Optimized PSD Envelope for Nonstationary Vibration

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Introduction - Nonstationary Flight Data

- Liftoff Vibroacoustics
- Transonic Shock Waves
- Fluctuating Pressure at Max-Q
Electronic components in vehicles are subjected to shock and vibration environments.

Practical flight accelerometer time histories are nonstationary and non-Gaussian.

A single PSD with an implied stationary, normal distribution time history must be derived for design and test purposes - use rainflow fatigue!
Endo & Matsuishi 1968 developed the Rainflow Counting method by relating stress reversal cycles to streams of rainwater flowing down a Pagoda.

• Endo & Matsuishi, Rainflow Cycle Counting Method, 1968

• T. Dirlik, Application of Computers in Fatigue Analysis (Ph.D.), University of Warwick, 1985


• K. Ahlin, Comparison of Test Specifications and Measured Field Data, Sound & Vibration, 2006

• Scot McNeill, Implementing the Fatigue Damage Spectrum and Fatigue Damage Equivalent Vibration Testing, SAVIAC Conference, 2008


Conclusions

SDOF Model

- Assume component behaves as single-degree-of-freedom (SDOF) system

- *Avionics are typically black boxes for mechanical engineering purposes!*

- Unknowns
  - Component natural frequency
  - Amplification factor $Q$
  - Fatigue exponent $b$

- Perform fatigue damage calculation on each response for permutations of the three unknowns

- This adds conservatism to the final PSD envelope

- The fatigue calculation can be performed starting with either a time history or PSD base input
• A relative fatigue damage index can be calculated from the rainflow cycles using a Miners-type summation

\[ D = \sum_{i=1}^{m} A_i^b n_i \]

where

- \( A_i \) is the acceleration response amplitude from the rainflow analysis
- \( n_i \) is the corresponding number of cycles
- \( b \) is the fatigue exponent

• The damage index \( D \) becomes the *Fatigue Damage Spectrum (FDS)* metric as a function of: natural frequency, amplification factor \( Q \) and fatigue exponent \( b \)
A PSD envelope can be derived for nonstationary flight data using rainflow cycling counting and the relative fatigue damage index.

The enveloping is justified using a comparison of Fatigue Damage Spectra between the candidate PSD and the measured time history.

The derivation process can be performed in a trial-and-error manner in order to obtain the PSD with the least overall GRMS level which still envelops the flight data in terms of fatigue damage spectra.

Could also seek to minimize overall displacement, velocity, peak G^2/Hz level, etc.

Or minimize weighted average of these metrics.
Conclusions

Enveloping Approach (cont)

• The Dirlik semi-empirical method can be used to calculate the FDS for each candidate PSD in the frequency domain

• The immediate output of the Dirlik method is a “rainflow cycle probability density function (PDF)”

• The rainflow PDF can be converted to a cumulative histogram

• The cumulative histogram can be converted into individual cycles with their respective amplitudes

• Compare the fatigue spectra of the candidate PSD to that of the flight data for each Q & b case of interest

• Scale candidate PSD so that it barely envelops the flight data in terms of FDS

• Include some convergence option along the way

• Select the candidate which has the least overall GRMS level, or some other criteria
Dirlik method calculates rainflow cycle cumulative histogram from response PSD.

The Dirlik equation is based on the weighted sum of the Rayleigh, Gaussian and exponential probability distributions.

Uses area moments of the response PSD as weights.

- Sample base input and SDOF response
Conclusions

• The response analysis for the nonstationary time history is performed using the Smallwood, ramp invariant digital recursive filtering relationship, for each fn & Q

• Perform rainflow cycle count on response time history

• Calculate the damage index $D$ for each fn, Q & b

• The damage for each permutation is then plotted as function of natural frequency, as an FDS
• Derive a 60-second PSD to envelope the flight data

• Consider 800 candidate PSDs formed by random number generation, with four coordinates each
# Permutations

### Q & b Values for Fatigue Damage Spectra

<table>
<thead>
<tr>
<th>Case</th>
<th>Q</th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>9</td>
</tr>
<tr>
<td>3</td>
<td>30</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>30</td>
<td>9</td>
</tr>
</tbody>
</table>

**For Reference Only**

- **Natural Frequencies:** 20 to 2000 Hz

- All cases will be analyzed for each successive trial.
The PSD with the least overall GRMS which envelops the flight data via fatigue damage spectra.
FATIGUE DAMAGE SPECTRA  $Q=10$  $b=4$

- **PSD Envelope**
- **Measured Data**
FATIGUE DAMAGE SPECTRA  $Q=30 \quad b=4$

- **PSD Envelope**
- **Measured Data**

**NATURAL FREQUENCY (Hz)**

**DAMAGE INDEX**
Conclusions

FATIGUE DAMAGE SPECTRA  Q=10  b=9

PSD Envelope
Measured Data

DAMAGE INDEX

NATURAL FREQUENCY (Hz)

FDS Comparison 3
Conclusions
Maximum Envelope is traditional piecewise stationary method, but its PSD need further simplification.
Conclusions

• An optimized PSD envelope was derived for nonstationary flight data using the fatigue damage spectrum method

• The FDS case with both the highest Q & b values drove the PSD derivation for the sample flight data

• Still recommend using permutations because other cases may be the driver for a given time history

• The method can be used more effectively if the natural frequency, amplification factor, and fatigue exponent are known

• The method is flexible

• The PSD duration can be longer or shorter than the flight vibration duration

• Could require the candidate PSDs to each have a ramp-plateau-ramp shape

• *A similar method could be used for deriving force & pressure PSDs*
Software

VIBRATIONDATA
Shock & Vibration Software & Tutorials

• Software and papers for applying this method are freely given at:

  http://vibrationdata.wordpress.com/

• Will also cover in a future Shock & Vibration Webinar
Thank you

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