SIMPLE SHOCKS HAVE SIMILAR SHOCK SPECTRA WHEN PLOTTED AS PVSS ON 4CP

By Howard A. Gaberson, Ph.D., P.E.
Consultant
234 Corsicana Dr.; Oxnard, CA 93036; (805) 485-5307
hagaberson@att.net

ABSTRACT

The pseudo velocity shock spectrum plotted on four coordinate paper (PVSS on 4CP) emphasizes shock motion severity. This paper applies this fact to the theoretical half sine and other simple shocks. By adding the drop acceleration and comparing simple drop table shock machine shocks with the same velocity change during the impact, I show all have essentially the same PVSS. But when the calculations are applied to shaker generated simple shock motions, they turn out not similar and they seriously lack low frequency content. I also consider how simple shocks can be related to actual explosive transients.

INTRODUCTION

Previous work has shown the traditional four coordinate pseudo velocity shock spectrum (PVSS on 4CP) is the best format for violent machine transient foundation motion analysis. [1] The pseudo velocity shock spectrum plotted on four coordinate paper emphasizes shock motion severity. This paper applies this fact to the theoretical half sine and other simple shocks. By adding the drop acceleration and comparing simple drop table shock machine shocks with the same velocity change during the impact (which means same drop height), I show all have essentially the same PVSS. The trick turns out to be inclusion of pre and post shock motion in the calculation and comparing shocks with the same impact velocity change. But when the calculations are applied to shaker generated simple shock motions, they turn out not similar; they seriously lack low frequency content. I also show how the simple shocks can be related to actual explosive transients. Focusing on the velocity change during impact helps.

I'll say there are five simple shocks, and they are the half sine, haversine, initial and final peak saw tooth, and trapezoidal shocks. You will also find analyses of triangular shocks with various asymmetries, and square shocks. Many pioneers of our business went on and on calculating positive and negative, initial and residual acceleration shock spectra of the simple shocks, and the called them shock response spectra (SRS's). The mistake they made was to consider only the SRS, and to normalize all of the spectra to peak acceleration. Typical references include Mindlin[2], Jacobsen and Ayre[3], Ayre[4], Rubin[5], all the famous of shock analysis authors. Their analyses no longer need to be studied. Lalanne [6] did a sensible careful analyses of the kinematics, as he terms it, of the simple shocks, but never tried to calculate PVSSs of them; at the time he wrote his book, he was not emphasizing PVSS theory. One author pair, Gertel and Holland [7], got it correct but everybody ignored them. Anyhow, to try to set matters straight, this paper shows you why the simple shocks are all basically the same, and equally good test shocks. This paper is an expansion of a paper I wrote in 2004. [8]

To make the point of simple shock PVSS similarity, I have to calculate the PVSS on 4CP (pseudo velocity shock spectrum plotted on four coordinate paper) for all the simple shocks. The way I show them similar is to show the PVSSs separately and then superpose all of their spectra on a single composite plot. Below I will describe the characteristics of each shock and the compromises I had to make to establish the truth of the title of this paper. I have assumed no rebound in the following simple shock analyses. Experts have told me that this is generally not the case. However, to keep things uncomplicated I will proceed with the no rebound analyses. Rebound affects the low frequency asymptote and this is carefully discussed in an appendix available from me.

HALF SINE SHOCK

Half sine shocks are usually specified by a peak acceleration and a duration, such as an 11 millisecond 30g half sine. To develop a formula, I'll say the shock has peak acceleration, $\ddot{x}_{max}$, and duration t_d. The frequency associated with the half sine duration has a period of twice this duration, and since the frequency is one over the period, the frequency associated with the half sine is given by Eq (1a).

$$f_d = \frac{1}{2t_d} \tag{1a}$$

The formula for our half sine shock is given by Eq (1).

$$\ddot{x} = \ddot{x}_{\max} \sin 2\pi f_d t \tag{1}$$

The velocity change during the half sine pulse is very important. Assume zero initial velocity and integrate over the half cycle to get the final velocity, which is the velocity change.

$$\Delta\dot{x} = \int_0^{t_d} \ddot{x}_{\max} \sin 2\pi f_d t dt = \ddot{x}_{\max} \left[\frac{-1}{2\pi f_d} \cos 2\pi f_d t \right]_0^{t_d} = \frac{-\ddot{x}_{\max}}{2\pi f_d} \left[\cos\pi - 1 \right] = \frac{\ddot{x}_{\max}}{\pi f_d} \tag{2a}$$

I'll express the result in terms of duration using Eq (1a) in Eq (2).

$$\Delta\dot{x} = \frac{2\ddot{x}_{\max} t_d}{\pi} \tag{2}$$

This result is important. It relates three important pulse properties: velocity change, peak acceleration, and duration. Two of these, $\Delta\dot{x}$, $\ddot{x}_{\max}$, can be read off the pseudo velocity shock spectrum. The shock spectrum of this pulse by itself is what is usually computed. A package is on the table and something delivers a half sine shock upward in a zero gravity situation and the package and table continue going off into space at constant velocity. This is unrealistic, and leaves the low frequency asymptote or displacement limit out. Yet we find this in every book on shock that presents the half sine shock spectrum. I want you to see why this is incomplete.

I'm going to plot many shock spectra in what follows and I'm going to select fairly severe parameters so you can get familiar with severe shock spectra. I will use a velocity change of 100 ips and peak acceleration level of 200 g. Let's plot the above shock along with its two integrals, and then calculate the PVSS on 4CP for this unrealistic half sine shock. Using Eq (2) we find the duration to be 2.034 ms. Figure 1 shows the acceleration shock and its two integrals. The PVSS for it undamped and with damping of 5%, is given in Figure 2.

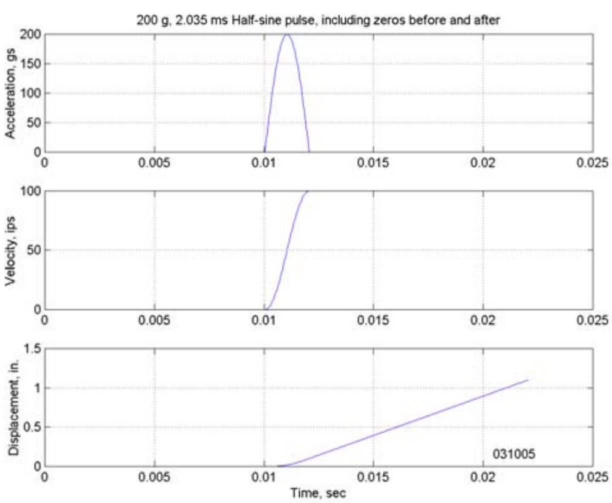


Figure 1. Acceleration, velocity, and displacement of a half sine acceleration shock preceded and followed by zeros. The shock has a velocity change of 100 ips; and continues at this velocity forever. This is an unrealizable shock.

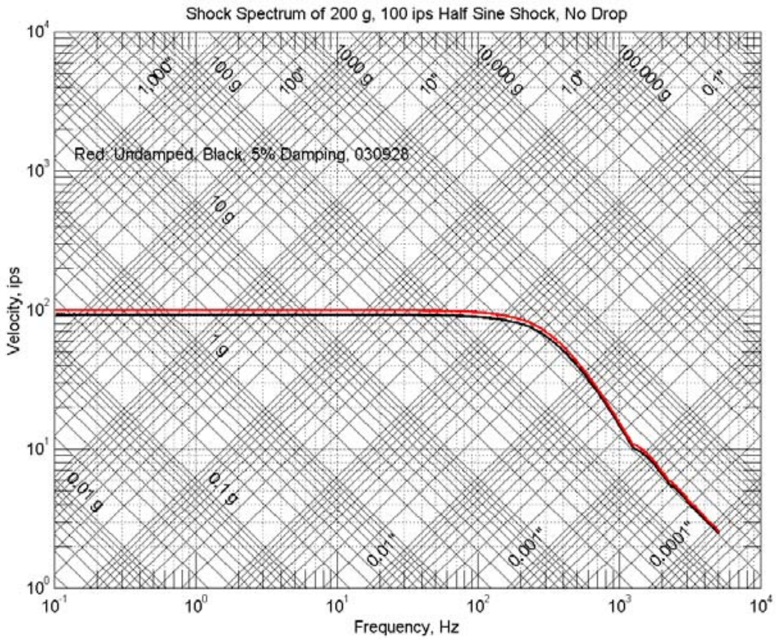


Figure 2. The PVSS of a 200 g 100 ips half sine acceleration shock. In the lower right corner the spectra are asymptotic to 200 g. From about 100 Hz down to 0.1 Hz the undamped spectrum shows a constant velocity of 100 ips.

In Figure 2, the constant velocity of 100 ips down to 0.1 Hz probably did not happen. The PVSS plotted on 4CP indicates that the shock would cause the 0.1 Hz SDOF a deflection of 160 inches or 13.3 ft. Assuming it was done in drop table shock machine; we would seldom have 13 ft available, and if so a 13 ft drop would cause a much greater velocity change than 100 ips. .This what you are liable to see, and I don't like. The velocity change of 100 ips shows up as it should and the acceleration level is asymptotic to 200 g's, as expected. The plot doesn't show how low a frequency could be excited to 100 ips. It's an excellent plot of an analysis of a unreal pulse.

Now we consider a realistic half sine shock. We drop it through a distance, such that a shock programmer delivering our half sine just brings it to rest. The velocity begins and ends at zero, thus the shock acceleration will have a zero mean. To calculate the drop height, consider a 1 g drop for a time t_{drop} so that the area (g times t_{drop}) equals the impact velocity change, thus Eq (3a) will give t_{drop}.

$$g t_{drop} = \Delta \dot{x}, \;\; or \;\; t_{drop} = \frac{\Delta \dot{x}}{g} \tag{3a}$$

For a freely falling body starting from rest, the velocity will be gt, and the displacement will be $h = \frac{1}{2} g t^2$.

