Statistical Energy Analysis Parameters
Revision AA

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Introduction

Statistical energy analysis (SEA) is a vibratory energy-flow technique which provides prediction procedures that are suitable for high frequencies. The application of SEA requires that the system be divided into a set of coupled subsystems. Each subsystem represents a group of modes with similar characteristics. Parameters such as the coupling loss factors and modal densities represent ensemble average quantities.

The statistical prediction gives energy averages over spatial locations and bands of frequency. The bandwidths are typically one-third octave. Velocity, acceleration, pressure, stress and other response parameters can be calculated from the subsystem energy.

Input Variables

There are a number of input parameters needed for SEA including:

1. Modal density
2. Dissipation loss factors (equals twice the viscous damping ratio)
3. Coupling loss factors
4. Driving Point impedance and mobility
5. Characteristic impedance of air or gas
6. Radiation efficiency
7. Transmission loss
8. Critical frequency
9. Ring frequency for cylindrical shell
10. Subsystem mass
11. Wave speed
12. External power inputs

Some of these parameters are related to one another. Some are needed for the primary analysis. Others are needed to calculate secondary response variables from the total energy. Equations for these variables are given in the appendices.

Assumptions

SEA makes certain assumptions, including:

1. The subsystems in SEA are finite, linear, elastic structures or fluid cavities.

2. For a system with two subsystems, the energy flow is proportional to the acoustic or vibrational energies of the two subsystems.
3. Subsystem modes in each band must be uncoupled from one another or have equal energies.
4. Subsystems have small modal damping, equal for all modes in a given frequency band.
5. The primary response is resonant.
6. Acoustical fields are either diffuse or turbulent boundary layer.
7. Acoustic volumes have much higher modal density than the structures in models.
8. A cylindrical shell behaves as a flat plate above its ring frequency.
9. Traditional SEA has assumed steady-state incoherent broadband random excitation.
10. Transient SEA methods have also been developed.
11. Boundary conditions become less relevant at higher frequencies.
12. A circular plate has the same modal density as a rectangular plate of the same surface area.

Another important assumption depends on the modal overlap value, which describes dissipation in the subsystems of an SEA model. It is defined as the ratio of the damping bandwidth to the average separation of the natural frequencies of the modes. It measures the 'smoothness' of the frequency response function. A high modal overlap factor implies either high damping or high modal density, or both. The modal overlap M_{OV} is

$$M_{OV} = n \eta f \tag{1}$$

where

n	Modal density (modes/Hz)
η	Loss factor
f	Frequency (Hz)

Statistical energy analysis is suitable if the modal overlap is ≥ 1. Otherwise, deterministic methods, such as the finite element or boundary element method may be performed. Further information is given in Appendix J.

Power Flow Equation for One System

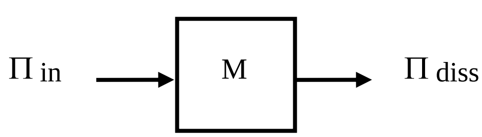

Figure 1.

The velocity-power equation is

$$M\eta < v^2 > = \frac{1}{\omega}\Pi_{in} \qquad (2)$$

where

M	Mass		$<v^2>$	Spatial average mean velocity squared
η	Loss factor		ω	Frequency (rad/sec)
Π_{in}	Power input			

The left-hand side of equation (2) represents the dissipated power.

Power Flow Equation for Two Subsystems

A diagram for a system consisting of two subsystems is shown in Figure 2. The arrows indicate power flows. The flow between subsystems actually occurs in both directions. There are three types of power terms.

$\Pi_{in,i}$	Power input to subsystem i
$\Pi_{diss,i}$	Power dissipated by subsystem i
Π_{ij}	Power transferred from subsystem i to j

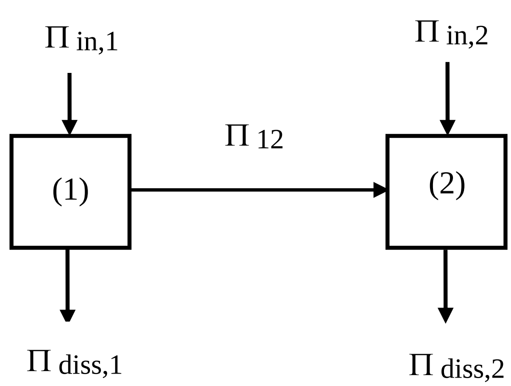

Figure 2.

The total energy $\langle E_i \rangle$ in subsystem i is calculated via

$$\begin{bmatrix} \eta_1 + \eta_{12} & -\eta_{21} \\ -\eta_{12} & \eta_2 + \eta_{21} \end{bmatrix} \begin{Bmatrix} \langle E_1 \rangle \\ \langle E_2 \rangle \end{Bmatrix} = \frac{1}{\omega} \begin{Bmatrix} \Pi_{in,1} \\ \Pi_{in,2} \end{Bmatrix} \tag{3}$$

where

η_i	Dissipation loss factor in subsystem i
η_{ij}	Coupling loss factor from subsystem i to j
ω	Band center frequency (rad/sec)

Note that the (2 x 2) coefficient matrix is typically nonsymmetrical. The velocity for each system can then be calculated as a post-processing step.

$$< E_i > = M_i < v_i^2 > \tag{4}$$

where

M_i	Mass of subsystem i
$< v_i^2 >$	Spatial average mean square velocity in subsystem i

Power Flow Equation for Four Subsystems in Series

A diagram for a system consisting of four subsystems is shown in Figure 3.

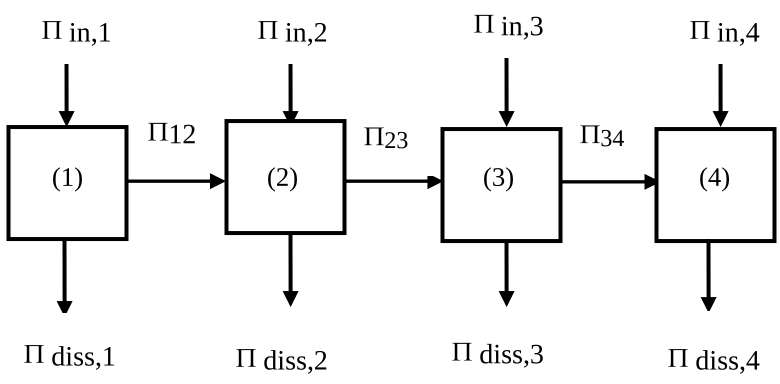

Figure 3.

The total energy $\langle E_i \rangle$ in subsystem i is calculated via

$$\begin{bmatrix} \eta_1+\eta_{12} & -\eta_{21} & 0 & 0 \\ -\eta_{12} & \eta_2+\eta_{21}+\eta_{23} & -\eta_{32} & 0 \\ 0 & -\eta_{23} & \eta_3+\eta_{32}+\eta_{34} & -\eta_{43} \\ 0 & 0 & -\eta_{34} & \eta_4+\eta_{43} \end{bmatrix} \begin{Bmatrix} \langle E_1 \rangle \\ \langle E_2 \rangle \\ \langle E_3 \rangle \\ \langle E_4 \rangle \end{Bmatrix} = \frac{1}{\omega} \begin{Bmatrix} \Pi_{in,\,1} \\ \Pi_{in,\,2} \\ \Pi_{in,\,3} \\ \Pi_{in,\,4} \end{Bmatrix} \tag{5}$$

<u>Reference</u>

J. Wijker, Random Vibrations in Spacecraft Structure Design, Springer, New York, 2009.

Appendix Index

Appendix	Topic
A	Structural Wave Speeds, Wavelengths & Wavenumbers
B	Critical Frequency & Ring Frequency
C	Radiation Efficiency & Resistance
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O	Baffled Homogeneous or Honeycomb Sandwich Panel Response to Diffuse Acoustic Pressure Field
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Appendix A

Structural Wave Speeds, Wavelengths & Wavenumbers

Variables for Homogeneous Beams and Plates

B	Flexural Rigidity
C_L	Longitudinal wave speed
C_S	Shear wave speed
C_B	Bending phase speed
C_G	Bending group speed
E	Elastic modulus
G	Shear modulus, $G=E / (2+2\nu)$
I	Area moment of inertia

h	Plate thickness
k_B	Wavenumber
ρ	Mass density (mass/volume)
m'	Mass/length
m''	Mass/area
ν	Poisson ratio
ω	Frequency (rad/sec)
λ	Wavelength

Wave Characteristics

Wave Type	Characteristic	Group & Phase Relationship
Compression	Non-dispersive	Equal
Shear	Non-dispersive	Equal
Bending	Dispersive	$C_G = 2C_B$

Compression Wave Speed

Beam or Rod $$C_L = \sqrt{E/\rho} \qquad \text{(A-1)}$$

Plate $$C_L = \sqrt{\frac{E}{\rho\left(1-\nu^2\right)}} \qquad \text{(A-2)}$$

Shear Wave Speed

Beams $$C_S = \sqrt{G/\rho} \qquad \text{(A-3)}$$

Bending Wave Speed

Beam $$C_B = \left(\frac{EI}{m'}\right)^{1/4} \sqrt{\omega}$$ (A-4)

Plate $$C_B = \left(\frac{B}{m''}\right)^{1/4} \sqrt{\omega}$$ (A-5)

Flexural Rigidity $$B = \frac{E h^3}{12\left(1-\nu^2\right)}$$ (A-6)

Equations (A-1) through (A-6) are taken from Reference A.1. See also Reference A.2.

Bending Wavelengths & Wavenumbers

Beam $$k_B = \frac{2\pi}{\lambda} = \sqrt{\omega}\left[\frac{12\rho}{E h^2}\right]^{1/4}$$ (A-7)

Plate $$k_B = \frac{2\pi}{\lambda} = \sqrt{\omega}\left[\frac{12\rho\left(1-\nu^2\right)}{E h^2}\right]^{1/4}$$ (A-8)

Equation (A-7) & (A-8) are taken from Reference A.3, Equations (3.11) & (3.18), respectively.

