Use of Damping in Pseudo Velocity Shock Analysis

by Howard A. Gaberson, Ph.D., P.E.
Consultant, (Retired Civilian Navy)
234 Corsicana Dr.; Oxnard, CA 93036; (805) 485-5307
hagaberson@att.net

Nomenclature

4CP	four coordinate paper, tripartitie plot
c	damping coefficient
c_c	critical damping coefficient, $c_c = 2\sqrt{km}$
g	acceleration of gravity (386.087 in/sec^2)
h	drop height
ips	inches per second
k	spring constant
m	mass
PV	pseudo velocity
PVSS	pseudo velocity shock spectrum
R	ratio of shock plateau damping depression
SDOF	single degree of freedom
SDOFs	single degree of freedom system
SRS	shock response spectrum
t	time
x	absolute displacement of the SDOF mass
$\ddot{x}$	absolute acceleration
$\ddot{x}_{\max}$	maximum acceleration
y	shock displacement of bogey or wheeled foundation.
$\ddot{y}$	foundation shock acceleration
z	relative displacement, x - y.
$\dot{z}$	relative velocity
z_0, $\dot{z}_0$	initial values of $z, \dot{z}$
ζ	critical damping ratio, $\zeta = \dfrac{c}{c_c}$
η	$\eta = \sqrt{1-\varsigma^2}$
τ	integration time variable
ω	frequency in radians per unit time
ω_d	damped natural frequency, $\eta\omega$
ω_n	natural frequency, radians/sec, $\omega_n = \sqrt{\dfrac{k}{m}}$

Abstract
Pseudo velocity shock spectrum (PVSS) values indicate potential equipment modal velocity values which are proportional to maximum modal stress. That's why the PVSS shows shock severity. The damped pseudo velocity shock spectrum indicates modal velocity developed in similarly damped structure. Additionally damping in the analysis strongly reduces peak values due to resonant build up in multicycle shocks. Such shocks are synthesized for shaker use to meet a spectrum requirement. Modest damping can be required to assure a low frequency content. Damping also reduces the severe plateau levels of the simple shocks. General shocks can be expected to have their severe levels reduced by at least as much due to damping. Finally shock polarity, the comparison of the maximum magnitudes the positive and negative pseudo velocity shock spectra, is only observable in moderate to heavy damped spectra. The paper presents theory and examples of these effects.

Introduction
Damping is almost always in used in shock spectrum SRS and PVSS on 4CP calculations because at low levels it will relate more accurately to actual structural response. One cannot expect a zero damped response in actual structure. The best collection of current damping thinking is Lalanne [1]. He wrote a very comprehensive handbook like text on shock but only treats acceleration SRS technology. He does not treat the PVSS although it is mentioned, and he shows only a few illustrations of 4CP. Lalanne shows large differences in positive and negative SRS's due to a pyroshock zero shift and gives a few references that may substantiate the fact. Piersol [2] and [3] treat damping and the positive and negative spectrum. They show the use of a positive and negative SRS to weed out anomalies in pyroshock data. The Shock and Vibration Handbook [4, 5, 6] also mentions damped shock spectrum analysis. Here I use the positive and negative PVSS to evaluate polarity.

Pseudo velocity shock spectrum values roughly indicate possible equipment modal velocity the shock can induce. Maximum modal velocity values are proportional to maximum modal stress; that's why the PVSS shows shock severity.[7,8,9,10] The damped pseudo velocity shock spectrum indicates modal velocity developed in similarly damped structure. Additionally damping in the analysis strongly reduces peak shock spectrum values due to resonant build up in multicycle shocks. Such shocks are synthesized for shaker use to meet a spectrum requirement. Modest damping can be required to help assure a sufficiently severe shock from multicycle synthesized shocks. Damping very predictably reduces the severe plateau levels of the simple shocks. We can calculate this reduction. General shocks can be expected to have their severe levels reduced by up to as much as the simple shock damping plateau reduction. Finally shock polarity, the directionality comparison of the maximum magnitudes of the positive and negative pseudo velocity shock spectra, is only observable in moderate to heavy damped negative - positive PVSS spectra. The paper presents theory and examples of these effects. The examples document a gallery of posneg spectra.

Understanding the PVSS Plateau

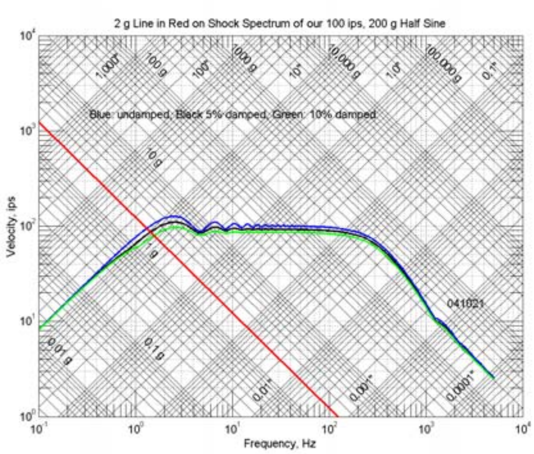

Figure 1. This is the PVSS on 4CP of a half sine shock with a velocity change on impact of 100 ips and a maximum acceleration of 200 g's. It occurs on a drop table shock machine with a 13 inch drop height and no rebound and with the table impacting a pad that causes a peak acceleration of 200 g's.

If you already understand the PVSS and its plateau, you can skip the next four pages, and go right to the meat of what I want to discuss. Otherwise, the PVSS on 4CP is a specific way to plot the relative displacement shock spectrum [9, 10]. It is very helpful for understanding shock severity. The PVSS is the peak displacement of many different frequency single degree of freedom systems excited by the shock, multiplied by the resonant frequency in radians.

The PVSS-4CP plateau is often the highest region of the spectrum. In simple shocks it is flat and is the top of a flattened hill, shape. Figure 1 shows the PVSS on 4CP of a half sine shock with a velocity change on impact of 100 ips and a maximum acceleration of 200 g's. It occurs on a drop table shock machine with no rebound and a about a 13 inch drop height. We include the drop in the analysis.

We call this a simple shock because it consists of one of the classic shapes: half sine, trapezoidal, haversine, initial and terminal peak sawtooths. These have all been historically studied. This is also a zero mean shock that begins and ends with zero velocity. We include the drop in the shock time history so this is the case.

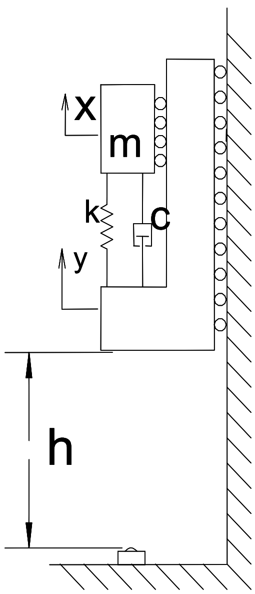

Figure 2, The shock table wheeled bogey with a single dwegree of freedom system

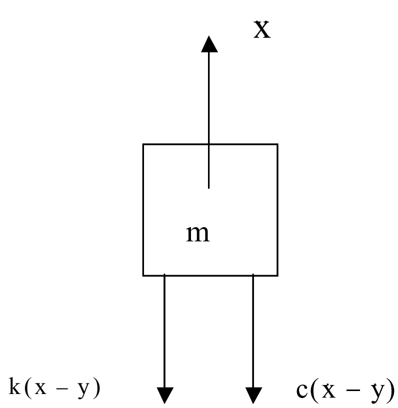

Figure 3, The free body diagram of the mass.

All PVSS's have a plateau to some extent, and this is the region where the shock is most severe so you have to understand it. Sometimes it's very short and sometimes long. Collision, kick off, and general non zero mean shocks don't begin and end with zero velocity, and are almost all plateau. I'll explain the plateau by thinking of the drop table shock machine and simple shocks. I need the Shock Spectrum equations to explain things; so quickly bear with me while we review them. To explain the damping effect on the simple shock, I need you to understand the plateau and simple shock PVSS general shape. So, I'll go over that first. Then we look at damping.

Shock Spectrum Equation: Figure 2, is the SDOFs (single degree of freedom system) model to explain the shock spectrum where:

y is the shock motion applied to the bogey or heavy wheeled foundation.
x is the absolute displacement of the SDOF mass
z is the relative displacement, x - y, positive when the spring stretches.

