CONDITIONS UNDER WHICH DISPLACEMENT, VELOCITY, OR ACCELERATION SHOULD BE USED FOR DIAGNOSTIC MONITORING

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Abstract:
Stress is proportional to velocity under many conditions, and because stress cannot be exceeded without failure, velocities lie in a limited range independent of machine size. Extremely high vibration velocities do not occur. The paper reports examples of monitoring using velocity and explains how velocity advocates have convinced others to use it. The theoretical proof that stress is proportional to velocity is presented, and examples are given of applications of the results. New uses for the stress velocity relation have been observed and these are discussed.

Key Words: modal analysis, monitoring, stress, velocity

Introduction:
In elastic structure definitely vibrating in a mode, maximum stress is proportional to the maximum vibratory velocity. I can prove this clearly for bending stress in uniform beams and axial stress in rods. What I'm finding now is that many believe the stress velocity relationship goes well, beyond a single mode and simple structures. Velocity is a common vibration property to measure and monitor, and with very good reason. Its use is increasing and the topic deserves examination. If you are not already convinced, I want to convince you that stress equals ρcv, where ρ is the mass density, c is the velocity of sound, and v is the maximum modal velocity. Chalmers and I discovered that [1] after several others. It has made a huge difference in my life. I remember Prof Carl Freberg, a consultant to our group saying, "Howie, I think you've got something there." Stress velocity I do almost full time right now; I have to tell you about it. It's a little complicated to prove, but more and more people are becoming aware and incorporating it into their analyses. Most of the monitoring charts are in velocity.

The Monitoring guidelines that I have been taught for low frequency and journal bearings include relative displacement and seismic case or bearing housing measurement. You can't allow the shaft to touch journal so that displacement of the shaft relative to the bearing must be monitored. In a smaller machinery situation, with rolling element bearings requires seismic acceleration and velocity measurement. Velocity will have upper limits because it's caused by stress which can only go so high. However the very high frequency impacting with ball bearings, gears require careful acceleration analysis; very high frequency weak acceleration signals emphasize high frequency. And while the consistent acceleration levels may not be cause for alarm, they carry information that can be interpreted to indicate machinery problems, often by their time history characteristics.

Three References That Show How the Stress Velocity Ideas Developed:
MAXWELL [2] tried to organize the monitoring problem, but mainly treated large journal bearing machinery. He says very little on rolling element bearing in his paper, and still doesn't come to any firm conclusion. He tries to organize the monitoring decision according to low frequency, mid frequency and high frequency. His thinking being that low frequency would be a displacement situation and mid frequency for velocities. In the velocity region he explains that maximum strain energy occurs at maximum deflection and zero velocity, and that maximum kinetic energy occurs as the system is passing thru zero deflection. Equating the two energies shows that maximum stress is proportional to maximum velocity. His reference to Plummer [3], is where it is well explained. Although he also references Wachel [4] and Ungar[5], who also deal with the stress velocity proportionality. Wachel informally and Ungar analytically.

He points out that common standards, Rathbone, Yates, and Military Specifications, indicate displacements at low frequency and velocity at higher frequencies. As examples of vibration standards that are exceptions to the common tolerance charts are conditions where stress is proportional to velocity, relative displacement to protect against contact between rotating and stationery, and acceleration for gears, rolling elements bearings, turbine blade passing frequencies. He shows a Dresser Clark chart that starts at constant velocity and goes to constant displacement at high shaft speeds. He cautions against the use of overall rms measurements because they can miss important low frequency information of .5x, and 1x.

Plummer (1978) [3] wrote an ASME nuclear power section paper advocating a change from displacement to velocity monitoring for In-Service pump inspection, and his article makes good points. He cites a reasoning that I have seen is several places that goes something like the following. Vibration and alternating stress are related as follows. In a simple vibration system the maximum kinetic energy is reached at the instant of maximum velocity when the system is passing through zero displacement. The velocity, v, is related to the maximum displacement y in single frequency sine wave motion because the velocity is 2πf times the displacement. ($v = 2\pi fy$) The kinetic energy is given by Equation (1).

$$\text{Kinetic energy} = k_1 \frac{1}{2} \rho V v^2 \quad (1)$$

where:

k_1 = a constant depending on the shape of the deflection curve and the point position
ρ = mass density
V = volume
f = frequency
y = vibration amplitude
v = velocity

The maximum strain energy occurs when the velocity is zero and the deflection is at a maximum; it is given by Equation (2).