Variables for Honeycomb Sandwich Panels

c	Wave speed
c_s	Shear speed
c_b	Bending speed, overall panel
c_{bf}	Bending speed, individual face sheet
B	Flexure rigidity, overall panel
B_f	Flexural rigidity, individual face sheet

ν	Face sheet Poisson ratio
G_c	Core shear modulus
t_c	Core thickness
t_f	Individual face sheet thickness
M	Overall mass density (mass/area)
ω	Frequency (rad/sec)

$$\left(c_s^2 / c_b^4\right) c^6 + c^4 - c_s^2 c^2 - c_{bf}^4 = 0 \qquad \text{(A-9)}$$

$$c_b = \sqrt{\omega}\,\left(B / M\right)^{1/4} \qquad \text{(A-10)}$$

$$c_s = \sqrt{G_c\, t_c / M} \qquad \text{(A-11)}$$

$$c_{bf} = \sqrt{\omega}\,\left(2 B_f / M\right)^{1/4} \qquad \text{(A-12)}$$

$$B = \frac{E\, t_f \left(t_c + t_f\right)}{2\left(1 - \nu^2\right)} \qquad \text{(A-13)}$$

Equations (A-9) through (A-12) are taken from References A.4 & A.5. Equation (A-13) is from Reference A.6.

Honeycomb Sandwich Panel Transition Frequencies

The following summary is taken from References A.4 and A.5.

Range	Characteristic
Low Frequencies	Bending of the entire structure as if were a thick plate
Mid Frequencies	Transverse shear strain in the honeycomb core governs the behavior
High Frequencies	The structural skins act in bending as if disconnected

The transition for global bending-to-shear motion is considered to occur at the frequency at which the global bending phase speed equals the core shear speed, as an idealization.

The global bending-to-shear transition frequency ω_1 is

$$\omega_1 = \frac{G_c\, h_c}{\sqrt{B M}} \qquad \text{(A-14)}$$

The transition frequency ω_2 for shear-to-face sheet bending motion is considered to occur at the frequency at which the core shear speed equals the face sheet bending phase speed.

$$B_f = \frac{E_f t_f^{\,3}}{12\left(1-\nu^2\right)} \qquad \text{(A-15)}$$

$$\omega_2 = \frac{G_c h_c}{\sqrt{2 B_f M}} \qquad \text{(A-16)}$$

References

A.1 Beranek & Ver, editors; Noise and Vibration Control Engineering Principles and Applications, Wiley, New York, 1992. Table (9.1)

A.2 L. Cremer and M. Heckl, Structure-Borne Sound, Springer-Verlag, New York, 1988. Page 101, Equation (85)

A.3 M. Brink, The Acoustic Representation of Bending Waves, M.Sc. Thesis, Delft, 2002.

A.4 D. Yuan, N. Roozen, O. Bergsma, A. Beukers, Sound Insulation of Composite Cylindrical Shells: a Comparison between a Laminated and a Sandwich Cylinder, Acoustics 2012, Hong Kong

A.5 H. Kurtze BGW, New Wall Design for High Transmission Loss or High Damping, J Acoustical Society America, 31, 1959 739-748.

A.6 J. Wijker, Random Vibrations in Spacecraft Structure Design, Springer, New York, 2009. Page 283

Appendix B

Critical Frequency & Ring Frequency

Introduction

σ_{rad}	Radiation efficiency		h	Panel thickness
ω	Frequency (rad/sec)		E	Elastic modulus
f	Frequency (Hz)		G	Shear modulus
f_{cr}	Critical frequency		ρ	Mass density (mass/volume)
c_o	Speed of sound in surrounding gas		ν	Poisson ratio

Homogeneous Thin Panel or Large Diameter, Thin-wall Cylinder

The critical frequency is the frequency at which the speed of the free bending wave in a structure becomes equal to the speed of the airborne acoustic wave.

$$f_{cr} = \frac{{c_o}^2}{2\pi h} \sqrt{\frac{12(1-\nu^2)\rho}{E}} \qquad \text{(B-1)}$$

The critical frequency formula is taken from Reference B.1. Note that a large diameter, thin-wall cylinder tends to behave as a flat, thin plate above its ring frequency.

Thick Panel

N	Shear rigidity		$\hat{k}$	Shear factor, $\hat{k} = \sqrt{5/6}$
B	Plate stiffness factor		m	Mass per area

The thick panel equations are taken from References B.1 & B.2.

$$f_{cr}{}^2 = \frac{1}{(2\pi)^2}\left[\frac{c^4 m}{B}\right]\left[\frac{1}{1-\left(c^2 m/N\right)}\right] \quad \text{for} \quad \left(c^2 m/N\right) < 1 \qquad \text{(B-2)}$$

$$N = \hat{k}\, G\, h \qquad \text{(B-3)}$$

$$B = \frac{E h^3}{12\left(1-\nu^2\right)} \qquad \text{(B-4)}$$

Honeycomb Sandwich Panel

E	Face sheet elastic modulus		H	Core thickness
G	Core shear modulus		t_f	Face sheet thickness, individual
ν	Poisson ratio		S	Shear Stiffness
m	(Total Mass)/area		D	Plate stiffness factor

The honeycomb sandwich equations are taken from Reference B.1.

$$f_{cr}^2 = \frac{1}{(2\pi)^2}\left[\frac{c_o^4 m}{D}\right]\left[\frac{1}{1-\left(c_o^2 m/S\right)}\right] \quad \text{for} \quad \left(c^2 m/S\right) < 1 \tag{B-5}$$

$$S = Gh\left(1 + (t_f / h)\right)^2 \tag{B-6}$$

$$D = \frac{E t_f \left(h + t_f\right)^2}{2\left(1-\nu^2\right)} \tag{B-7}$$

Composite Panel

m	(Total Mass)/area		G	Shear modulus (isotropic assumed)
S	Shear stiffness		H	Thickness

The composite panel equations are taken from Reference B.1.

$$f_{cr} = \frac{c^2\sqrt{m/\overline{D}}}{2\pi\sqrt{\frac{3+\alpha}{4}}} \quad \text{thin composite panel} \tag{B-8}$$

$$f_{cr} = \frac{c^2\sqrt{m/\overline{D}}}{2\pi\sqrt{\frac{3+\alpha}{4}-\frac{c^2 m}{S}}} \quad \text{thick composite panel} \tag{B-9}$$

$$\alpha = \frac{D_{12} + 2D_{66}}{\overline{D}} \tag{B-10}$$

$$\overline{D} = D_{11} = D_{22} \tag{B-11}$$

$$S = G h \tag{B-12}$$

The bending stiffness coefficients are taken from the moment-rotation gradient relationship.

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{21} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix} \begin{Bmatrix} \partial\phi_x / \partial x \\ \partial\phi_y / \partial y \\ \left(\partial\phi_x / \partial y\right) + \left(\partial\phi_y / \partial x\right) \end{Bmatrix} \tag{B-13}$$

Homogeneous Cylinder, Ring Frequency

The cylindrical shell moves radially outward and the radially inward at the ring frequency if the cylinder has infinite length. The ring frequency from Reference B.3 is

$$f_r = \frac{1}{2\pi R}\sqrt{E / \rho} \tag{B-14}$$

where R is the radius

Honeycomb Sandwich Cylinder, Ring Frequency

Equation (B-14) can also be used for a honeycomb sandwich cylinder by using the properties of the outermost skin, per Reference B-4.

References

B.1 Beranek and Ver, Noise and Vibration Control Engineering Principles and Applications, Wiley, New York, 1992. Table 4.4, Equations (9.84) & (9.216), Appendix J

B.2 Renji, Nair, Narayanan, Critical and Coincident Frequencies of Flat Panels, Journal of Sound and Vibration, (205)(1), 1997.

B.3 E. Szechenyi, Modal Densities and Radiation Efficiencies of Unstiffened Cylinders using Statistical Methods, Journal of Sound and Vibration, 1971. Appendix I.

B.4 T. Irvine, Honeycomb Sandwich Ring Mode, Vibrationdata, 2018.

Appendix C

Radiation Efficiency & Resistance

Common Variables

σ_{rad}	Radiation efficiency		ρc_o	Characteristic acoustic impedance of gas
ω	Frequency (rad/sec)		h	Panel thickness
f	Frequency (Hz)		E	Elastic modulus
f_c	Critical frequency (from Appendix B)		ρ	Mass density (mass/volume)
c_o	Speed of sound in surrounding gas		ν	Poisson ratio

The radiation efficiency σ_{rad} relates the radiated sound power to the spatially averaged vibration. The radiation depends on the critical frequency among other variables.

Panel, Homogeneous, Baffled

L_{max}	Maximum of length & width		λ_c	$= c_o / f_c$
L_{min}	Minimum of length & width		P	perimeter
β	$= \sqrt{f / f_c}$			

$$\sigma_{rad} = \frac{2\lambda_c}{S}\left[g_1(\beta) + \frac{P}{2\lambda_c} g_2(\beta) \right] C_1 \quad \text{for } f < fc \tag{C-1}$$

$$\sigma_{rad} = 0.45\sqrt{P/\lambda_c}\left(L_{min} / L_{max}\right)^{1/4} \quad \text{for } f = fc \tag{C-2}$$

$$\sigma_{rad} = \left(1 - f_c / f\right)^{-1/2} \quad \text{for } f \geq 1.3\ fc \tag{C-3}$$

$$g_1(\beta) = \begin{cases} \left(\dfrac{4}{\pi^4}\right)\left[\left(1-2\beta^2\right) \Big/ \beta\left(1-\beta^2\right)^{1/2}\right], & \text{for } f < 0.5 f_c \\ 0, & \text{for } f \geq 0.5 f_c \end{cases} \tag{C-4}$$

$$g_2(\beta) = \left(\frac{1}{4\pi^4}\right)\left\{\left[\left(1-\beta^2\right)\ln\left(\frac{1+\beta}{1-\beta}\right) + 2\beta\right] \Big/ \left(1-\beta^2\right)^{3/2}\right\} \quad \text{(C-5)}$$

$$C_1 = \begin{cases} 1, & \text{for simply supported edges} \\ \beta^2 \exp(10\lambda_c / P), & \text{for clamped edges} \end{cases} \quad \text{(C-6)}$$

Equations (C-1) through (C-6) are taken from Reference C.1.