Substituting the t_{drop} value from Eq (3a), we get Eq (3b) for the necessary drop height. The time history plot and the integrals are shown in Figure 3.

$$h = \frac{\Delta \dot{x}^2}{2g} \tag{3b}$$

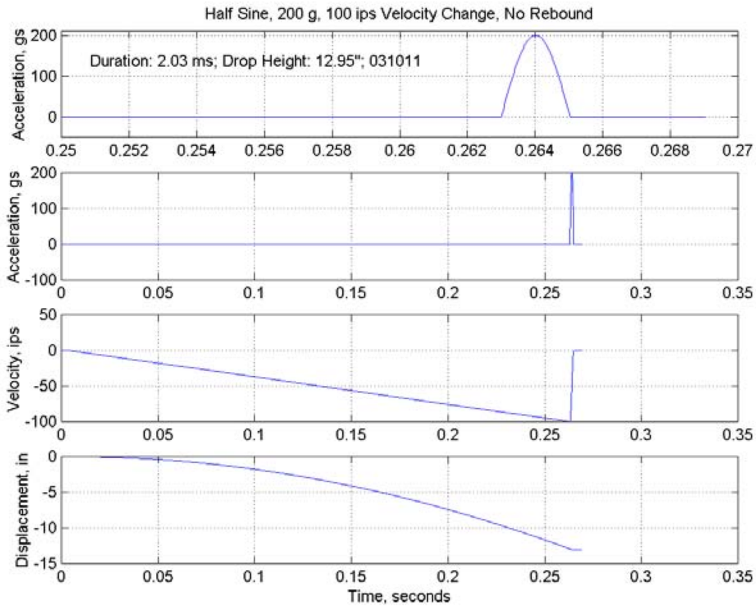


Figure 3 A 200 g, 100 ips half sine shock preceded by a 12.95 inch drop, and its two integrals. The top subplot has an expanded x-axis to show the actual pulse shape.

This is a shock that could occur. Its undamped and 10% damped PVSS on 4CP are shown in Figure 4. Notice the low frequency limit of the 5% damped shock spectrum is about 2 Hz, and the high frequency limit is 200 Hz. This is the frequency range for which the shock is severe. Notice also, as the spectrum droops downward to the right heading for the acceleration asymptote, that for a short range of frequencies, 300 - 600 Hz, that the acceleration nearly doubles over the 200 g asymptote level. This occurs to some extent on all five simple shock PVSSs. I will refer to this as doubling in the droop zone, and will bring it up four more times.

SIMPLE SHOCK PVSS CHARACTERISTICS

Figure 4 illustrates important concepts that I emphasize when teaching shock data analysis. The PVSS on 4CP of a simple shock has a shape like a flattened hill. The top of the hill, the plateau, is the impact velocity change for an undamped PVSS, somewhat less in a damped PVSS analysis. The hill slopes down and to the right with an asymptote equal to the peak acceleration of the shock. The high frequency asymptote is the maximum shock acceleration. The hill slopes down and to the left with an asymptote equal to the maximum shock displacement. The low frequency asymptote is the maximum displacement. You will see this on all simple shocks. I carefully explain this in [1]. Simple shocks begin and end with zero velocity, and have a maximum displacement. If they didn't end with zero velocity the displacement would increase without bound. That is why we will see that all simple shocks have similar PVSS of 4CP when scaled to the same impact velocity change. That's what I'm going to show you, and is the main point of the paper.

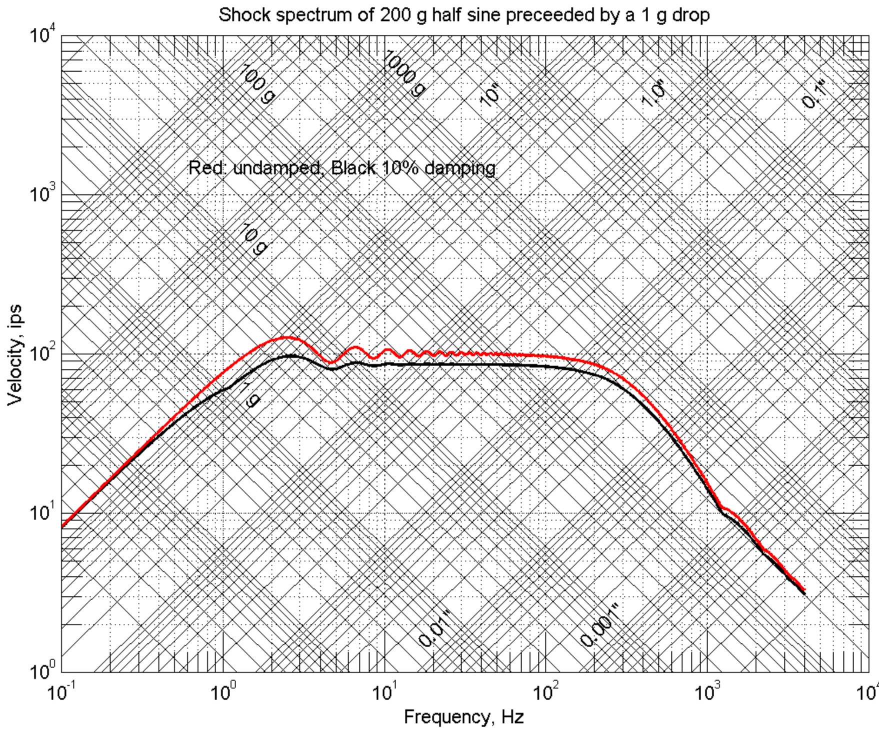


Figure 4. PVSS of 200 g, 100 ips half sine, preceded by a 1 g drop. Notice now we see the 13" asymptote at the low frequencies. The 100 ips velocity change and the 200 g peak asymptote clearly. Notice however, as the spectrum droops downward to the right heading for the acceleration asymptote, that for a short range of frequencies, 300 - 600 Hz, that the acceleration nearly doubles over the 200 g asymptote level. This occurs on all five shock PVSS's. I call this as doubling in the droop zone.

TRAPEZOIDAL SHOCK

Abrupt rise times, while interesting to the academic community, don't occur in shocks. Trying to deal with these practically requires discussion and I'll deal with that here. As I hope you realize, the PVSS on 4CP shows the potential velocity or stress the shock can develop in test items. Thus our tool for evaluating slight changes in simple shock shapes is the effect on the PVSS.

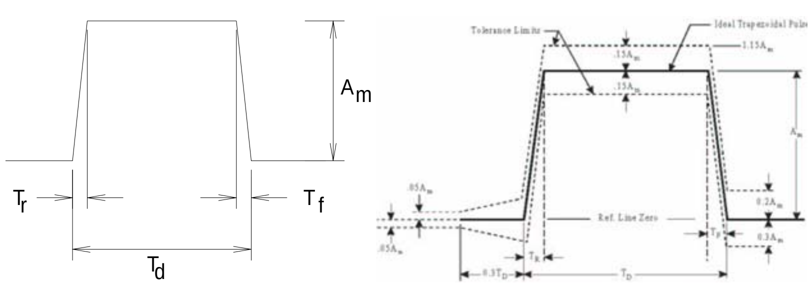


Figure 5. This is a trapezoidal shock of duration T_d, and maximum acceleration amplitude, A_m, expressed in g. It has a rise time, T_r, and a fall time of T_f. Next to it is a copy of the drawing from Mil Std 810 F. [9]

Figure 5 shows a trapezoidal shock of duration T_d, and maximum acceleration amplitude, A_m, expressed in g. It has a rise time, T_r, and a fall time of T_f. These symbols and definitions are taken from MIL-STD 810F, [9] Figure 516.5-11, on page 516.5-16 which is reproduced above. The area of this pulse is equal to the velocity change it causes. Lets add up these two triangular areas and the rectangular center in Eq (4a) and take three lines to show the simplifications. Here I'll take g_0 to mean the numerical value 386.087 in/sec^2, and A_m to mean the acceleration in g.

$$\Delta V = \tfrac{1}{2} A_m g_0 T_r + A_m g_0 \left(T_d - T_r - T_f\right) + \tfrac{1}{2} A_m g_0 T_f \;= A_m g_0 \left(T_d - \tfrac{1}{2} T_r - \tfrac{1}{2} T_f\right) \qquad (4a)$$

For the ideal case we are calculating we will take the rise and fall times equal and given by ϕ times the pulse duration. This substitution of, ϕ, times the shock duration, T_d makes the result tidy as shown in Eq (4b).

$$\Delta V = A_m g_0 \left(T_d - \tfrac{1}{2}\phi T_d - \tfrac{1}{2}\phi T_d\right) = (1-\phi) A_m g_0 T_d \qquad (4b)$$

Using better symbols we have the linear ramp trapezoidal equation relating velocity change, peak acceleration, and duration given in Eq (5a).

$$t_d = \frac{\Delta \dot{x}}{\left(1-\phi\right)\ \ddot{x}_{\max}} \qquad (5a)$$

The acceleration equations for the three time intervals are given in Eqs (5 b, c, d.)

$$\begin{aligned} \ddot{x} &= \frac{\ddot{x}_{\max}}{\phi t_d} t, \quad 0 \le t \le \phi t_d, \\ \ddot{x} &= \ddot{x}_{\max}, \quad \phi t_d \le t \le (1-\phi) t_d, \\ \ddot{x} &= \frac{\ddot{x}_{\max}}{\phi t_d}\left(t_d - t\right), \quad (1-\phi) t_d \le t \le t_d \end{aligned} \qquad (5\ b,c,d)$$

I wrote a linear ramp trapezoid shock MATLAB script, halftrapz.m, to calculate the time history preceded by 200 zeros, the 1 g drop to attain the 100 ips, the ramp up, the flat portion, and the ramp down followed by 200 zeros. (I wrote programs to do this for all 5 simple shocks. I'll be happy to send them to you.) This time history for $\phi = 0.1$ acceleration and its two integrals are given in Figure 6 Notice that the top subplot has an expanded time scale to show the shock shape.

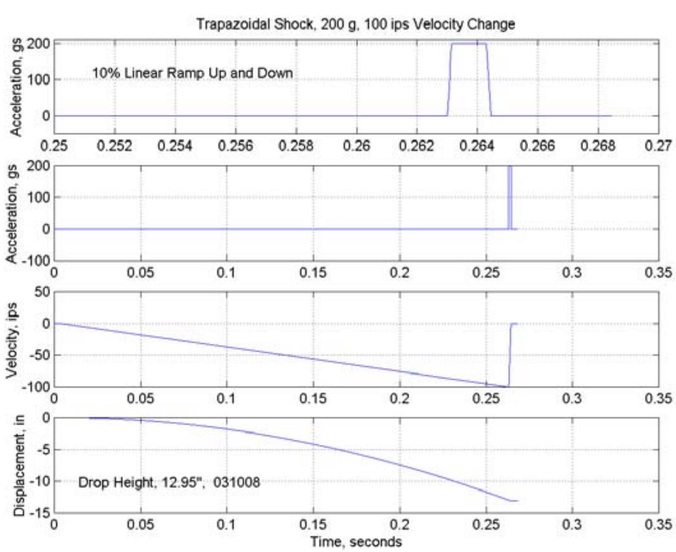


Figure 6. This is a time history for a 10% linear ramp trapezoidal shock. The drop is 12.95 inches. The pulse duration turned out to be 1.44 mili-seconds.

