Experimental Verification of the Dirlik Fatigue Cycle Method

Revision A

By Tom Irvine
Email: tom@vibrationdata.com

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Introduction

The rainflow method is a method for counting fatigue cycles from a response time history, as shown in Reference 1. The fatigue cycles are stress-reversals. The rainflow method allows the application of Miner's rule in order to assess the fatigue life of a structure subject to complex loading. This method may also be used to calculate a Miner-like relative damage index for acceleration response cycles, as shown in Appendix A. Furthermore, it may be used for either stationary or nonstationary data.

The fatigue cycles may also be calculated from a response PSD using the semi-empirical Dirlik method from References 2 and 3. This method effectively assumes that the PSD has a corresponding time history which is stationary with a normal distribution.
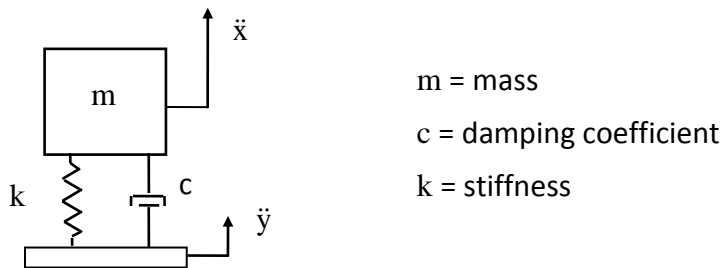
SDOF System



m = mass

c = damping coefficient

k = stiffness

Figure 1. Single-degree-of-freedom System

Assume a single-degree-of-freedom (SDOF) system response with an amplification factor Q=10 and a fatigue exponent b=6.4. The natural frequency is variable. Use this system for the following examples to compare the time domain fatigue results with the Dirlik results.

Base Input PSD
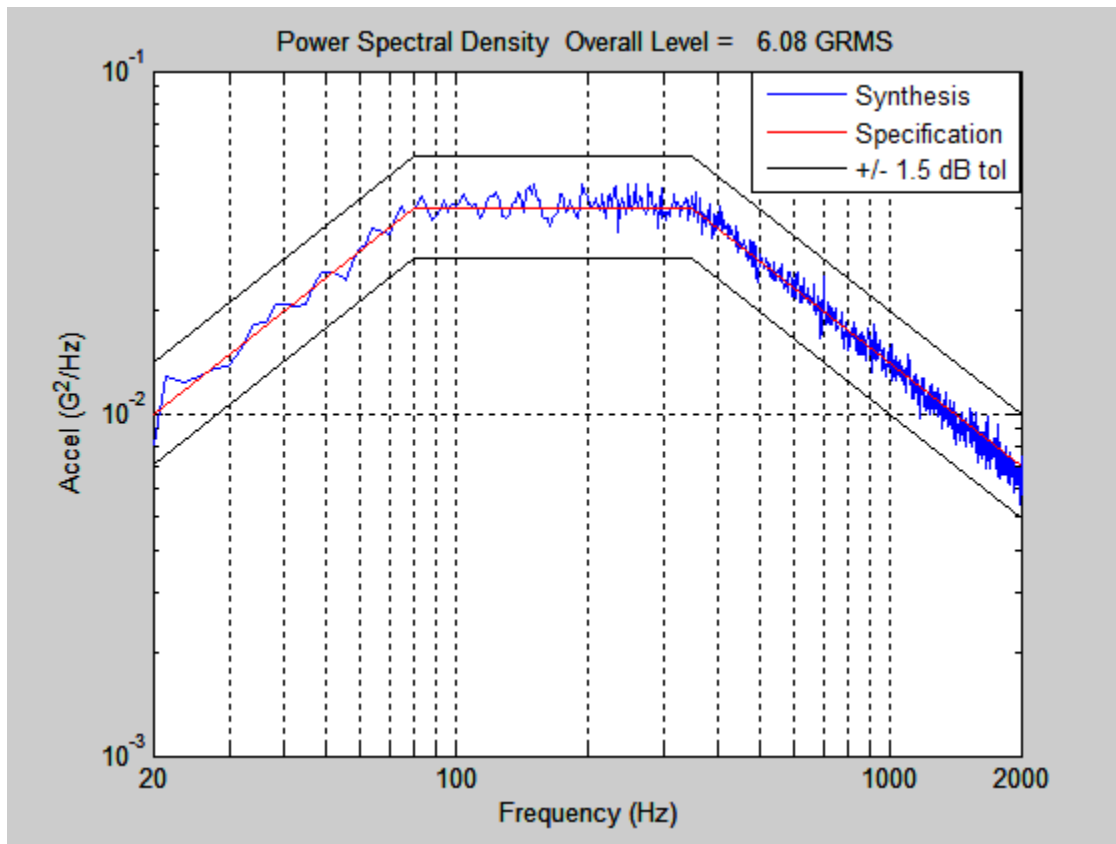


Figure 2.

