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RAINFLOW CYCLE COUNTING AND ACOUSTIC FATIGUE ANALYSIS TECHNIQUES FOR RANDOM LOADING

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ABSTRACT

Traditional fatigue analysis uses a technique called rainflow cycle counting to decompose a variable amplitude time signal of stress into fatigue cycles. The damage from each cycle is computed using a Wöhler curve and the damage over the entire time signal is calculated by summing the damage from all the individual cycles. In the case of acoustic and high-cycle fatigue, a long duration of time signal is often required in order to obtain a statistically representative fatigue estimate. FE-based design analysis of dynamic components using such long time signals is often rendered impractical on account of a computationally intensive forced response analysis. In most cases the loads are found to be random (or sine-on-random) in nature, and PSD are often used to describe the random loads in a more efficient manner. This representation also facilitates a very efficient FE analysis technique based on harmonic response rather than the forced response approach which is required for time signals. This paper introduces several techniques for rainflow cycle counting directly from a resultant PSD of stress. It compares these techniques with traditional rainflow analysis and concludes with a case study showing how the PSD-based analysis was used during the design of an exhaust muffler for a new city bus.

1 INTRODUCTION

According to Battelle in 1982, between 80-90% of all structural failures occur through a fatigue mechanism. Since the advent of digital computers, engineers have developed methods for estimating the fatigue life of components to minimise the risk of in-service failure. With the advent of Rainflow cycle counting in 1968, most of these approaches have relied on a time signal of stress or strain from the component; however, in many cases it is impractical and statistically inappropriate to use time signals.

A time signal represents a deterministic event in time. It is ideal for components where fatigue damage is attributable to high transient loads; such as aircraft undercarriages, and automobile suspension, chassis and steering components. However, it is not suitable for components where fatigue damage is attributable to high-cycle vibration or acoustic loads over a long period of time. In these cases the loads are stochastic and it is common to express these random loads in terms of a PSD. PSD are used to drive vibration test rigs and also offer input to FE analysis programs. The PSD approach is able to utilise a very efficient FE algorithm based on the linear transfer function (or harmonic analysis) approach making it more desirable than equivalent transient-based analysis in the time domain. The FE program can output PSD of stress at each node/element directly, or output a more versatile transfer function matrix from which a PSD of stress can be calculated.

This paper describes techniques for performing rainflow cycle analysis directly from a PSD of stress. It compares the results of each method with traditional time series rainflow cycle counting. Comparisons are first of all made on analytical data to show the equivalence of the direct PSD-based rainflow methods. A simple laboratory test validation is then described which correlates FE-based fatigue life estimates with real fatigue under ideal laboratory conditions. The paper concludes with a case study showing how these techniques were used for fatigue assessments of an exhaust muffler for a new city bus. Excellent damage correlation is shown in all cases.

2 INTRODUCTION TO DIRECT METHODS OF PSD-BASED RAINFLOW ANALYSIS

This section describes four approaches for computing fatigue life, or damage, directly from a PSD of stress as opposed to a time signal. For more background information the reader is referred to Bishop and Sherratt [1] and Halfpenny [2],[3].

2.1 Narrow-band approach (Bendat and Rice)

In 1964 Bendat [4] proposed the first significant step towards a method for determining fatigue life from PSD. Earlier, in 1954 Rice [5] showed that the probability density function (pdf) of peaks for a narrow-band signal tended towards a Rayleigh distribution as the bandwidth reduced. Using this assumption Bendat reasoned that the pdf of rainflow range (i.e. stress range) would also tend to a Rayleigh distribution. To complete his solution, Bendat used a series of equations derived by Rice [5] to estimate the expected number of cycles using moments of area under the PSD. Bendat’s narrow-band solution for the rainflow range histogram is therefore determined from the pdf expressed in Equation (1).

$$N(S) = E[P]\cdot T\cdot\left\{\frac{S}{4\cdot m_0}\cdot e^{-\frac{S^2}{8\cdot m_0}}\right\} \tag{1}$$

The term in brackets in Equation (1) is the Raleigh probability distribution. N is the expected number of cycles of stress range S occurring in T seconds. m_0 is the zeroth moment of area of the PSD (i.e. area beneath the curve), and $E[P]$ is the expected number of peaks obtained by Equation (2).

$$E[P] = \sqrt{\frac{m_4}{m_2}} \tag{2}$$

m_4 and m_2 are the 4th and 2nd moments of area of the PSD respectively: where the nth moment of area is defined as $m_n = \int f^n \cdot G(f)df$, and $G(f)$ is the value of the single sided PSD at frequency f Hz.

