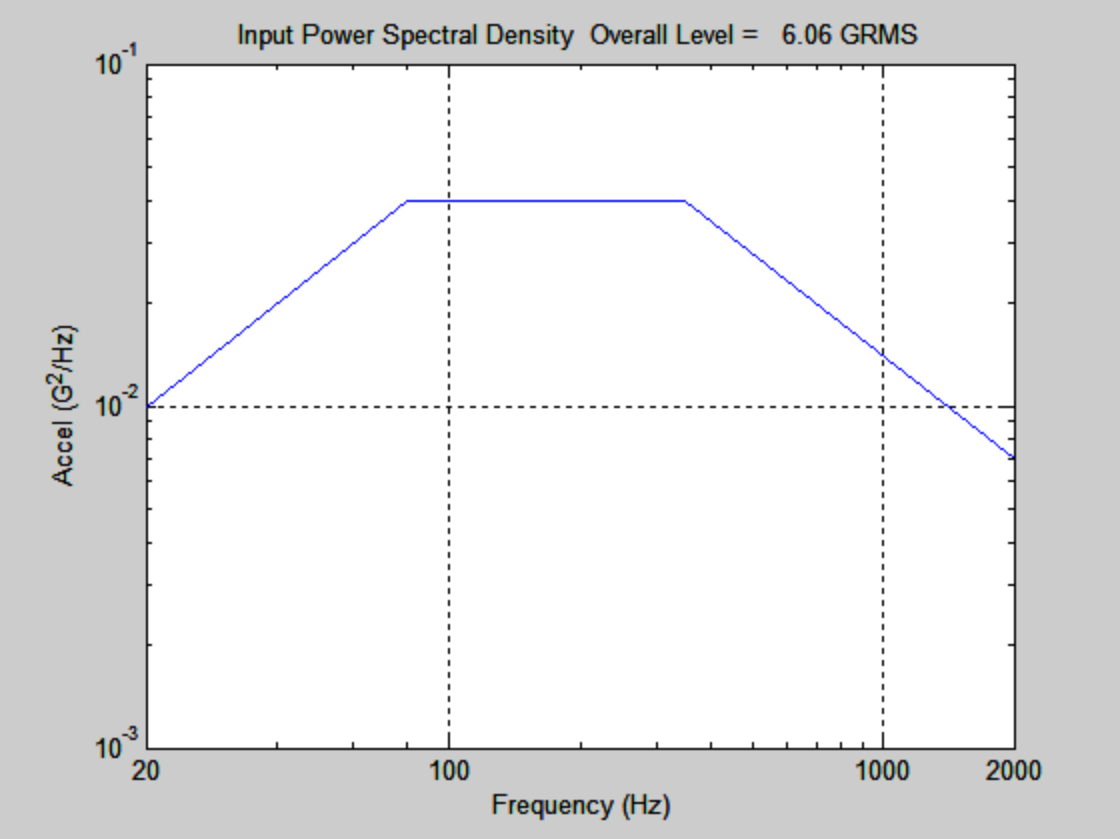


Figure 1.

Example

Assume that a test item must be subjected to the NAVMAT P-9492 base input in Figure 1 for 60 seconds, as applied on a shaker table.