| PSD Specification, 6.1 GRMS | |
| --- | --- |
| Freq (Hz) | Accel (G^2/Hz) |
| 20 | 0.010 |
| 80 | 0.040 |
| 350 | 0.040 |
| 2000 | 0.007 |

Synthesize a time history to satisfy the base input PSD using the method in Reference 4, with a duration of 60 seconds. The PSD of the synthesis is shown in Figure 2 along with the specification. The time history itself is shown in Figure 3. It has a normal distribution as shown in Figure 4.
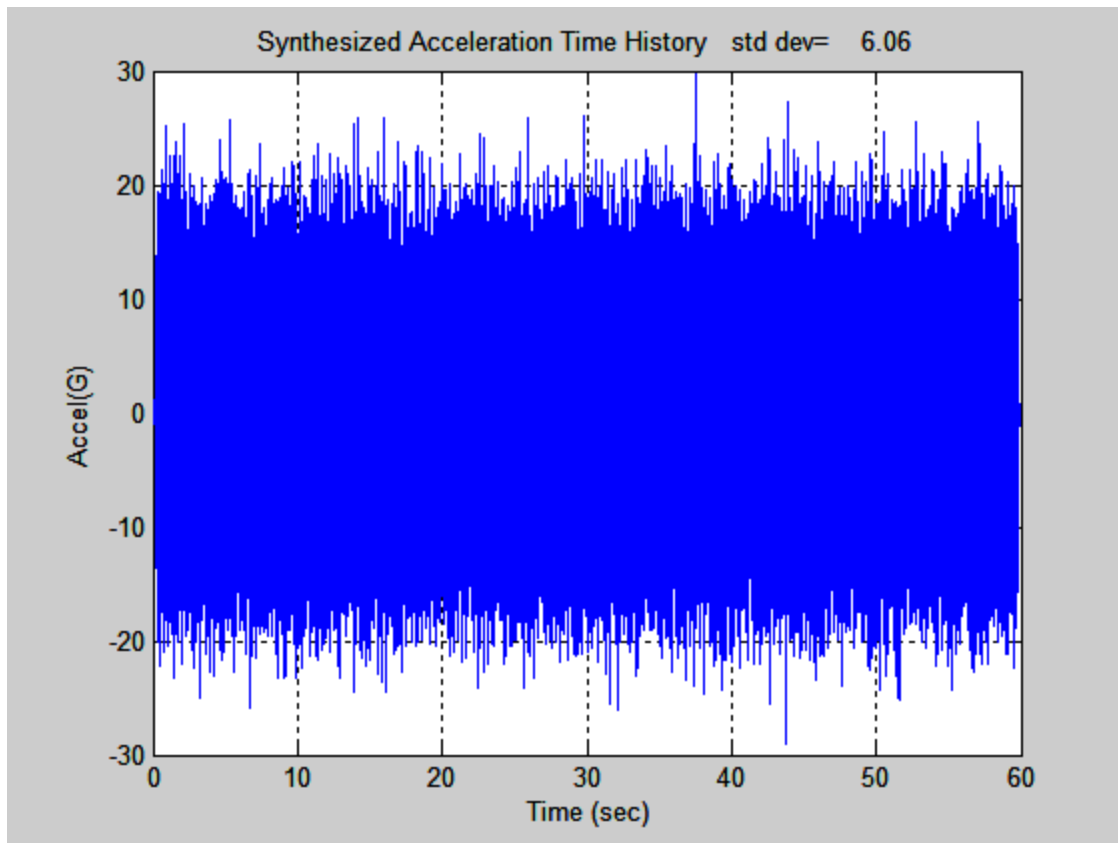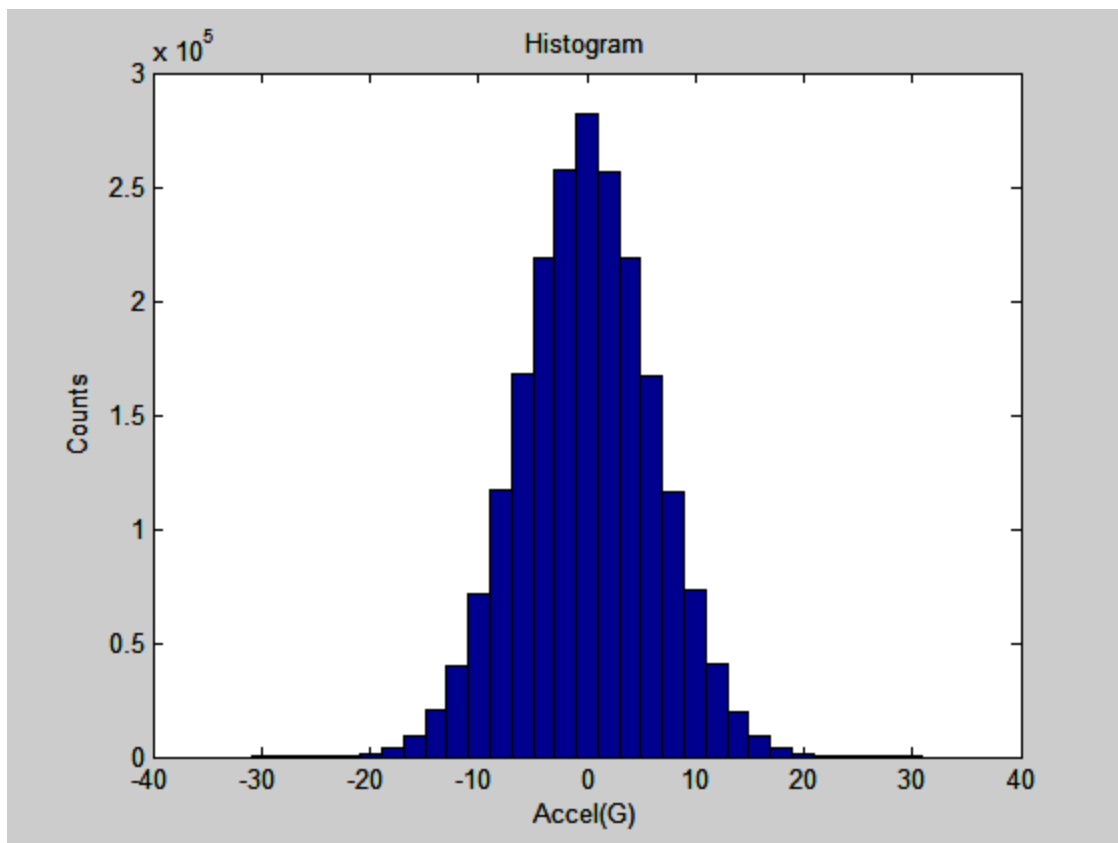
Figure 3.