.The free body diagram of the mass is shown in Figure 3.
Applying F = ma to the FBD (free body diagram) of Figure 3 gives us Eq. (1).

$$-c(\dot{x}-\dot{y})-k(x-y)=m\ddot{x} \qquad (1)$$

Using relative coordinates, defined as: z = x - y, gives (Eq. (2)):

$$
\begin{aligned}
&-c\dot{z} - kz = m(\ddot{z} + \ddot{y}), \quad or, \\
&m\ddot{z} + c\dot{z} + kz = -m\ddot{y}
\end{aligned} \tag{2}
$$

Dividing by "m," and substituting the definitions and symbols of Eq. (3a) give Eq. (3).

$$
\omega_n = \sqrt{\frac{k}{m}}, \quad \zeta = \frac{c}{c_c}, \text{ and } c_c = 2\sqrt{km} \tag{3a}
$$

$$
\ddot{z} + 2\zeta\omega_n\dot{z} + \omega_n^{\;2} z = -\ddot{y} \tag{3}
$$

Equation (3) is the shock spectrum equation, and the shock spectrum is our tool for understanding shock. In Eq. (3), $\ddot{y}$ is the shock. I give the solution in [11] explicitly with initial conditions as follows in Eq. (4).

$$
\begin{aligned}
z = {} & z_0 e^{-\zeta\omega t}\left(\cos\eta\omega t + \frac{\zeta}{\eta}\sin\eta\omega\, t\right) + \frac{\dot{z}_0 e^{-\zeta\omega t}}{\eta\omega}\sin\eta\omega t \\
& -\frac{1}{\eta\omega}\int_0^t \ddot{y}(\tau)e^{-\zeta\omega(t-\tau)}\sin\eta\omega(t-\tau)\,d\tau
\end{aligned} \tag{4}
$$

Where: $z_0, \dot{z}_0 =$ initial values of $z, \dot{z}$

$\omega_d =$ damped natural frequency, $\eta\omega$

$\eta = \sqrt{1-\zeta^2}$

$\tau =$ integration time variable

The derivative of Eq (4) is Eq (4a).

$$
\begin{aligned}
\dot{z} = {} & -\frac{z_0\omega}{\eta}e^{-\zeta\omega t}\sin\omega_d t + \frac{\dot{z}_0 e^{-\zeta\omega t}}{\eta}\left(-\zeta\sin\omega_d t + \eta\cos\omega_d t\right) \\
& -\frac{1}{\eta}\int_0^t \ddot{y}(\tau)e^{-\varsigma\omega(t-\tau)}\left[-\varsigma\sin\omega_d(t-\tau) + \eta\cos\omega_d(t-\tau)\right]d\tau
\end{aligned} \tag{4a}
$$

Equations (4) and (4a) are the solution of the shock spectrum equation. We need the first two terms of each which are the free vibration response and completely give the motion when $\ddot{y} = 0$.

Now I can explain why the simple shock plateau occurs. Think with me in the following way. We'll do the undamped case first. Think of an instantaneous shock. Look at Figure 2, to be clear on the meanings of x, y, and z. The bogey, is way heavier than the mass, like the table on a drop table shock machine. The bogey is released and falls from a height, h, and hits a shock pad that stops it to zero velocity with one of the simple shock impacts that has a peak acceleration, $\ddot{y}_{\max}$. Both the bogey and the mass fall together so that during the fall x = y, and z = 0. Both x and y attain a peak velocity of $\dot{x} = \dot{y}_i = -\sqrt{2gh}$. Just after the impact, the bogey velocity, $\dot{y}$, suddenly becomes zero, but $\dot{x}$, the mass velocity, hasn't yet changed and is still $\dot{x} = -\sqrt{2gh}$. Since $\dot{z} = \dot{x} - \dot{y}$, and, $\dot{y}$ has just become zero, just after the impact, which now I define to be t = 0, suddenly we have $\dot{z}_0 = \dot{x}_o = -\sqrt{2gh}$, and we have the initial velocity case for undamped free vibration of the mass attached to the motionless bogey. Repeating: the bogey and the mass fall together, and the shock is over before the spring does any compressing. The bogey suddenly stops and then the mass starts vibrating. This is undamped initial value free vibration of Eq. (5). The equation for this condition can be taken from Eq (4) with $\varsigma = 0, \eta = 1, z_0 = 0, and\ \ddot{y} = 0$, which is Eq (5). We have Equation (5) start immediately after impact when we take $z_0 = 0,$, but still, $\dot{z}_0 = -\sqrt{2gh}$. so

$$z = \frac{\dot{z}_0}{\omega}\sin\omega t = \frac{-\sqrt{2gh}}{\omega}\sin\omega t\,, \tag{5}$$

Since it's undamped, this goes on forever, and the maximum pseudo velocity is.

$$\omega z = \sqrt{2gh} \tag{5a }$$

Now, as simple as that is, that's how and why we get a plateau. All SDOFs, with half periods much longer than the impact duration, end up vibrating with the same peak velocity, the impact velocity change, no matter what their natural frequency. In this undamped sinusoidal motion, the relative velocity and the pseudo velocity have the same maximum values; they both continue forever with this peak velocity, the impact velocity. This is why we see the plateau; the shock spectrum of a simple shock will have a constant PV plateau for quite a wide frequency interval.

UNDAMPED PVSS'S OF SIMPLE DROP TABLE SHOCKS HAVE A FLAT CONSTANT PV PLATEAU AT THE VELOCITY CHANGE THAT TOOK PLACE DURING THE SHOCK.

PVSS Asymptotes and Plateau Limits

The high frequency asymptote is the constant acceleration line at the peak acceleration. There are limits to the frequencies at which this plateau can continue. In the very high frequency region, think of the mass as very light and the spring very stiff; so stiff that the mass exactly follows the input motion. The acceleration of the mass is equal to the acceleration of the foundation. In this region the maximum relative deflection, z, is given by the maximum force in the spring over its stiffness, k. The maximum force is the ma force, $m\ddot{x}$, and $\ddot{x}_{\max} = \ddot{y}_{\max}$. Thus the maximum spring stretch is:

$$\begin{aligned} z_{\max} &= \frac{m\ddot{x}_{\max}}{k} = \frac{1}{\omega_n^2}\ddot{x}_{\max} = \frac{1}{\omega_n^2}\ddot{y}_{\max}. \\ z_{\max} &= \frac{1}{\omega_n^2}\ddot{y}_{\max}. \end{aligned} \tag{6}$$

We multiply this by ω to get the PV which is Eq (6a).

$$PV = \omega z_{\max} = \frac{\ddot{y}_{\max}}{\omega_n} \tag{6a}$$

The very high frequency pseudo velocity asymptote is the peak acceleration divided by the natural frequency. On log log paper this is a straight line sloping down to the right. This is the 4CP constant acceleration line at the peak acceleration. I have calculated and plotted all of the simple shocks. I've found that on the RHS (right hand side) of the PVSS on 4CP, near the intersection of the acceleration asymptote and the plateau, the PVSS begins its downward slope at a higher acceleration than the asymptote but does not exceed twice a_{max}.

THE HIGH FREQUENCY LIMIT OF THE PLATEAU OF THE UNDAMPED PVSS OF SIMPLE SHOCKS IS SET BY THE MAXIMUM ACCELERATION OF THE SHOCK.

The low frequency asymptote of a zero mean simple shock is a constant displacement line at the peak displacement. Now on the low frequency end of the plateau, imagine the SDOFs this way: the mass is heavy and the spring is extremely soft, so the mass won't even start to move until the bogey has fallen, stopped, and the impact is over. The spring has stretched h, and then the mass starts vibrating with amplitude “h” forever. The deflection cannot exceed the drop height. Thus, on the left side of the PVSS on 4CP, z_{max} = h and the PV will be:

$$\omega z_{\max} = \omega h \tag{6b}$$

And that’s a line sloping down and to the left at a constant deflection, “h.”

Continue thinking of the very low frequency SDOFs. For every shock, the mass just sits there (doesn't start moving) until the shock is over. The calculating algorithm is recording the relative displacement as the shock goes by, and finally the mass starts vibrating at that or some lesser value. Think of a quick up and down table or bogey jolt, and think of the very low frequency SDOFs. The algorithm will pick off the peak and record that. It might not move at all until the shock is over. However the recorded maximum z will be the maximum table motion.

NOTICE: THE LOW FREQUENCY LIMIT OF THE PLATEAU OF THE PVSS ON 4CP OF A ZERO MEAN SIMPLE SHOCK IS SET BY THE MAXIMUM DEFLECTION OF THE SHOCK.

A summarizing thought is the following. I want to remind you of Figure 1, the example 800 g half sine shock. Please notice that there is no net velocity change; it starts at zero velocity and ends at zero velocity; however, there was a sudden 100 ips velocity change during the impact. No net velocity change means the acceleration time trace has a zero integral, or in fact a zero mean or average value.

