THE SPANN VIBROACOUSTIC METHOD
Revision A

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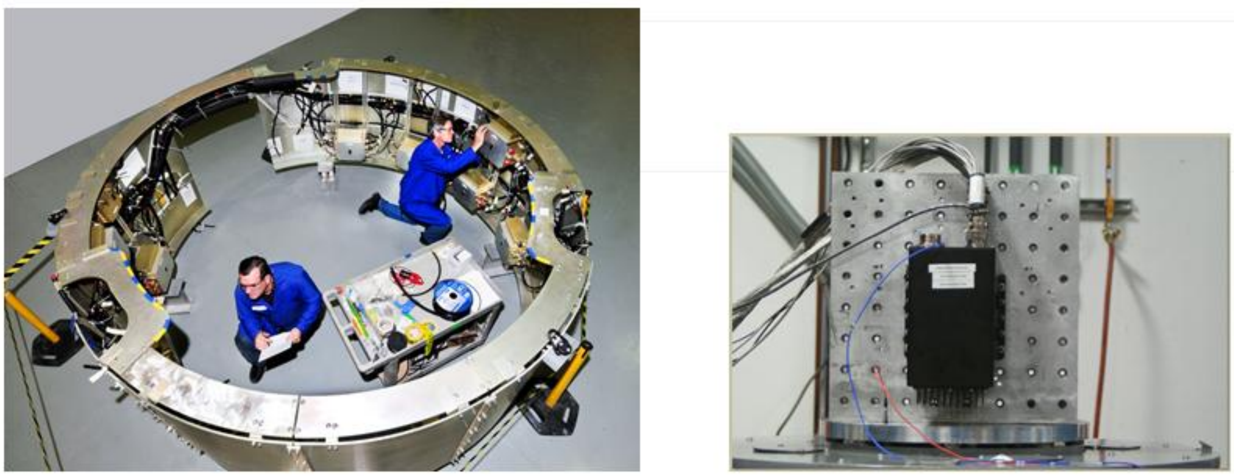

Figure 1. Avionics Installation and Testing

Introduction

Avionics components in aircraft and launch vehicles may be mounted to surfaces which are exposed to high intensity acoustic excitation. The external acoustic pressure field causes the panel and shell surfaces to vibrate. This vibration then becomes a base input to any component mounted on the internal side. Components must be designed and tested accordingly.

The component vibration input levels can be derived via analysis and testing for a given sound pressure level.

Acoustic testing of the structure can be performed in a reverberant chamber or using a direct field method. There is some difficultly in testing, however, because the simulated acoustic field in the lab facility may be different in terms of spatial correlation and incidence than that of the flight environment even if the sound pressure level can be otherwise replicated.

The vibroacoustic analysis techniques include finite element and boundary element methods, as well as statistical energy analysis. These are powerful tools, but they require numerous assumptions regarding external acoustic pressure field type, coupling loss factors, modal density, impedance, radiation efficiency, critical and coincident frequencies, distinguishing between acoustically fast and slow modes, etc.

As an alternative, simple empirical methods exist for deriving the structural vibration level corresponding to a given sound pressure level. Two examples are the Franken and Spann techniques. These methods may be most appropriate in the early design stage before hardware becomes available for lab testing and before more sophisticated analysis can be performed.

The Franken method is given in References 1 and 2. The Spann equation will be covered in this paper.

The Spann method provides a reasonable estimate of the acoustically excited component vibration environments when only the areas exposed to the acoustic environment and mass are known, according to Reference 3.

Spann Method

This section describes the steps required to derive component vibration test specifications for typical aerospace structures subjected to high-intensity acoustic environments. The application of this prediction method is based on two conditions:

1. Definition of the acoustic environment
2. An adequate general understanding of structure and components to obtain estimates of mass and areas exposed to acoustic excitations

Step 1

The method begins with an external one-third octave band sound pressure level (dB).