2.2 Broad-band approach (Steinberg)

Bendat's narrow-band solution tends to be conservative for broad-band signals. A reliable measure of bandwidth was offered by Rice[5] as a ratio of the number of zero up-crossings in a time signal, to the number of peaks. This ratio is often known as the 'irregularity factor' and is given by Equation (3). For narrow-banded signals the irregularity factor tends to unity whereas broad-banded signals progressively tend to zero.

$$\gamma = \frac{m_2}{\sqrt{m_0 \cdot m_4}} \quad (3)$$

Rice found that for narrow-banded signals, the pdf of peaks tended towards a Rayleigh distribution while broad-band signals tend to a Gaussian distribution. Steinberg [6] assumed that for broad-band signal the pdf of rainflow range would also tend to a Gaussian distribution and proposed a solution based on discrete multiples of the RMS (Root Mean Square) amplitude. This approach is used extensively in the field of electronics testing and the Steinberg equation is shown in Equation (4).

$$N(S) = E[P] \cdot T \cdot \left| \begin{array}{l} 0.683 \times 2RMS \\ +0.271 \times 4RMS \\ +0.043 \times 6RMS \end{array} \right. \quad (4)$$

2.3 General approach for all band-widths (Dirlik)

Rice concluded that for a signal of arbitrary bandwidth, the pdf of peaks could be obtained from the weighted sum of the Rayleigh and Gaussian distributions. However, Dirlik [7] reasoned that the pdf of peaks is not the same as the pdf of rainflow range. In 1985 he proposed an empirical solution to estimate the pdf of rainflow range following extensive computer simulations using the Monte Carlo technique. The Dirlik formulation is given in Equation (5) and (6).

$$N(S) = E[P] \cdot T \cdot p(S) \quad (5)$$

Where, $N(S)$ is the number of stress cycles of range S N/mm^2 expected in time T *sec.* $E[P]$ is the expected number of peaks obtained by Equation (2).

$$p(S) = \frac{\frac{D_1}{Q} \cdot e^{\frac{-Z}{Q}} + \frac{D_2 \cdot Z}{R^2} \cdot e^{\frac{-Z^2}{2 \cdot R^2}} + D_3 \cdot Z \cdot e^{\frac{-Z^2}{2}}}{2 \cdot \sqrt{m_0}}$$

$$D_1 = \frac{2 \cdot (x_m - \gamma^2)}{1 + \gamma^2} \qquad D_2 = \frac{1 - \gamma - D_1 + D_1^2}{1 - R} \qquad D_3 = 1 - D_1 - D_2$$

$$Z = \frac{S}{2 \cdot \sqrt{m_0}} \qquad Q = \frac{1.25 \cdot (\gamma - D_3 - D_2 \cdot R)}{D_1} \qquad R = \frac{\gamma - x_m - D_1^2}{1 - \gamma - D_1 + D_1^2}$$

$$\gamma = \frac{m_2}{\sqrt{m_0 \cdot m_4}} \qquad x_m = \frac{m_1}{m_0} \cdot \sqrt{\frac{m_2}{m_4}} \tag{6}$$

The Dirlik equation is based on the weighted sum of the Rayleigh, Gaussian and exponential probability distributions. In terms of accuracy, Dirlik's empirical formula for Rainflow ranges has been shown to be far superior to the earlier methods.

2.4 Lalanne /Rice approach

In contradiction to Dirlik, Lalanne [8] reasoned that over a sufficiently long period of time the pdf of rainflow range would tend to the pdf of peaks and thereby demonstrated that Rice's original formula (based on a simple weighted sum of the Rayleigh and Gaussian distributions) would also suffice for rainflow ranges. The Lalanne/Rice formula is given in Equation (7).