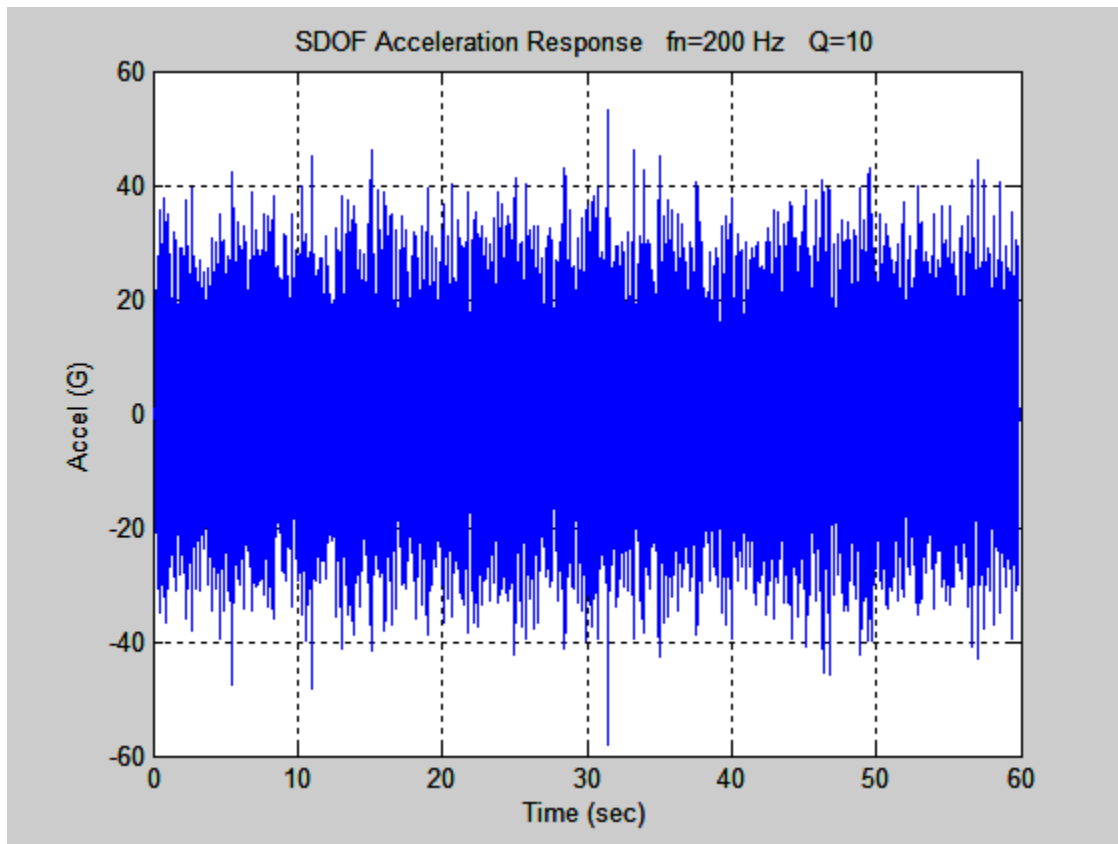Figure 4.