Panel, Homogeneous, Freely-Suspended

P	Perimeter	S	Surface area

$$\sigma_{rad} = \frac{P c_o}{\pi^2 S f_c}\sqrt{\frac{f}{f_c}} \quad \text{for } f \le f_b, \quad f_b = f_c + \frac{5 c_o}{P} \quad \text{(C-7)}$$

$$\sigma_{rad} = \frac{1}{\sqrt{1-(f_c/f)}} \quad \text{for } f > f_b \quad \text{(C-8)}$$

Equations (C-7) and (C-8) are taken loosely from Reference C.1. The results seem to agree with Reference C.2.

Panel, Honeycomb Sandwich

k_a	Acoustic wave number	c_p	Wave speed from equation (A-7)
k_p	Unloaded panel wave number		

$$\sigma_{rad} = \begin{cases} 0.47\left(k_a / k_p\right), & \text{for } k_a < 1.5 k_p \\ 1, & \text{for } k_a \ge 1.5 k_p \end{cases} \quad \text{(C-9)}$$

$$k_a = \omega / c_o \quad \text{(C-10)}$$

$$k_p = \omega / c_p \quad \text{(C-11)}$$

Equations (C-9) through (C-11) are taken from Reference C.3.

Panel, Ribbed

A_p	Panel surface area		L	Total rib length
σ_p	Panel radiation efficiency		$\hat{R}_{rad}$	Radiation resistance per length

The radiation resistance R_{rad} is

$$R_{rad} = (R_{rad})_{panel} + (R_{rad})_{ribs} \tag{C-12}$$

$$(\hat{R}_{rad})_{ribs} = \rho c_o \ \lambda_p \ g_3(f / f_c) \tag{C-13}$$

$$\lambda_p = \sqrt{c_o{}^2 / f_c \, f} \tag{C-14}$$

$$(R_{rad})_{ribs} = \left[(\hat{R}_{rad})_{ribs}\right] L \tag{C-15}$$

$$(R_{rad})_{panel} = \rho c_o \ A_p \ \sigma_p \tag{C-16}$$

Equations (C-12) are taken from References (C-4) and (C-5).

Cylinder, Homogeneous

f_r	Ring frequency (Hz) See Equation (B-14)		F	1.122 for one-third octave bands 1.414 for octave bands
μ	Poisson ratio			

The following equation is taken from Reference C.6.

$$\sigma_{rad} = \frac{\nu_o{}^{3/2} \ f_r / f_c}{2B(F - 1/F)} \left(1 - \nu_o \sqrt{1 - \nu_o{}^2 (f_r / f_c)^2}\right) \left[\frac{1}{\sqrt{(1/F) - \nu_o}} - \frac{1}{\sqrt{F - \nu_o}}\right] \sqrt{12(1 - \mu^2)} \tag{C-17}$$

$$\nu_o = f / f_r \tag{C-18}$$

$$B = 2.5 \sqrt{\nu_o} \quad \text{for } \nu_o \le 0.48 \tag{C-19}$$

$$B = 3.6 \nu_o \quad \text{for } 0.48 < \nu_o \le 0.83 \tag{C-20}$$

$$B = 2 + \frac{0.23}{(F - 1/F)}\left[F\cos\left(\frac{1.745}{F^2 \nu_o{}^2}\right) - \frac{1}{F}\left(\frac{1.745\, F^2}{\nu_o{}^2}\right)\right] \quad \text{for } \nu_o > 0.83 \tag{C-21}$$

Equation (C-17) is valid if either of the following conditions is met. Otherwise, use Reference C.6.

i) $\nu_o < 1/F$ and $\nu_o < 0.65 \log_{10}(3 f_c / f_r)$

ii) $(f_r / f_c) > 1.5$ and $\nu_o < (f_c / f_r)$

Cylinder, Honeycomb Sandwich

k	Acoustic wave number
k_m	Axial wave number, index m
k_n	Circumferential wave number, index n

L	Length
d	Diameter

The natural frequencies and wave numbers are calculated per the method in Reference C.8. The corresponding radiation efficiencies are calculated via Reference C.6. See also Reference C.4.

Cylinder modes must be categorized as either acoustically fast (AF) or acoustically slow (AS) in order to determine their ability to interact with sound waves.

The distinction between these two classes is that an AF mode has a structural wavenumber smaller than the acoustic trace, according to Reference C.6.

$$k = \omega / c_o \tag{C-22}$$

$$k_m = m\pi / L \tag{C-23}$$

$$k_n = \frac{n\pi}{\pi d} = \frac{n}{d} \tag{C-24}$$

For acoustically fast (AF) modes

$$k^2 \geq k_m{}^2 + k_n{}^2 \tag{C-25}$$

The radiation efficiency for AF modes is

$$\sigma_{rad} = \sqrt{1 - \frac{(k_m{}^2 + k_n{}^2)}{k^2}} \tag{C-26}$$

$$\sigma_{rad} \approx 1 \quad \text{for} \quad k^2 >> k_m{}^2 + k_n{}^2 \tag{C-27}$$

Relationship between Radiation Efficiency & Resistance for a Panel

R	Radiation resistance
η_{pa}	Coupling loss factor, panel-to-air

A	Panel surface area
M	Panel mass

The following equations are taken from Reference C.3.

The radiation resistance R is

$$R = \rho c_o A \sigma_{rad} \tag{C-28}$$

The coupling loss factor η_{pa} is

$$\eta_{pa} = \frac{R}{M\omega} \tag{C-29}$$

References

C.1 Beranek and Ver, Noise and Vibration Control Engineering Principles and Applications, Wiley, New York, 1992. Table (9.8) & Equation (9.84)

C.2 Jordi Villar Venini, Vibroacoustic Modelling of Orthotropic Plates, Master Thesis, Universitat Politècnica de Catalunya, BarcelonaTech, 2011. Figure 3.28

C.3 J. Wijker, Random Vibrations in Spacecraft Structure Design, Springer, New York, 2009. Equations (4.133) , (4.134) & (4.144)

C.4 H. Maidanik, Response of Ribbed Panels to Reverberant Fields, Journal of the Acoustical Society of America, Volume 34, Number 6, June 1962. Equations (2.16), (2.64) & (2.66)

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C.6 F. Szechenyi, Modal Densities and Radiation Efficiencies of Unstiffened Cylinders using Statistical Methods, Journal of Sound and Vibration, 1971. Equations (10) & (12) & (Section 5.1, page 73)

C.7 R. Lyon, Machinery Noise and Diagnostics, Butterworth-Heinemann, Boston, MA, 1987. Figure (5.37)

C.8 Bing-ru, et al, Study on Applicability of Modal Analysis of Thin Finite Length Cylindrical Shells using Wave Propagation Approach, Journal of Zhejiang University SCIENCE, 2005.

Appendix D

Driving Point Impedance & Mobility

Introduction

Y	Mobility
Z	Mechanical impedance
E	Elastic modulus

ρ	Mass density (mass/volume)
ν	Poisson ratio

$$Z = 1 / Y \tag{D-1}$$

Thin Plate

Let h = plate thickness.

Force at Middle Point
$$Z = 8\sqrt{B \rho h} \tag{D-2}$$

Force at Edge Point
$$Z = 3.5\sqrt{B \rho h} \tag{D-3}$$

$$B = \frac{Eh^3}{12\left(1 - \nu^2\right)} \tag{D-4}$$

Equations (D-1) through (D-4) are taken from Reference D.1.

Unstiffened Cylindrical Shell

f_1	Fundamental frequency
f_r	Ring frequency See Equation (B-14)
f	Center frequency (Hz)
ω	Center frequency (rad/sec)

R	Radius
L	Length
h	Thickness

The fundamental frequency is

$$f_1 = \frac{0.375}{L}\sqrt{\frac{E\,h}{\rho R}} \qquad \text{(D-5)}$$

The ring frequency is

$$f_r = \left[1/\left(2\pi R\right)\right]\sqrt{E/\rho} \qquad \text{(D-6)}$$

The impedance is

$$Z = 2.5\,E\,h\left(\frac{R}{L}\right)^{1/2}\left(\frac{h}{R}\right)^{1.25}\left(\frac{1}{\omega}\right) \qquad \text{for } f \le f_1 \qquad \text{(D-7)}$$

$$Z = \left(4/\sqrt{3}\right)\rho h^2\sqrt{\frac{E}{\rho R}}\left(\frac{E}{\rho}\right)^{1/4}\frac{1}{\sqrt{\omega}} \qquad \text{for } f_1 \le f \le f_r \qquad \text{(D-8)}$$

$$Z = \left(4/\sqrt{3}\right)h^2\sqrt{E\rho} \qquad \text{for } f \le f_r \qquad \text{(D-9)}$$

Equations (D-5) through (D-9) are taken from Reference D.2.

Beam or Rod, Longitudinal, Semi-infinite

Let A = cross-section area.

$$Z = A\sqrt{E\rho} \qquad \text{(D-10)}$$

Equation (D-10) is taken from Reference D.3.

General Structure

n	Modal density (modes/Hz)		M	Mass

$$Y = \frac{n}{4M} \tag{D-11}$$

Equation (D-11) is taken from Reference D.4.

References

D.1 Beranek and Ver, Noise and Vibration Control Engineering Principles and Applications, Wiley, New York, 1992. Table (9.3)

D.2 K. Change & H. Kao, Simplified Techniques for Predicting Vibro-Acoustic Environments, Wyle Laboratories. Huntsville, Alabama, 1975. Table 1. Available from NASA Technical Reports Server.

D.3 L. Cremer and M. Heckl, Structure-Borne Sound, Springer-Verlag, New York, 1988. Table IV.1, page 317.

D.4 R. Lyon & R. DeJong, Theory and Application of Statistical Energy Analysis, Second Edition, Lyon Corp, Cambridge, MA, 1998. Equation (8.5.2)

Appendix E

Mass Ratios for Structures with Equipment

<u>Variables</u>

$\left\langle a_b^2(f) \right\rangle$	Structural loaded with equipment, spatial average mean square acceleration
$\left\langle a_i^2(f) \right\rangle$	Unloaded structure, spatial average mean square acceleration
f	Center frequency (Hz)

M_i	Bare structure mass
M_b	Added equipment mass
m_i	Bare structure [mass/(surface area)]
m_b	Equipment [mass/(footprint area)]

The acceleration terms may be either power spectra or power spectral densities, as long as they are consistent.