To illustrate "doubling in the droop zone", I want to first show the undamped PVSS of a 5% linear ramped trapezoid in Figure 7a. This is what I mean by the difficulty with abrupt rise times. The 100 ips flat portion is severe from about 1.5 - 200 Hz. Notice, however, that the acceleration is mostly at 400 g from about 400 Hz to about 4000 Hz, instead of the 200 g asymptote I want you to expect. The spectrum is beginning its dive to 200 at just about 4000 Hz. I'll show you that this occurs on all the calculated abrupt rise time simple shocks I've done. I'm sure this is due to the impulsive rise time of the trapezoidal acceleration.

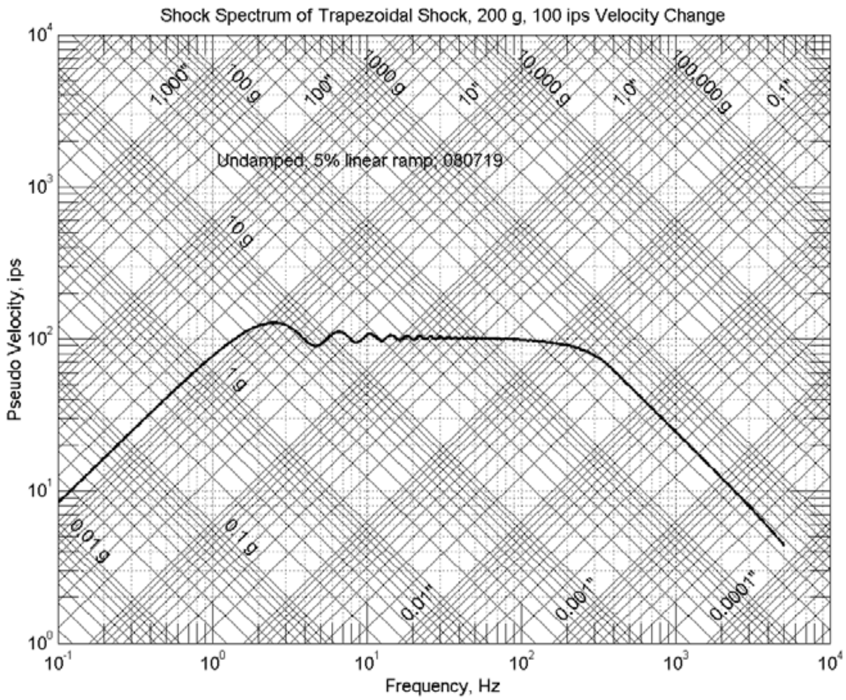


Figure 7a. Undamped shock spectrum for 200g, 100 ips velocity change trapezoidal shock with a 5% linear ramp. Notice that the acceleration asymptote is a 400 g most of the way.

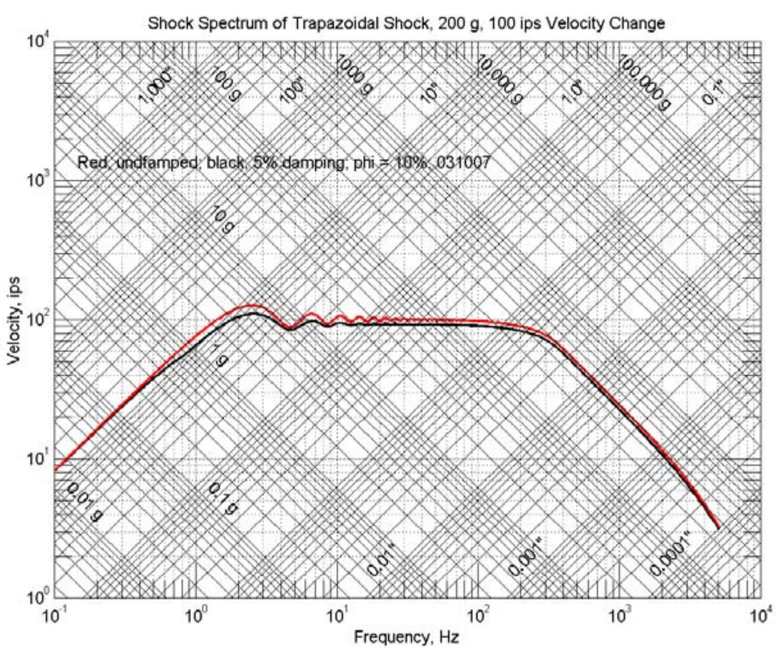


Figure 7. The 0 and 5% damped shock spectra for the 10% linear ramp trapzoidal shock of Figure 6.

The damped and undamped shock spectra for Figure 6, the 10% linear ramp trapezoidal shock are given in Figure 7. The 100 ips flat portion is severe from about 1.5 - 200 Hz. Notice that the acceleration asymptote is mostly at 400 g from about 400 Hz to 2000 Hz, instead of the 200 we are expecting. This trapezoidal shock had a 10% linear rise and fall, and now at 5000 Hz it is almost down to 200 g's. Even with ϕ changed to 10%, things improve but we are still not down to 200 g's at 5000 Hz.

Now I redo the trapezoidal shock with a half cosine ramp up; again let the ramp duration be ϕt_d. Figure 8 shows the portion of the cosine I used.

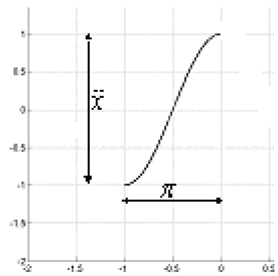


Figure 8. Cosine ramp geometry.

The frequency of this cosine ramp has to be such that during the rise time interval, 0 to ϕt_d, the argument of the cosine increases by π. Thus the frequency of the ramp has to given by Eq 6.

$$2\pi f_r \phi t_d = \pi, \quad \text{or} \quad f_r = \frac{1}{2\phi t_d} \tag{6}$$

The acceleration during the ramp interval is given by Eq (7). The argument of the cosine has to look like $2\pi ft$; using f_r from Eq (6) , we develop Eq (7).

$$\ddot{x} = \ddot{x}_{max}\frac{1}{2}\left(1-\cos\frac{\pi t}{\phi t_d}\right), \quad \text{for} \quad 0 \le t \le \phi t_d \tag{7}$$

The area (or the velocity change) during the cosine ramp is found by integrating Eq (7) in Eq (8a).

$$\frac{1}{\ddot{x}_{max}}\int_0^{\phi t_d}\ddot{x}dt = \frac{\Delta\dot{x}_c}{\ddot{x}_{max}} = \frac{1}{2}\int_0^{\phi t_d}\left(1-\cos\frac{\pi t}{\phi t_d}\right)dt = \frac{1}{2}\left[t - \frac{\phi t_d}{\pi}\sin\frac{\pi t}{\phi t_d}\right]_0^{\phi t_d} \tag{8a}$$

Evaluating the limits yields the simple answer shown in Eq (8b)

$$\frac{\Delta\dot{x}_c}{\ddot{x}_{max}} = \frac{1}{2}\left[\phi t_d - 0 - \frac{\phi t_d}{\pi}(0-0)\right] = \frac{\phi t_d}{2}; or\ \frac{\Delta\dot{x}_c}{\ddot{x}_{max}} = \frac{\phi t_d}{2} \tag{8b}$$

Thus the total velocity change is (with a cosine ramp on the beginning and the end of the trapezoid) is evaluated in Eq (8c).

$$\Delta\dot{x} = 0.5\phi t_d \ddot{x}_{\max} + \ddot{x}_{\max}\left(t_d - \phi t_d - \phi t_d\right) + 0.5\ddot{x}_{\max}\phi t_d = \Delta\dot{x} = t_d \ddot{x}_{\max}\left(1-\phi\right) \tag{8c}$$

This is actually the same as Eq (5a) which can be seen by rearranging to obtain Eq (8d). The cosine ramp has the same integral as the linear ramp.

$$t_d = \frac{\Delta\dot{x}}{\left(1-\phi\right)\ \ddot{x}_{\max}}; \quad or \quad \Delta\dot{x} = \left(1-\phi\right) t_d \ddot{x}_{\max} \tag{8d}$$

The acceleration equations for the cosine ramp up, and the flat center region are given in Eqs (8 e). I didn't try to figure out the relation for the ramp down because Matlab has a flip left right function, and I just flipped the ramp up and appended it to that flat portion.

$$\ddot{x} = \ddot{x}_{\max}\frac{1}{2}\left(1-\cos\frac{\pi t}{\phi t_d}\right), \quad 0 \le t \le \phi t_d \ , \tag{8 e}$$
$$\ddot{x} = \ddot{x}_{\max}\ , \quad \phi t_d \le t \le \left(1-\phi\right) t_d$$

Again I wrote a cosine ramp trapezoid shock MATLAB script, halftrapcos.m, to calculate the time history preceded by 200 zeros, the 1 g drop to attain the 100 ips, the ramp up and the flat portion. The ramp down is obtained by flipping the ramp up left right and appending to the flat portion, followed by the 200 zeros.

Now to justify how much cosine ramp is reasonable, I graphically placed a 30% ramp trapezoid shock within the confines of the IEC Specification, according to Figure 3, on p 40. [10] I did this in Matlab and show the result in Figure 9.