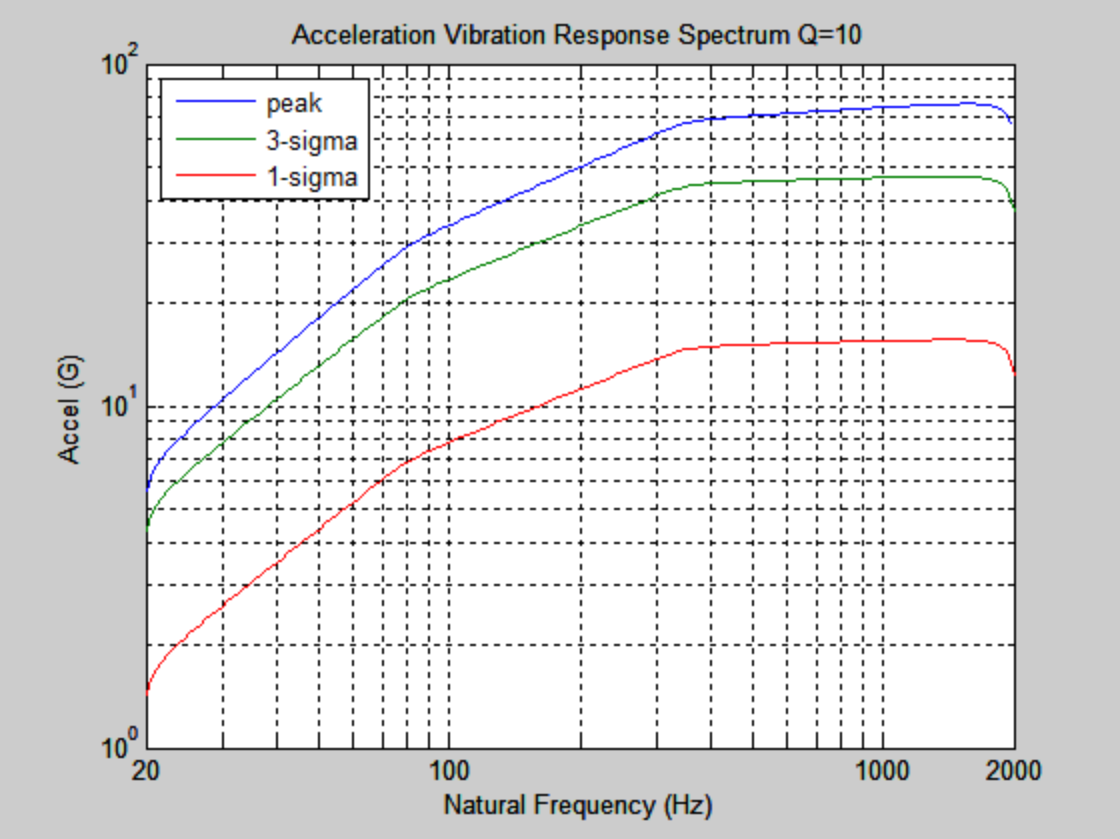


Figure 2.

The peak value VRS is shown in Figure 2, as calculated using the methods in References 1 and 2. The peak value curve accounts for duration. It allows for peaks higher than 3-sigma. It is essentially the shock response spectrum (SRS) of the base input PSD.

The calculation was performed using Matlab script: VRS.m

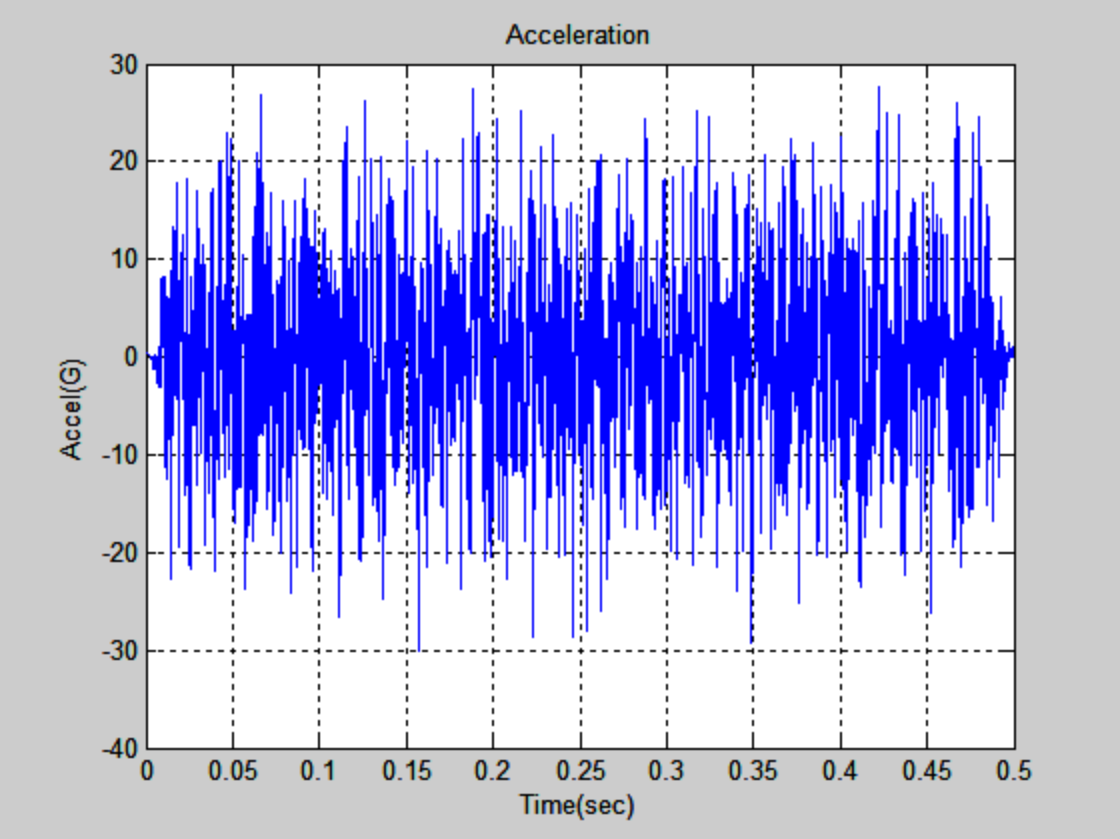


Figure 3.