The 2g Line, The undamped no rebound simple drop table shock machine shock plateau low frequency limit is the 2g line. On the undamped PVSS on 4CP of a simple no rebound drop table shock machine shock, the shock machine drop height will be given by the intersection of the plateau and the 2g line. This is explained as follows. The low frequency, no rebound asymptote is the drop height constant displacement line. The PV everywhere on this line is ωh. The undamped PV plateau is a horizontal line on the velocity change at impact or $\omega z = \sqrt{2gh}$. Thus, the LF asymptote intersects the velocity plateau line where $\omega h = \sqrt{2gh}$. Squaring both sides we have the intersection at:

$$\begin{aligned} \omega^2 h^2 &= 2gh, \quad or \\ \omega^2 h &= 2g \end{aligned} \tag{7}$$

$\omega^2 h$ is an acceleration. The undamped no rebound simple shock PV plateau intersects the drop height asymptote at an acceleration of 2g’s. I have drawn in the 2g line on Figure 1. No rebound must be stated in the 2g line definition. I had to say no rebound because a rebound increases the velocity change during impact, or for a given velocity change a rebound reduces the needed drop height, and will reduce the low frequency asymptote. The line is not a hard fast rule, but just a handy approximation to the low frequency limit of drop table shocks.

Damping

Damping in PVSS analysis is the reason for this paper, and here we are on page 6 before we start. I'm sorry; I think we needed all that preliminary. We use damping in the PVSS on 4CP to see what velocities develop in damped equipment, because the velocities relate to stress. I will discuss this effect, and the use of damping in evaluating polarity. We begin with the effect of damping on simple shocks which we can carefully analyze.

Damping Reduces the Simple Zero Mean Shock Plateau Level.

The way I established the simple shock undamped plateau was with Eq (4) and $\varsigma = 0, \eta = 1, and\ \ddot{y} = 0$, which is Eq (5). We thought of the bogey and the mass falling together and the bogey suddenly stopping. I explained the initial displacement was zero and the initial velocity was the impact velocity, or the velocity change at impact. To do the same problem with damping, we need the damped Eq (4) with $\ddot{y} = 0, and\ z_0 = 0,$, which is Eq (8).

$$z = \frac{\dot{z}_0 e^{-\zeta\omega t}}{\omega\eta}\sin\eta\omega t = \frac{-\sqrt{2gh}e^{-\zeta\omega t}}{\omega\eta}\sin\eta\omega t \tag{8}$$

I have to inject the idea that a shock in the positive direction leads to an initial negative change in z. Go back and look at Figure 2, and imagine a sudden positive y motion. This will cause spring compression, which is negative z. In the plateau region, finding the maximum relative displacement "z" is again an initial velocity problem. At time equal to zero, the initial displacement is 0, and we have an initial velocity (where $\dot{z}_0$ = initial velocity, $= -\sqrt{2gh}$, if no rebound; actually the impact velocity change). Equation (8) is a decaying sine wave with an initial velocity. Now with this negative initial velocity, we'll get a negative minimum and a positive maximum in the first period, The product of these and the natural frequency will be the negative and positive pseudo velocity plateau shock spectrum values. I want to calculate both because we will ultimately want them. Notice the way we have defined the system will make the magnitude of negative minimum greater than the positive value. These maxima occur when $\dot{z} = 0$, in Eq (4a) with both $z_0 = 0$, and $\ddot{y} = 0$. I have written this as Eq (9).

$$\dot{z} = \frac{\dot{z}_0}{\eta} e^{-\varsigma\omega t}\left[-\varsigma \sin\eta\omega t + \eta cos\eta\omega t\right] \tag{9}$$

A minimum and a maximum occur in the first cycle when the bracketed RHS factor in Eq. (9) is zero. The larger first value will be negative and the second value positive. I want to calculate the ratio of the maximum and minimum pseudo velocity to the impact velocity for a set of dampings. I will call these R_{neg} and R_{pos}. To get these we divide Eq. (8) by the impact velocity, $\dot{z}_0$, and multiply it by ω. This is Eq (10). The R values are given by the two $\eta\omega$t values from Eq. (9) substituted in Eq. (10).

$$R = \frac{\omega z}{\dot{z}_0} = \frac{e^{-\zeta\omega t}}{\eta}\sin\eta\omega t \tag{10}$$

I wrote a Matlab script to do this and some results are listed in Table 1:

Table 1. Plateau Depression Due to Damping

ζ	R_{neg}	R_{pos}	R_{pos}/R_{neg} Polarity
0	1.0000	-1.0000	1.0000
0.0050	0.9922	-0.9767	0.9844
0.0100	0.9845	-0.9541	0.9691
0.0200	0.9695	-0.9104	0.9391
0.0300	0.9548	-0.8689	0.9100
0.0400	0.9406	-0.8294	0.8818
0.0500	0.9267	-0.7918	0.8545
0.1000	0.8626	-0.6290	0.7292
0.1500	0.8062	-0.5005	0.6209
0.2000	0.7561	-0.3982	0.5266
0.2500	0.7115	-0.3162	0.4443
0.3000	0.6715	-0.2500	0.3723
0.3500	0.6355	-0.1965	0.3092
0.4000	0.6029	-0.1530	0.2538
0.4500	0.5733	-0.1177	0.2053
0.5000	0.5463	-0.0891	0.1630

The above and Table 1 shows that damping, in a very predictable way, lowers the PVSS plateau of simple shocks by the amount given in Table 1, in the R_{neg} column. We can expect all shock PVSS's to be reduced by damping, and that is the major effect of damping. We're only looking at the plateau, but that is the severe region of the shock. We'll also observe that damping has a much greater effect on multicycle shocks, and this is a valuable property.

The fact that damping ruins the plateau (being at $\sqrt{2gh}$) is disappointing, but true. I cannot teach that the simple shock machine shock PVSS plateau. is at $\sqrt{2gh}$. It's only true for the undamped case. From Table 1, the plateau level, R_{neg}, is down to 93% for 5% damping and in the positive direction it only gets to 80%. For 10% damping it's down to 86% and 63%, and for 25% it's 71% and 32%.

Damping Makes the Zero Mean Simple Shock 2g Line Approximate*.* The 2g line, a cute concept, is only good for undamped, no rebound simple shocks. It's still handy because it approximately indicates the low frequency limit of the plateau, and indicates an approximate drop height.

Non Zero Mean Shocks

Collision and Kick Off Shocks**:** are two common non-zero mean shocks. The low frequency asymptote of each is a constant PV line at the velocity change of the shock if undamped, and close to it if damped.

The collision shock has the bogey moving with a constant initial velocity and coming to rest during the collision. The kick off shock has the bogey initially sitting with zero velocity and, after the shock, continuing forever with the kick off velocity. In each case the extremely low frequency SDOFs begin with a zero relative displacement and velocity and receive an initial velocity as a result of the shock; both continue with a free vibration beginning with an initial velocity. Again think of our heavy, very low frequency SDOFs that doesn't even begin to move until the shock is over. All of a sudden, in the kick off case, the SDOFs notices that the bogey is moving with an initial velocity equal to the final velocity. The collision shock is again reasoned out by thinking of a very low frequency SDOFs traveling at constant velocity on the bogey which suddenly stops. It ends up vibrating with the velocity equal to the initial collision velocity. The peak velocity damped or undamped is obtained from Equation (8) with the initial displacement, $z_0 = 0$, and $\ddot{y} = 0$,.

$$z = \frac{\dot{z}_0 e^{-\zeta\omega t}}{\eta\omega} \sin \eta\omega t \qquad (8)$$

The maximum is reduced the values in the R_{neg} of Table 1.

General Non Zero Mean Shock The general non zero mean shock other than a kick off or a collision is one that starts at one velocity and ends at another. Again imagining and extremely low frequency SDOFs traveling along with no relative displacement until the velocity changes, the suddenly it will start vibrating as in Eq (8) and will have a low frequency asymptote of a constant PV at the velocity change or just below it if damped.

DAMPING REDUCES THE POSITIVE AND NEGATIVE PLATEAUS FOR SIMPLE SHOCK PVSS'S ACCORDING TO THE VALUES IN TABLE 1.

Damping and Simple Shock Polarity

The positive and negative PVSS. Up until now we have been looking at the overall or maximax PVSS's. The way I calculate the maximax PVSS is to calculate both the positive and negative maximums and select the numerically greater value. It is a small program change to have the program collect and plot both. Up to now I have been calling it the posneg spectrum and will continue that here out of habit. The axis orientation in Figure 2, makes the negative spectrum have a higher plateau for a shock in the positive direction; thus I might say negpos is a better idea. But that's only because of how I drew the free body diagram. Thinking of the spring, we could call it the compress-stretch PVSS, or vice versa. I have friends who call it the max-min PVSS, and we could say min-max. To save me time and trouble, I will continue with the term posneg, but I have no strong feelings about what we should call it. I have to plot a lot of them for the remainder of the paper.