Example 1, fn=200 Hz



Figure 5.

The response time history for a natural frequency of 200 Hz is shown in Figure 5, as calculated by the method in Reference 5. It has a normal histogram as shown in Figure 6.

The rainflow cycles were then calculated using the method in Reference 1.
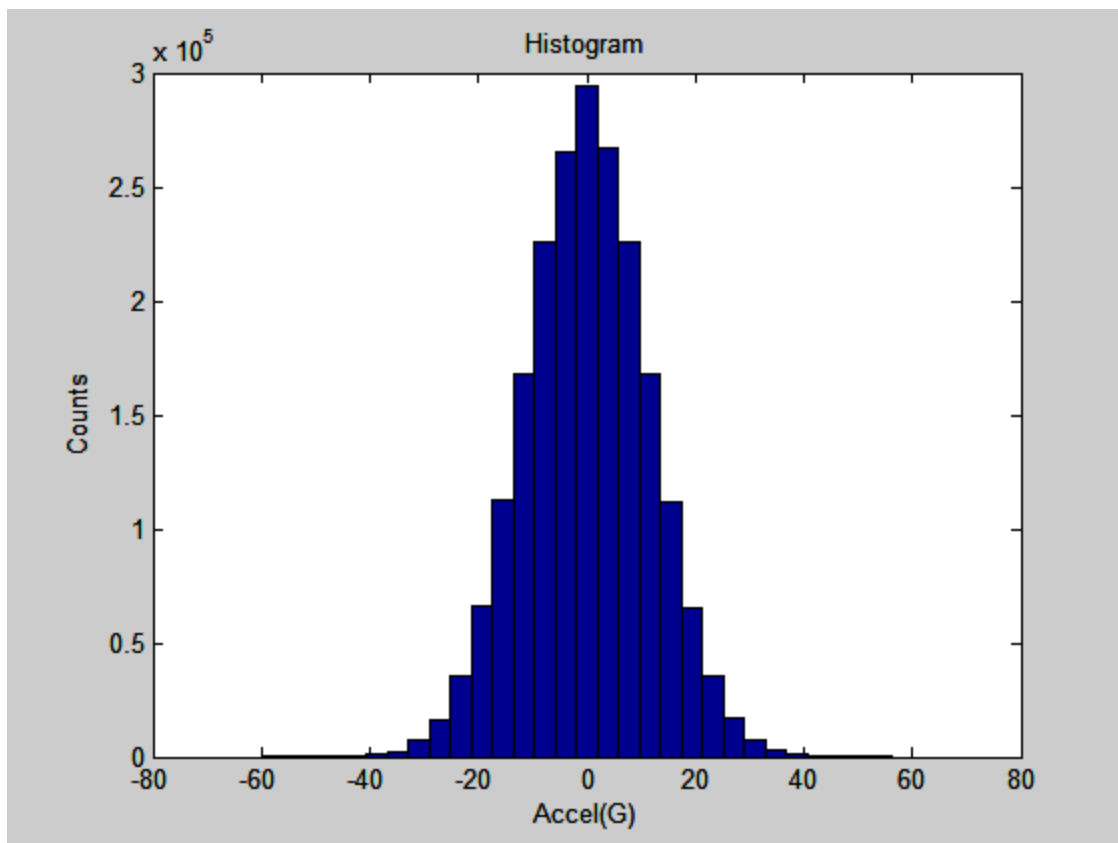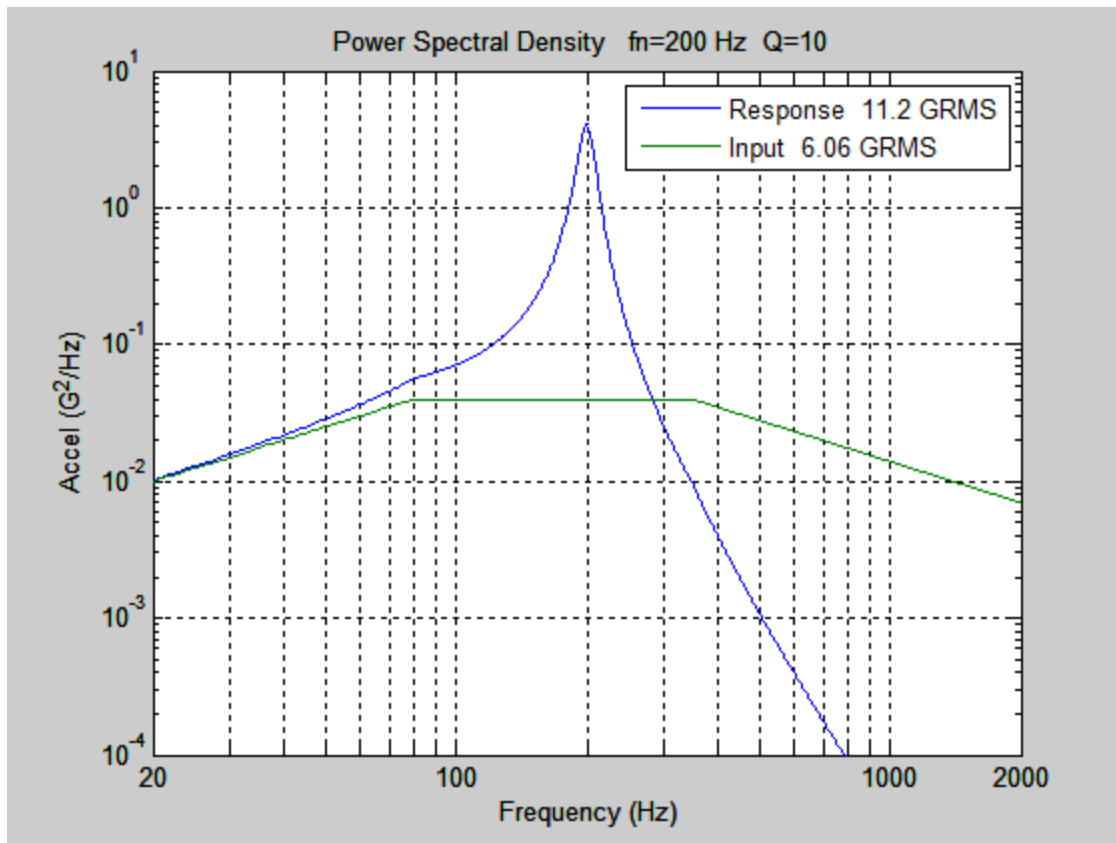
Figure 6.

Figure 7.

The response PSD is calculated from the specification using the method in Reference 6. The rainflow cycles were then calculated using the Dirlik method in References 2 and 3.

The relative damage index from the rainflow cycles for each method is shown in Table 1, as calculated by applying the rainflow cycles to the formula in Appendix A.

| Table 1. Damage Comparison, fn =200 Hz | | |
|---|---|---|
| Method | Damage Index | Cycles |
| Time Domain | 4.788e+12 | 13839 |
| Dirlik | 4.003e+12 | 13928 |

The damage index for the time domain is 20% higher than that of the Dirlik method, which is actually small for a fatigue calculation given the exponent applied to the amplitude.