The sound pressure level $SPL(f_c)$ for band center frequency f_c is converted into a pressure spectral density via the following equation.

$$W_P(f_c) = p_{ref}^2 \left[\frac{10^{SPL(f_c)/10}}{\Delta f_c} \right] \quad Pa^2/Hz \tag{1}$$

where

p_{ref}	is the zero dB reference pressure (Pa)
$SPL(f_c)$	is the sound pressure level (dB)
Δf_c	is the frequency bandwidth (Hz)

The typical zero dB reference is

$$p_{ref} = 20\ \mu Pa \tag{2}$$

$$p_{ref}^2 = \left(20\ \mu Pa\right)^2 \tag{3}$$

The RMS pressure in $p_{rms}(f_c)$ each band is

$$p_{rms}^2(f_c) = p_{ref}^2\left[10^{\,SPL(f_c)/10}\right] \tag{4}$$

The pressure PSD is thus

$$W_P(f_c) = \frac{p_{rms}^2(f_c)}{\Delta f_c} \tag{5}$$

The one-third octave bandwidth is

$$\Delta f_c = 0.2316\, f_c \tag{6}$$

Preferred center frequencies are given in Appendix A.

Step 2

Estimate area A (m^2) of the component and spacecraft support structure supporting the component exposed to acoustic excitation.

Step 3

Calculate total mass M (kg) of the component and support structure included in the above estimate in step 2.

Step 4

Calculate, at each one-third-octave band frequency, the equivalent acceleration response $W_A(f_c)$ (G^2/Hz) using the following equation and the values of the pressure power spectral density $W_P(f_c)$ (Pa^2/Hz), area A (m^2) and mass M (kg) calculated in steps 1, 2 and 3.

The acceleration power spectral density in one-third octave format for the component base input is

$$W_A(f_c) = \beta^2 Q^2 \left(\frac{A}{gM} \right)^2 W_P(f_c) \qquad (G^2/Hz) \tag{7}$$

where

β	is recommended as 2.5 per experimental data
Q	is the amplification factor recommended as 4.5
g	is the gravitational constant 9.81 m/sec^2

Note that the suggested values for β and Q are taken from Reference 3.

The derivation of equation (7) is given in Appendix B.

Step 5

Iterate steps 2, 3 and 4, using different values of the input parameters to determine maximum $W_A(f_c)$ response curve.

Example

An example is given in Appendix C

References

1. Summary of Random Vibration Prediction Procedures, NASA CR-1302, 1969.

2. T. Irvine, Vibration Response of a Cylindrical Skin to Acoustic Pressure via the Franken Method, Revision H, Vibrationdata, 2008.

3. Spacecraft Mechanical Loads Analysis Handbook, ECSS-E-HB-32-26A, Noordwijk, The Netherlands, February 2013.

4. T. Irvine, Steady-State Vibration Response of a Plate Simply-Supported on all Side Subjected to a Uniform Pressure, Rev C, Vibrationdata, 2014.

APPENDIX A

Preferred One-Third Octave Bands

1/3 Octave Bands		
Lower Band Limit *(Hz)*	Center Frequency*(Hz)*	Upper Band Limit *(Hz)*
14.1	16	17.8
17.8	20	22.4
22.4	25	28.2
28.2	31.5	35.5
35.5	40	44.7
44.7	50	56.2
56.2	63	70.8
70.8	80	89.1
89.1	100	112
112	125	141
141	160	178
178	200	224
224	250	282
282	315	355
355	400	447
447	500	562
562	630	708
708	800	891
891	1000	1122
1122	1250	1413
1413	1600	1778
1778	2000	2239
2239	2500	2818
2818	3150	3548
3548	4000	4467
4467	5000	5623
5623	6300	7079
7079	8000	8913
8913	10000	11220
11220	12500	14130
14130	16000	17780
17780	20000	22390

The previous table is taken from:

http://www.engineeringtoolbox.com/octave-bands-frequency-limits-d_1602.html

Note the following relationships.

f_l	Lower band frequency
f_c	Band center frequency
f_u	Upper frequency

$$f_c = \sqrt{f_u f_l} \tag{A-1}$$

$$f_u = 2^{1/6} f_c \tag{A-2}$$

$$f_l = f_c / 2^{1/6} \tag{A-3}$$

For two consecutive bands,

$$f_{c,\,i+1} = f_{c,\,i} \tag{A-4}$$

The preferred frequencies in the previous table approximately satisfy this set of formulas.

APPENDIX B

<u>Derivation</u>

The following is taken from Reference 4. The baffled simply-supported plate in Figure B-1 is subjected to a uniform pressure.