Mass Ratio Method

$$\left\langle a_b^2(f) \right\rangle = \left\langle a_i^2(f) \right\rangle \frac{M_i}{M_i + M_b} \tag{E-1}$$

Mass Area Density Ratio Method

$$\left\langle a_b^2(f) \right\rangle = \left\langle a_i^2(f) \right\rangle \frac{m_i}{m_i + m_b} \tag{E-2}$$

Note that the Mass Ratio Method is more conservative than the Mass Area Density Ratio Method.

<u>References</u>

E.1 J. Wijker, Random Vibrations in Spacecraft Structure Design, Springer, New York, 2009. Equations (4.218) & (4.219)

E.2 R. Barrett, NASA TN E-1836, Techniques for Predicting Localized Vibratory Environments of Rocket Vehicles, 1963. Page 22

Appendix F

Dissipation Loss Factor

Introduction

The dissipation loss factor is η. The frequency is f.

Panel

Pivot frequency $f_p = 2500$ Hz.

$$\eta = \begin{cases} 0.050 , & f \le 80\,\text{Hz} \\ 1.8 / f^{0.87} , & 80\text{ Hz} < f < f_p \\ 0.002 , & f \ge f_p \end{cases} \tag{F-1}$$

Equation (F-1) is taken from Reference F.1.

Sandwich Panel

Pivot frequency $f_p = 500$ Hz.

Bare Sandwich Panel

$$\eta = 0.3 / f^{0.63} \tag{F-2}$$

Built-up Sandwich Panel

$$\eta = \begin{cases} 0.050 , & f < f_p \\ 0.050\sqrt{f_p / f} , & f \ge f_p \end{cases} \tag{F-3}$$

The Sandwich Panel equations are taken from Reference F.1.

Stowed Solar Array

Pivot frequency f_p = 250 Hz.

$$\eta = \begin{cases} 0.050\,, & f < f_p \\ 0.050\sqrt{f_p / f}\,, & f \geq f_p \end{cases} \tag{F-4}$$

Equation (F-4) is taken from Reference F.1.

Cylindrical Shell

$$\eta = \begin{cases} 0.002 \text{ to } 0.03\,, & f < 3000 \text{ Hz} \\ 0.004 \text{ to } 0.006\,, & f \geq 3000 \text{ Hz} \end{cases}$$

(F-5)

Equation (F-5) is taken from Reference F.1, page 272.

Acoustic Room

T_R is the reverberation Time (sec) for a 60 dB decrease relative to starting energy level.

$$\eta = \frac{2.2}{f\, T_R} \tag{F-6}$$

Equation (F-5) is taken from Reference F.1.

Reference

F.1 J. Wijker, Random Vibrations in Spacecraft Structure Design, Springer, New York, 2009. Equations (4.117) through (4.122) and page 272.

Appendix G

Coupling Loss Factor

Introduction

The coupling loss factor for power flow from subsystem i to j is η_{ij}.

L-Beam

c_{bi}	Bending phase speed in transmitting beam
c_{li}	Longitudinal wave speed in transmitting beam
τ_{ij}	Transmission coefficient

ω	Center frequency (rad/sec)
L_i	Length of transmitting beam

The following beam equations are taken from Reference G.1.

Coupling loss factor for propagation from beam i to j.

$$\eta_{ij} = \frac{c_{bi}\,\tau_{ij}}{\omega L_i} \qquad \text{(G-1)}$$

Bending-to-Bending

$$\tau_{bb} = \frac{2\beta^2 + 1}{9\beta^2 + 6\beta + 2} \qquad \text{(G-2)}$$

Bending-to-Longitudinal & vice versa

$$\tau_{bl} = \tau_{bl} = \frac{8\beta^2 + 5\beta}{9\beta^2 + 6\beta + 2} \qquad \text{(G-3)}$$

Longitudinal-to-Longitudinal

$$\tau_{ll} = \frac{\beta^2}{9\beta^2 + 6\beta + 2} \qquad \text{(G-4)}$$

$$\beta = c_{bi}/c_{li} \qquad \text{(G-5)}$$

L-Shaped Plates

L	Junction length
$C_{B,1}$	Bending phase speed in transmitting plate
$A_{p,1}$	Surface area of transmitting plate
ω	Frequency (Hz)
h_j	Thickness of plate j

τ_{12}	Wave transmission coefficient from plate 1 to 2
$\tau_{12}(0)$	Normal transmission coefficient from plate 1 to 2
$C_{L,j}$	Longitudinal wave speed in plate j
ρ_j	Mass density (mass/volume) in plate j

The coupling loss factor equations for power flow from transmitting plate 1 to receiving plate 2 are taken from Reference G.1.

$$\eta_{12} = \frac{2}{\pi}\frac{C_{B,1}L}{\omega A_{p,1}}\tau_{12} \qquad \text{(G-6)}$$

$$\tau_{12} = \tau_{12}(0)\frac{2.754\,X}{1+3.24\,X} \qquad \text{(G-7)}$$

$$\tau_{12}(0) = 2\left[\sqrt{\psi} + \frac{1}{\sqrt{\psi}}\right]^{-2} \qquad \text{(G-8)}$$

$$\psi = \frac{\rho_1\, C_{L,1}^{3/2}\, h_1^{5/2}}{\rho_2\, C_{L,2}^{3/2}\, h_2^{5/2}} \qquad \text{(G-9)}$$

$$X = h_1 / h_2 \qquad \text{(G-10)}$$

Point Bridge between Two Mechanical Structures

Z_i	Mechanical impedance subsystem *i*
$n_i(\omega)$	Modal density in subsystem *i* (modes / (rad/sec))

ω	Center frequency (rad/sec)

The following equation is taken from Reference G.2.

$$\eta_{ij} = \frac{2}{\pi \omega n_i(\omega)} \frac{\mathrm{Re}(Z_i)\mathrm{Re}(Z_j)}{\left| Z_i + Z_j \right|^2} \qquad \text{(G-11)}$$

Bolted Joints between Two Plates

N	Number of bolts, studs, or point impedances
f	Center frequency (Hz)
S_i	Surface area of plate i
h_i	Thickness plate i
ρ_i	Mass density of plate i (mass/volume)
$c_{L i}$	Longitudinal wave speed in plate i

The coupling loss factor η_{ij} for propagation from plate i to plate j, as taken from Reference G.3.

$$\eta_{ij} = \frac{4N}{S_i \sqrt{3}} \left(\frac{h_i c_{Li}}{2\pi f} \right) \frac{\left(\rho_j h_j^2 c_{Lj}\right)\left(\rho_i h_i^2 c_{Li}\right)}{\left(\rho_j h_j^2 c_{Lj} + \rho_i h_i^2 c_{Li}\right)^2} \qquad \text{(G-12)}$$

Line Joints between Two Plates, Same Material

E	Elastic modulus
ν	Poisson ratio
c_{gi}	Bending wave group velocity
c_b	Bending wave phase velocity
m''	Mass/area
L_c	Junction length

h_i	Thickness of plate i
A_i	Area of plate i
ω	Frequency (rad/sec)
τ	Transmission coefficient
B	Flexural rigidity

The coupling loss factor η_{ij} for propagation from plate i to plate j is taken from References G.7 and G.8.

$$\eta_{ij} = \frac{c_{gi} L_c}{\omega \pi A_i} \tau_{ij} \qquad \text{(G-13)}$$

The group velocity is twice the phase velocity for bending waves.

$$c_{gi} = 2c_{bi} = 2\left(\frac{B_i}{m''}\right)^{1/4}\sqrt{\omega} \quad , \quad B_i = \frac{E h_i^3}{12\left(1-\nu^2\right)} \qquad \text{(G-14)}$$

The transmission coefficient is

$$\tau_{ij} = \frac{2}{\sigma^{-5/4} + \sigma^{5/4}} \, , \quad \sigma = h_j / h_i \qquad \text{(G-15)}$$

See Appendix P for alternate transmission coefficient formulas.

Panel-to-Acoustical Space

R	Radiation resistance
η_{pa}	Coupling loss factor, panel-to-air
σ_{rad}	Radiation efficiency (see Appendix C)
ρc	Characteristic impedance of the gas

A	Panel surface area
M	Panel mass
ω	Center frequency (rad/sec)

The following panel-to-acoustic equations are taken from Reference G.1. The radiation resistance R is

$$R = \rho c A \sigma_{rad} \qquad \text{(G-16)}$$

The coupling loss factor η_{pa} is

$$\eta_{pa} = \frac{R}{M\omega} = \frac{\rho c A \sigma_{rad}}{M\omega} \qquad \text{(G-17)}$$

CLF from External to Interior Acoustic Space via a Fairing Wall

$\eta_{int,ext}$	Coupling loss factor, interior to exterior
$\eta_{plf,int}$	Coupling loss factor, fairing to interior
$\eta_{plf,ext}$	Coupling loss factor, fairing to exterior
R_m	Mass Law Transmission loss (dB)
τ	Transmission coefficient
ω	Frequency (rad/sec)
ρc	Characteristic impedance of the air

S	Surface area
V	Internal air volume
c	Speed of sound
f	Frequency (Hz)
S	Surface area
m	Mass per area of fairing
σ_{rad}	Radiation efficiency

Equation (G-18) is the CLF due to the non-resonant mass-law, as taken from Reference G.6, equations (6.62) and (6.65). The mass law transmission loss R_m equations are given in Appendix P for three incidence options.

Equation (G-19) is taken from Reference G.4, equation (4-43). See also Reference G.5, equation (9).

$$\eta_{int,ext} = \frac{cS}{8\pi f V}\tau_{int,ext} \quad , \quad \tau_{int,ext} = 10^{\wedge}\left(R_m / 10\right) \tag{G-18}$$

$$\eta_{plf,int} = \frac{\rho c}{m\omega}\sigma_{rad} \tag{G-19}$$

$$\text{Assume} \quad \eta_{plf,ext} = \eta_{plf,int} \tag{G-20}$$

CLF Reciprocity

The following reciprocity formula is taken from Reference G.1, equation (4.95).