$$\text{Strain energy} = k_2 \frac{1}{2} \frac{V\sigma^2}{E} \quad (2)$$

k_2 = constant as before
σ = max stress
E = modulus of elasticity

Equating kinetic energy and strain energy

$$k_1 \frac{1}{2} \rho V v^2 = k_2 \frac{1}{2} \frac{V\sigma^2}{E} \quad (3)$$

So that:

$$\sigma = v\sqrt{\frac{k_1}{k_2}}\sqrt{E\rho} \quad (3a)$$

This is how he reasons that the maximum stress is proportional to the product of geometric constants and material properties times the velocity. He continues and cites four references that recommend velocities for monitoring, presents a copy of Rathbone's graph, and presents a chart showing the similarity of the 4 scales. I'll translate his chart to Table 1.

Table 1

Velocity (ips)	YATES[6]	RATHBONE[7]	VDI[8]	HEINS[9]
2	Immediate investigate			danger shutdown
1.5				Acute fault, investigate, shutdown
0.9	Unsat			
0.5	slightly rough	very rough, action		Fault, fix when convenient
0.3				minor fault
0.2	normal	rough, should fix	inadmissible	no fault
0.1	very good	fair	improve desired	
0.048	spins like top	good	sat	
0.028		very smooth	good	

He shows the table of displacement inspection values, and present a suggested replacement table of velocities referenced to the installed pump velocities and in some cases considers 0.45 to 1.5 ips to be high values and cause for concern.

Maten [10] in 1984 also wrote an article explaining why we should be monitoring pipe vibration velocity. He cites his own experience and gives Table 2 with suggested velocity standards.

Table 2

Severity rating	Machinery bearing housing (ips)	Piping span center (ips)
Acceptable up to	0.2	0.6
Fair to rough (correction needed)	0.2 to 0.5	0.6 to 1.5
Very rough (danger, consider shutdown)	0.5 to 1	1.5 to 3

He then develops an approximate proof where he assumes a uniform load as opposed to the true inertial load on a simply supported beam of schedule 40 steel pipe, and takes the handbook natural frequency value [11] to form a ratio of the vibration stress to the velocity. His answer is Equation (4)

$$\sigma = 1.46 \rho c v \tag{4}$$

This 1.46 will be called the K value for a beam. If we put in the ρc values for steel we find Equation (4a)

$$\frac{\sigma}{v} = 216 \tag{4a}$$

These two values, the K value and the stress per velocity value are important and will be considered further.

WACHEL, [4] while his monitoring studies are mainly concerned with large piping vibrations, makes a dual case for both displacement and velocity. Wachel and his co-authors are a very important part of the stress velocity literature, although they deal completely with pipe vibrations. I want to briefly mention some of the ideas he presents. Of his papers I have read four [12, 13, 14]. Maybe the most representative is [4], where he almost develops a piping vibration handbook dealing with natural frequency calculation, stress calculation based both on deflection and velocity, stresses from high frequency local turbulent fluid motion, stress that can be inferred from the adjacent sound level, and a nice discussion of the source of piping vibrations. I'm not sure but he may prefer measuring piping displacement to velocity. But, that's not my interest here. In his development of the velocity stress relation he points out that the radius of gyration for typical pipe is 0.34D, a factor that I have found useful.

His stress velocity equation is Equation (5).

$$\sigma = K_v v\, SCF$$
$$K_v = K\rho c \tag{5}$$

If you consider an allowable stress of, σ_{allow} = 13,000 psi; K_v = 318, SCF = 5, (stress concentration factor), SF = 2, (safety factor), you get an allowable velocity of

$$v_{allow} = 4 \text{ ips} \tag{5a}$$

K_v which is the stress velocity ratio, (σ/v), values range from 160 - 620 (K = 1:4.2). We will need to manipulate sonic velocity, mass density, and modulus of elasticity frequently, so please notice the relations in Equation (6).

$$c = \sqrt{\frac{E}{\rho}}, \quad \rho c^2 = E, \quad \rho^2 c^2 = E\rho\, , or$$
$$\sqrt{E\rho} = \rho c \tag{6}$$

For steel [E = 30000000; ρ = .283/386.09]
ρc = sqrt(E*ρ) = 148.2892 lbf-sec/in^3

In [12] which is downloadable from his company web site "endyn.com", he presents a development that appears to show that stress is proportional to displacement, which is not true; what he means is if you know the span and the pipe diameter, and that it's in the first mode the formulas work, and if you read it carefully he does state that, but a trivial read might make you think that stress proportional to the displacement vice velocity.