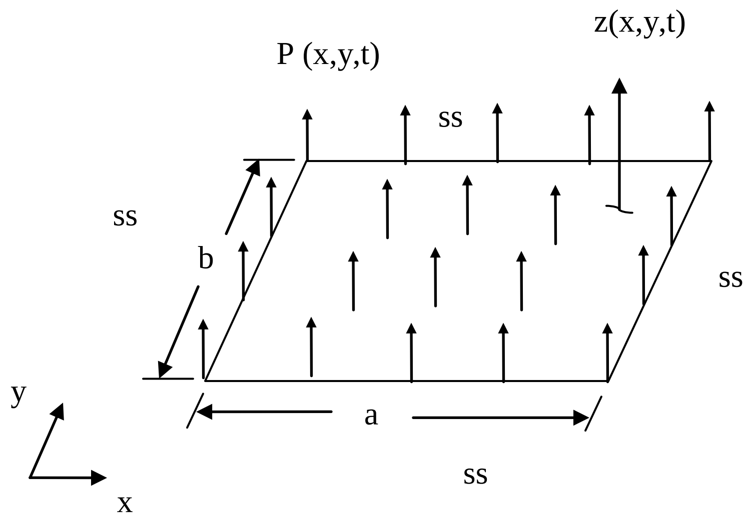

Figure B-1.

The following equations are taken from Reference 4.

The governing differential equation is

$$D\left(\frac{\partial^4 z}{\partial x^4} + 2\frac{\partial^4 z}{\partial x^2 \partial y^2} + \frac{\partial^4 z}{\partial y^4}\right) + \rho h \frac{\partial^2 z}{\partial t^2} = P(x, y, t) \qquad \text{(B-1)}$$

The plate stiffness factor D is given by

$$D = \frac{Eh^3}{12\left(1 - \mu^2\right)} \qquad \text{(B-2)}$$

where

E	is the modulus of elasticity
μ	Poisson's ratio
h	is the thickness
ρ	is the mass density (mass/volume)
P	is the applied pressure

Now assume that the pressure field is uniform such that

$$W(t) = P(x, y, t) \tag{B-2}$$

The differential equation becomes

$$D\left(\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}\right) + \rho h \frac{\partial^2 w}{\partial t^2} = W(t) \tag{B-3}$$

The mass-normalized mode shapes are

$$Z_{mn} = \frac{2}{\sqrt{\rho a\, b\, h}} \sin\left(\frac{m\pi\, x}{a}\right) \sin\left(\frac{n\pi\, y}{b}\right) \tag{B-4}$$

The natural frequencies are

$$\omega_{mn} = \sqrt{\frac{D}{\rho h}} \left(\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right) \tag{B-5}$$

The participation factors for constant mass density are

$$\Gamma_{mn} = \rho h \int_0^b \int_0^a Z_{mn}(x, y)\, dx dy \tag{B-6}$$

$$\Gamma_{mn} = \left(\frac{2\sqrt{\rho\, a\, b\, h}}{m\, n\, \pi^2} \right) [\cos(n\pi) - 1][\cos(m\pi) - 1] \tag{B-7}$$

The displacement response Z(x, y, ω) to the applied force is

$$Z(x, y, \omega) = \frac{1}{\rho h} W(\omega) \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left\{ \frac{\Gamma_{mn} Z_{mn}(x, y)}{\left(\omega_{mn}^2 - \omega^2 \right) + j 2 \xi_{mn} \omega \omega_{mn}} \right\} \tag{B-8}$$

$$Z(x, y, \omega) = \frac{1}{\rho h} W(\omega) \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left\{ \frac{\left(\frac{2\sqrt{\rho\, a\, b\, h}}{m\, n\, \pi^2} \right) [\cos(n\pi) - 1][\cos(m\pi) - 1] \frac{2}{\sqrt{\rho a\, b\, h}} \sin\left(\frac{m\pi\, x}{a} \right) \sin\left(\frac{n\pi\, y}{b} \right)}{\left(\omega_{mn}^2 - \omega^2 \right) + j 2 \xi_{mn} \omega \omega_{mn}} \right\} \tag{B-9}$$

$$Z(x, y, \omega) = \frac{4}{\rho h\, \pi^2} W(\omega) \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left\{ \left(\frac{1}{m\, n} \right) \frac{[\cos(n\pi) - 1][\cos(m\pi) - 1] \sin\left(\frac{m\pi\, x}{a} \right) \sin\left(\frac{n\pi\, y}{b} \right)}{\left(\omega_{mn}^2 - \omega^2 \right) + j 2 \xi_{mn} \omega \omega_{mn}} \right\} \tag{B-10}$$