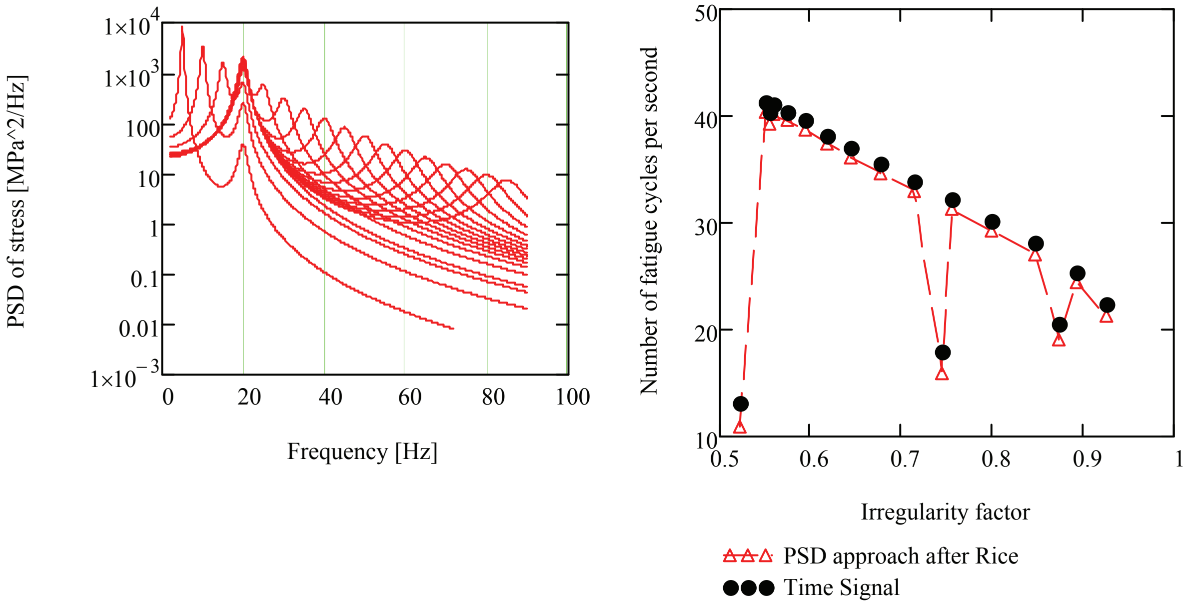
$$N(S) = \frac{1}{rms} \cdot \frac{\sqrt{1-\gamma^2}}{\sqrt{2\pi}} \cdot e^{\frac{-S^2}{2rms^2(1-\gamma^2)}} + \frac{S \cdot \gamma}{2rms} \cdot \left[1 + erf\left(\frac{S \cdot \gamma}{rms\sqrt{2(1-\gamma^2)}}\right)\right] \tag{7}$$

Where γ is the irregularity factor determined from Equation (3), and $erf(x)$ is error function defined by: $erf(x) = \frac{2}{\sqrt{\pi}} \cdot \int_0^x e^{-t^2} dt$.

The Lalanne/Rice approach is equally robust as the Dirlik method. It gives similar results in most cases and offers the advantage of being less empirical. In this paper both Dirlik and Lalanne/Rice are shown to give excellent correlation with traditional time-domain rainflow cycle counting.

3 COMPARISON OF PSD-BASED ANALYSIS WITH TIME-BASED RAINFLOW ANALYSIS

In this section the fatigue damage and life estimations obtained using PSD data are compared with those derived using time signal reconstruction and rainflow cycle counting. The analytical comparisons consider the response of a typical structural component with two excited modes (bi-modal response) to a broad-band random excitation. This case is useful because the band-width can be controlled by adjusting the relative frequencies of each mode thereby offering a range of comparisons between broad and narrow-banded response. A fixed frequency mode is introduced at 20Hz along with a varying mode ranging between 5-85 Hz. The test PSD is illustrated in Figure 1a.



a b

Figure 1 a)Test PSD used during study
b) number of fatigue cycles predicted by both methods

For comparison purposes, several random time signals were used of varying durations and sample rates. A sample rate of 20 times Nyquist was adopted to ensure adequate amplitude resolution. Any sample rate below this would cause significant 'clipping' of the stress peaks and compromise the time signal accuracy.

The duration of the time signal is also found to affect the quality of the comparison. If the time signal is too short then the fatigue damage varies significantly between each successive time signal reconstruction. A study into the required time signal duration was performed and concluded that the number of points required depends on the frequency range, the band-width, the slope of the SN curve, and the complexity of the SN curve. Using a single-slope SN curve, a time signal of at least 1 million data points was required to achieve statistically consistent results for the examples proposed. With more complex SN curves, significant inconsistencies were still observed when over 10 million data points were used. The duration of time signal data required would therefore render the time signal reconstruction approach unfeasible for practical design analysis and demonstrates the desire for using the fast direct PSD approach. As the time signal duration was increased, the observed results converged to the direct PSD solutions in all cases.

3.1 Estimating the number of fatigue cycles

The expected number of fatigue cycles is determined using Rice's formula given in Equation (2). This is compared directly with the estimate taken from the time signal reconstruction and plotted over a range of bandwidths. The results are plotted in Figure 1b and demonstrate excellent correlation with the time signal for all PSD irrespective of bandwidth. The 4 lowest values in the plot relate to the first 4 test spectra where the varying modal frequency is less-than or equal-to the fixed mode frequency. In these cases the frequency of both modes are quite low and therefore the expected number of cycles is also low. When these results are arranged in terms of increasing bandwidth (irregularity factor γ) they give rise to the distinctive 'dips' in the plots.