To me the interesting thing that I want you to take from these papers is that a lot of people know, or have the feeling, that stress is proportional to modal velocity and show this fact can be useful. It ties things together. So let do the Stress is Kρcv proof, and see how the knowledge ties a lot of this into a nice picture.

The Stress Velocity Proof:
For long thin rods, the proof can be outlined as follows. Reference [15, page 7.7] gives the solutions we need, and is a readily available reference. The solution of the differential equation for plane or axial vibration in a long thin rod is:

$$u = (C\cos\frac{\omega x}{c} + D\sin\frac{\omega x}{c})\cos(\omega t + \phi) \tag{7}$$

where

u = displacement of rod cross sectional plane at distance x and time t
x = distance down the rod
$\omega_n = f_n/2\pi$ = frequency in radians/sec; f_n = is frequency in Hz. The subscript n implies the equation only applies at the natural frequencies which we will determine in a few lines
c = wave speed = $(E/\rho)^{1/2}$
E = Young's modulus
ρ = mass density; mass per unit volume
t = time

We'll just derive it for the case of a post or a hanging rod with one end fixed and the other end free. If it is fixed or built in at x = 0, the displacement would have to be zero for all time at x = 0. The only way this can be true is if the constant, C = 0. If C is zero we might as well call D, u_{max} and Eq (1) becomes

$$u = u_{max} \sin\left(\frac{\omega x}{c}\right)\cos\left(\omega t + \phi\right) \tag{7a}$$

The strain is $\partial u/\partial x$ or (again see [15])

$$\varepsilon = \frac{\partial u}{\partial x} = u_{max} \frac{\omega}{c} \cos\left(\frac{\omega x}{c}\right)\cos\left(\omega t + \phi\right) \tag{8}$$

At the free end, where x = L, stress or strain is zero. This means that at x = L, $\partial u/\partial x$ has to be zero for all time. Therefore the cos(ωL/c) has to be zero, which occurs whenever (ωL/c) equals π/2, 3π/2, 5π/2. This means that the allowed or natural frequencies are

$$\frac{\omega_n L}{c} = (2n-1)\frac{\pi}{2}, \; for \quad n = 1,2,3,4,\ldots \tag{8a}$$

ω_n is the natural frequency in radians per second; each value of n gives us one natural frequency. As can be seen the frequency increases with n, as does the number of undulations or nodes and antinodes.

This explains the possible modal vibrations of a long thin rod built in at one end. The modal frequencies are found by solving Equation (8a) for ω and dividing by 2π or

$$f_n = (2n-1)\frac{c}{4L} \tag{8b}$$

The maximum strain, from Equation (8), is

$$\varepsilon_{max} = u_{max} \frac{\omega}{c} \tag{9}$$

The maximum stress is E times the maximum strain, and it occurs where x = 0, and in the higher modes, wherever $\cos(\omega_n x/c)$ is +/- 1. We rearrange E, c, and ρ, as discussed in Equation (6).

$$\sigma_{max} = u_{max} \frac{E\omega}{c} = \omega u_{max} \sqrt{E\rho} = \omega u_{max} \rho c, \tag{9a}$$

where c is the wave speed, and is given by

$$c = \sqrt{E/\rho} \tag{9b}$$

We could differentiate Eq (7a) with respect to time to get the velocity, but we know its motion is sinusoidal so the velocity has to be ωu_{max}, so Equation (9a) may be written:

$$\sigma_{max} = \rho c v_{max} \tag{9c}$$

We're talking about motion in a single mode so the motion is sinusoidal. At the antinode or peak velocity point, the displacement is given by v/ω and the maximum acceleration is also given by vω; thus the maximum stress is also proportional the acceleration and displacement and can also be written as

$$\sigma_{\max} = \rho c \omega u_{\max} = \rho c \frac{a_{\max}}{\omega} \qquad (10)$$

But notice, when expressed in terms of acceleration or displacement frequency now enters the equation and peak displacement or peak acceleration alone does not indicate high stress. You have to also state the frequency along with the maximum displacement or acceleration of vibration to indicate a severe vibration. Velocity doesn't involve frequency; so we can just say 400 ips is bad no matter what the frequency is. However with displacement I might say .010 inches displacement is terrible in a vibrating rod, and you would have to come back and say that it depends upon the frequency; similarly with an acceleration level of this or that many g's.