The velocity response V(x, y, ω) to the applied force is

$$V(x,y,\omega)=\frac{4}{\rho h\,\pi^2}\,W(\omega)\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left\{\left(\frac{j\omega_{mn}}{m\,n}\right)\frac{[\cos(n\pi)-1][\cos(m\pi)-1]\sin\left(\frac{m\pi\,x}{a}\right)\sin\left(\frac{n\pi\,y}{b}\right)}{\left(\omega_{mn}^2-\omega^2\right)+j\,2\xi_{mn}\omega\omega_{mn}}\right\}$$

(B-11)

The acceleration response A(x, y, ω) to the applied force is

$$A(x,y,\omega)=\frac{4}{\rho h\,\pi^2}\,W(\omega)\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left\{\left(\frac{-\omega_{mn}{}^2}{m\,n}\right)\frac{[\cos(n\pi)-1][\cos(m\pi)-1]\sin\left(\frac{m\pi\,x}{a}\right)\sin\left(\frac{n\pi\,y}{b}\right)}{\left(\omega_{mn}^2-\omega^2\right)+j\,2\xi_{mn}\omega\omega_{mn}}\right\}$$

(B-12)

The acceleration response $A_c(\omega)$ due to the first mode only at the center is

$$A_c(\omega)=\frac{16\,\omega_{11}{}^2}{\rho h\,\pi^2}\left\{\frac{1}{\left(\omega_{11}^2-\omega^2\right)+j\,2\xi_{11}\omega\omega_{11}}\right\}W(\omega) \tag{B-13}$$

Assume resonant excitation and response for conservatism.

$$\omega=\omega_{11} \tag{B-14}$$

$$\left|\frac{\omega_{11}{}^2}{\left(\omega_{11}^2-\omega^2\right)+j\,2\xi_{11}\omega\omega_{11}}\right|=\frac{1}{2\xi_{11}}=Q \tag{B-15}$$

$$A_c(\omega)=\frac{16\;Q}{\rho h\,\pi^2}W(\omega) \tag{B-16}$$

$$\rho h = \frac{gM}{A} \tag{B-17}$$

$$A_c(\omega) = \frac{16\ Q}{\pi^2}\left(\frac{A}{gM}\right)W(\omega) \tag{B-18}$$

For band center frequency f_c.

$$A_c(f_c) = \frac{16\ Q}{\pi^2}\left(\frac{A}{gM}\right)W(f_c) \tag{B-19}$$

Let

$$\beta = \frac{16\ Q}{\pi^2} \tag{B-20}$$

$$A_c(f_c) = \beta Q\left(\frac{A}{gM}\right)W(f_c) \tag{B-21}$$

$$[A_c(f_c)]^2 = \beta^2 Q^2\left(\frac{A}{gM}\right)^2[W(f_c)]^2 \tag{B-22}$$

$$\frac{[A_c(f_c)]^2}{\Delta f_c} = \beta^2 Q^2\left(\frac{A}{gM}\right)^2\frac{[W(f_c)]^2}{\Delta f_c} \tag{B-23}$$