Damping in the PVSS on 4CP shows the polarity of the shock on the posneg PVSS: I'll define simple shock polarity to be the ratio, R_{pos} / R_{neg} . Thus a $\varsigma = 0.25$, simple shock will have a polarity of 44%. Its positive plateau will be 44% of its negative. Polarity is the ratio of positive to negative PV content in the plateau region of the PVSS. The simple shock tests have the strongest polarity I can imagine. This is accounted for in MIL-STD 810 [12] and the IEC [13] specifications as both require three shocks in the positive and negative directions. Unfortunately, the undamped PVSS of simple shocks shows equal positive and negative amplitudes in the plateau region, and this has lead many to wrongly conclude a shock has equal content in both directions because they don't use heavy damping. Because the SDOFs being undamped or lightly damped, rings with almost equal positive and negative amplitudes. The damping affects the severe velocity plateau region, but not the asymptotes. The 4th column of Table 1, the polarity, is the ratio of the positive to negative plateaus. These are only for simple shocks. Since stress is proportional to the plateau levels, simple shock machine shocks cause a reduced stress level in the opposite direction.

Damping Examples
I apologize for using color and making it more expensive to copy. It is much easier to make this clear with color.

Simulated Half Sine Shock with the Drop Included; It takes heavy damping to show positive and negative charateristics of the shock. The simple shocks like the half sine are as "polarized" as a shock can get. For an example I show the time history and integrals of a 100 ips, 200 g half sine shock in Figure 4a. I included the drop to show the low frequency plateau limits. Figure 4b shows its maximax PVSS for 0%, 5%, and 10% damping. Notice that the plateaus drop as calculated in Column 2, of Table 1.

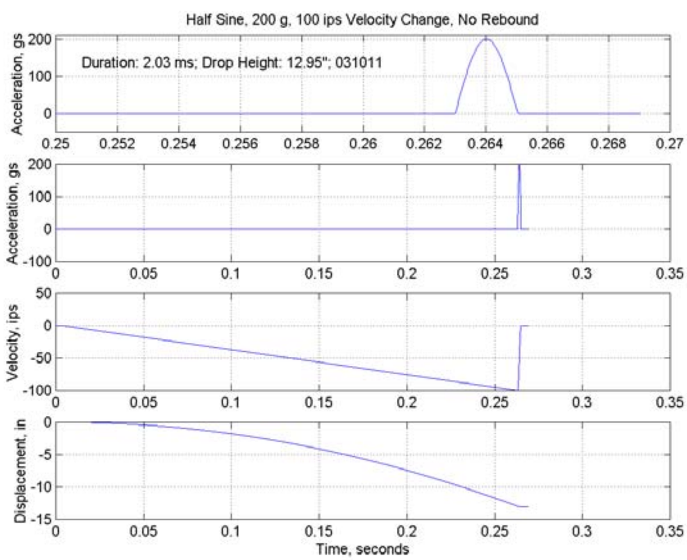

Figure 4a. Time history and integrals for a 100 ips, 200g, drop table half sine shock. The uppermost subplot is an expanded version of the second subplot, included to show detail of the half sine shock.

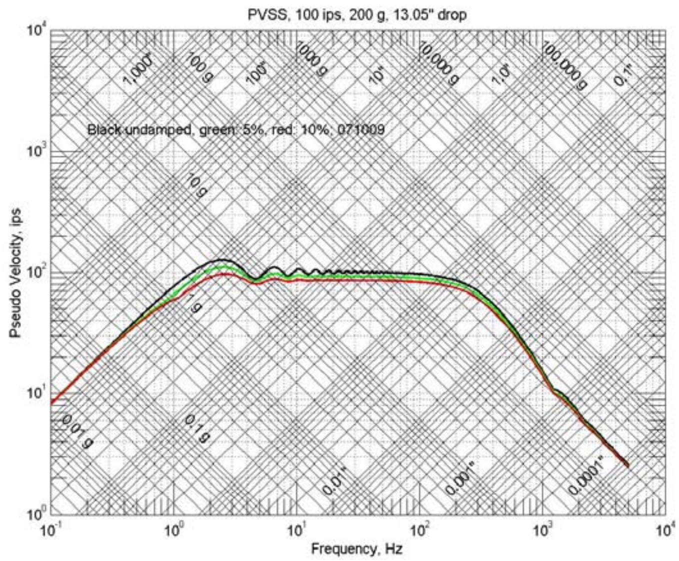

Figure 4b. Effect of Damping on the simple half sine shock. The undamped plateau, in black, is right on 100 ips. The 5% green plateau is at 93% or 93 ips. The 10% red plateau is at 86% or 86 ips. The low frequency asymptote is a constant displacement line at 13 inches, and the high frequency asymptote is on the 200 g constant acceleration line.

In Figure 4b, I want to mention that damping reduces the PVSS everywhere except the asymptotes. The 10% damped PVSS lies under the 5% PVSS; and the 5% lies under the undamped. Many years ago, I asked George O'Hara if I would ever see a case where a higher damped PVSS would ever exceed a lower damped PVSS, and he said no. I have seen nothing in my experience to violate this idea.

Figure 4c is the undamped posneg PVSS of this shock. In the undamped posneg PVSS, we only see positive and negative differences in the high frequency region where the shock is no longer severe.

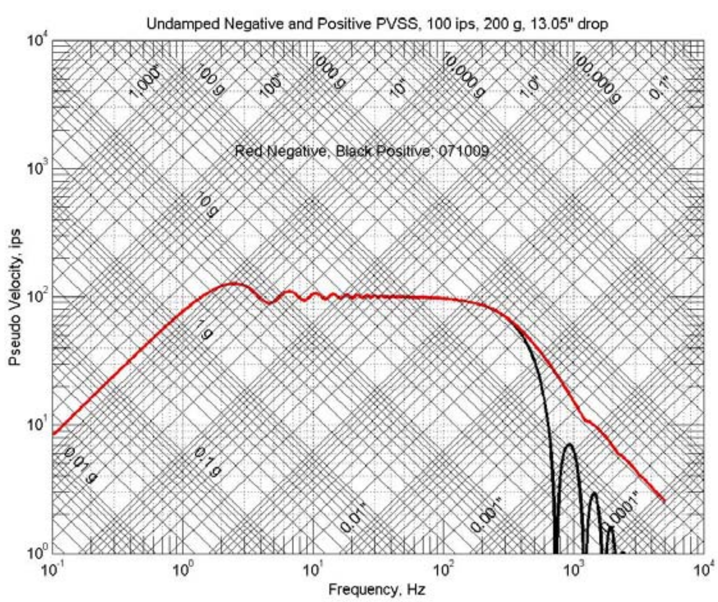

Figure 4c. Positive and negative PVSS for the undamped 100 ips, 200g, half sine shock of Figure 4a.

In Figure 4c, the negative spectrum only exceeds the positive at high frequencies where the PV is low.

The peak negative acceleration is 1 g, so, possibly, the high frequency asymptote of the black positive PVSS will have an asymptote at 1g. The figures seem to show it tending that way.

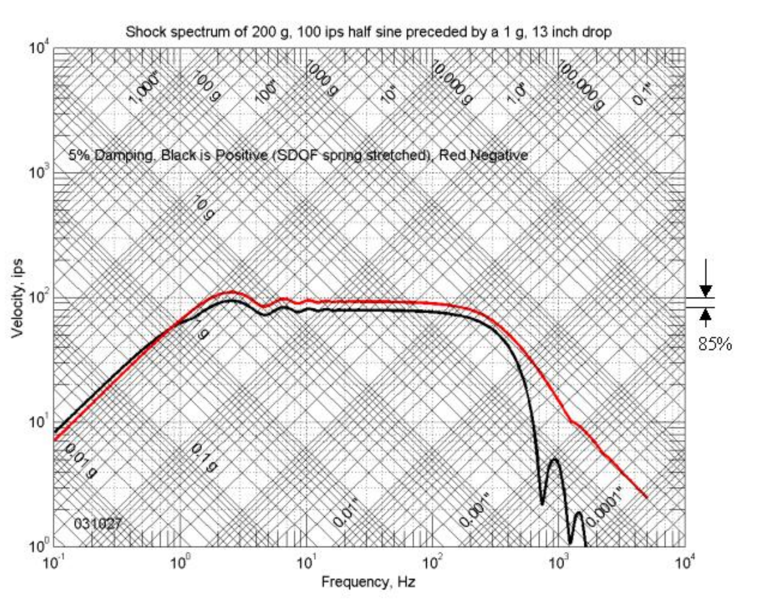

Figure 4d. Positive and negative PVSS for a 5% damped 100 ips, 200g, half sine. The negative spectrum exceeds the positive everywhere except at low frequencies during the drop.