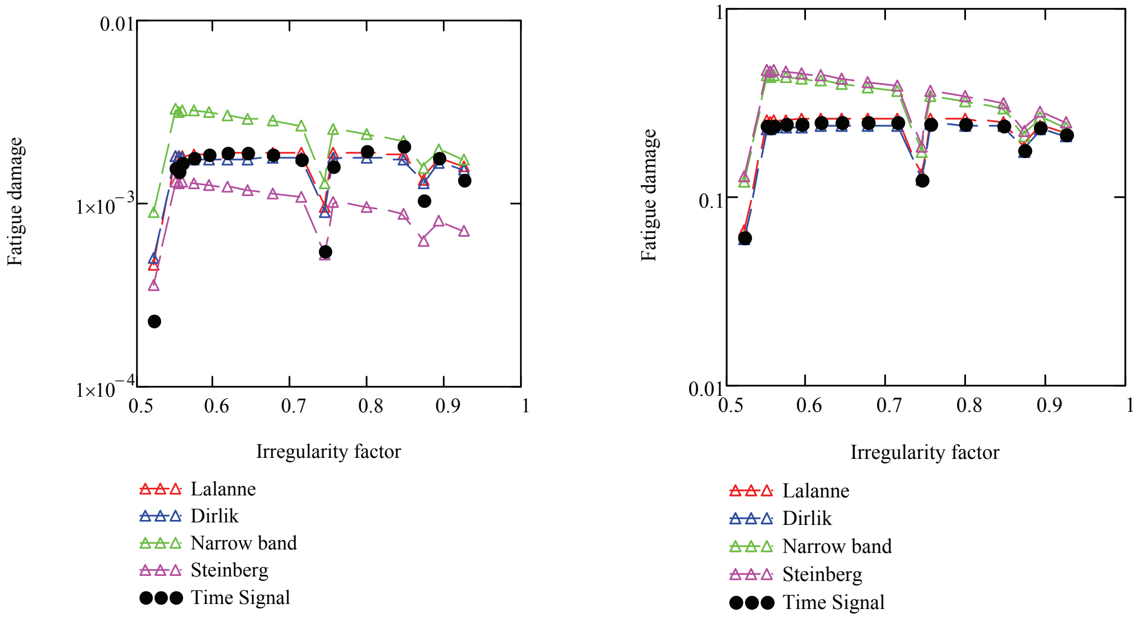
3.2 Estimating the fatigue damage

The fatigue damage is calculated for each PSD using all the methods described in Section 2. The comparable damage is also calculated by rainflow cycle counting the reconstructed time signals. Figure 2 shows a comparison of fatigue damage values using two single slope SN curves.

Figure 2a pertains to a Basquin exponent of 13. This shallow value is typical of fatigue failures in un-notched components. The results clearly show the Narrow-band solution offering an upper bound to the fatigue analysis, whilst the Steinberg approach offers a lower bound. The Dirlik and Lalanne approaches both demonstrate a trend that ranges between Steinberg (Gaussian) for broad-band signals and Rayleigh at the narrow-band region. The time signal reconstruction closely follows the Dirlik and Lalanne estimates.

Figure 2b pertains to a Basquin exponent of 4. This relatively steep value is typical of fatigue failures adjacent to notches and welds. In this case the Steinberg approach yields the most severe damage estimation on account of its discrete stress range probability distribution. The Dirlik and Lalanne again offer excellent correlation with the time signal reconstruction.

Excellent correlation was demonstrated over the entire range of SN curves up to a Basquin exponent of 18. These very shallow curves show increasing conservatism when compared with time signal reconstruction. However, it was impossible to conclude whether this discrepancy is due to inaccuracies in the tail of the pdf, or an inability to generate sufficiently long time signal reconstructions. The Dirlik approach is more conservative than Lalanne in this respect on account of the additional exponential distribution which is used in the tail.



a

b

Figure 2 Fatigue damage comparisons for SN curves of slope 13 and 4 respectively

Studies were also considered with frequencies into the acoustic range. These also showed the same excellent levels of convergence; however, they were limited by the shear length of the reconstructed time signal required which was several hundred million points.