Let

W_A = acceleration power spectral density at the center of the plate

W_p = uniform pressure power spectral density

$$W_A = \frac{[A_c(f_c)]^2}{\Delta f_c} \tag{B-24}$$

$$W_p = \frac{[W(f_c)]^2}{\Delta f_c} \tag{B-25}$$

$$W_A(f_c) = \beta^2 Q^2 \left(\frac{A}{gM}\right)^2 W_p(f_c) \tag{B-26}$$

APPENDIX C

<u>Example</u>

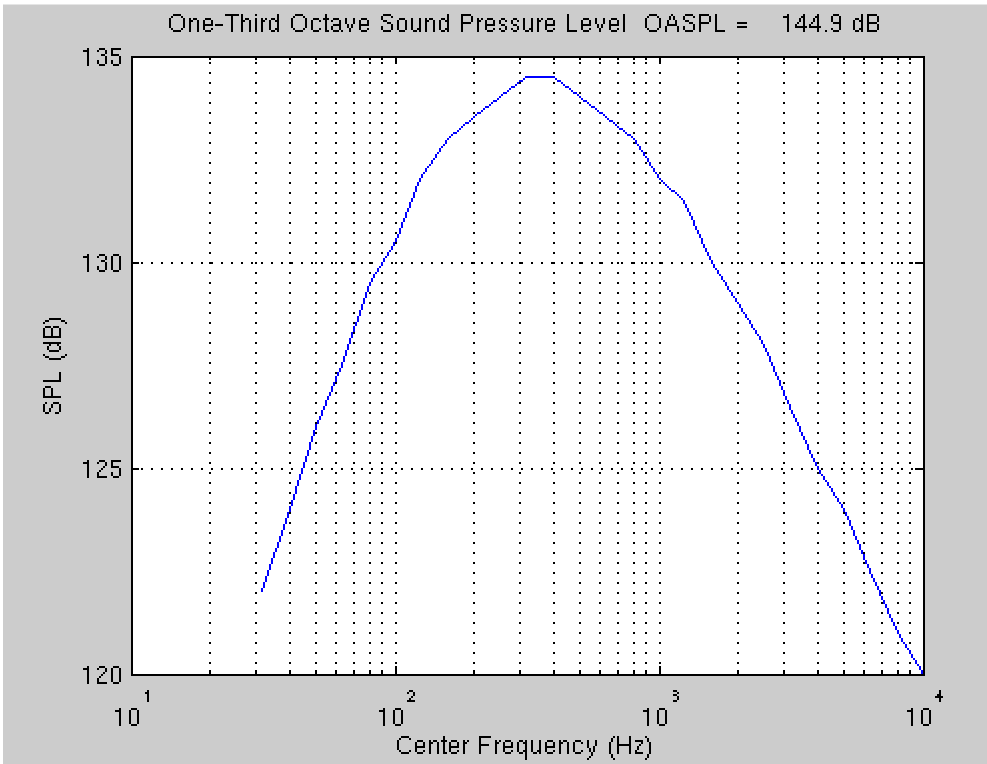

Figure C-1.

An avionic box is mounted to a surface with the following parameters.

β	2.5
Q	4.5
M	10 lbm
A	400 in^2

The external mounting surface is subjected to the sound pressure level in Figure C-1.

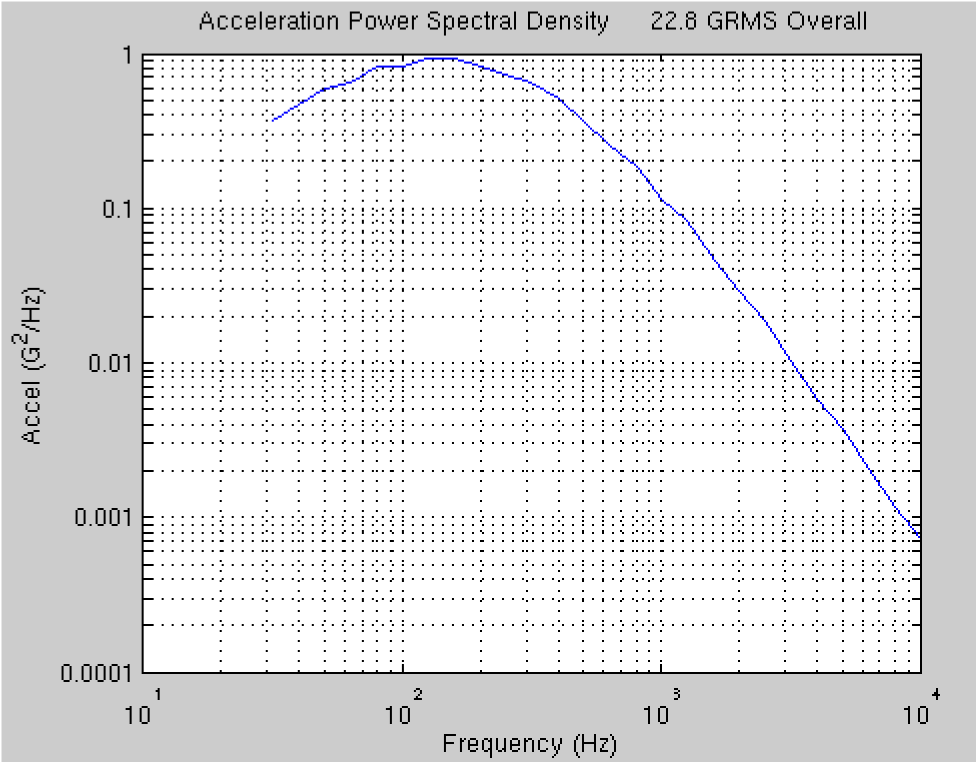

Figure C-2.

The resulting acceleration PSD is shown in Figure C-2. This would be the base input for the avionics component.