The coupling loss factor for power flow from subsystem j to i is

$$\eta_{ji} = \eta_{ij}\left(\frac{n_i}{n_j}\right) \tag{G-21}$$

where n_i is the modal density for subsystem i.

The modal densities may be in units of (modes/Hz) or (modes/rad) as long as consistency is maintained-

References

G.1 J. Wijker, Random Vibrations in Spacecraft Structure Design, Springer, New York, 2009. Equations (4.95), (4.124) through (4.134)

G.2 D. Bies, C. Hansen, Engineering Noise Control: Theory and Practice, Fourth Edition, CRC Press, 2009.

G.3 J. Wilby and T. Scharton, Acoustic Transmission through a Fuselage Sidewall, NASA-CR-132602, 1973.

G.4 NASA-HDBK-7005 Dynamic Environmental Criteria, 2001. Equations (4.41) & (4.43)

G.5 Hyun-Sil Kim, Jae-Seung Kim, Seong-Hyun Lee and Yun-Ho Seo, A Simple Formula for Insertion Loss Prediction of Large Acoustical Enclosures using Statistical Energy Analysis Method, Int. J. Nav. Archit. Ocean Eng. (2014). Equation (9)

G.6 M. Norton & E. Karczub, Fundamentals of Noise and Vibration Analysis for Engineers, Second Edition Cambridge University Press, 2003.

G.7 R. Panuszka, J. Wiciak, M. Iwaniec, Experimental Assessment of Coupling Loss Factors of Thin Rectangular Plates, Archives of Acoustics, 30, 4, 2005. Equations (4) and (5)

G.8 A. Nilsson & B. Liu, Vibroacoustics, Volume 2, Springer, 2013. Equation (16.73).

Appendix H

Acoustic Cavity Modal Density

Variables

n	Modal density (modes/Hz)
c	Speed of sound in gas
f	Band center frequency (Hz)
L	Length
W	Width

H	Height
V	Volume
R	Radius
A	Area
P	Total perimeters

1D Pipe

The modal density for long slender pipes where the wavelength of sound is greater than any of the cross-dimensions from Reference H.1 is

$$n = \frac{2L}{c} \tag{H-1}$$

2D Rectangle

The modal density for 2D cavities where the wavelength of sound is at least twice the depth from Reference H.1 is

$$n = \frac{2\pi f A}{c^2} + \frac{P}{c} \tag{H-2}$$

3D Rectangular Prism

The modal density from Reference H.1 is

$$n = \frac{4\pi f^2 V}{c^3} + \frac{\pi f A}{2c^2} + \frac{P}{8c} \tag{H-3}$$

$$V = L W H \tag{H-4}$$

$$A = 2(LW + LH + WH) \tag{H-5}$$

$$P = 4\,(L + W + H) \tag{H-6}$$

<u>3D Cylinder</u>

The equivalent rectangular room approximation from Reference H.2 is

$$n = \frac{4\pi f^2 V}{c^3} \tag{H-7}$$

$$V = \pi R^2 H \tag{H-8}$$

<u>3D Sphere</u>

The equivalent rectangular room approximation from Reference H.2 is

$$n = \frac{4\pi f^2 V}{c^3} \tag{H-9}$$

$$V = \frac{4}{3}\pi R^3 \tag{H-10}$$

<u>3D Other</u>

The equivalent rectangular room approximation from Reference H.2 is

$$n = \frac{4\pi f^2 V}{c^3} \tag{H-11}$$

<u>References</u>

H.1 M. Norton & E- Karczub, Fundamentals of Noise and Vibration Analysis for Engineers, Second Edition Cambridge University Press, 2003. Equations (6.33) through (6.35).

H.2 NASA-CR-102876, The Response of Cylindrical Shells to Random Acoustic Excitation over Broad Frequency Ranges Final Report, 1970. Equation (24).

This reference is available online from NASA Technical Reports Server.

Appendix I

Structural Modal Density

Variables

$n(\omega)$	Modal density (modes/(rad/sec))
$n(f)$	Modal density (modes/Hz)
f	Center frequency (Hz)
ω	Center frequency (rad/sec)

E	Elastic modulus
ρ	Mass density (mass/volume)
ν	Poisson ratio
C_L	Beam longitudinal wave speed

$$n(f) = 2\pi n(\omega) \tag{I-1}$$

$$C_L = \sqrt{E/\rho} \quad \text{for a beam} \tag{I-2}$$

$$C_L = \sqrt{\frac{E}{\rho\left(1-\nu^2\right)}} \quad \text{for a plate or cylindrical shell} \tag{I-3}$$

Beam

κ	Radius of Gyration
L	Length
A	Cross section area
I	Area moment of inertia

The following beam equations are taken from Reference I.1.

$$n(f) = \frac{L}{\sqrt{2\pi f \, \kappa C_L}} \tag{I-4}$$

$$\kappa = \sqrt{I/A} \tag{I-5}$$

Rectangular Plate, Bending

A	Length
B	Width
S	Surface area, S = a b
H	Thickness
M	Mass Density (mass/area)

The following plate equations are taken from References I.2 and I.3.

Generic BCs

$$n(\omega) = \frac{S}{4\pi}\sqrt{\frac{m}{B}} \tag{I-6}$$

Simply-Supported

$$n(\omega) = \frac{S}{4\pi}\sqrt{\frac{m}{B}} - \frac{1}{4}\left(\frac{m}{B}\right)^{1/4}\left(\frac{a+b}{\pi}\right)\frac{1}{\sqrt{\omega}} \tag{I-7}$$

Free

$$n(\omega) = \frac{S}{4\pi}\sqrt{\frac{m}{B}} + \frac{1}{2}\left(\frac{m}{B}\right)^{1/4}\left(\frac{a+b}{\pi}\right)\frac{1}{\sqrt{\omega}} \tag{I-8}$$

Fully Clamped

$$n(\omega) = \frac{S}{4\pi}\sqrt{\frac{m}{B}} - \frac{1}{2}\left(\frac{m}{B}\right)^{1/4}\left(\frac{a+b}{\pi}\right)\frac{1}{\sqrt{\omega}} \tag{I-9}$$

$$B = \frac{Eh^3}{12\left(1-\nu^2\right)} \tag{I-10}$$

Rectangular Plate, In-plane

Let a and b be the length and width, respectively. The following plate equation is taken from Reference I.4.

$$n(\omega) = \frac{a\,b\,\omega}{2\pi C_L} \tag{I-11}$$

<u>Circular Plate</u>

d	diameter		h	Thickness
S	Surface area		m	Mass Density (mass/area)

The circular plate bending modal density is calculated from that of a rectangular plate of equal area, per References I.5 & I.6. The equations for generic boundary conditions are

$$n_{rect}(\omega) = \frac{S}{4\pi}\sqrt{\frac{m}{B}} \qquad \text{(I-12)}$$

$$S = \pi d^2 / 4 \qquad \text{(I-13)}$$

$$n_{circ}(\omega) = \frac{8}{\pi^2} n_{rect}(\omega) \qquad \text{(I-14)}$$

<u>Honeycomb Sandwich Panel</u>

E	Face sheet elastic modulus		t_f	Face sheet thickness, individual
G	Core shear modulus		S	Shear Stiffness
ν	Poisson ratio		D	Plate stiffness factor
A	Surface Area		m	(Total Mass)/area
h	Core thickness			

The honeycomb sandwich panel equations are taken from References I.5 & I.6.

Rectangular

$$n_{rect}(f) = \frac{\pi A m f}{S}\left\{1 + \left(m^2\omega^4 + \frac{4m\omega^2 S^2}{D}\right)^{-1/2}\left(m\omega^2 + \frac{2S^2}{D}\right)\right\} \qquad \text{(I-15)}$$

$$S = Gh\left(1 + (t_f / h)\right)^2 \qquad \text{(I-16)}$$

$$D = \frac{E t_f (h + t_f)^2}{2(1 - \nu^2)} \qquad \text{(I-17)}$$

Circular

The circular plate bending modal density is calculated from that of a rectangular plate of equal area.

$$n_{circ}(f) = \frac{8}{\pi^2} n_{rect}(f) \tag{I-18}$$

Unstiffened Cylinder

f_{ring}	Ring Frequency
L	Cylinder length
H	Thickness
D	Diameter
C_L	Longitudinal wave speed

The following cylinder equations are taken from Reference I.7.

$$n(f) \ = \ B\,L \Big/ \left(\pi h f_{ring}\right) \tag{I-19}$$

$$f_{ring} \ = \ \frac{C_L}{\pi d} \tag{I-20}$$

$$\nu_o = f / f_{ring} \tag{I-21}$$

$$B = 2.5\sqrt{\nu_o} \quad \text{for } \nu_o \le 0.48 \tag{I-22}$$

$$B = 3.6\nu_o \quad \text{for } 0.48 < \nu_o \le 0.83 \tag{I-23}$$

$$B = 2 + \frac{0.23}{(F - 1/F)}\left[F\cos\left(\frac{1.745}{F^2 {\nu_o}^2}\right) - \frac{1}{F}\left(\frac{1.745F^2}{{\nu_o}^2}\right)\right] \quad \text{for} \quad \nu_o > 0.83 \tag{I-24}$$

F = 1.122 for one-third octave bands, (I-25)
1.414 for octave bands

References

I.1 R. Lyon & R. DeJong, Theory and Application of Statistical Energy Analysis, Second Edition, Lyon Corp, Cambridge, MA, 1998. Equation (8.1.10)

I.2 H. Xie, E-J. Thompson and D.J.D. Jones, The Influence of Boundary Conditions on the Mode Count and Modal Density of Two-Dimensional Systems, ISVR Technical Memorandum No 894,October 2002. Equations (3.2) to (3.4)

Note that Reference I.2 had a typo error which placed $\sqrt{\omega}$ *in the numerator. This was corrected to the denominator in Reference I.3.*

I.3 H. Xie, E-J. Thompson and D.J.D. Jones, A Modeling Approach for Extruded Plates, Tenth International Congress on Sound and Vibration, Stockholm, Sweden, 2003.