Now we consider transverse beam vibrations. The maximum modal velocity also indicates beam vibration severity. We consider the simplest type of beam vibrations for which the vibration wavelength is long compared to the beam depth. This neglects the rotary inertia and shear effects. Reference [15, page 7.14] gives the solution for the free vibrations of these beams as

$$y = \begin{bmatrix} A(\cos kx + \cosh kx) + B(\cos kx - \cosh kx) \\ +C(\sin kx + \sinh kx) + D(\sin kx - \sinh kx) \end{bmatrix} \cos(\omega t + \phi) \qquad (11)$$

where k and ω are related by

$$k^4 = \frac{\omega^2}{\eta^2 c^2} \qquad (11a)$$

and where

- y = deflection of neutral surface
- x = distance down the beam
- A, B C, D, = arbitrary constants.
- ρ = density, (mass per unit volume)
- E = modulus of elasticity
- η = radius of gyration = $(I/A)^{1/2}$
- I = cross-sectional area moment of inertia about beam neutral axis
- A = cross –sectional area
- ω = frequency in radians per unit time
- k = frequently called a wave number
- c = longitudinal wave speed $(E/\rho)^{1/2}$

I consider the symbols of Reference [15] clumsy but his Equation (7.16) is the same as Equation (11) above. His Table 7.3, gives the results for five common beam vibration problems. The simplest case of a simply supported beam of length L is good enough to prove the point. Here both the curvature and the deflection have to be zero at x = 0 and L. This says

$$\begin{aligned} y = y'' = 0 \;\; at \;\; x = 0,\; and \\ y = y'' = 0 \;\; at \;\; x = L \;. \end{aligned} \qquad (12)$$

The second derivative of Eq (11) with respect to x is

$$y'' = k^2 \begin{bmatrix} A(-\cos kx + \cosh kx) + B(-\cos kx - \cosh kx) \\ +C(-\sin kx + \sinh kx) + D(-\sin kx - \sinh kx) \end{bmatrix} \cos(\omega t + \phi) \qquad (13)$$

Looking up definition of cosh and sinh we find that cosh(x) = 1, at x = 0, sinh(x) = 0, at x = 0, so we find that the constants: C = D, and A and B are zero. For this case then Eq. (11) becomes

$$y = 2C \sin kx \cos(\omega t + \phi), \qquad (14)$$

The maximum displacement is 2C, and we'll call it y_{max}. The curvature from Eq (13) becomes

$$y'' = -y_{max} k^2 \sin kx \cos(\omega t + \phi) \qquad (15)$$

From Roark [16, pp 94,95], the maximum stress for a beam where h is the distance from the neutral axis to the outer fiber, is

$$\sigma_{max} = \frac{Mh}{I}, \text{ and } M = EIy'', \text{ so} \qquad (16)$$
$$\sigma_{max} = Ehy''$$

Put the maximum value of the curvature from Eq (15) and the value k^2 from Eq(11a) into Eq (16) and we have

$$\sigma = Ehy_{max} \frac{\omega}{\eta c} = \frac{h}{\eta} \rho c \quad \omega y_{max} = \frac{h}{\eta} \rho c \quad v_{max} \qquad (17)$$

and the point is proved again. For a simply supported vibrating beam, stress is proportional to the peak modal velocity and it doesn't matter what the frequency is. You get the same answer for all beams support configurations. We will write Eq (17) as

$$\sigma_{max} = K_b \rho c v_{max} \text{.....where...} K_b = \frac{h}{\eta} \qquad (17a)$$

Here K_b is a beam shape factor: distance from the neutral axis to the outer fiber, over η the radius of gyration. Values of K_b [1] are given in Table 3. Dick Chalmers and I [1] did the above in 1969. Hunt [17] gives an academic presentation of the derivation and provided the first published proof. Ungar [5] published on the topic in 1962 In that sense Hunt did all of the above and more in 1960, but few know it because of the academic presentation. Hunt also did it for thin rectangular plates, tapered rods

Table 3

Cross-Section Beam	K_b, h/η
Solid rectangle	1.73
Solid round bar	2
Solid triangle	2.83
Thin hollow tube	1.41
Pipe	1.46
Thin hollow square	1.22

and wedges. He felt strongly that for practical situations the shape factor stays under "half an order of magnitude." For the rest of this paper let's consider Hunt's Equation to be Equation (18) and this is stretching because academics hate specifics; he wrote the equation in terms of strain because it's nondimensional, (a very un-engineering approach.)

$$\sigma_{max} = K \rho c v_{max} \tag{18}$$

And let's refer to K and Hunt's constant. Again the same set of observations about acceleration and displacement apply. Some example severe velocities values are given in Table 4. These are peak velocities to attain the indicated stress, not counting any stress concentrations, nonuniformities, other configurations. For long term and random vibration, fatigue limits as well as the stress concentrations, and the actual configuration would make the velocity values much lower.