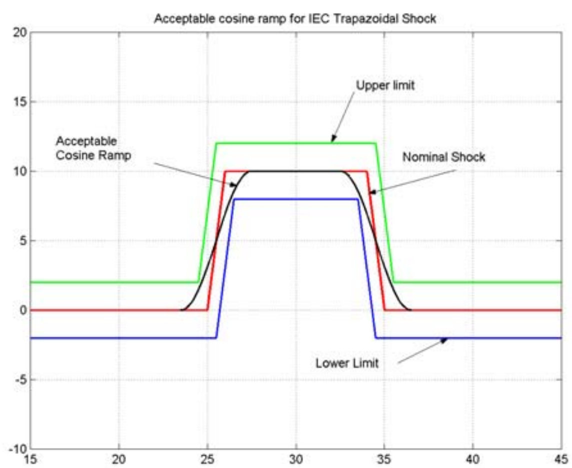


Figure 9, The figure shows how 30% cosine ramped trapezoid fits in the limits of the IEC shock specification. [10]

Now studying this picture, their nominal trapezoid has a linear ramp with phi = 0.1; the blue and green limits allow one to increase t_d by 0.4, and then use a cosine with a $\phi = 0.3$. It seems like rather than get all confused by the nominal and a permissible duration, the I just use this as proof that it is acceptable in some circumstances to use a $\phi = 0.3$, and see what this yields in the spectrum. Lets look at the time history of a 30% trapezoid shock in Figure 10. This time history for phi = 0.3 and its two integrals are given in Figure 10. Notice that the top subplot has an expanded time scale to show the shock shape.

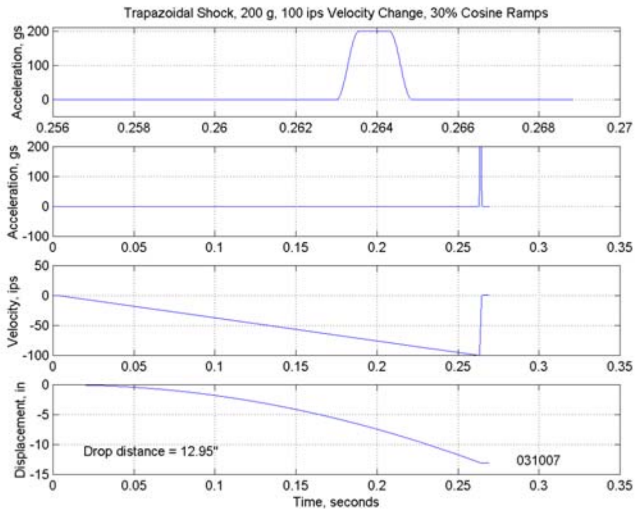


Figure 10. Time history and integrals for a 30% cosine ramp trapezoidal shock. The time duration for this pulse came out to be 1.85 mili-seconds.

Now this is sufficient rise time to be able to see the 200 g asymptote of the PVSS shown below in Figure 11.

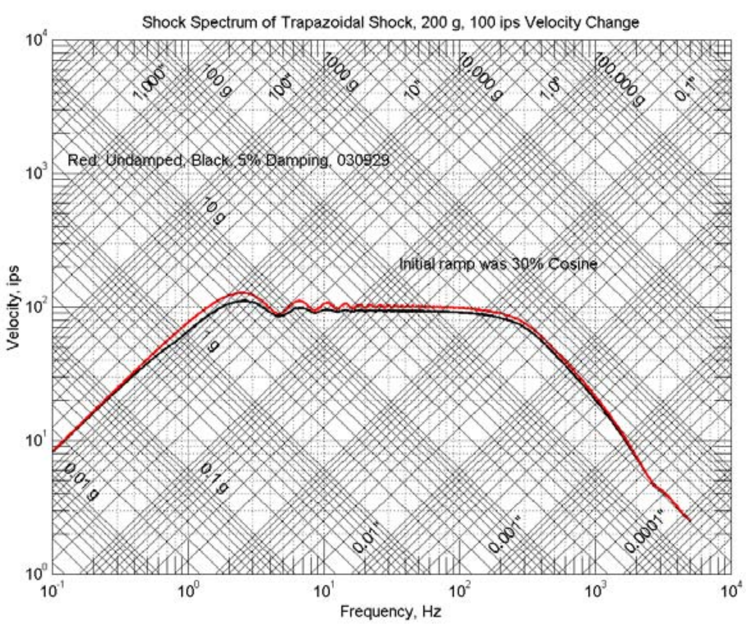


Figure 11. PVSS for a 30 % cosine ramp trapezoidal shock. Notice that finally it reaches its 200 g asymptote at 2500 Hz. Notice that finally it reaches its 200 g asymptote at 2500 Hz.

Notice that this 30% ramp smoothes it enough to permit it to dive down to 200 g at about 2500 Hz. The five percent damped spectrum is now as expected. I have gone over this to show that all the simple shocks have the expected simple shock asymptotes, The abrupt rise times double the peak g level over a range before the peak g asymptote appears.

HAVERSINE SHOCK

The haversine shock (also called a versed sine) is a raised cosine shape. It is shown in Figure 12. The haversine shock is another classical simple shock. It happens to be a cosine ramped trapezoid shock with ϕ = 50%, so I can use my Matlab script halftrapcos.m with ϕ = 50%, to generate it. The equation relating peak acceleration, velocity change, and duration is obtained from Eq (8d) with $\phi = 0.5$, and is shown in Eq (8f).

$$t_d = \frac{2\Delta\dot{x}}{\ddot{x}_{\max}}$$
$$\Delta\dot{x} = \frac{t_d\ddot{x}_{\max}}{2} \tag{8f}$$

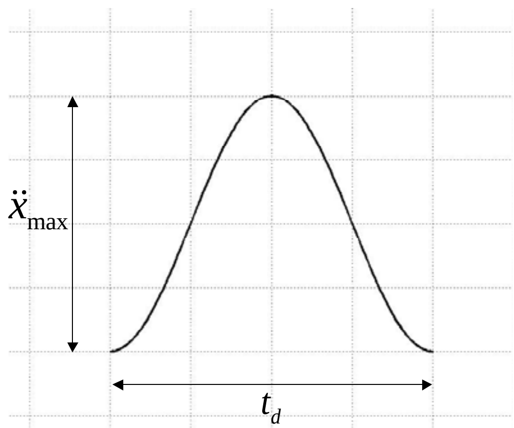


Figure 12. Drawing of Haversine, or versed sine shock.

Figure 13. shows a time history of the 100 ips, 200 g haversine shock we will use for comparison. Finally the PVSS of this haversine shock is shown in Figure 14. It is what I expect from the more gentle initial rise than the half sine.

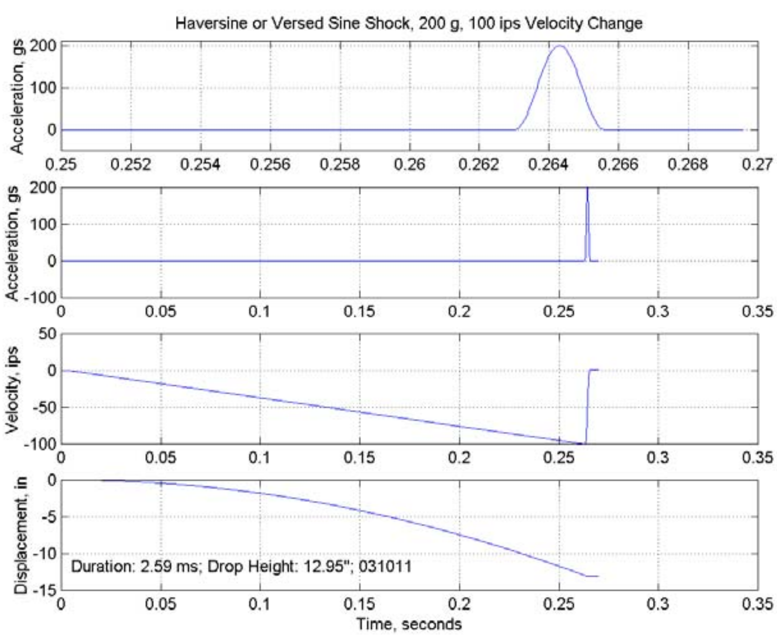


Figure 13. Time history and integrals of haversine shock. Top subplot shows expanded time scale to see shock shape detail.

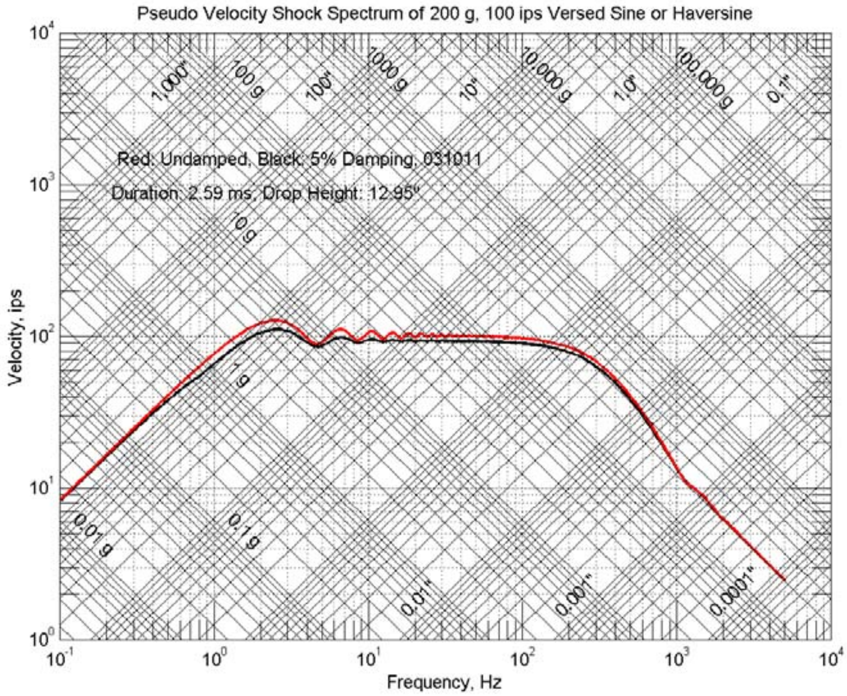
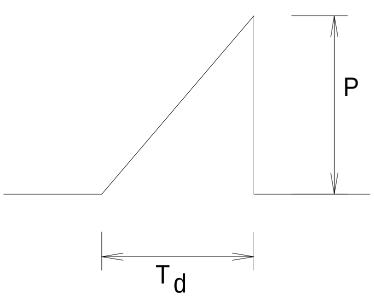


Figure 14. PVSS of haversine shock. It comes back to its 200 g asymptote at just over 1000 Hz.

Comparing Figure 14, the haversine PVSS, and Figure 4 the half sine PVSS, the haversine PVSS has a little less doubling in the droop zone, and comes to the 200 g asymptote slightly sooner than the half sine.