I.4 AutoSEA Theory and Quality Assurance Manual, Vol 1, Vibro-Acoustic Sciences Limited, Australia, 1991-1995. Equation (3.32)

I.5 J. Wijker, Random Vibrations in Spacecraft Structure Design, Springer, New York, 2009. Page 286 & Table 4.1, Renji method-

I.6 NASA CR-1773, Compendium of Modal Densities for Structures, 1971. Page 35. *This reference is available online from NASA Technical Reports Server (NTRS).*

I.7 F. Szechenyi, Modal Densities and Radiation Efficiencies of Unstiffened Cylinders using Statistical Methods, Journal of Sound and Vibration, 1971. (Section 4)

Appendix J

Modal Overlap

Variables

n	Modal density (modes/Hz)		f	Frequency (Hz)
η	Loss factor			

Modal overlap is defined as the ratio of the damping bandwidth to the average separation of the natural frequencies of the modes. It measures the 'smoothness' of the frequency response function. A high modal overlap factor implies either high damping or high modal density, or both.

The modal overlap M_{ov} is

$$M_{OV} = n \, \eta \, f \tag{J-1}$$

Deterministic methods, such as finite element or boundary element method, can be used for $M_{ov} < 1$.

Statistical energy analysis can be used for $M_{OV} > 1$.

Reference

J.1 J. Wijker, Random Vibrations in Spacecraft Structure Design, Springer, New York, 2009. Equation (4.179)

Appendix K

Equivalent Power for Acoustic Fields, Panels & Cylinders

Panel & Cylinder Excitation, Diffuse Field

$< \Pi_{p,in} >$	Power input
c_a	Speed of sound in air
U_c	Convection velocity (Appendix T)
σ_{rad}	Radiation efficiency
$\left\langle p_a^2 \right\rangle$	Spatial average of mean square pressure in acoustic field
L_s	Distance between simple supports
a_1, a_2	Empirical constants

f	Center frequency (Hz)
$\overline{\delta f_p}$	Average separation between adjacent modal frequencies (Hz). This is the inverse of the modal density (modes/Hz)
ρ_p	Mass density (mass/volume)
h_p	Panel or cylinder wall thickness
A_p	Panel mass per area
c_p	Panel phase speed

The diffuse field acoustic pressure can be converted into equivalent power. The power input for a single frequency band per Reference K.1 is

$$\Pi_{p,in} = \frac{c_a^{\,2} \; \sigma_{rad} \left\langle p_a^2 \right\rangle}{4\pi f^{\,2} \; \overline{\delta f_p} \; \rho_p h_p} \qquad \text{(K-1)}$$

The acoustic modal density is assumed to be much greater than that of the panel or cylindrical shell.

Panel & Cylinder Excitation, Turbulent Boundary Layer, Lyon & DeJong Method

The turbulent boundary layer case from Reference K.1 gives the following power input.

The power for the hydrodynamically slow case is

$$\Pi_{p,in} = \frac{A_p \left\langle p_a^2 \right\rangle}{\pi^2 f \; \rho_p h_p} \left(\frac{U_c}{c_p} \right), \quad U_c > c_p \qquad \text{(K-2)}$$

The power for the hydrodynamically fast case is

$$\Pi_{p,in} = \frac{A_p \left\langle p_a^2 \right\rangle}{2\pi f\ \rho_p h_p} \left(\frac{U_c}{c_p} \right)^3 \left[\frac{a_1}{6} + a_2 \left(\frac{U_c}{2\pi f\, L_s} \right)^2 \right], \quad U_c < c_p \qquad \text{(K-3)}$$

The coefficients a1 and a2 are constants approximately equal to one for the idealized case of smooth flow over a rectangular, simply-supported plate, but may vary by factors of 3 or more depending on the details of the turbulent flow and on the plate mode shapes.

Panel & Cylinder Excitation, Turbulent Boundary Layer, Corcos Method

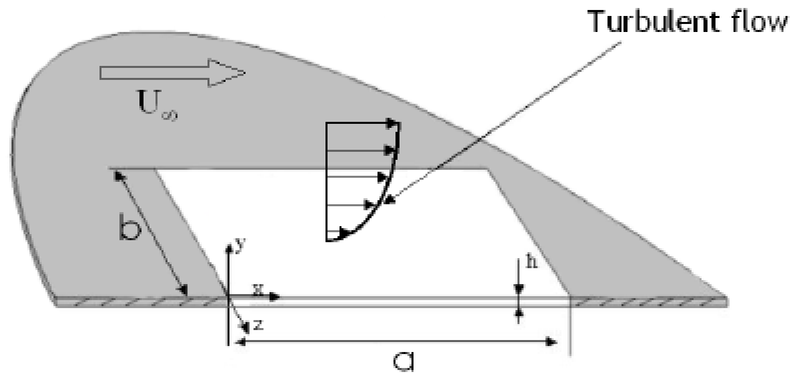

Figure K-1. Turbulent Flow over a Plate

$\left\langle P_{inj}(\omega) \right\rangle$	Injected power
$\left\langle S_{pp}(\omega) \right\rangle$	Pressure Power Spectrum
a, b	Length, width
A	Surface area
a_x , a_z	Corcos coefficients
$L_x(\omega)$, $L_z(\omega)$	Correlation lengths

ω	Excitation frequency
ω_c	Aerodynamic Coincidence frequency
M	Mass per area
D	Plate Bending Stiffness
U_c	Convection speed (Appendix T)

The power injected from the flow to the plate is

$$\left\langle P_{inj}(\omega) \right\rangle = \frac{U_c^2}{a_x\, a_z\, \pi \sqrt{M D}\, \omega^2} \frac{A}{2} \Psi c \left(\frac{\omega}{\omega_c} \right) \left\langle S_{pp}(\omega) \right\rangle \qquad \text{(K-4)}$$

The aerodynamic coincident frequency is

$$\omega_c = U_c^2 \sqrt{M/D} \tag{K-5}$$

The Corcos function Ψ_C is

$$\Psi c\left(\frac{\omega}{\omega_c}, \frac{\omega a}{U_c}, \frac{\omega b}{U_c}\right) = \frac{\omega a}{U_c}\frac{\omega b}{U_c} a_x a_z \int_{U_c \pi/(\omega b)}^{\sqrt{\frac{\omega_c}{\omega} - \left(\frac{U_c \pi}{\omega a}\right)^2}} \frac{1}{\sqrt{\frac{\omega_c}{\omega} - X^2}} F_1\left(\frac{\omega b}{U_c}\left(-a_z + iX\right)\right)$$

$$\cdot \left[F_2\left(\frac{\omega a}{U_c}\left(-a_x + i\left(\sqrt{\frac{\omega_c}{\omega} - X^2} - 1\right)\right)\right) + F_2\left(\frac{\omega a}{U_c}\left(-a_x + i\left(\sqrt{\frac{\omega_c}{\omega} - X^2} + 1\right)\right)\right) \right] \mathrm{d}X$$

(K-6)

The associated functions are

$$F_1(\boldsymbol{z}) = -\frac{\Re(\boldsymbol{z})}{|\boldsymbol{z}|^2} + \frac{\Re\left(\boldsymbol{z}^{*2}\left(e^{\boldsymbol{z}} - 1\right)\right)}{|\boldsymbol{z}|^4} + \frac{\Im\left(\boldsymbol{z}^*\left(e^{\boldsymbol{z}} - 1\right)\right)}{\left(\omega b/U_c\right) X |\boldsymbol{z}|^2} \tag{K-7}$$

$$F_2(\boldsymbol{z}) = -\frac{\Re(\boldsymbol{z})}{|\boldsymbol{z}|^2} + \frac{\Re\left(\boldsymbol{z}^{*2}\left(e^{\boldsymbol{z}} - 1\right)\right)}{|\boldsymbol{z}|^4} + \frac{\Im\left(\boldsymbol{z}^*\left(e^{\boldsymbol{z}} - 1\right)\right)}{\frac{\omega a}{U_c}\sqrt{\frac{\omega_c}{\omega} - X^2}\, |\boldsymbol{z}|^2} \tag{K-8}$$

$\Re$ is the real component. $\Im$ is the imaginary component. * indicates complex conjugate.

The correlation lengths are related to the Corcos coefficients by

$$L_x(\omega) = U_c / \left(\alpha_x \omega\right) \tag{K-9}$$

$$L_z(\omega) = U_c / \left(\alpha_z \omega\right) \tag{K-10}$$

Equations (K-4) through (K-10) are taken from Reference K.2.

Sample Corcos coefficient values for a turbulent boundary layer are given as follows from Reference K.3.

Source	a_x	a_z
Willmarth	0.12	0.70
Efimstov	0.10	0.77
Robert	0.13	0.83
Blake	0.12	0.70
Finnveden	0.116	0.70

Acoustic Energy-Pressure Relationship

$< E_{av} >$	Average subsystem acoustical energy
V	Volume
$< p^2 >$	Spatial average of the mean square pressure
ρ	Gas density
c	Gas speed of sound

$$< E_{av} > = \frac{V}{\rho c^2} < p^2 > \qquad \text{(K-11)}$$

Equation (K-11) is taken from Reference K.4.

References

K.1 R. Lyon & R. DeJong, Theory and Application of Statistical Energy Analysis, Second Edition, Lyon Corp, Cambridge, MA, 1998. Equations (11.2.2) to (11.2.4).

K.2 Totaro, Robert, Guyader, Frequency Averaged Injected Power under Boundary Layer Excitation: An Experimental Validation, ACTA ACUSTICA, 2008

K.3 A. Nilsson, B. Liu, Vibro-Acoustics, Volume 2, Springer, Science Press, Beijing, 2016.