Table 4. Severe Velocities

Material	E (psi)	σ (psi)	ρg (lb/in^3)	v_{max} (ips) rod $\sigma/(\rho c)$	v_{max} Beam Rectangular	v_{max} Plate
Douglas fir	1.92×10^6	6,450	0.021	633	366	316
Aluminum 6061-T6	10.0×10^6	35,000	0.098	695	402	347
Magnesium AZ80A-T5	6.5×10^6	38,000	0.065	1015	586	507
Steel Structural	29×10^6	33,000 100,000	0.283	226 685	130 394	113 342

What I want you to realize is that vibration velocity is a parameter that proportional to stress, and as such indicates the severity of the vibration in machinery and structure. Ungar [5] understands it and wrote on the topic in 1962 unaware of Hunt's paper, Ungar's work includes noise excitation and damping so his results are harder to understand. Lyon [18] (1975) had no doubt about Hunt's work and started applying it in Statistical Energy Analysis (SEA) and shows its use it in his book. From what I am seeing this area of application is growing. [23, 24]. There are lots of new applications to come, so it's important that you are convinced that maximum modal stress is proportional to maximum modal velocity. Now one might well gamble that vibratory velocity is going to increasingly become the important monitoring quantity. It will be used in ways I can't imagine.

Hunt's Constant Can be much Larger Than He Thought:

Crandall [19] commented on Hunt's article in 1962, and started a line of thinking that I will conclude with. He marveled that it took until 1960 for this (stress velocity) truth to be discovered, but went on to argue, in a manner difficult for me, that there can be wide increases in the value of K. I thought he was groping for exceptions to appear scholarly, but no. People are now showing this is correct as I will explain. Crandall's argument that Hunt underestimated maximum K values turned out to be important. We continue with large K values in Hunt's equation.

Now TULK (1989) [20] (of the Research Department at Ontario Hydro, Toronto) comes into the picture. He forms the equal kinetic and strain energy idea as "the maximum strain energy during each cycle is equal to the maximum kinetic energy during each cycle." Strain energy is proportional to the square of the strain integrated over the structure, and the maximum kinetic energy is proportional to the square of the velocity integrated over the structure. He gives Hunt's equation as

$$\frac{\sigma_{peak}}{E} = K \frac{v_{max}}{c} \tag{19}$$

To show that this is the same as ρcv, write Eq (19) and manipulate as shown in Eq (6).

$$\sigma_{peak} = K\frac{E}{c}v_{\max}$$
$$c^2 = \frac{E}{\rho} \quad \Rightarrow \quad \rho c = \frac{E}{c} \qquad (20)$$
$$\sigma_{peak} = K\rho c v_{\max}$$

I might even say Eq (19) is a better formulation ratio of stress to modulus is K times ratio of velocity to sonic velocity. He points out that σ_{max} and v_{max} occur at different spots on the structure. He also cites two Canadian Electrical Association reports that show that K values lie in the range of 1.5 to 4 for many simple structure configurations, and that K equals two for a round bar. He comments that if it can be shown that K has an upper bound than velocity can be used as a screening factor for piping inspection.

He cites 3 Wachel references including [14] as proposing velocity based vibration screening of piping systems, as the basis for ANSI/ASME OM3-1982 "Requirements for Preoperational and Initial Start-up Vibration Testing of Nuclear Power Plant Piping Systems", ASME New York, 1982. This standard defines an allowable peak velocity defined as follows.

$$v_{allow} = \frac{C_1 C_4}{C_3}\frac{3.65x10^{-3}\left(0.8\right)}{C_2 K_2}\sigma_{allow} \qquad (21)$$

where

v_{allow} *allowable velocity (ips)*

σ_{allow} *endurance strength (psi)*

$C_{1,2,3,4}, K_2$ *compensating factors*

The factors allow for geometric discontinuities, weight of fluid and insulation, concentrated masses between supports, tees and elbows.. The factors make the effective K = 70, which he believes is overly conservative. (Later he says 50 is OK.)