TERMINAL PEAK SAWTOOTH

The saw tooth shocks are also common shocks, terminal peak being the most common. Papers were written claiming this was a very good test shock for various reasons; they are unimportant



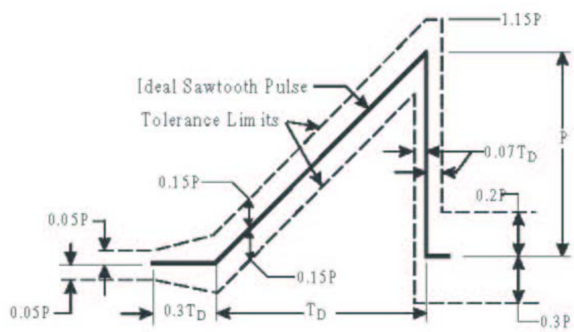


Figure 15a. Terminal peak saw tooth drawing with symbols from Mil-S-810-F, Method 516.5, SHOCK, and Figure 516.5-10. Figure 15b. This is the exact figure of the terminal peak shock reproduced from Mil Std 810F [9]

The velocity change associated with the ideal shock is its area, one half of the height times the duration as expressed in Eq (9). (g_0 = 386.087 in/sec^2)

$$\Delta V = 0.5 P g_0 T_d \tag{9}$$

I'll write this relation in terms I like better in Eq (9a).

`
$$t_d = \frac{2\Delta\dot{x}}{\ddot{x}_{\max}}$$
$$\Delta\dot{x} = \frac{t_d\ddot{x}_{\max}}{2} \qquad (9a)$$

One minor point is that this can't be exact because when you digitally draw it, there has to be a tiny additional area of the little triangle from the peak down to the first zero sample. I'll get out of this clumsy complication by adding either a straight line ramp or a cosine ramp from $\ddot{x}_{\max}$ down to zero. Notice that the initial peak sawtooth will also have the same velocity change. The equation for the ideal acceleration of a terminal peak sawtooth shock from 0 to $\ddot{x}_{\max}$, is shown in Eq (10).

$$\ddot{x} = at + b,\ @\ \ t = 0,\ \ddot{x} = 0;\ b = 0;\quad @\, t = t_d,\ \ddot{x} = \ddot{x}_{\max},\ a = \frac{\ddot{x}_{\max}}{t_d}$$
$$\ddot{x} = \frac{\ddot{x}_{\max}}{t_d} t \qquad (10)$$

For the initial peak sawtooth we get the result in Eq (10a).

$$\ddot{x} = at + b,\ @\ \ t = 0,\ \ddot{x} = \ddot{x}_{\max};\ b = \ddot{x}_{\max};\quad @\, t = t_d,\ \ddot{x} = 0 = at_d + \ddot{x}_{\max},\ a = -\frac{\ddot{x}_{\max}}{t_d}$$
$$\ddot{x} = \ddot{x}_{\max}\left(1 - \frac{t}{t_d}\right) \qquad (10a)$$

Now let's construct a half cosine fall off from the peak since the instantaneous drop to zero is practically unattainable. We will have the fall off occur in a time interval ϕt_d. Here t_d means the total pulse duration including the linear ramp up from 0 to max acceleration and the cosine fall off back down to zero. We want to insert a half cosine of duration, ϕt_d; this cosine will have a frequency, f_c defined by Eq (11). The argument of a half of a period of cosine is π.

$$2\pi f_c \phi t_d = \pi, \quad \text{or}$$
$$f_c = \frac{1}{2\phi t_d} \qquad (11)$$

The acceleration during this cosine fall off is given by Eq (12).

$$\ddot{x} = \ddot{x}_{\max}\frac{1}{2}\left(1 + \cos 2\pi\frac{1}{2\phi t_d}t\right) = \ddot{x}_{\max}\frac{1}{2}\left(1 + \cos\frac{\pi t}{\phi t_d}\right), \quad for\ \ 0 \le t \le \phi t_d \qquad (12)$$

Integrating this to velocity over the cosine duration gives Eq (12a).

$$\Delta\dot{x}_{\cos} = \ddot{x}_{\max}\frac{1}{2}\left[t + \frac{\phi t_d}{\pi}\sin\frac{\pi t}{\phi t_d}\right]_0^{\phi t_d} = \ddot{x}_{\max}\frac{1}{2}\left[\phi t_d + \frac{\phi t_d}{\pi}(0-0)\right]$$
$$\Delta\dot{x}_{\cos} = \frac{1}{2}\ddot{x}_{\max}\phi t_d \qquad (12a)$$

This velocity change (over the interval ϕt_d) must be added to the velocity change of Eq (9a) (over the interval $\left(t_d - \phi t_d\right)$), which gives Eq (13).

$$\Delta\dot{x} = \frac{1}{2}\ddot{x}_{\max}\left(t_d - \phi t_d\right) + \frac{1}{2}\ddot{x}_{\max}\phi t_d = \frac{1}{2}\ddot{x}_{\max}\phi t_d\left(1 - \phi + \phi\right),\ or$$
$$\Delta\dot{x} = \frac{\ddot{x}_{\max}t_d}{2} \qquad (13)$$

Eq (13) gives the velocity change for a terminal peak sawtooth shock with a cosine fall off of ϕt_d Analysis of Figure 16, shows that a $\phi = 0.1$ easily fits the tolerance levels of Mil Std 810. [9]

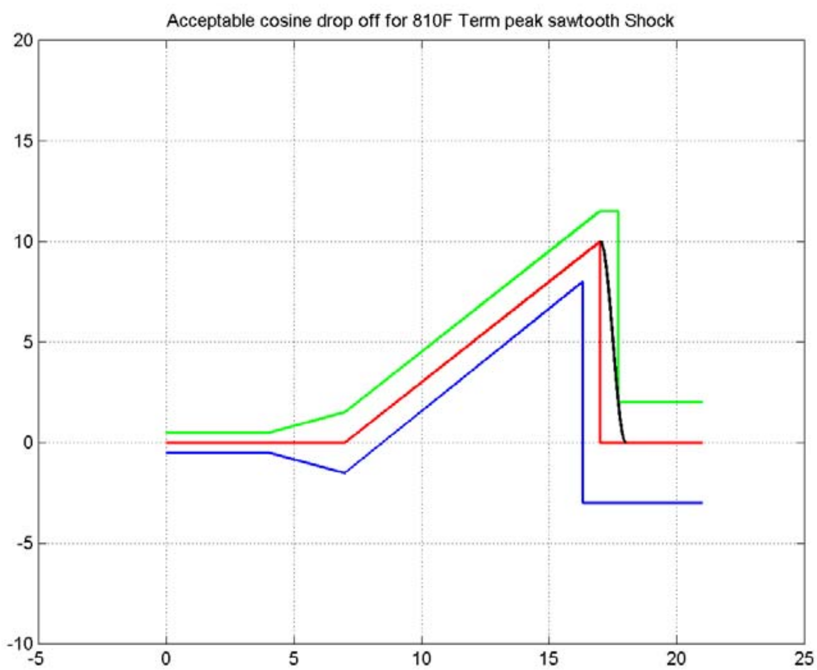


Figure 16. Scaled drawing from Figure 516.5 of Mil STD 810F. [9] The black cosine fall off ramp is a 10% ramp. Notice that I can slide the red curve with the black cosine fall off to the left much more, certainly enough to allow for a 20% fall off cosine ramp which we will need for the analysis of the initial peak shock.

I'll do the analysis of terminal peak and initial peak shocks with this $\phi = 0.1$ cosine ramp to save trouble. Now using Eq (13) to define the t_d for our 100 ips velocity change, 200 g shock, and Eqs (10) and (12) for the shock shape, we get the time history, and its two integrals shown in Figure 17. Again, I wrote a Matlab program, halftermpkcos.m, that generates the acceleration time histories shown in Figures 17 and 19. The PVSS for the 10% cosine ramp is shown in Figure 18.

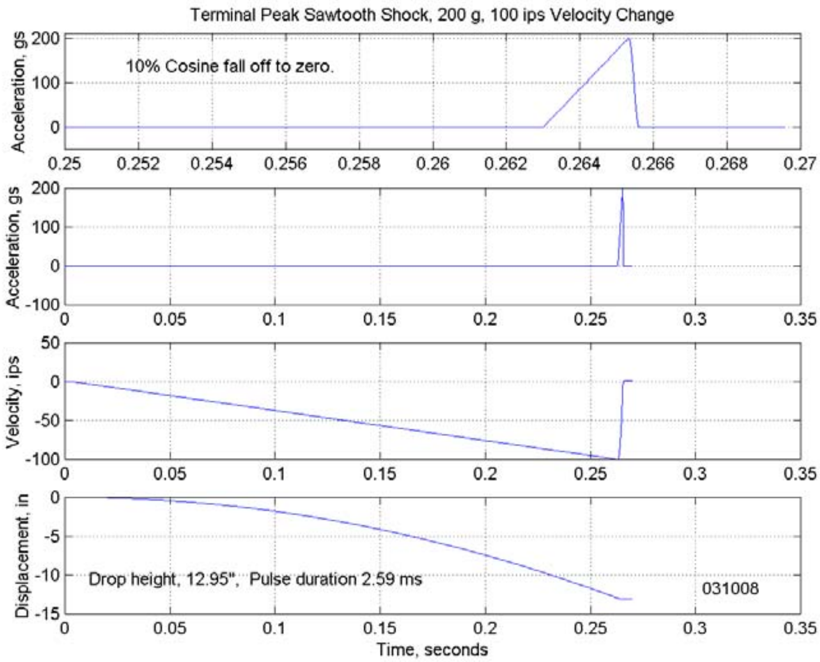


Figure 17. The second subplot shows a 200 g, 100 ips terminal peak sawtooth shock with a 10% cosine fall off to zero, preceded by a 12.95 inch drop. The drop and shock are in the second subplot, and its two integrals are in the third and fourth subplots.