Notice in Figure 4d, that the positive (black) plateau is at 85% of the negative plateau agreeing with the polarity value from Table 1.

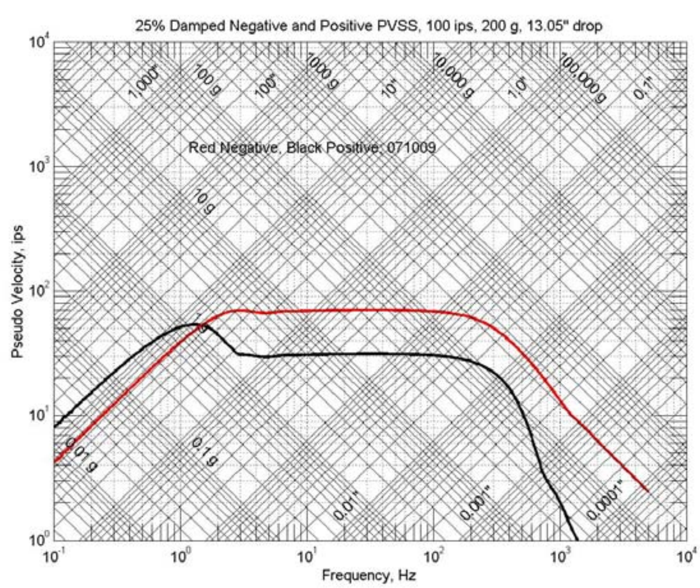

Figure 4e. Positive and negative PVSS for a 25% damped 100 ips, 200 g, half sine. The negative spectrum strongly exceeds (over twice) the positive in the high PV plateau.

In Figures 4d and 4e, let's think about why the positive PVSS exceeds the negative PVSS at low frequencies. This is because the heavy mass supported by the very soft spring, just sits there until the drop is over, and then starts to move. Imagining in Figure 2 with a very heavy mass, when the table drops the very heavy mass will start stretching the spring, which is a positive PV and then after the bogey comes to rest, the mass will start unstretching the spring, but won't compress the spring the full drop height because of the damping. We can always expect to see in the simple shock low frequency range a PVSS sign reversal, where the positive exceeds the negative.

When we use damping to evaluate shock polarity, keep in mind the 4th column of Table 1, the ratio of the positive to negative plateaus of simple shocks . Notice that 5% damping makes the negative plateau 85% of the positive, 10% makes it down 73%, and 25% pushes it down 44%. If one is testing with a complicated shock, and not going to test in both the positive and negative directions, a heavily damped posneg PVSS might be helpful for polarity comparison with a simple shock.

The question of how realistic is a 25% damped PVSS, or do we ever encounter such a situation, I don't know. At failure levels many non linearities set in. Joints slip, members yield, and parts impact each other. I have always thought that the collective effect could be somewhat approximated by heavy damping, but I have not seen any data to back up that idea.

Damping in a Simulated Earthquake Shock: Figure 5a, is a time history and integrals of a simulated earthquake from [14]. This is a simulated earthquake that was developed to provide a high PV level at low frequencies for shaker testing. Its damped PVSS plot in Figure 5b, shows it develops 20 inch PVSS displacement at 1 Hz with 5 % damping with only the 5 inch peak shaker displacement. The shaker displacement is given in Figure 5a. It is a multi cycle time history using resonant buildup to attain a high PV. These shocks are very sensitive to damping. Undamped it develops an 800 ips peak, while at 5% it's down to 160 ips, as shown in Figure 5b. In Table 1, I calculated that a simple shock only drops 7% due to 5% damping, and this shock is dropping 80%.

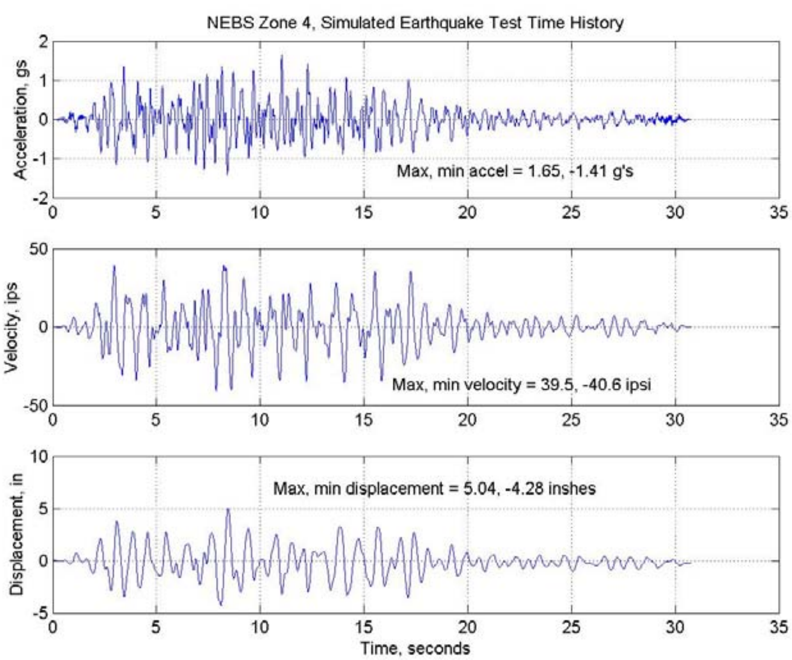

Figure 5a. Time history and integrals of the NEBS Zone 4 Earthquake test time history [14].

The shock is given as an ascii file in [14]. Figure 5a, shows a plot of the acceleration and its two integrals The undamped, 5% and 10% damped PVSS is shown in Figure 5b. Notice the strong effect damping has on the shock.

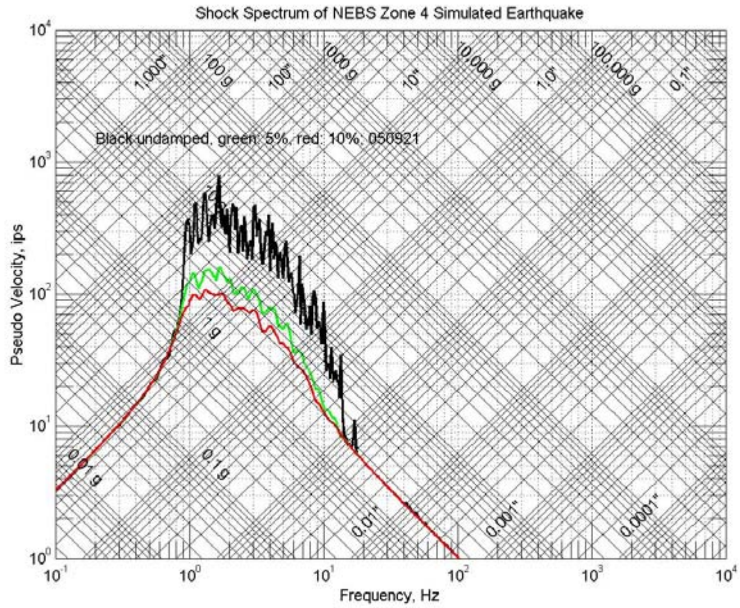

Figure 5b,   The undamped, 5% and 10% damped PVSS of the NEBS Zone 4 Earthquake test time history [14].

Considering the polarity of this shock from an examination of its time history, Figure 5a, the shock seems to have roughly equal positive and negative excursions in acceleration, velocity, and displacement. I would expect it is not very polarized. The undamped posneg PVSS is shown in Figure 5c..