K.4 J. Wijker, Random Vibrations in Spacecraft Structure Design, Springer, New York, 2009. Equation (4.192)

Appendix L

Homogeneous Panel Response to a Diffuse Sound Field

Limp Panel, Non-Resonant Response

$\langle v^2 \rangle$	spatial average mean square velocity
p	Pressure rms

m	Mass per area
ω	Center Frequency (rad/sec)

The spatial mean square velocity $\langle v^2 \rangle$ from Reference L.1 is

$$\langle v^2 \rangle = \frac{2\langle p^2 \rangle}{m\,\omega^2} \tag{L-1}$$

Freely Hung Panel

$\langle p^2 \rangle$	spatial mean square pressure
c_{air}	Speed of sound in air
$(\rho c)_{air}$	Characteristic impedance of air
h	Panel thickness
c_L	Longitudinal wave speed

ρ_s	Surface mass density (mass/area)
ω	Center frequency (rad/sec)
η	Loss factor
σ_{rad}	Radiation efficiency

The spatial mean square velocity $\langle v^2 \rangle$ from Reference L.2 is

$$\langle v^2 \rangle = \langle p^2 \rangle \frac{\sqrt{12}\,\pi\, c_{air}{}^2}{2(\rho c)_{air}\, h\, c_L\, \rho_s \omega^2} \left\{ \frac{1}{1+\left[\dfrac{\rho_s \omega \eta}{2(\rho c)_{air}\,\sigma_{rad}}\right]} \right\} \tag{L-2}$$

References

L.1 J. Wijker, Random Vibrations in Spacecraft Structure Design, Springer, New York, 2009. Equation (4.203)

L.2 Beranek and Ver, Noise and Vibration Control Engineering Principles and Applications, Wiley, New York, 1992. Equation (9.142)

Appendix M

Homogeneous Panel Response to a Point Force

<u>Variables</u>

$< v^2 >$	mean square velocity
F	Driving Point Force RMS in frequency band
Y	Mobility (velocity/force)

M	System mass
η	Dissipation loss factor
ω	Center frequency (rad/sec)

$$< v^2 > = \frac{F^2 \, \mathrm{real}\{Y\}}{M \eta \omega} \qquad \text{(M-1)}$$

<u>Reference</u>

M.1 R. Lyon & R. DeJong, Theory and Application of Statistical Energy Analysis, Second Edition, Lyon Corp, Cambridge, MA, 1998. Equations (2.4.5, 2.4.8, 8.5.2)

Appendix N

Homogeneous Panel Excited by Point Force, Radiation into Acoustic Space

Variables

Π_{dp}	Drive point radiation
Π_{rad}	Resonant modes radiation
Π	Total acoustic power radiated
F_{rms}	Drive point force
$\left\langle v^2 \right\rangle$	Spatial average of mean square velocity
f	Center frequency (Hz)
f_c	Panel critical frequency (Hz)

S	Surface Area
c	Gas speed of sound
ρ_o	Gas mass density (mass/volume)
ρ_s	Panel mass density (mass/area)
σ	Panel radiation efficiency
η	Total panel loss factor (dissipation plus radiation)

$$\Pi_{dp} = \frac{\rho_o F_{rms}^2}{2\pi c \rho_s^2} \quad \text{for } f < f_c \tag{N-1}$$

$$\Pi_{dp} = 0 \quad \text{for } f \geq f_c \tag{N-2}$$

$$\Pi_{rad} = \frac{\rho_o f_c F_{rms}^2 \sigma}{8 c f \rho_s^2 \eta} \quad \text{for a free field} \tag{N-3}$$

$$\Pi_{rad} = \rho_o c S \left\langle v^2 \right\rangle \sigma \quad \text{for a reverberant room} \tag{N-4}$$

$$\Pi = \Pi_{dp} + \Pi_{rad} \tag{N-5}$$

The previous power equations are taken from Reference N.1. Note that the radiation efficiency depends on whether the panel is baffled or freely suspended, but the power formulas are otherwise the same for each case.

Reference

N.1 M. Norton & D. Karczub, Fundamentals of Noise and Vibration Analysis for Engineers, Second Edition Cambridge University Press, 2003. Equations (3.56) & (6.78)

Appendix O

Baffled Homogeneous or Honeycomb Sandwich Panel
Response to Diffuse Acoustic Pressure Field

Variables

$\langle E_S \rangle$	Total time-average energy of structural vibration in the bandwidth
$\langle p_0^2 \rangle$	Mean square acoustic pressure in the bandwidth
$\langle v_s^2 \rangle$	Mean square velocity in the structure
c	Speed of sound in gas
ρ_o	Gas density (mass/volume)
M	Total panel mass

$n_s(\omega)$	Modal density of the structure (modes/(rad/sec))
ω	Band center frequency (rad/sec)
R_{rad}	Resistance due to acoustic radiation, per equation (C-28)
R_{int}	Resistance due to dissipation effects other than acoustic radiation
η_{int}	Loss factor due to dissipation effects other than acoustic radiation

$$\langle E_S \rangle = \left\{ \frac{2\pi^2 c\, n_s(\omega)}{\rho_o \omega^2} \left\langle \frac{R_{rad}}{R_{int} + R_{rad}} \right\rangle \right\} \left\langle p_0^2 \right\rangle$$ (Reference O.1) (O-1)

$$R_{int} = M \omega \eta_{int}$$ (Reference O-2) (O-2)

$$\langle E_S \rangle = M \left\langle v_s^2 \right\rangle$$ (Reference O-2) (O-3)

$$\left\langle v_s^2 \right\rangle = \left\{ \frac{2\pi^2 c\, n_s(\omega)}{\rho_o M \omega^2} \left\langle \frac{R_{rad}}{R_{int} + R_{rad}} \right\rangle \right\} \left\langle p_0^2 \right\rangle$$ (O-4)

References

O.1 F. Fahy & P. Gardonio, Sound and Structural Vibration, Radiation, Transmission and Response, Second Edition, Academic Press, New York, 2007. See equation (6.37)

O.2 J. Wijker, Random Vibrations in Spacecraft Structure Design, Springer, New York, 2009. See equations (4.112) & (4.133)

Appendix P

Transmission Loss

Transmission Loss for a Panel via the Mass Law

Transmission Loss Normal-incidence R_N

$$R_N \approx 10 \log\left[1+\left(\frac{\rho_s \omega}{2\rho_o c_o}\right)^2\right] \quad \text{dB} \tag{P-1}$$

Valid for $f << f_{cr}$

Transmission Loss Field-incidence R_{field}

$$R_{field} \approx R_N - 5 \text{ dB} \tag{P-2}$$

Field-incidence approximates a diffuse incident field with a limiting angle of about 78°.

Transmission Loss Random-incidence R_{random}

$$R_{random} \approx R_N - 10 \log\left(0.23 R_N\right) \quad \text{dB} \tag{P-3}$$

Random incidence covers 0° to 90°.

R_N	Transmission Loss Normal-incidence
R_{random}	Transmission Loss Random-incidence
ω	Frequency (rad/sec)
f	Frequency (Hz)
f_{cr}	Critical frequency (Hz)
ρ_s	Panel mass per area
$\rho_o c_o$	Characteristic impedance of the gas, assume the same on both sides

Transmission Loss through Composite Panel

Consider a panel with two surface area sections. Each transmission loss is in terms of dB.

S_1	Section area with the highest transmission loss
S_2	Section area with the lowest transmission loss
K	Area ratio

TL_1	Transmission loss for section with highest loss
TL_2	Transmission loss for section with lowest loss
TL_C	Composite Transmission loss

$$K = \frac{S_2}{S_1 + S_2} \tag{P-4}$$

$$TL_C = TL_1 - 10\log\left[1 - K + K10^{(TL_1 - TL_2)/10}\right] \tag{P-5}$$

Transmission Loss through a Payload Fairing to the Interior Acoustic Space

f	Frequency (Hz)
V	Volume
c	Speed of sound, interior
S	Fairing surface area

$\eta_{int,ext}$	Coupling loss factor, interior to exterior
$\eta_{plf,int}$	Coupling loss factor, fairing to interior
$\eta_{plf,ext}$	Coupling loss factor, fairing to exterior
$\eta_{plf,d}$	Dissipation loss factor, fairing
α	Average absorption coefficient, as calculated from the area-weighted section absorption coefficients for the case of added blankets

The transmission coefficient τ is

$$\tau = \frac{8\pi f V}{cS}\left[\eta_{int,ext} + \frac{\eta_{plf,int}\,\eta_{plf,ext}}{\eta_{plf,d}}\right] \tag{P-6}$$

The transmission loss TL is

$$\mathrm{TL(dB)} = 10\log\left(\frac{1}{\tau}\right) \tag{P-7}$$

The noise reduction NR is

$$\mathrm{NR(dB)} = 10\log\left(1+\frac{\alpha}{\tau}\right) \tag{P-8}$$

Transmission Loss across Junction of Two Connected Plates

The following is take from Reference P-4.

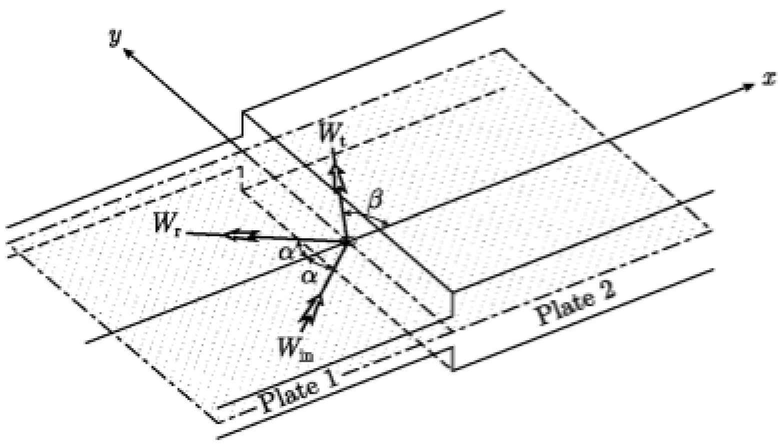

Figure P-1.