3.3 Comparison of fatigue estimate on a real test specimen

A vibration test was performed on several test specimens made of aluminium alloy 6082 in the T6 condition. The test specimens consisted of a plate with a reduced section as illustrated in Figure 3. The specimen is clamped to the test rig at the left-hand end and excited by a random acceleration in the vertical direction. The observed fatigue lives are listed in Table 1 and show a range of lives within a factor of 2. The geometry, loading definition and physical test results were kindly provided by the Thales group. Material SN data was obtained from the HBM-nCode Materials Testing Laboratory.

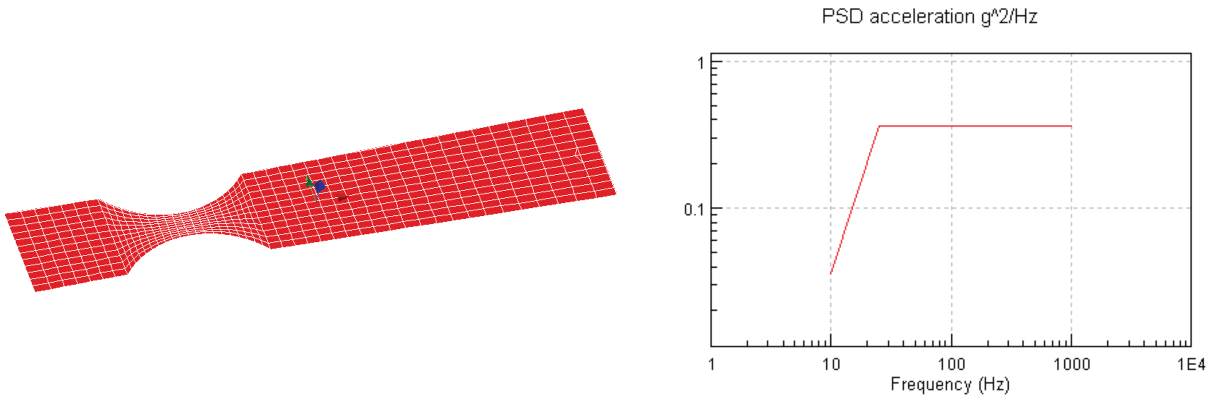


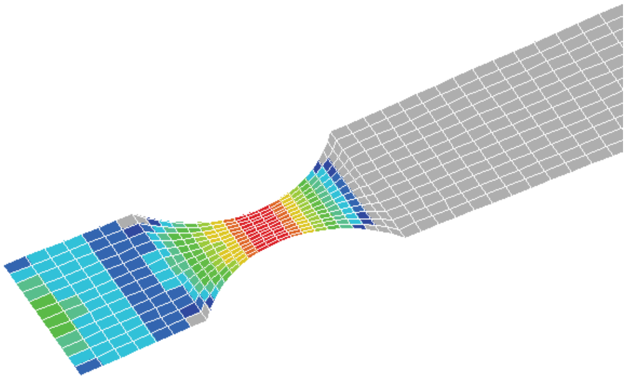
Figure 3 Simple vibration test specimen

A comparable analysis using the Finite Element program MSC.Nastran† was performed. A transfer function solution was run to establish the component response and this was used by the HBM-nCode DesignLife‡ program. A comparison between the measured life and the theoretical fatigue life estimates are listed in Table 1.

These results demonstrate excellent correlation and all, apart from Steinberg, are within a factor of 2 of the observed life. The Steinberg approach is not ideally suited to this application because the SN curve is modelled in a complex fashion whereas the Steinberg rainflow pdf only contains discrete stress values at 3 multiples of the RMS stress range. The Steinberg approach is therefore only suitable when a single gradient SN curve is used.

The Dirlik estimate is slightly more conservative than the Lalanne in this instance on account of the exponential term in the distribution as $\gamma < 0.6$. This effect is made more pronounce as the material SN curve has a relatively shallow slope of 13 in the high-cycle region.

† MSC.Nastran is a registered trademark of the MSC Software Corporation.
‡ nCode, GlyphWorks and DesignLife are registered trademarks of HBM.



Specimen Id	Test Life (seconds)
1	1620
2	1020
3	1560
4	1560
5	1800
6	1860
7	2100
Average test life	1646
Irregularity	0.55
Lalanne	1980
Dirlik	1010
Narrow band	1080
Steinberg	5050

Table 1 Fatigue life comparisons for test specimen

4 CASE STUDY TO ESTIMATE FATIGUE LIFE OF AN EXHAUST MUFFLER

4.1 Vibration test configuration

Physical tests were performed on an electrodynamic shaker as illustrated in Figure 4 and Figure 5. These are driven using a specified acceleration PSD and vibrate the component against its own inertia. The fatigue stresses incurred are attributed to the product of component mass and the applied acceleration. The input PSD was derived using HBM-nCode's Accelerated Testing package. The resultant PSD for each perpendicular axis of vibration is shown in Figure 6.