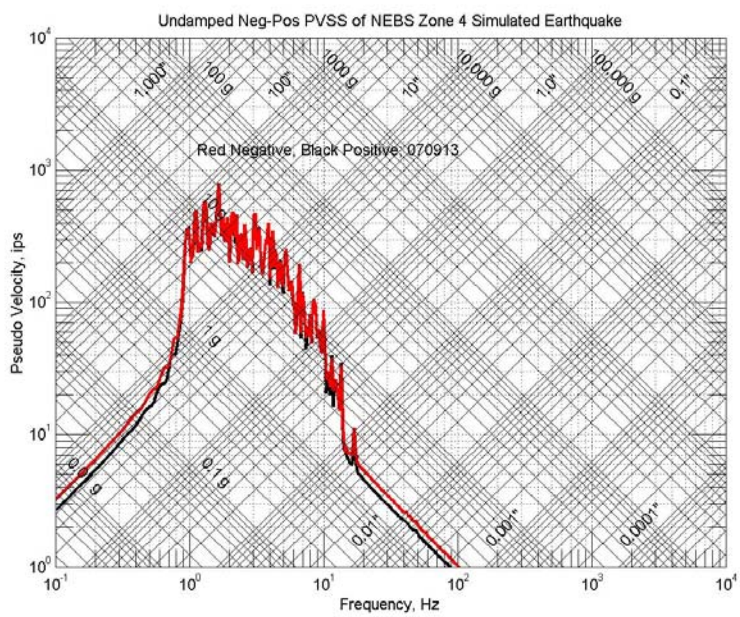

Figure 5c. The undamped posneg PVSS of the NEBS Zone 4 acceleration of Figure 5a.

In Figure 5c, the high frequency asymptotes tend to the peak acceleration. Notice that the negative acceleration exceeds the positive as it should since the maximum positive acceleration in the time history exceeds the negative as labeled in Figure 5a. Also notice at low frequencies, since the low frequency asymptotes tend to the peak displacement, the negative asymptote exceeds the positive because the positive maximum displacement exceeds the negative maximum displacement as shown in Figure 5a.

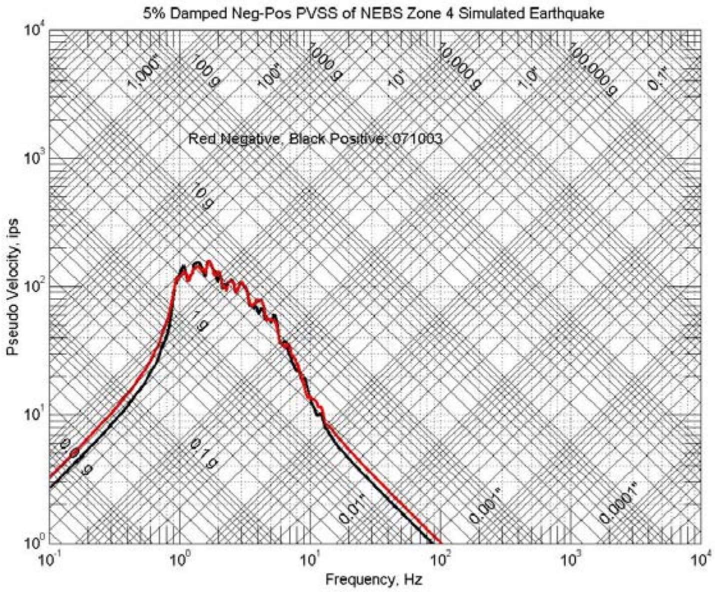

Figure 5d. The 5 % damped posneg PVSS of the NEBS Zone 4 acceleration of Figure 5a.
Now 25% damping

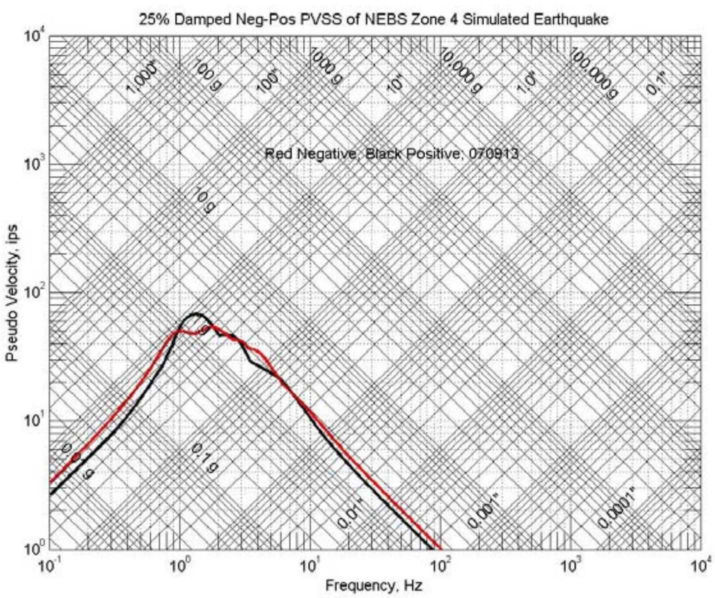

Figure 5e. The 25 % damped posneg PVSS of the NEBS Zone 4 acceleration of Figure 5a

Figure 5e shows very little difference between the positive and negative PVSS's, indicating very low polarity. The asymptotes are unchanged.

Damping in an Actual Pyroshock Example. Now we turn to some pyroshock data. Figure 6a shows a detrended segment of some excellent data, with its integrals and with the peak values labeled

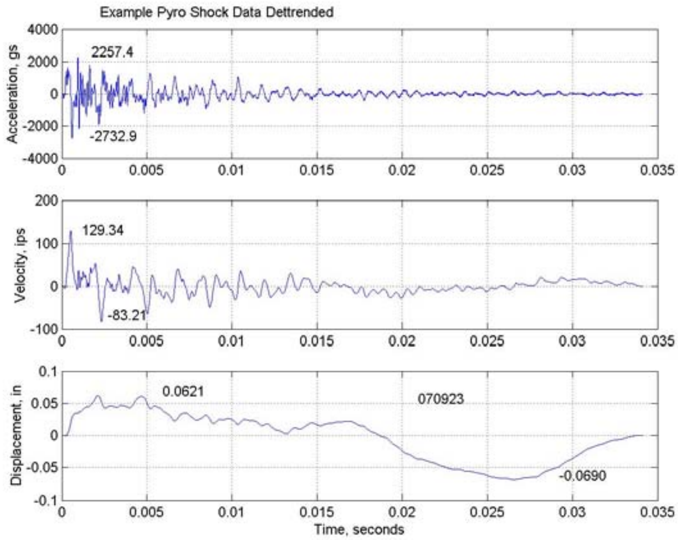

Figure 6a. The is a pyroshock detrended data segment with peak values printed on graph.

In Figure 6a, the displacement has somewhat symmetrical values. However notice that the peak negative acceleration exceeds the peak positive acceleration, and the peak negative displacement exceeds the peak positive displacement. This will show up later on the asymptotes of the posneg PVSS's.

Figure 6b shows the PVSS for three damping values. Notice in Figures 6b to 6e, that I have shifted the x axis to one decade higher frequencies to accommodate the high frequency content of the pyroshock.

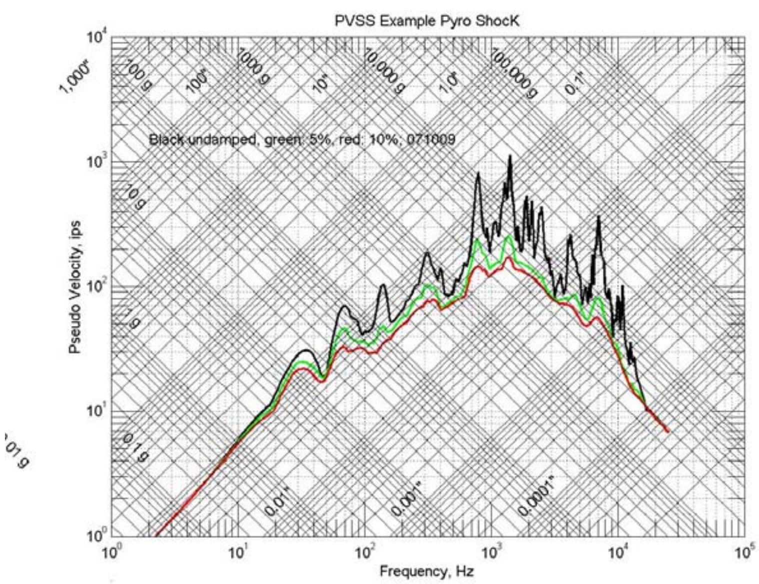

Figure 6b The example pyroshock PVSS with 0%, 5%, and 10% damping.

In Figure 6b the high narrow peaks are due to ringing and are greatly reduced by the damping. In the case of the highest undamped peak at about 1100 ips, it is reduced to about 250 ips by 5% damping. Figure 6b is also another example which shows that the higher damped PVSS's always lie under the lower damped PV SS's; notice that none of the spectra cross.

Next we examine the undamped posneg PVSS in Figure 6c. Notice that there are only one or two small areas where the positive and negative spectra do not overlay each other. My thinking is that in the residual region, the spring compression should be about the same as the stretch, and the residual positive and negative spectra should overlay. Maybe wherever they don't overlay those areas have to be occurring in the primary or during region. I don't know how significant that is, and invite your ideas.