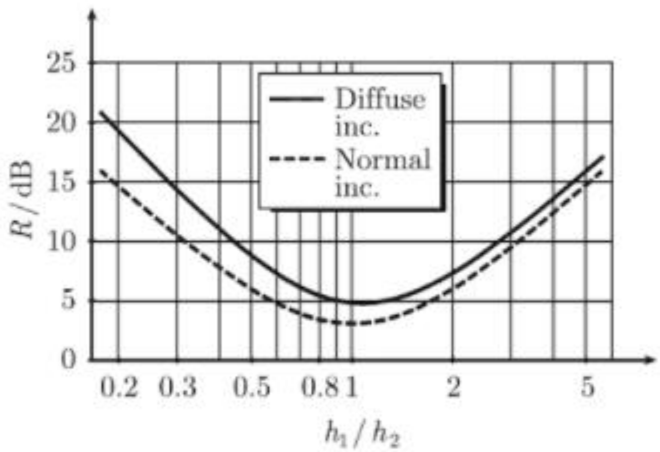

Figure P-2.

R is the same as TL in equation (P-7). h1/h2 is the thickness ratio.

The normal transmission coefficient is

$$\tau(0) = \frac{2Z^5}{\left(Z^5+1\right)^2}\ , \quad Z = \sqrt{h_1/h_2} \tag{P-9}$$

Sandwich Panel Junction Transmission Coefficient

Here is an approximate approach based loosely on References P.4 and P.5.

Select R from Equation P-2. Convert R to a transmission coefficient τ_1 via equation (P-7). Use this value a constant for frequencies up to 500 Hz. Assume a -3 dB/octave slope for frequencies above 500 Hz.

$$\tau = \tau_1 \left(\frac{f}{500}\right)^{-1} \quad \text{for f > 500 Hz} \tag{P-10}$$

References

P.1	Beranek and Ver, Noise and Vibration Control Engineering Principles and Applications, Wiley, New York, 1992. Equations (9.80c), (9.99), (9.100)
P.2	George Diehl, Machinery Acoustics, Wiley-Interscience, New York, 1973. See section (6.3)

P.3	NASA-HDBK-7005 Dynamic Environmental Criteria, 2001. Equations (4.36) & (4.41)
P.4	A. Nilsson & B. Liu, Vibroacoustics, Volume 1, Springer, 2015. Equation (5.130)
P.5	S. Hambric, et al, Experimental Vibro-Acoustic Analysis of Honeycomb Sandwich Panels Connected by Lap and Sleeve Joints, Inter Noise, Osaka, Japan, 2011.

Appendix Q

Noise Reduction

Noise Reduction into an Enclosure with Transmission Loss and Absorption

α	Average absorption coefficient, as calculated from the area-weighted section absorption coefficients
τ	Transmission coefficient

The noise reduction NR is

$$\mathrm{NR(dB)} = 10 \log\left(1 + \frac{\alpha}{\tau}\right) \qquad \text{(Q-1)}$$

The transmission loss TL is

$$\mathrm{TL(dB)} = 10 \log\left(\frac{1}{\tau}\right) \qquad \text{(Q-2)}$$

References

Q.1 NASA-HDBK-7005 Dynamic Environmental Criteria, 2001. Equation (4.36)

Q.2 George Diehl, Machinery Acoustics, Wiley-Interscience, New York, 1973. Equation (6.1)

Appendix R

Acoustic Blankets

Insertion Loss

The insertion loss should be obtained from test measurements of the blanket. The following table is typical data.

Freq (Hz)	Loss (dB)
63	2
80	2
100	2
125	2
160	4
200	6
250	7.7
315	11.9
400	16.1
500	19.6

Freq (Hz)	Loss (dB)
630	23.1
800	25.2
1000	27.3
1250	28
1600	28
2000	28
2500	28
3150	28
4000	28
5000	28

Acoustic Absorption Coefficients

The absorption coefficients should be obtained from test measurements of the blanket. An empirical method is given in Reference 1 for preliminary analysis. The peak absorption coefficient α_{peak} for a blanket with thickness t (inches) is

$$\alpha_{peak} = (t/3) + 0.0005 \quad \text{with an upper limit of } \alpha_{peak} = 1 \tag{R-1}$$

Compute the peak frequency f_{peak} and round to the nearest one-third octave band center frequency,

$$\log\left(f_{peak}\right) = -0.201\ t + 3.302 \tag{R-2}$$

Construct the curve for frequencies f

$f < f_{peak}$ using $\alpha = \alpha_{peak} \, (f / f_{peak})$ (R-3)

$f > f_{peak}$ using $\alpha = \alpha_{peak} \, (f_{peak} / f)$ (R-4)

Transmission Coefficient

The average transmission coefficient τ_{ave} for a fairing lined with blankets is

$$\tau_{ave} = \frac{8\pi f V}{c S}\left[\eta_{int,ext} + \frac{\eta_{plf,int}\eta_{plf,ext}}{\eta_{plf,d}}\left[(1-B) + B\,\tau_{blanket}\right]\right] \tag{R-5}$$

where

$\tau_{blanket}$ Blanket transmission coefficient from insertion loss

B Ratio of surface area covered by blankets

Equation (R-5) is taken from Reference (R-2).

The relationship between the insertion loss IL(dB) and the blanket transmission coefficient is

$$IL(dB) = 10 \log\left(\frac{1}{\tau_{blanket}}\right) \tag{R-6}$$

Reference

R.1 K. Weissman, M. McNelis, W. Pordan, *Implementation of Acoustic Blankets* in Energy Analysis Methods with Application to the Atlas Payload Fairing, Journal of the IES, July, 1994.

R.2 NASA-HDBK-7005, Dynamic Environmental Criteria, 2001.

APPENDIX S

Statistical Response Concentration

<u>Velocity Response Amplification</u>

Pure Tone:	Broadband:
$\frac{V_{max}^2}{V_{rms}^2} = \frac{\pi \eta f}{2\overline{\delta f}} \psi_{max}^2 , \qquad \psi_{max}^2 = 2^D$	$\frac{V_{max}^2}{V_{rms}^2} = 1 + \left(\frac{\pi \eta f}{2\overline{\delta f}} \psi_{max}^2 - 1 \right) \frac{\pi \eta f}{2 \Delta f}$

f	Band center frequency
Δf	Bandwidth
$\overline{\delta f}$	Average modal frequency separation (modal density inverse)

η	Net loss factor
D	Subsystem Dimension: 1, 2 or 3

<u>Reference</u>

S.1 R. Lyon & R. DeJong, Theory and Application of Statistical Energy Analysis, Second Edition, Lyon Corp, Cambridge, MA, 1998. See equation (13.3.2)

APPENDIX T

Turbulent Boundary Layer Convection Velocity

The convection velocity U_C is the velocity at which the fluctuating pressure field propagates beneath a turbulent boundary layer. It is usually expressed as a fraction of the free-stream velocity U_∞.

U_C	Convection velocity
U_∞	Free-stream velocity

ω	Frequency (rad/sec)
δ^*	Displacement thickness

Relationship	Reference
$U_c \approx 0.75 U_\infty$	T.1
$U_c / U_\infty = 0.6 + 0.4 \exp\left(-2.2\, \omega \delta^* / U_\infty\right)$	T.2

The convection velocity may vary with frequency and flow per Reference T.3. This is shown in Figure T-1, which compares the convection velocities as a function of frequency as reported by different researchers. The Bies and Lowson models refer to attached boundary layers, whereas Cockburn and Robertson (C&R) refer to separated flow.

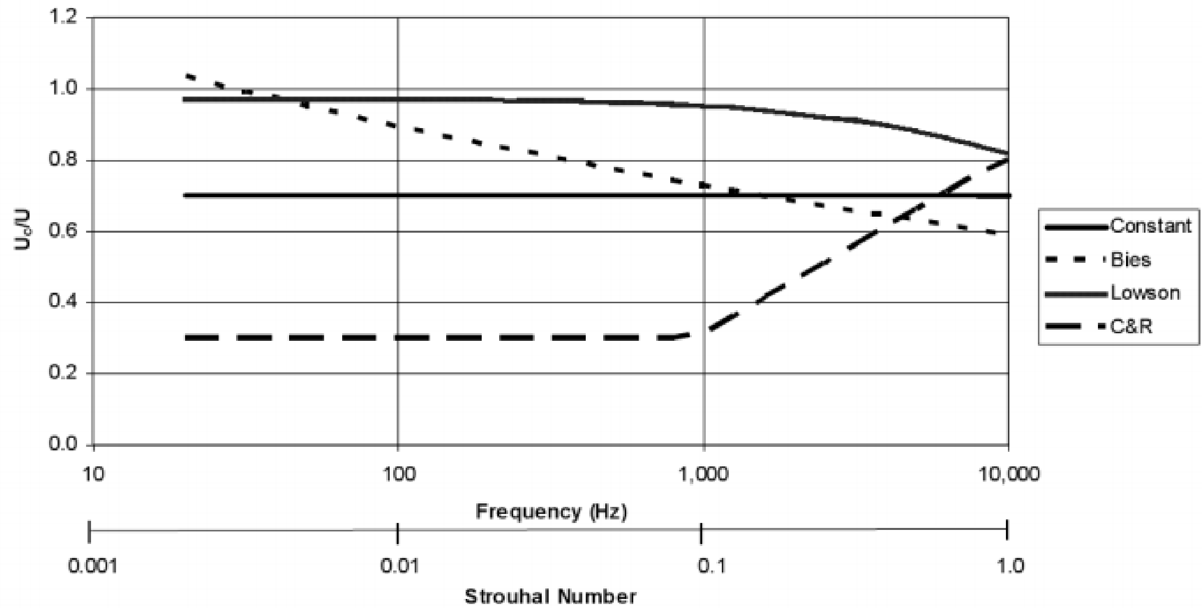

Figure T-1. Convection velocity vs. Frequency for a Typical Launch Vehicle Application

References

T.1 R. Lyon & R. DeJong, Theory and Application of Statistical Energy Analysis, Second Edition, Lyon Corp, Cambridge, MA, 1998. See page 208.

T.2 Totaro, Robert, Guyader, Frequency Averaged Injected Power under Boundary Layer Excitation: An Experimental Validation, ACTA ACUSTICA, 2008. See Equation (17).

T.3 M. Yang & J. Wilby, Derivation of Aero-Induced Fluctuating Pressure Environments for Ares I-X, 14th AIAA/CEAS Aeroacoustics Conference (29th AIAA Aeroacoustics Conference) Vancouver, British Columbia, Canada, 2008.