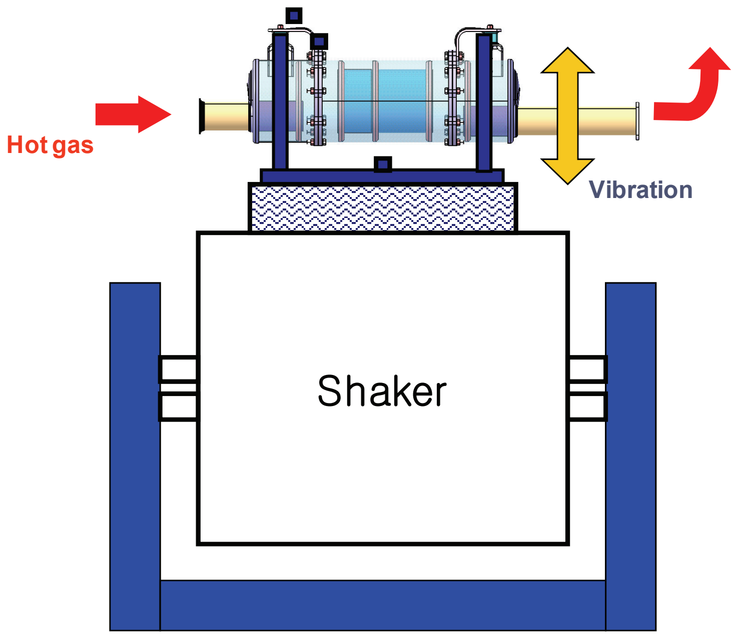


Figure 4 Illustration of vibration rig with muffler assembly

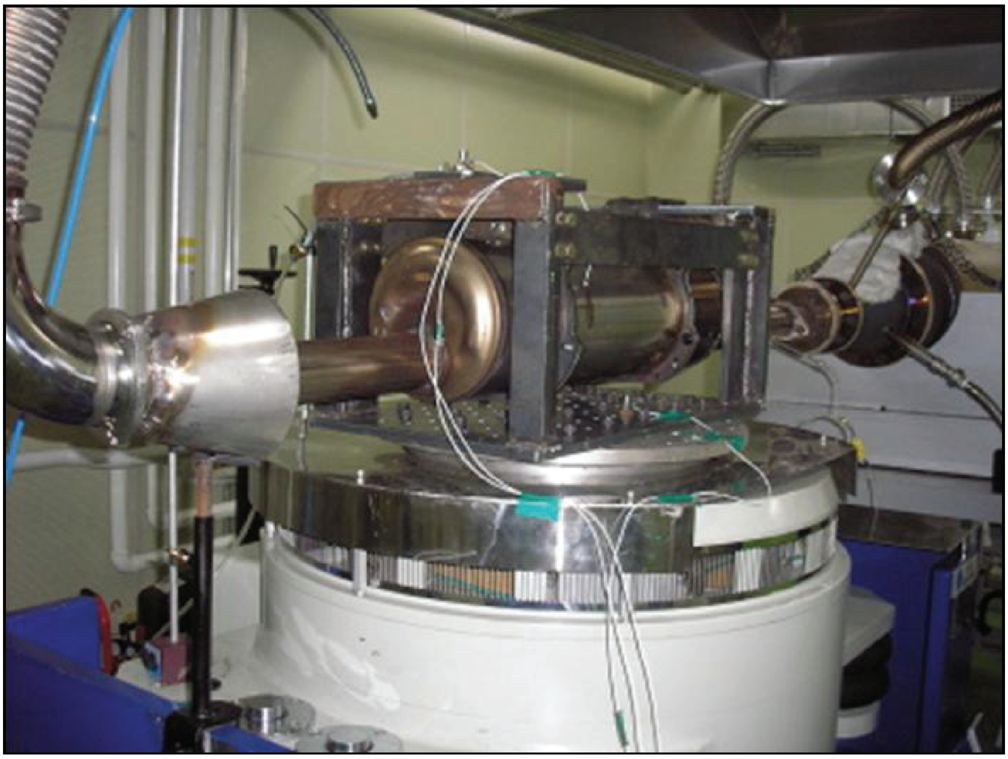


Figure 5 Photograph showing electro-dynamic vibration rig with muffler assembly

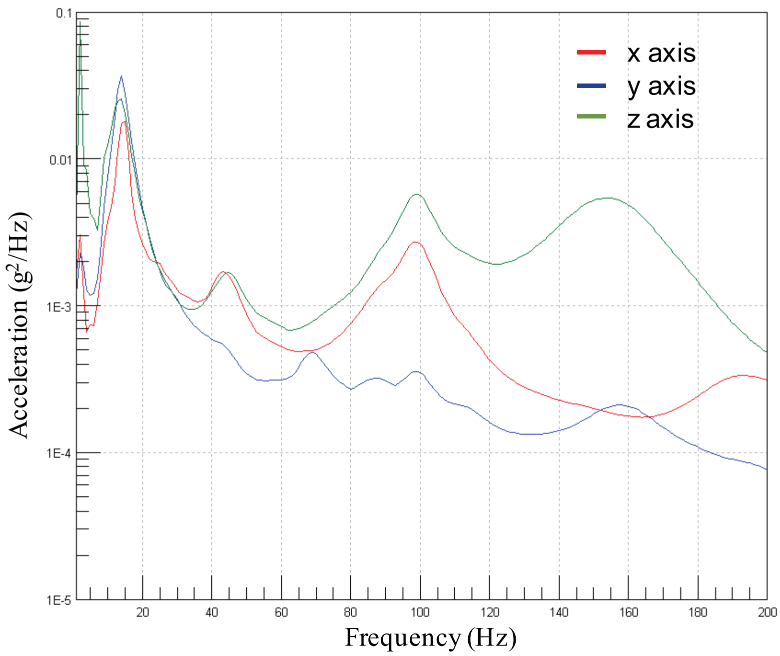


Figure 6 Input PSD used during vibration test

4.2 'Virtual shaker' test

CAE based analysis is usually performed using a Finite Element (FE) simulation. The simplest analyses use a quasi-static approach where the component is not sensitive to dynamic effects such as resonance and transients. If dynamic effects are important then more intensive analyses are required such as random vibration (or harmonic), transient and modal transient analyses.

Random vibration analysis is performed in the frequency domain with loads being specified in terms of a Power Spectral Density (PSD) function. This form of analysis is relatively quick. Transient analysis is performed in the time domain and requires significant computation time. Transient analysis is usually restricted to very short discrete events, and is not feasible when fatigue damage is attributable to long periods of high-cycle random vibration or acoustic loading.

A Random Vibration analysis using the Finite Element program MSC.Nastran[†] was performed. A transfer function solution was run to establish the component response and this was used by the nCode DesignLife[‡] program to estimate the fatigue life of the component. This analysis is equivalent to a single axis shaker table test as illustrated in Figure 4. The benefits of numerical simulation include:

- Pre-test validation – perform virtual durability assessment prior to the physical test to minimise risk of failure and avoid costly re-design and tooling changes
- Optimise the test configuration to ensure it maximises the damage potential in the least amount of time
- Ensure that the test PSD does not over-accelerate the test which could result in non-linear plasticity in the component
- Estimate residual life and safety margins for physical tests that do not run to destruction

nCode DesignLife™ is dedicated to fatigue analysis. Its features include:

- Random (PSD loading)
- Swept sine loading
- Static offset loads and residual stresses
- Temperature corrections
- Simple or complex, multi-segment SN curves

4.3 The FE analysis

Figure 7 shows the FE model with its boundary conditions. The PSD excitation is applied at the excitation point. Rigid elements link the Excitation point with the brackets.

4.4 Fatigue life estimates obtained

The results presented here are limited to the brackets for reasons of confidentiality. The FE-based fatigue analysis accurately identified all the failure locations and estimated fatigue lives were within a factor of 2 of observed test results. Both Dirlik and Lalanne approaches predicated similar lives.

Time domain reconstruction was only possible on a few selected elements due to computational requirements; however, these results correlated to within 10% of those estimated using Lalanne and Dirlik approaches.

[†] MSC.Nastran is a registered trademark of the MSC Software Corporation.
[‡] nCode, GlyphWorks and DesignLife are registered trademarks of HBM.