The difference may be due to even minor variations of the response time history from the ideal normal distribution.
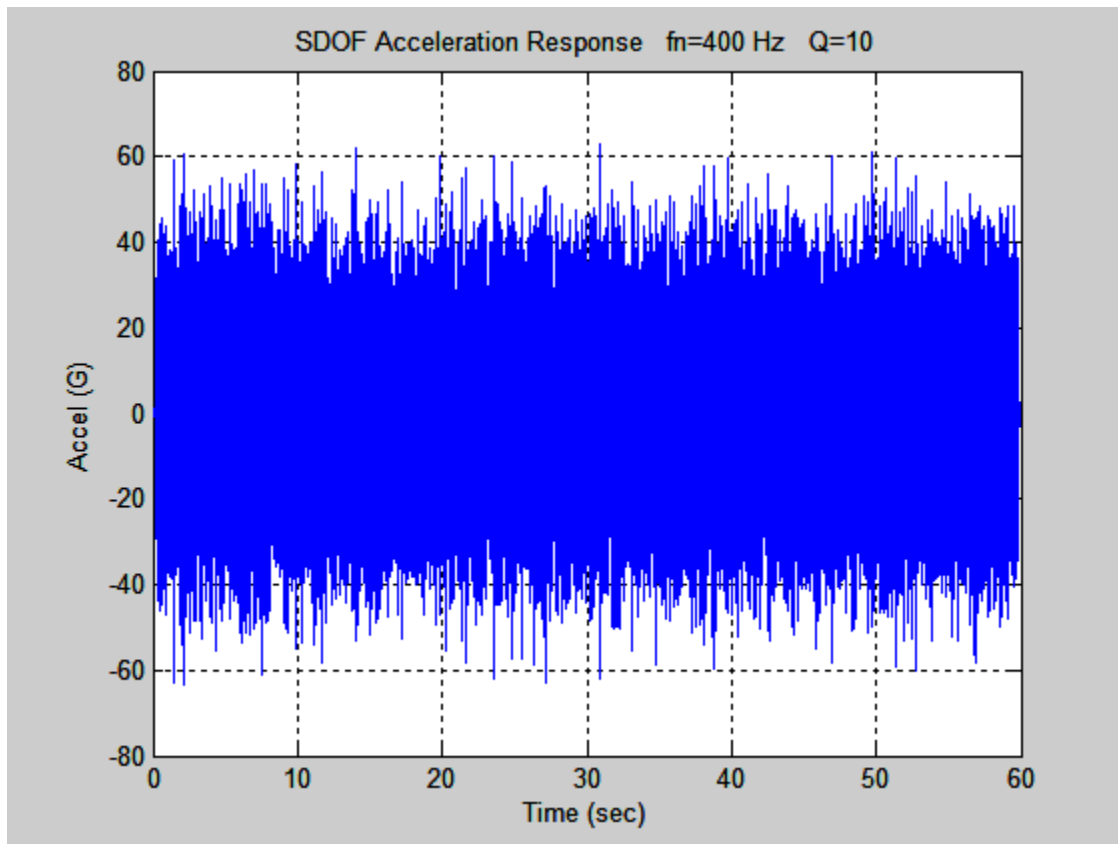
Example 2, fn=400 Hz



Figure 8.

The response time history for a natural frequency of 400 Hz is shown in Figure 8. It has a normal histogram as shown in Figure 9.