An abbreviated time history was synthesized 3 using a series of sinusoids with random phase as shown in Figure 3 to satisfy the peak VRS in Figure 2.

The time history is sample at 20 KHz. It has 4000 samples.

The synthesis was performed using Matlab script: srs_sine_syn.m

A series of trend removal and highpass filtering steps were performed so that the corresponding velocity and displacement would each oscillate about the zero baseline. This is important for calculating numerically stable relative displacements.

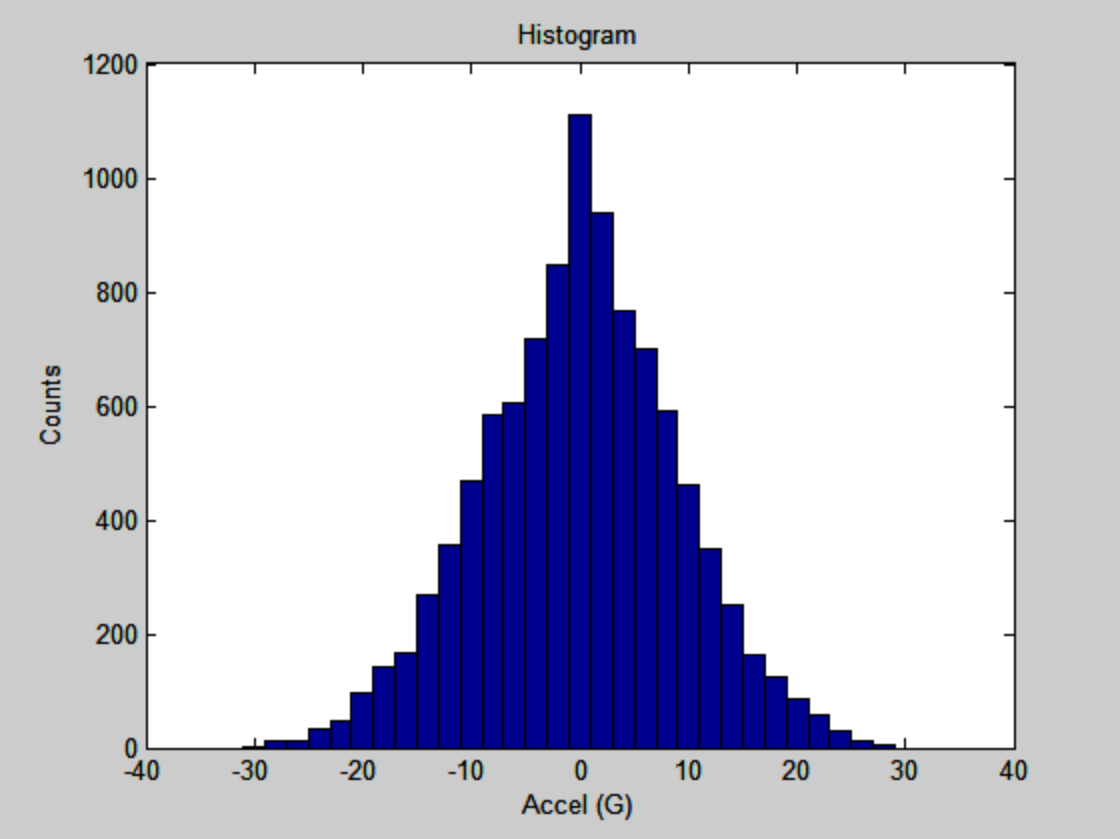


Figure 4.

The histogram of the synthesize acceleration time history is shown in Figure 4. It has a normal distribution expect that there are extra points near the zero on the x-axis due to the fade in and fade out.