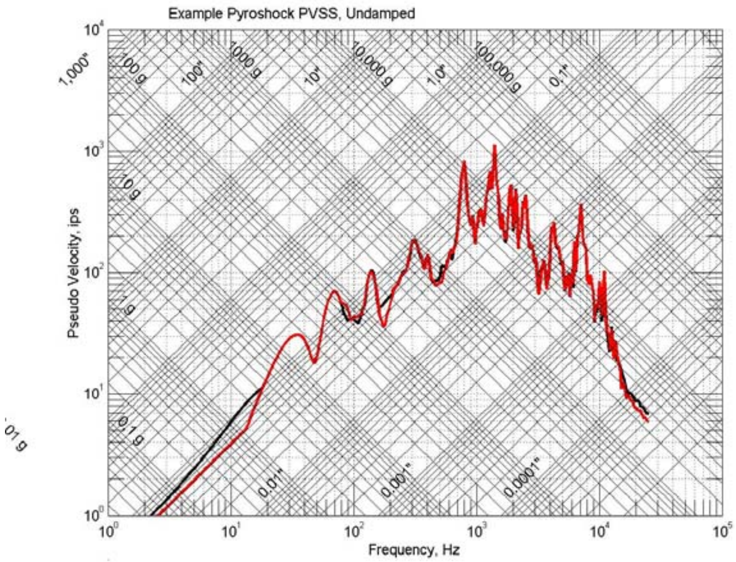

Figure 6c Pyroshock example undamped posneg PVSS.

In Figure 6c notice the low-frequency asymptotes; the red or the negative spectrum is under the black or positive spectrum because looking at the third subplot in Figure 6a we notice the negative displacement exceeds the positive displacement; again in Figure 6a in the top subplot we see the negative acceleration exceeds the positive acceleration. So in Figure 6c, we see the in the high-frequency asymptotes, the red negative line under the black positive line, because the negative acceleration exceeds the positive acceleration in the first subplot of Figure 6a.

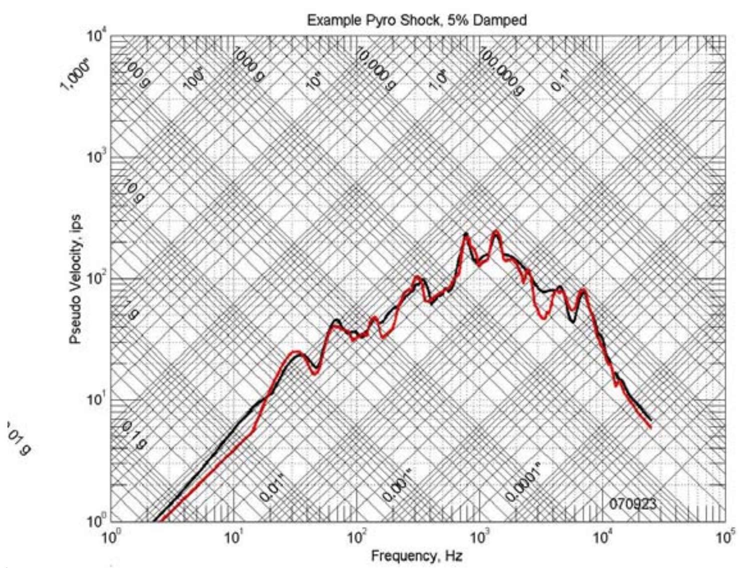

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Figure 6d Pyroshock example 5% damped posneg PVSS.

In Figure 6d, the 5% damped posneg PVSS, we notice a few more areas where the black and red curves do not overlay each other. Now we consider the 25% damped posneg PVSS.

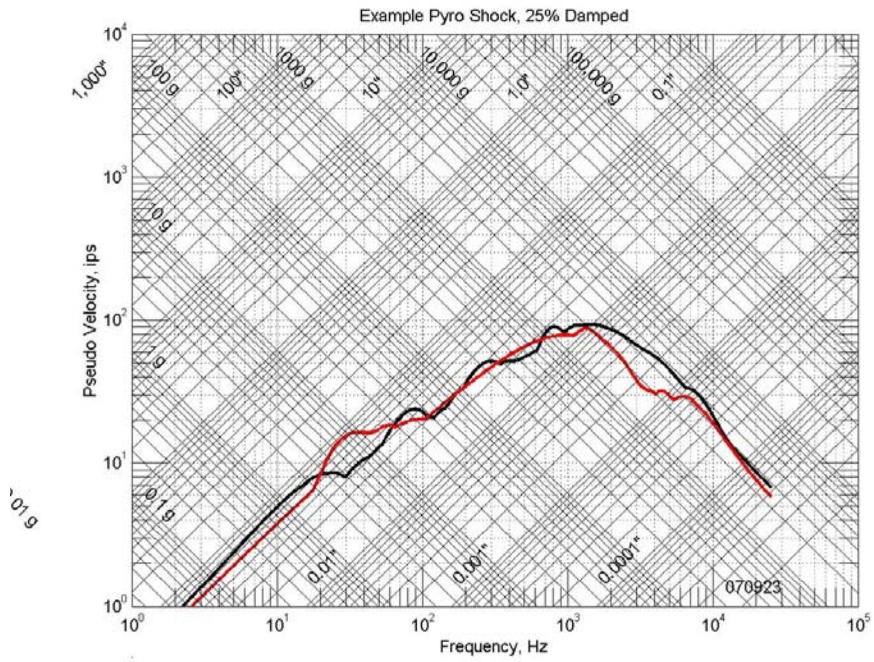

Figure 6e   Pyroshock example 25% damped posneg PVSS. There are two regions with a significant lack of polarity.

In Figure 6e, the 25% damped posneg PVSS, most of the spectrum, even with 25% damping shows very little polarity. But there is an area between 20 and 30 Hz in again between 2000 and 5000 Hz, where there is considerable polarity. Again the asymptotes remain the same.

Damping in a Shock from the Navy Medium Weight Shock Machine: The last example is data from a blow 3 Navy Medium Weight Shock Machine test. The machine consists of a 3000 pound swinging steel hammer striking upward a 4500 pound steel anvil table and transmitting the shock through some stiff channels to the test article.

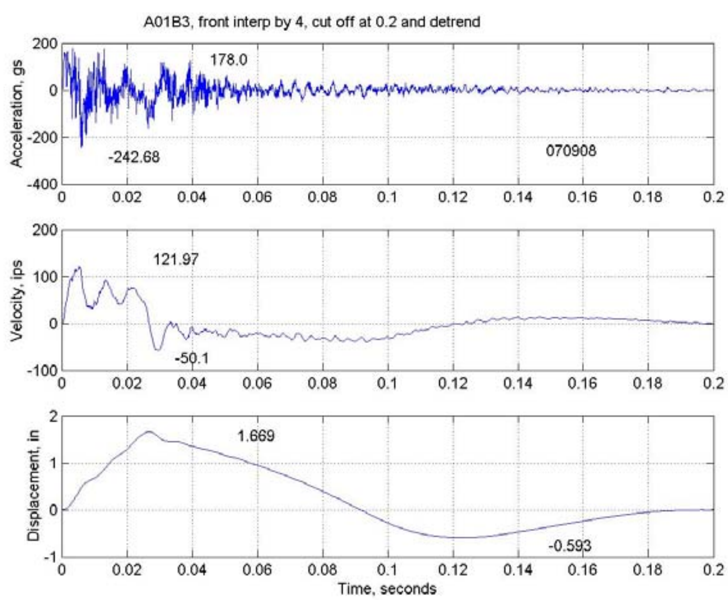

Figure 7a. The time history and integrals from Blow 3 of a Navy Medium Weight Shock Machine test.

The channels are arranged to have a natural frequency of about 100 Hz. The data was truncated and detrended. In Figure 7a, the third subplot, the anvil and equipment support, after being struck and thrown upward by the hammer continue for 1.5 inches and hit steel stops which impact it back downward. This can also be seen in the third subplot in Figure 7a. Thus there is a positive and negative aspect to the to the shock. However, looking at the third subplot in Figure 7a, the displacement starts downward and the anvil hits the stops at about 28 ms. The spike in acceleration at 28 ms is smaller than the initial spike when the hammer hits. The upward hit from the hammer is stronger than the downward hit from the stops. We have a case where the negative peak acceleration is larger than the positive, and the positive peak displacement is greater than the negative, which will show up in the posneg PVSS asymptotes..