The interesting thing he reports is a MSC NASTRAN finite element analysis of four large nuclear piping systems, with pipe diameters from 6 to 15 inches. The pipe runs include elbows, concentrated masses between supports, and tees. They analyzed 126 modes under 40 Hz, for maximum stress and calculated maximum velocity from maximum deflection times frequency. They calculated the K's. A total of 126 modes were analyzed. Calculations included factors for tees, elbows and welded joints. Velocities came from maximum deflection in each mode times frequency. Stresses were derived from element deflection data. K ranged from 4 to 35, with a mean value of 7.4, and standard deviation of 3.8. They decided a conservative screening K = 50; which leads to screening vibratory velocity of 35 mm/sec or 1.38 ips for piping inspection. OM3 only allows 0.5 ips

The next interesting finding is that he states that large K ratios seemed to occur with a combination of effects magnifying things, such as a valve body supported by a branch pipe. He suggests that you do this analysis at design stage, and use these high K's to eliminate high stress situations. The K ratio is larger for piping systems than for simple structural forms, because of geometric features that increase stress.

Finally Hartlen, [20] (who had been a sponsor from Ontario Hydo of Tulk's work)(now a consultant from, Plant Equipment Dynamics, Ontario, CANADA,) has put the idea from Tulk's paper into practice. He prepared a 'technical article' for SST Systems who is offering this analysis capability for their CAEPIPE piping analysis software package that performs the stress velocity ratio calculations of piping systems at the design stage. They call the analysis "The 'Dynamic Susceptibility' Method for Piping Vibration", and bill it as "A Screening Tool for Potentially-large Alternating Stresses". His article explains the strain and kinetic energy argument exactly as does Tulk.

They do a modal analysis of a piping system and add to the analysis the bending stresses in each mode. The dynamic susceptibility for any mode is the ratio of the maximum alternating bending stress to the maximum vibration velocity It is K with the ρc included. Using Hunt's equation, the stress velocity ratio is

$$\sigma = K\rho c v$$

$$\frac{\sigma}{v} = K\rho c \qquad (22)$$

He states the K can vary and a high K indicates a susceptibility of the mode to high stress. Maybe the concept is that all modes are excited with equal or random velocity levels, and the most susceptible modes ought to be analyzed and corrected in design. This is done from a finite element modal analysis and can be analyzed before being built. This seems like an interesting idea for any structural modal analysis. Their program uses the "Stress per Velocity" method, as an aid to screen the vibration modes of a system. It identifies which modes, if excited, could potentially cause large dynamic stresses. It shows maximum stress location and helps the designer evaluate features of the system layout and support that may be responsible for large dynamic stresses.

The stress velocity ratio, σ/v, will lie in a lower "baseline range" for uncomplicated systems such as classical uniform-beam configurations. For more complex systems, the ratio will increase due to complications such as three-dimensional layout, discrete heavy masses, changes of cross-section and susceptible branch connections. For example, steel pipe has a ρc = 146, and pipe has a K = 1.47, so the " baseline" ratio is 215. He gives Table 5, as an example of values from their screen program. The high stress velocity values were from cantilever pipe stubs with valves or masses and could be explained,

Table 5

Mode #	Frequency	Max V Node	Max σ node	σ/v
2	20.78	20A	50	649
7	129.2	70	70	594
8	132.0	70	70	589
4	31.23	70	80	526
3	27.75	60	80	521
1	14.51	20B	10	456
5+	47.41	40A	50	383
6	52.42	30	30	339
10	163.0	40A	40A	272
12	191.9	40A	40B	266
11	173.1	20B	40B	256
9	138.7	30	20A	104

Statistical Energy Analysis Application:
As mentioned Lyon [18] applied the stress velocity proportionality in statistical energy analysis and that is continuing. Internet searching on stress and velocity in piping systems leads to several articles that are studying the conditions under which analysis of field velocity measurements is as good as strain gage measurements for inspection. Karczub'a and Norton's [22] paper presents analysis of data from the shuttle launch structure where velocity measurement compares favorably with strain gage stress measurement. Finnveden and Pinnington [23] present a lengthy computer analysis verifying a concept that average strain energy density is proportional to the mean square velocity. They state that this velocity method may become engineering practice for estimating fatigue life.

Conclusions:
The reason velocity measurement is the most widely used vibration monitoring parameter is because it is proportional to stress; when the machinery stresses get bad, the velocity gets bad. The range of velocities to be measured is comparatively small because the stress values are limited. The practice of measuring vibratory velocity for evaluating machinery health has been evolving since before Rathbone [7] in 1939, and there is every indication that new uses and procedures for using velocity measurement will continue to be developed. Stress-velocity ratio evaluations from finite element analyses of complex structure look like a completely new tool for design.

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