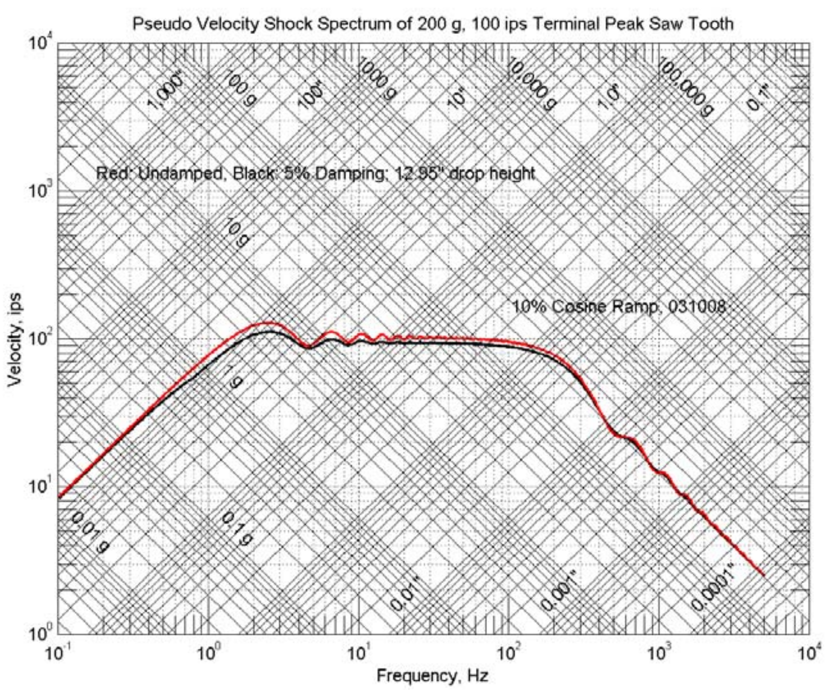


Figure 18. This is the PVSS of the terminal peak saw tooth shock with the 10% cosine fall off. It has a surprisingly small droop zone doubling region because of the gentle linear rise. It is very nicely behaved. It hits our asymptotes of 13 inches, 100 ips, and 200 g's. I don't think we would notice any difference if we had used a linear fall of instead of the cosine ramp fall off to zero.

In Figure 18 the 100 ips velocity change plateau and the 200 g peak asymptote show clearly. Notice the droop zone, (as the spectrum droops downward to the right heading for the acceleration asymptote,) covers a shorter range of frequencies, 180 - 320 Hz, and that the acceleration doesn't come close to doubling. For the terminal peak sawtooth shock we observe less than 1.5 times the peak g level in the droop zone.

INITIAL PEAK SAWTOOTH

Trouble develops though when we try the initial peak saw tooth. Figure 19 shows the initial peak time history with a 10% cosine ramp up to the initial peak.

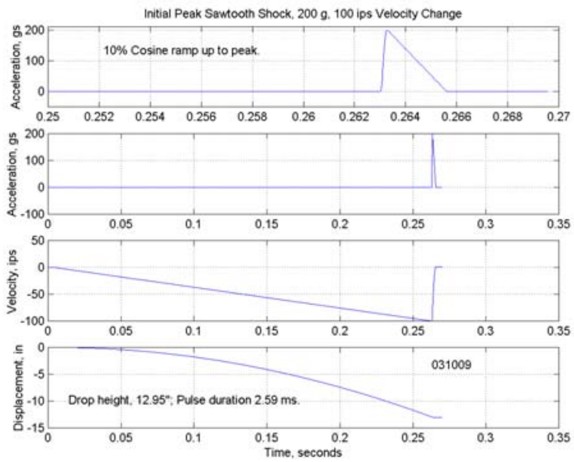


Figure 19. This is a 10% cosine ramp up initial peak saw tooth shock. The cosine is difficult to see but we'll proceed. We're going to have trouble with the shock spectrum because of the steep initial rise.

The PVSS for this 10% cosine ramp up initial peak sawtooth shock is shown in Figure 20. As the spectrum droops downward to the right heading for the acceleration asymptote, it doesn't quite get there even at 5000 Hz , and the acceleration does double. At 5000 Hz it is definitely heading for the 200 g line. If I increase ϕ to 0.2, we make it, as is illustrated below. The time history with the 20% cosine ramp is shown Figure 21.

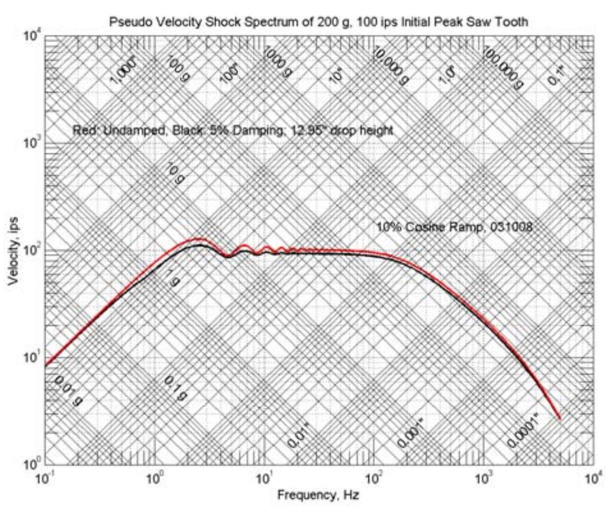


Figure 20. Initial peak sawtooth PVSS for the shock of Figure 19. We see the , 13 inch drop, 100 ips velocity change, but the 200 g maximum acceleration asymptote is not reached. The droop zone trouble develops with the initial peak because of steep initial rise.

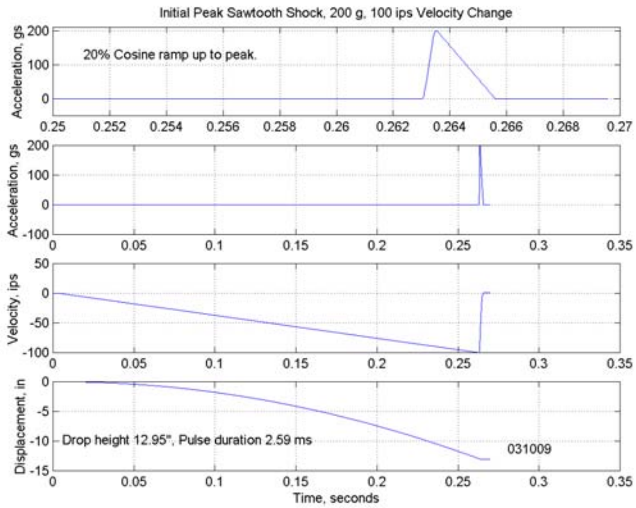


Figure 21. Initial peak sawtooth shock with a 20% cosine ramp to the peak.

Examining Figure 21, and comparing it to Figure 16, it is clear that a 20% rise is with in the specification [9]. The resulting PVSSs are shown in Figure 22. We get back down to 200 g at about 2500 Hz. This compromise of the initial peak sawtooth would fit well in the red and green limits of Figure 16, and is sufficient to permit the 200 Hz asymptote to appear, and completes the analysis of the five simple shocks. They all are similar.

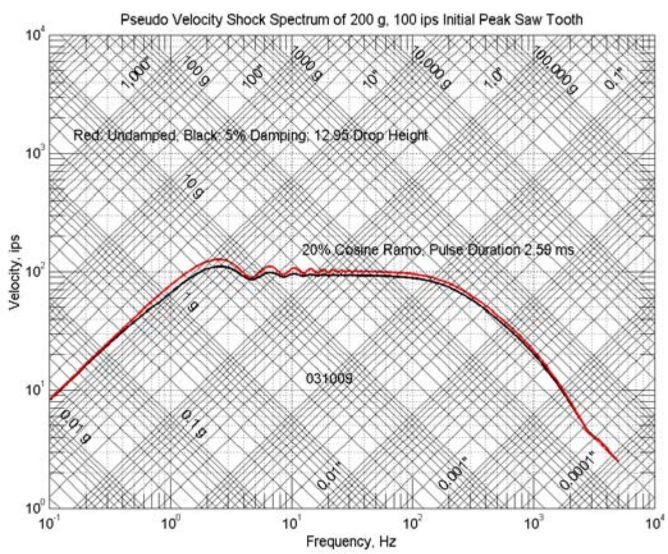


Figure 22. PVSS of the initial peak sawtooth with a 20% cosine ramp to the peak.

SIMPLE SHOCK COMPOSITE PVSS PLOTS

Now we have completed an examination of the PVSS of the simple pulses: the half sine, the trapezoid, the haversine, and the initial peak and the terminal peak saw tooth. We observed that both damped and undamped, the PVSS's are similar. The reason that we can see that they are similar is that I scaled them to have equal velocity change and peak acceleration. Figure 23 is the undamped composite plot. Next we do the 5% damped PVSS composites in Figure 24, which supports the same conclusion.

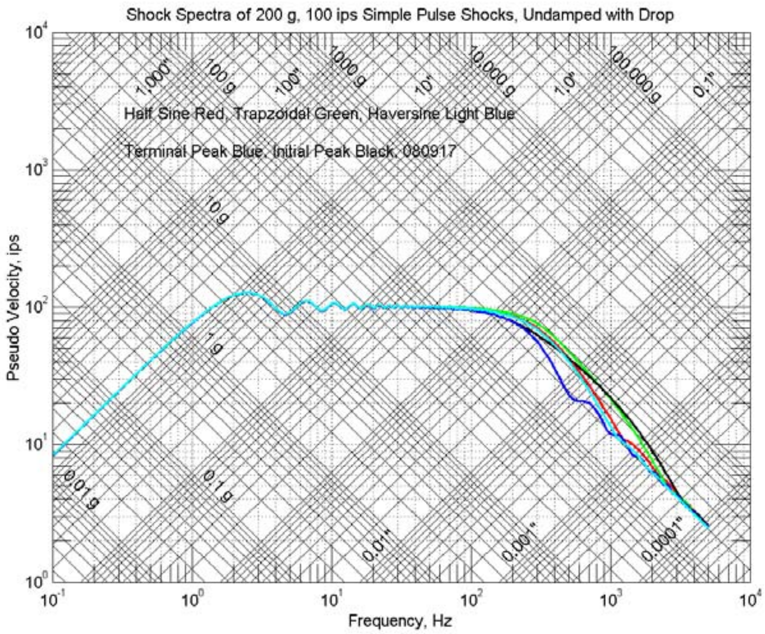


Figure 23. This is an undamped composite PVSS plot of the five simple shocks. Notice that the shocks only differ in the droop zone.