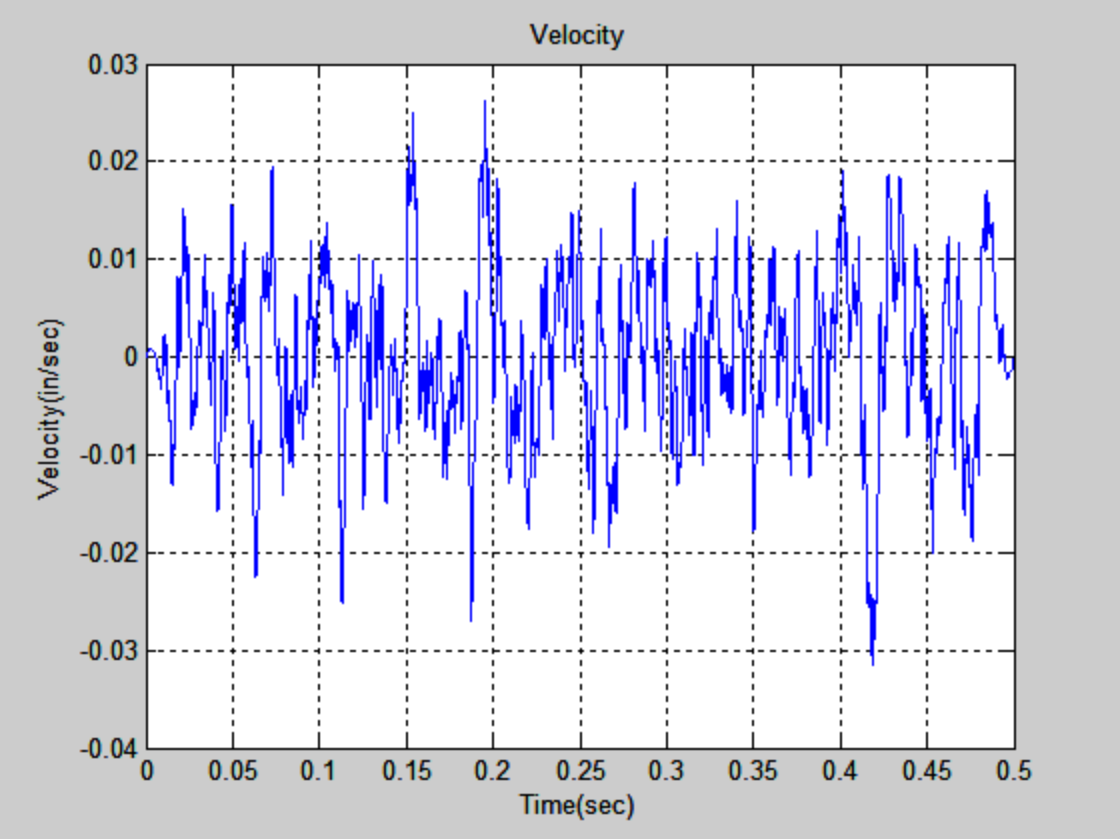


Figure 5.

The corresponding velocity of the synthesized time history is shown in Figure 5.