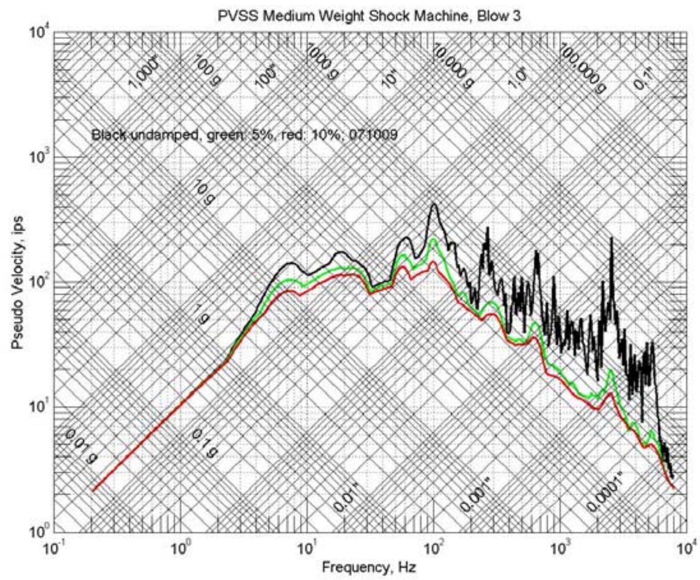

Figure 7b. The maximax PVSS of blow 3 on the Navy Medium Weight Shock Machine,.

In Figure 7b, again we see near 2500 Hz, a very high narrow peak, which is significantly reduced with 5% damping. In general notice the large effect damping has on the high narrow peaks. Now consider its undamped posneg PVSS.

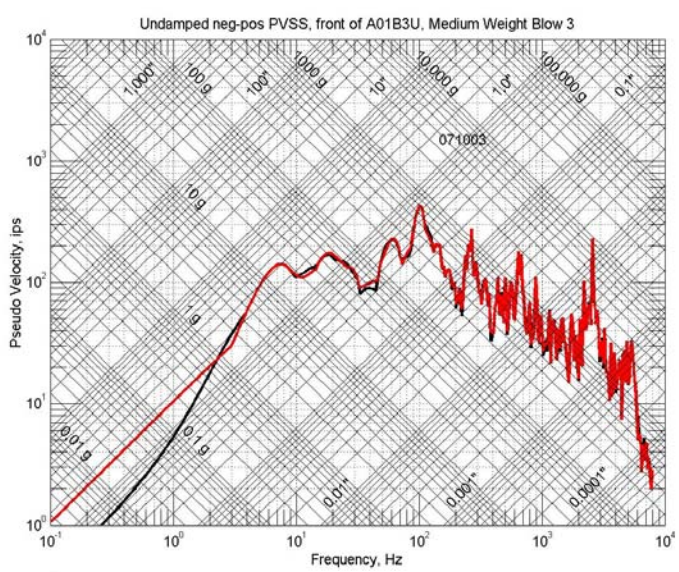

Figure 7c. The undamped posneg PVSS of blow 3 on the Navy Medium Weight Shock Machine.

Figure 7c is the undamped posneg PVSS. Here at low-frequency again we see the negative (red) PV spectrum at 1.6 inches and the positive PV down at 0.6 inches. Again, the negative is well above the positive, because the deflection is a 1.6 inches the positive direction in .6 in a negative direction in the third subplot of Figure 7a.

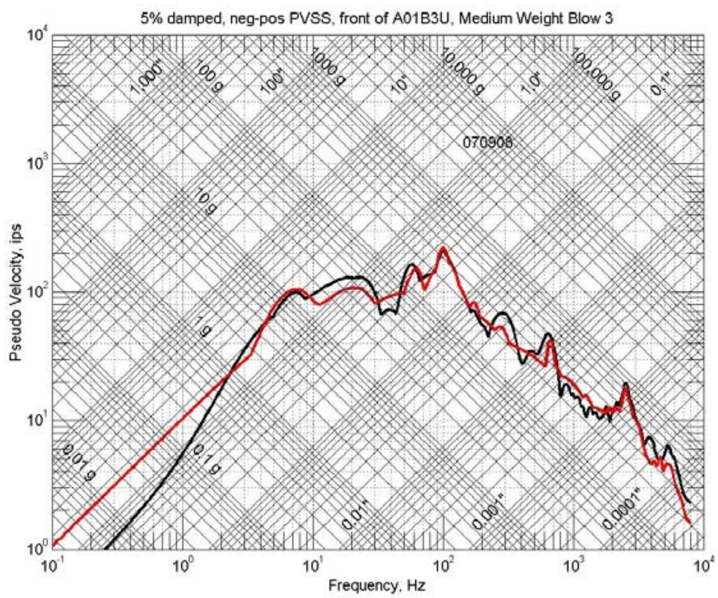

Figure 7d The 5% damped posneg PVSS of blow 3 on the Navy Medium Weight Shock Machine.

In Figure 7d, the 5% damped posneg PVSS, we see the high frequency asymptotes with the positive PVSS (black) greater than the negative, since the negative peak acceleration was greater in the first subplot of Figure 7a. The red and black PVSS's cross each other and stay together. And finally now, consider the 25% damped posneg PVSS

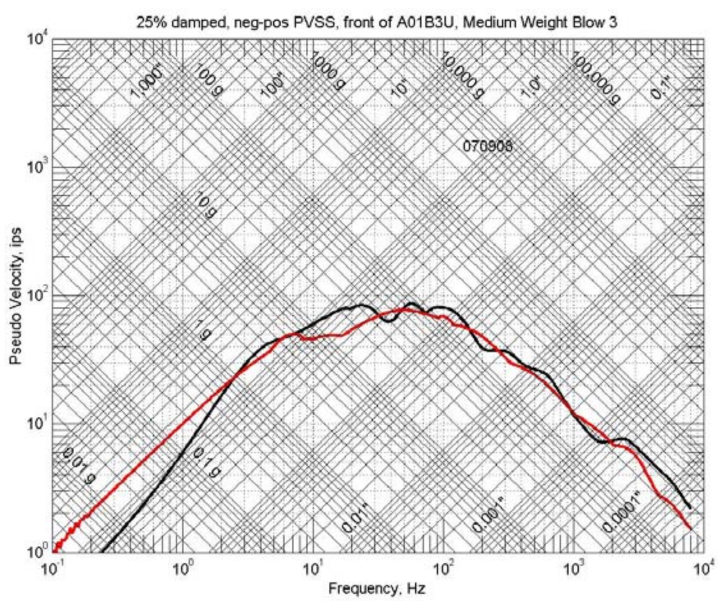

Figure 7e. The 25 % damped posneg PVSS of blow 3 on the Navy Medium Weight Shock Machine.

Figure 7e, the 25% damped posneg PV SS for the Medium Weight Shock Machine blow has much less polarity than the simple shock polarity of Figure 4e. There is a small region between 8 and 30 Hz with slight polarity. However the smoothness with which the positive and negative PV's intertwine indicates the medium weight shock machine does a good job of exciting the equipment in both the positive and negative directions.

Computing And Plotting

The options for making the calculations and plotting up the spectra are few. You have to calculate and plot the true pseudo velocity shock spectrum on 4CP. The pseudo velocity approximated from the maximum acceleration by dividing by the frequency, except for zero damping, does not give the true asymptotes, and only approximates the maximum displacement. My paper, [11], gives a FORTRAN program for calculating the PVSS. Since then I have translated that program to MATLAB® script and written a program to draw the 4CP. I have also modified that program to a version that overlays the positive and negative PVSS and gives the posneg spectrum. These programs were used here. The Navy's Underwater Explosives Research Division (UERD) has written and distributes a program called UERD Tools that does the calculations, They distribute the program to government agencies and contractors. The options I know are my MATLAB® programs (which are free from me), Navy's UERD Tools [15], and the Army Corps of Engineers, SRCW.[16] I don't think any of the analyzer companies offer this option. I realize that you cannot just instantly buy and learn MATLAB® and start using my programs. However, with a little coaching, it's possible. Government employees and government contractors can be issued copies of UERD Tools[15], and the Army's SCRW [16] program which do the calculations. SCRW outputs its data to a commercial $55.00 DPLOT program which plots and draws the 4CP.

Conclusions:

I'm not sure the paper makes a great contribution to shock technology. It has come up several times in my work and I wanted to take time to examine it, make some calculations and try to provide a feeling for what to expect. The resonant peaks of the PVSS are strikingly reduced by damping, bur that's not new. Polarity as a shock property is new. I've tried to examine it and present some ideas. Explosive and impact shocks do show some degree of polarity in localized regions. The random synthesized earthquake file shows almost no polarity. The simple shocks have the greatest polarity.

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