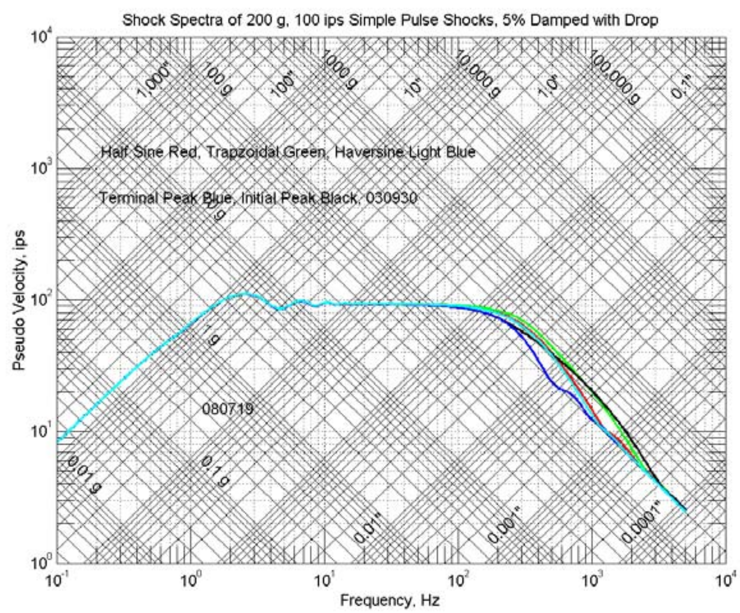


Figure 24. This is a 5 % damped composite PVSS plot of four of the simple shocks. Again notice that the shocks only differ in the droop region.

This is interesting and important. After a huge amount of calculating and plotting the PVSSs of the simple shocks show the similarity. ONLY THE PVSS ANALYSIS WITH THE TABLE DROP ACCELERATION INCLUDED AND SCALED TO VELOCITY CHANGE WITH THE SAME PEAK ACCELERATION SHOWS THE SIMPLE SHACKS SIMILAR. THIS FACT HAS ESCAPED ALMOST ALL SHOCK EXPERTS. I HAVE TO EMPHASIZE THIS BECAUSE IT IS GENERALLY NOT KNOWN. Only in the high frequency region, where the velocity(severity) levels are becoming less severe do they diverge. Gertel's [5] off the cuff comment that that all the simple pulses are similar is confirmed if not proven. The terminal peak saw tooth comes down to the 200 g asymptote faster than the other four because it has a more gradual rise. The velocity change during the pulse was the similar thing about them. They all required the same drop height so they all have the same low frequency asymptote.

All of the simple pulses developed on a drop table shock machine by a programmer that results in zero velocity when the pulse is over will have a velocity change of the square root of 2 g h. $\left(\sqrt{2gh}\right)$ They will all have the same drop height or maximum displacement, hence the same low frequency asymptote. Since they all have the same velocity change, they all have the same plateau region. Since I adjusted the pulses to have the same peak acceleration, they all must have the same high frequency asymptote. The only way their shock spectra can differ, are at the two corners and this can be seen in Figures 23 and 24. I had trouble getting the acceleration asymptotes to appear in the trapezoid and the initial peak saw tooth. These have abrupt rise times that cause a doubling of the peak accelerations in the droop zone. I had to decrease the rise time abruptness by using a half cosine ramp rise of 30% in the trapezoid and 20% in the initial peak wave form.

I'll summarize the formulas for peak acceleration, velocity change, and duration for convenience and comparison.

Half Sine Shock

$$\Delta\dot{x} = \frac{2\ddot{x}_{\max} t_d}{\pi} \tag{2}$$

Cosine ramp trapezoid shock (rise time: ϕt_d)

$$\Delta\dot{x} = \left(1-\phi\right)\ddot{x}_{\max} t_d \tag{8d}$$

Haversine shock

$$\Delta\dot{x} = \frac{\ddot{x}_{\max} t_d}{2} \tag{8f}$$

Cosine ramped saw-tooth shock

$$\Delta\dot{x} = \frac{\ddot{x}_{\max} t_d}{2} \tag{13}$$

I want to convince you of two other important ideas. Simple shocks are frequently simulated on a shaker, but the drop cannot be simulated, and the drop is very important. This ruins the frequency range over which the shock is severe. Also, the half sine, and the other simple shocks have a PVSS that can be thought of as similar to that of the explosive shocks, and can be used to simulate them.

BUT SHAKER GENERATED SHOCKS ARE NOT SIMILAR!

Shaker generated shocks have a severely limited low frequency asymptote and can have almost a negligible plateau. Half sine shock tests can be conducted on an electrically driven shaker but shakers have a limited displacement capability. The PVSS on 4CP of a shaker generated shock will reflect this with a greatly reduced low frequency capability.

Let's consider a half sine with rectangular pre and post pulses. The magnitude of the pre and post pulses is set at γ times the maximum acceleration. See Figure 25. The area of the pre and post pulses must equal the half sine velocity change we developed in Eq (2) which is shown in Eqs (14 a and b). This yields the pre and post pulse duration, t_p in Eq. (14b). Again I wrote a short Matlab program, halfsin4.m that generates the rectangular pre and post pulse halfsine shaker shock acceleration time history. The time history and its integrals are shown in Figure 26.

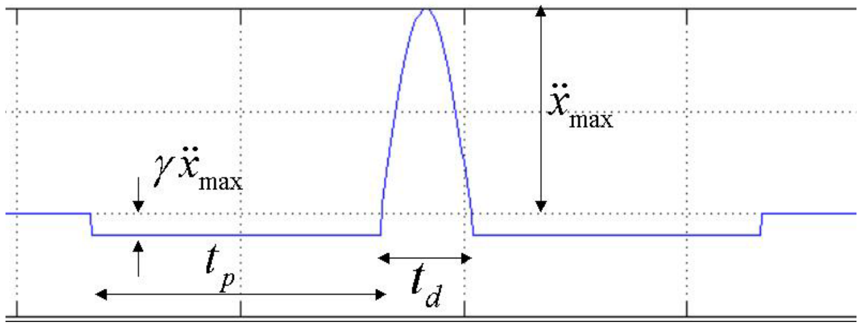


Figure 25. Shaker shock simplified acceleration geometry. The negative area of the pre and post flat regions must equal the area of the half sine.

$$2\gamma \ddot{x}_m t_p = \frac{2\ddot{x}_m t_d}{\pi}$$
$$t_p = \frac{t_d}{\pi\gamma} \qquad (14\ a,b)$$

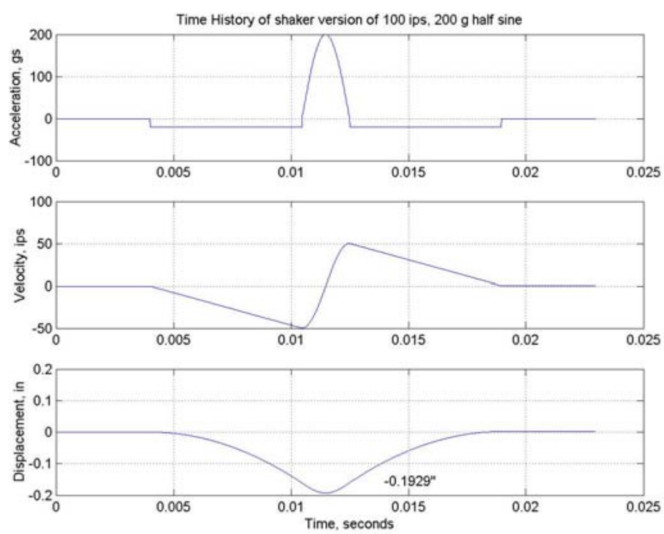


Figure 26. Time history and integrals for a rectangular pre and post pulse half sine shaker shock

Notice in Figure 26, that the peak displacement is only about 0.2 inch, and this will limit the low frequency severe portion of its PVSS plateau. The 5% damped shock spectrum is shown in Figure 27. The shock now is only severe from about 70 to 200 Hz. Also the shaker armature motion is all in one direction. Centering the shaker armature so it is returnd to zero, further reduces the displacement to about 0.11 inch and affects the plateau a little differently. This is very bad. A greayt deal of equipment has its first mode frequency below 70 Hz and that means that for this equipment the first mode would not be excitied by the shock test.

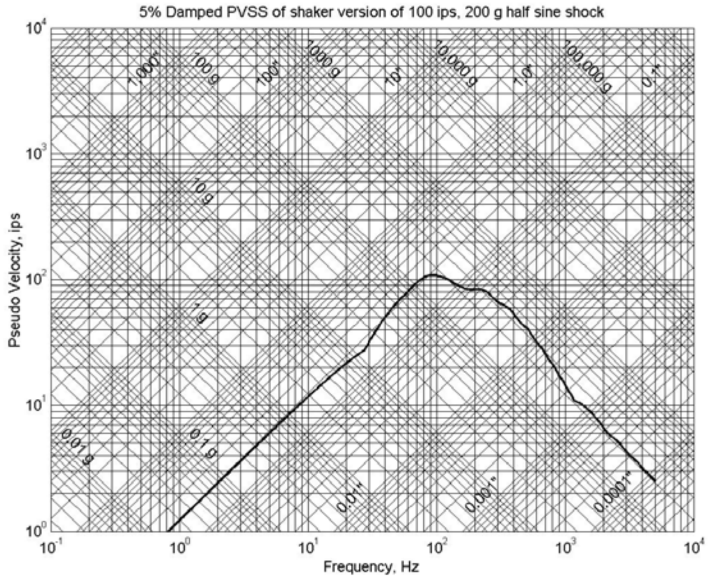


Figure 27 5% damped PVSS of the shaker shock version of out 100 ipsa 200 g half sine. Notice its almost non exisatant plateau. It has a severity of near 90 ips from 70 to 200 Hz.

Comparing shaker shocks with the drop table shocks in Figure 24, it is easy to see the reduced frequency range of the shaker shock plateau. However, there is no way this can be noticed unless the shock spectrum is plotted as a PVSS on 4CP, and compared to a drop table half sine with the drop included.   Shaker simulated half sines can be inadequate for machinery and equipment with lower modal frequencies. This is including the shocks synthesizing a shock spectrum with a collection of oscillatory motions. The beauty of shaker shock is that the direction of the shock or its polarity can be reversed without turning the equipment over. That convenience can not make up for the loss of low frequency plateau.