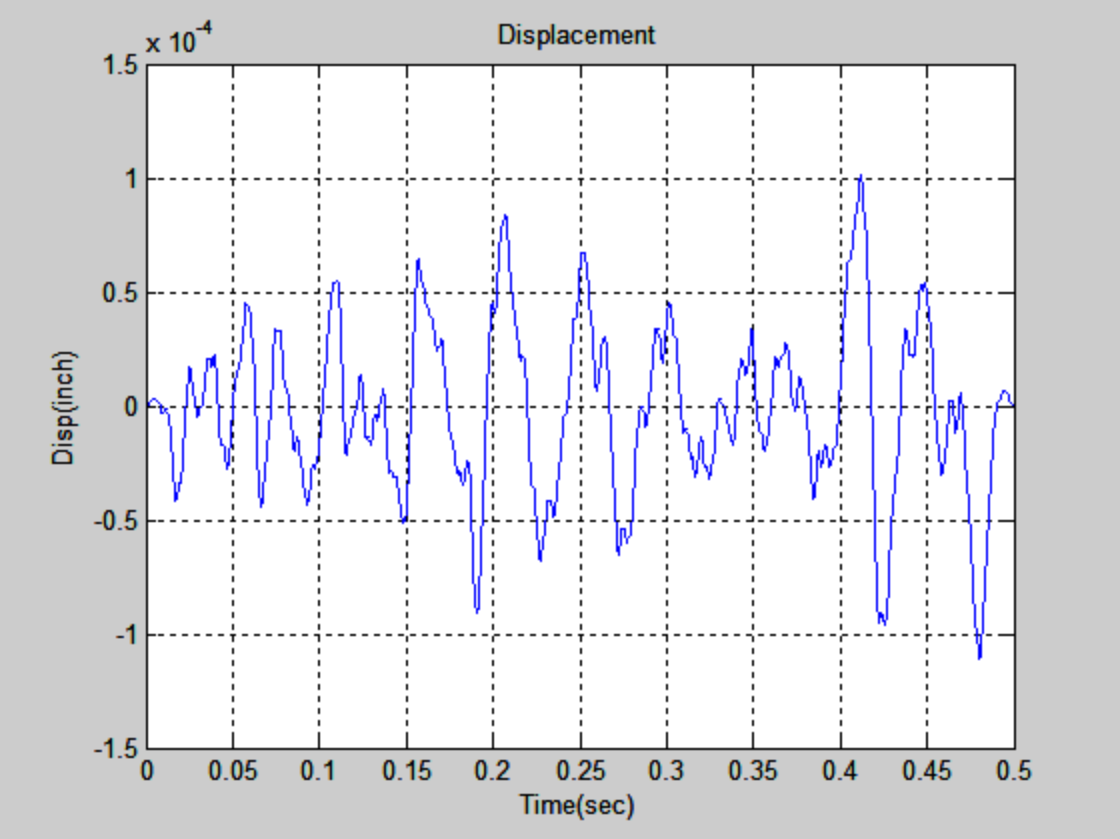


Figure 6.

The corresponding displacement of the synthesized time history is shown in Figure 5.

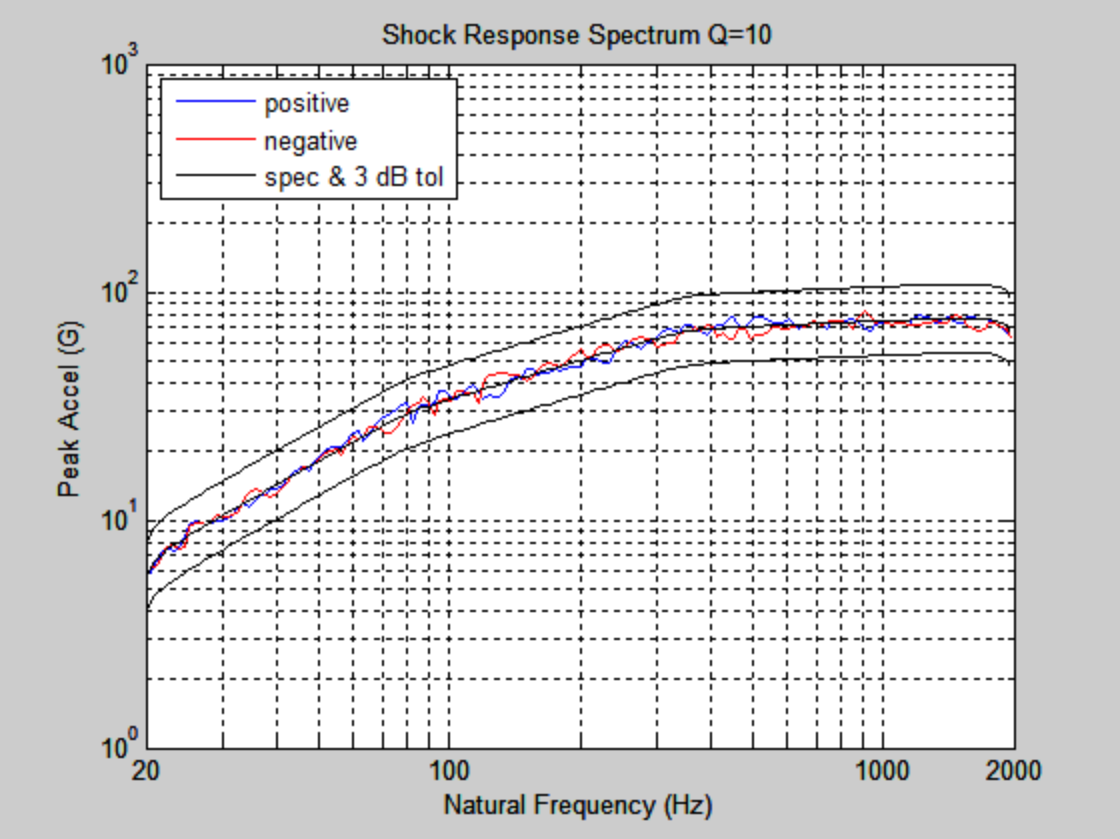


Figure 7.

The synthesized, abbreviated acceleration time history satisfies the peak response VRS within the tolerance bands. It thus can be applied to a finite element model to represent the full-duration PSD specification for peak response, non-fatigue analyses.

The method will be extended to fatigue in a future revision.

References

1. T. Irvine, An Introduction to the Vibration Response Spectrum, Revision D, Vibrationdata, 2009.

2. T. Irvine, Equivalent Static Loads for Random Vibration, Revision N, Vibrationdata, 2012.