The rainflow cycles were then calculated using the method in Reference 1.
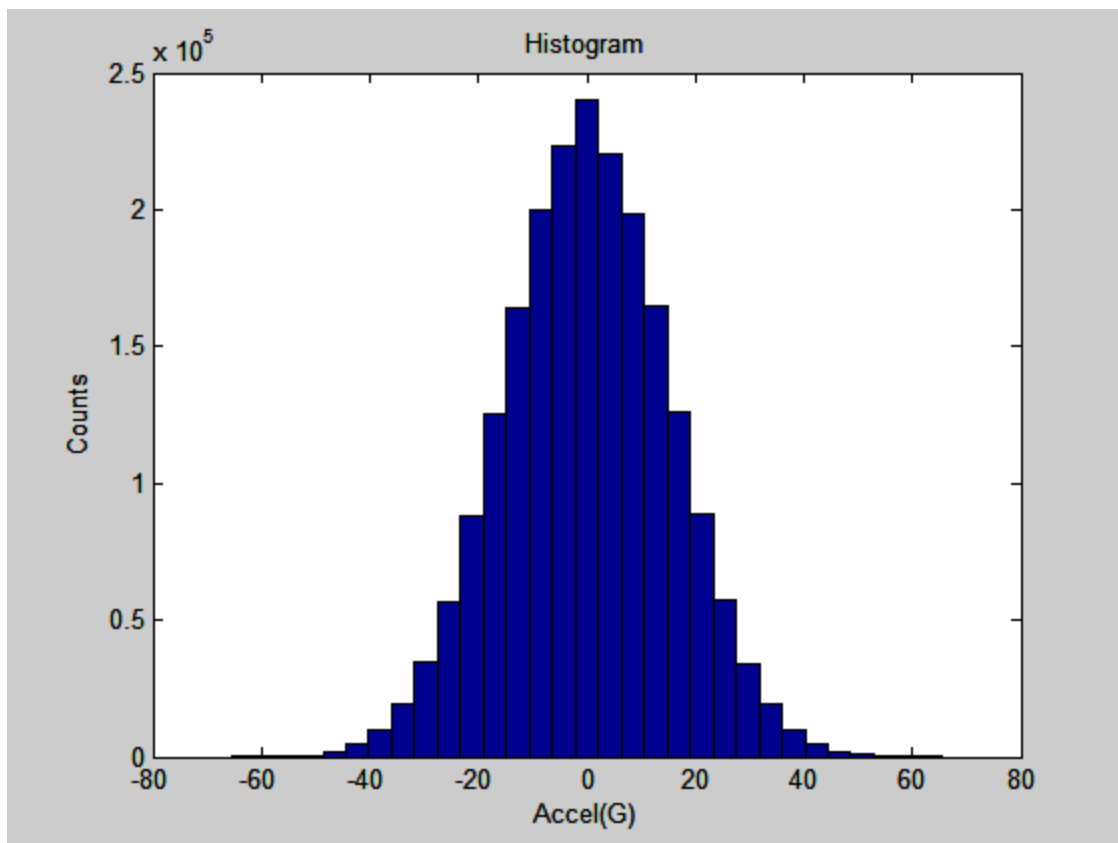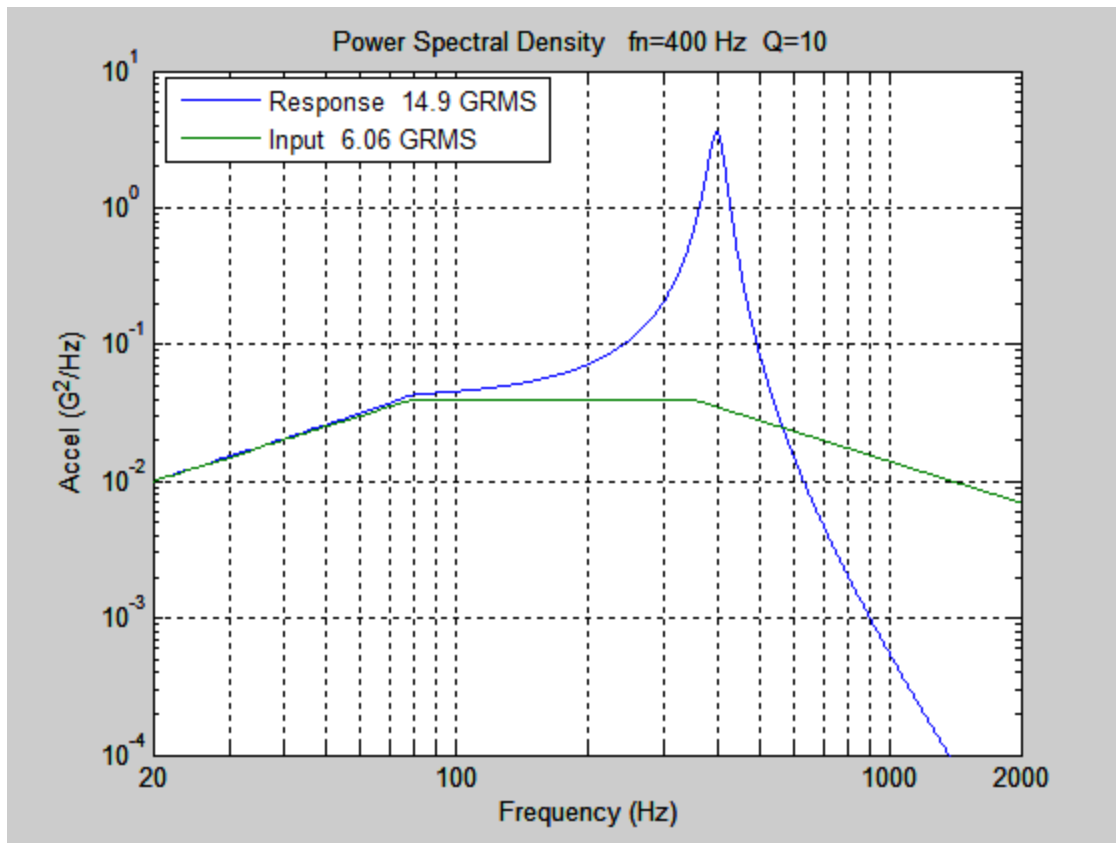
Figure 9.