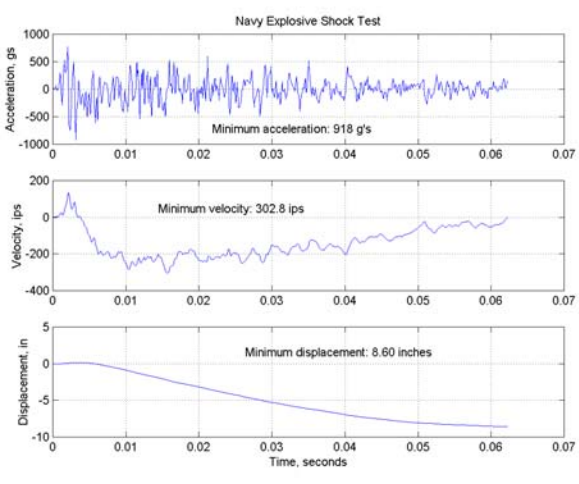


Figure 28. The acceleration time history and its two integrals for an example explosive shock.

SHOCK SPECTRA FROM EXPLOSIVE EVENTS ARE SIMILAR TO SIMPLE PULSE TESTS

One example will be given to explain the similarity. Figure 28 is an acceleration time history of an explosive event and its two integrals. The mean has been removed from the acceleration time history to assure that the velocity ends at zero. The fact the extreme values are all minima makes no difference. Figure 29 gives its shock spectra for 3 damping values.

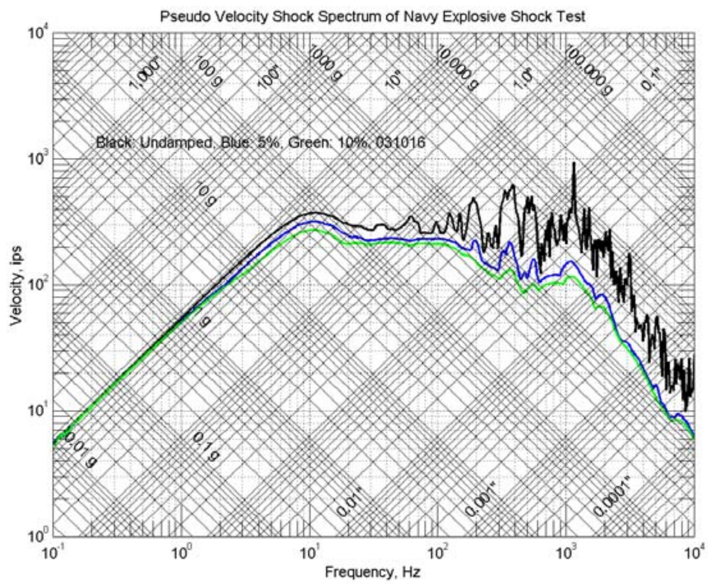


Figure 29. Shock spectra of an example explosive shock test.

Notice on the left all three curves are asymptotic to just under 9 inches, that the center severe region of the middle 5% damped curve is hovering just above 200 ips, and that at the highest frequencies the damped curves at least are heading for about 900 g's. I would have to calculate to higher frequencies to see the high frequency acceleration asymptote on the undamped spectrum. To use a half sine to approximate the 5% damped curve, I tried a 300 ips, 1500 g half sine with a coefficient of restitution of 1, to reduce the drop height, which is still too high. This is

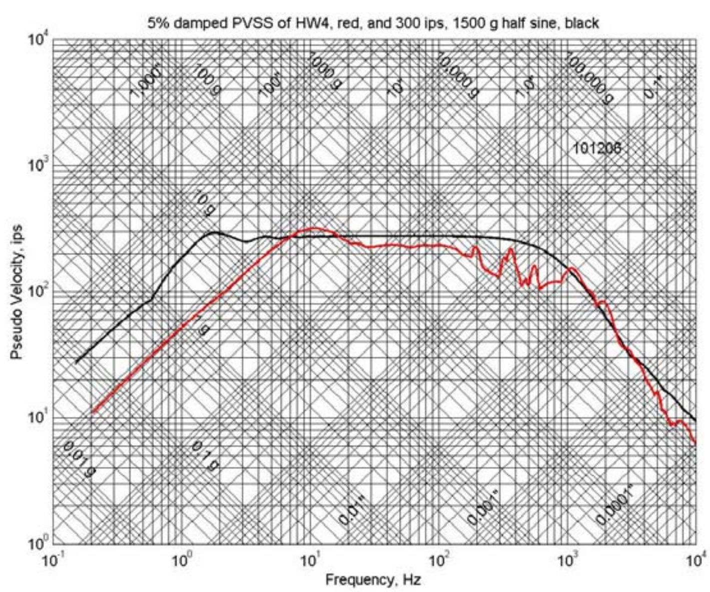


Figure 30. 5% Damped PVSS of HW4 in red and 300 ips, 1500 g half sine in black. I could increase g in my hafsinGV program to reduce the drop height and the low frequency asymptote. The droop zone doubling of the 1500 g halfsine does a nice job of almost enveloping HW4.

shown in Figure 30. The point here is that a simple pulse with a peak g level 1500 g, and a velocity change of 300 ips, has a shock spectrum matching the severe and high frequency regions of HW4. It would provide a test of equal severity. To attain the velocity change with a only 9 inch drop would require an artificial high g level in the program, which could be done. In practice I think we could obtain the 300 velocity change 9 inches with bungee cord. This would give the 9 inch displacement low frequency asymptote. In this sense I make the statement that the simple shock tests are similar to explosive shock spectra. Since the PVSS of the shock shows its capacity to induce modal velocities in equipment test items, the test has the same severity as the explosive shock.. The simple shock would have to be applied in both the positive and negative directions because the simple shocks are the most highly directional or polarized.

CONCLUSIONS

The very important final point I'll make is that all the simple shocks are similar when plotted on the same page and scaled to have the same velocity change and peak acceleration. Undamped and damped composite plots of the five shocks are given in Figures 23 and 24 to show how similar they are. All simple shocks have that same PVSS on 4CP; and this means that there is no sense in using different simple shock shapes. The half sine is fine. If you are making a high frequency high acceleration shock, and the half sine deteriorates into a haversine, that's fine too. I repeat what I emphasized on page 18: ONLY THE PVSS ANALYSIS WITH THE TABLE DROP ACCELERATION INCLUDED AND SCALED TO VELOCITY CHANGE WITH THE SAME PEAK ACCELERATION SHOWS THE SIMPLE SHACKS SIMILAR. THIS FACT HAS ESCAPED ALMOST ALL SHOCK EXPERTS. I HAVE TO EMPHASIZE THIS BECAUSE IT IS GENERALLY NOT KNOWN.

Generating a simple shock on a shaker sacrifices a huge amount of low frequency content, even though that is extremely convenient (convenient because you can changes the direction without turning the object over). You won't have any low frequency content and will likely allow meeting shock specification that couldn't be met on a drop table shock machine. Compare Figures 27 and 24 to see the shocking difference.

The PVSS on 4CP of all zero mean shocks are shaped like a hill ands have a high PV plateau. The simple shock PVSS on 4CP is very simple; its 5% damped plateau is at 93% of the impact velocity change, its high frequency asymptote slopes down and to the right on a constant acceleration line at the peak shock acceleration, and the plateau low frequency asymptote slopes down and to the left on a constant displacement line at the peak shock displacement. The plateau identifies the severe frequency range of the shock. To test for an explosive shock PVSS shaped like a hill, and enveloped by $|y|_{\max}$, $|\Delta\dot{y}|_{\max}$, $|\ddot{y}|_{\max}$ use a simple shock with a 93% velocity change that covers $|\Delta\dot{y}|_{\max}$, and a peak acceleration equal to $|\ddot{y}|_{\max}$, and a maximum displacement equal to $|y|_{\max}$.

There is a droop zone, right after the intersection of the plateau and the maximum acceleration asymptote. Abrupt rise time shocks can double the acceleration asymptote for a range of frequencies. A worst case is shown in Figure 7, where the doubling begins at about 70% of the plateau level, and doubling continues until to 10% of the plateau level. I consider this a minor point because it occurs well below the severe PV levels. This droop zone doubling region significance is what was exaggerated by the authors of the normalized simple shocks analyses in [2, 3, 4, 5].

The paper is a summary of results of many Matlab calculations. I wrote short programs to generate all the simple shocks, a plotting program that integrates and plots the shock acceleration, velocity and displacement. Other slightly longer programs were used to calculate and plot the PVSSs and craw the 4CP. I will make these available to any who are interested.

REFERENCES

1. Gaberson, H. A, "Pseudo Velocity Shock Spectrum Rules and Concepts", Proceeding of the Annual Meeting of the Mechanical Failure Prevention Technology Society [www.mfpt.org] April 19, 2007. Also published as Gaberson, H. A,, "Pseudo Velocity Shock Spectrum Rules for Analysis of Mechanical Shock"; IMAC XXV, Orlando, FL; Society of Experimental Mechanics; Bethel, CT, www.sem.org; Feb 2007; p 367

2. Mindlin, R.D., “Dynamics of Package Cushioning,” Bell System Technical Journal, vol 24, Jul-Oct 1945, pp 353-461.

3. Jacobsen, L. S., and Ayre, R.S., "Engineering Vibrations", McGraw- Book Co. 1958.

4. Ayre., R.S., "Transient Response to Step and Pulse Functions", Chapter 8 of "Harris' Shock and Vibration Handbook", 5th Ed. by Harris, C. M. and Piersol, A.G., 2002

5. Rubin, S. “Concepts in Shock Data Analysis”, "Harris' Shock and Vibration Handbook", 5th Ed. by Harris, C. M. and Piersol, A.G., 2002. pp 24.6-24.9

6. Lelanne, Christian, "Mechanical Shock", Volume II in "Mechanical Vibration and Shock", Hermes Science Publications, Paris 1999; English Edition, Hermes Penton Ltd. 2002. Taylor and Francis Books Inc., 29 West 35th Street, New York, NY, 10001

7. Gertel, Mike, and Holland, R.,"A Study of Selected Shock Analysis Methods", A Report Allied Research Associates, Inc, Concord, MA done under contract for U.S. Army, Quality Assurance Directorate, Frankford Arsenal, Philadelphia, PA April 1967; AD# AD814820

8. Gaberson, H.A. "Half Sine Shock Tests to Assure Machinery Survival in Explosive Environments". IMAC XXII, Dearborn, MI; Society of Experimental Mechanics, Jan 29, 2004

8. MIL-STD-810F: Method 516.5 – Shock; Jan 2000. [Polarity: para 2.3.3, p 516.5-16][Terminal peak sawtooth shock. p516.5-14, and Figure 516.5-10

10. IEC Shock Specification, 68-2-27; Basic environmental testing procedures; Part 2: Tests - Test Ea and guidance: Shock. Third Edition 1987