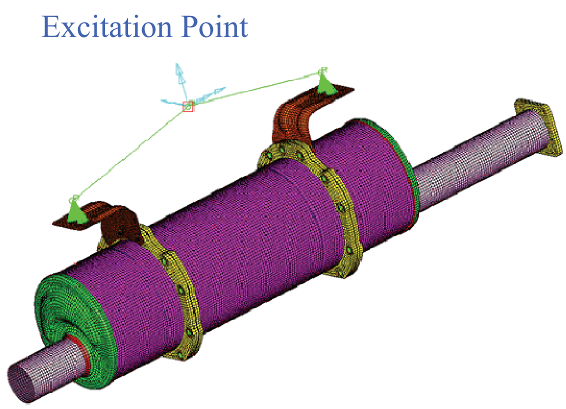


Figure 7 FE model from virtual vibration rig

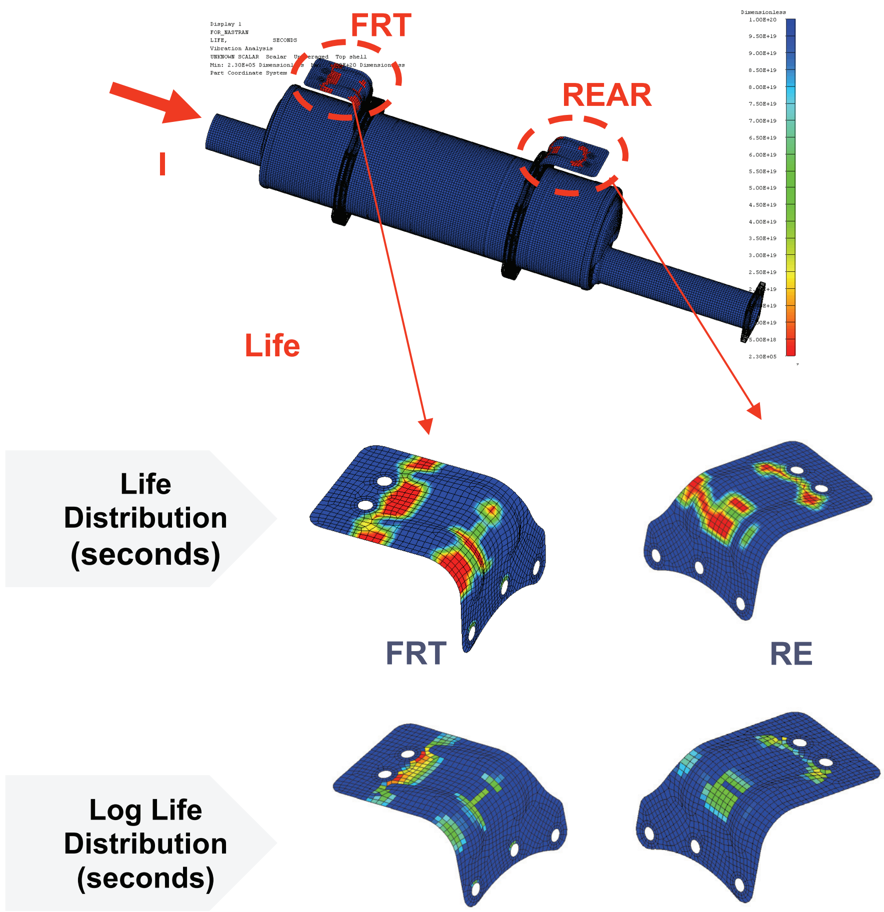


Figure 8 FE-based fatigue hot spots

5 CONCLUSIONS

This paper has presented a brief review of the most popular methods available for cycle counting directly from a PSD of stress. A full comparison between time signal and PSD cycle counting has been performed and the results demonstrate an excellent and robust correlation for the Dirlik and Lalanne methods. The Narrow-band method is generally too conservative for broad-band signals but demonstrates excellent correlation for narrow-band signals. The Steinberg method is very popular in the electronics industry and demonstrates good correlation for broad-band signals but is excessively non-conservative for narrow-band signals and is limited to single-slope SN curves. All the methods have been tested over a full range of Basquin slopes and are shown to give good correlation for all slope values below 18. For shallow SN curves of slope greater than 18, they show an increasing conservatism when compared with the time signal approach. It has been impossible to conclude whether this discrepancy is due to inaccuracies in the tail of the stress range histogram, or an inability to generate sufficiently long time signal reconstructions. The Dirlik approach is more conservative than Lalanne in this respect on account of the additional exponential distribution which is used in the tail.

The PSD techniques prove significantly more robust and efficient than the equivalent reconstructed time signals on account of the long time signal duration required to converge on a statistically representative rainflow result. A statistically representative PSD can be obtained from a very short time signal provided that the underlying process is 'Ergodic stationary Gaussian and random'; however, the rainflow cycle counting technique demands a very long time signal in order to achieve a statistically representative cycle count. In many practical cases it is simply impossible to obtain the required duration of time signal for rainflow counting and in these cases especially, the PSD-based approaches offer significantly better fatigue life estimates.

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