Figure 10.

The response PSD is calculated from the specification using the method in Reference 6. The rainflow cycles were then calculated using the Dirlik method in References 2 and 3.

The relative damage index the rainflow cycles for each method is shown in Table 1, as calculated by applying the rainflow cycles to the formula in Appendix A.

| Table 2. Damage Comparison, fn =400 Hz | | |
|---|---|---|
| Method | Damage Index | Cycles |
| Time Domain | 5.292e+13 | 25233 |
| Dirlik | 4.827e+13 | 25508 |

The damage index for the time domain is 10% higher than that of the Dirlik method, which is a small difference.

## Conclusions

The Dirlik and time domain methods gave reasonably similar results for two cases. Further cases should run by varying the amplification factor, natural frequency, fatigue exponent, duration, etc. Multi-degree-of-freedom response should also be considered.

## References

1. ASTM E 1049-85 (2005) Rainflow Counting Method, 1987.

2. Halfpenny & Kim, Rainflow Cycle Counting and Acoustic Fatigue Analysis Techniques for Random Loading, RASD 2010 Conference, Southampton, UK.

3. Halfpenny, A frequency domain approach for fatigue life estimation from Finite Element Analysis, nCode International Ltd., Sheffield, UK.

4. T. Irvine, A Method for Power Spectral Density Synthesis, Rev B, Vibrationdata, 2000.

5. David O. Smallwood, An Improved Recursive Formula for Calculating Shock Response Spectra, Shock and Vibration Bulletin, No. 51, May 1981.

6. T. Irvine, An Introduction to the Vibration Response Spectrum, Revision D, Vibrationdata, 2009.

## APPENDIX A

A relative damage index D can be calculated using

$$D = \sum_{i=1}^{m} A_i^b \, n_i \tag{A-1}$$

where

$A_i$     is the *response* amplitude from the rainflow analysis

$n_i$     is the corresponding number of cycles

$b$     is the fatigue exponent

Note that the amplitude convention for this paper is (peak-valley)/2.

Dirlik Method

The *n*th spectral moment $m_n$ for a PSD is

$$m_n = \int_0^\infty f^n \, G(f) \, df \qquad \text{(B-1)}$$

where

       $f$       is frequency

     $G(f)$     is the one-sided PSD

The expected peak rate $E[P]$ is

$$E[P] = \sqrt{m_4/m_2} \qquad \text{(B-2)}$$

The Dirlik histogram formula $N(S)$ for stress cycles ranges is

$$N(S) = E[P] \cdot T \cdot p(S) \qquad \text{(B-3)}$$

where

       T       is the duration

       S       is the stress cycle range (peak-to-peak)

The function $p(S)$ is

$$p(S) = \frac{\dfrac{D_1}{Q}\exp\left(-\dfrac{Z}{Q}\right) + \dfrac{D_2 \, Z}{R^2}\exp\left(\dfrac{-Z^2}{2R^2}\right) + D_3 \, Z \exp\left(\dfrac{-Z^2}{2}\right)}{2\sqrt{m_0}} \qquad \text{(B-4)}$$

The coefficients and variables are

$$D_1 = \frac{2\left(x_m - \gamma^2\right)}{1 + \gamma^2} \tag{B-5}$$

$$D_2 = \frac{1 - \gamma - D_1 + D_1^2}{1 - R} \tag{B-6}$$

$$D_3 = 1 - D_1 - D_2 \tag{B-7}$$

$$Z = \frac{S}{2\sqrt{m_0}} \tag{B-8}$$

$$Q = \frac{1.25\left(\gamma - D_3 - D_2 R\right)}{D_1} \tag{B-9}$$

$$R = \frac{\gamma - x_m - D_1^2}{1 - \gamma - D_1 + D_1^2} \tag{B-10}$$

$$\gamma = \frac{m_2}{\sqrt{m_0\, m_4}} \tag{B-11}$$

$$x_m = \frac{m_1}{m_0}\sqrt{\frac{m_2}{m_4}} \tag{B-12}$$

A cumulative histogram of the peaks can then be calculated from equation (B-3).

The stress range of individual cycles can then be obtained by interpolating the cumulative histogram.