

PSD Band-Splitting Example

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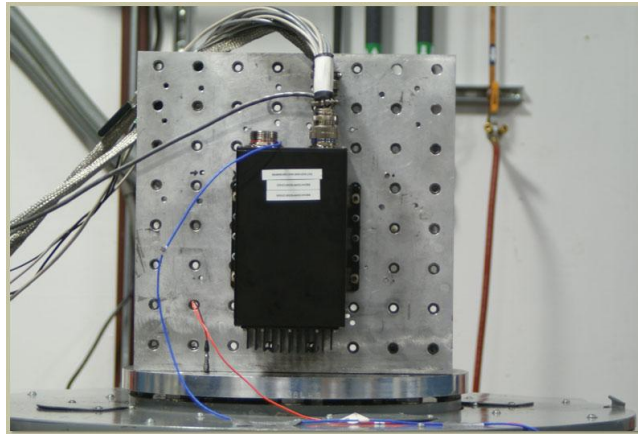


Figure 1. Avionics Component, Vibration Test

Introduction

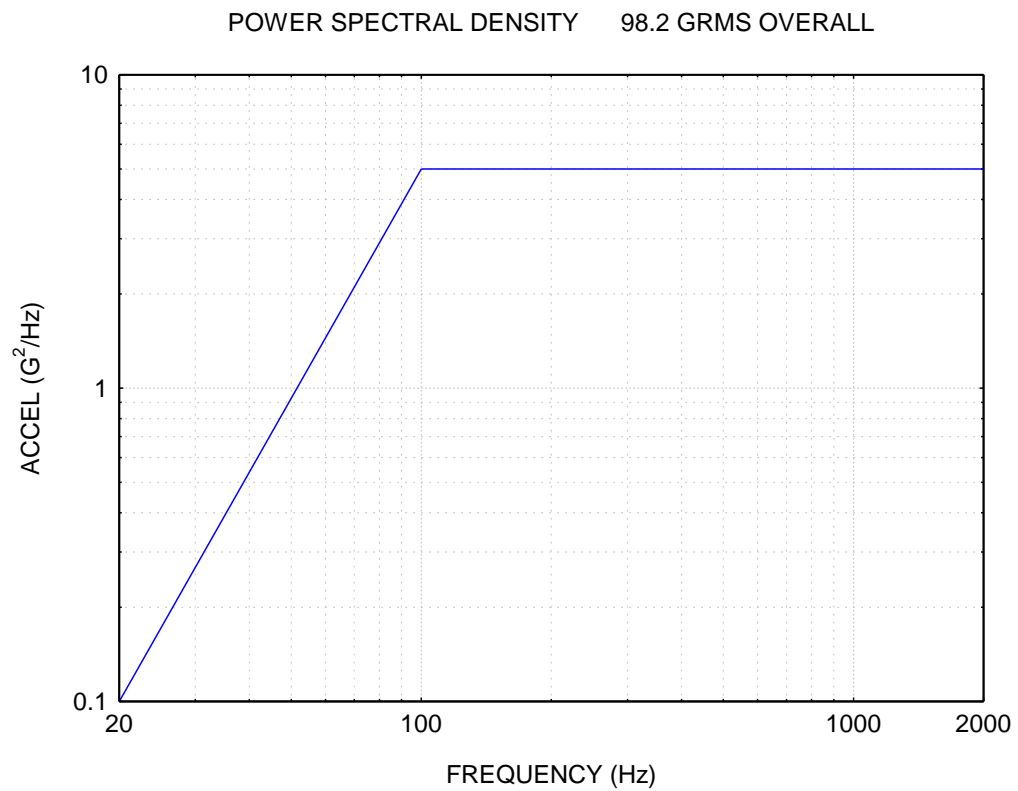
Avionics components are subjected to vibration tests on shaker tables to verify their workmanship and design. Random vibration is specified in terms of a power spectral density for this testing.

Some power spectral density specifications are too high in amplitude for a given shaker system. Band-splitting can be cautiously used in these cases per Reference 1.

The purpose of this paper is to show an example for a simple case.

Example

A sample PSD specification is shown in Figure 2. The overall level is 98.2 GRMS which is too high for a given test lab's shaker. A solution is to split the PSD into bands. Up to four bands are allowed per Reference 1. Two bands will be used for this example, each with the same overall level. Furthermore, the duration for each band will be the same as that for the original specification.



Freq (Hz)	Accel (G^2/Hz)
20	0.1
100	5
2000	5

Figure 2. PSD Specification, 360 sec/axis

Table 1. PSD, Band 1, 69.4 GRMS, 360 sec/axis	
Freq (Hz)	Accel (G ² /Hz)
20	0.1
100	5
1036	5

Table 2. PSD, Band 2, 69.4 GRMS, 360 sec/axis	
Freq (Hz)	Accel (G ² /Hz)
1036	5
2000	5

The two bands are shown in Tables 1 and 2, as calculated using Matlab script: bandsplit.m ver 1.1.

Note that the upper frequency of Band 1 is also the lower frequency of Band 2.

Response PSD Analysis

Model the avionics component as a single-degree-of-freedom system subject to base excitation as shown in Figure 3. Apply separately the original PSD specification to the system and the two band PSDs.

The response PSD is calculated using the method in Reference 3 for each of four natural frequency cases: 200, 400, 600, 800 Hz. The uniform amplification factor is $Q=10$. The response PSD plots are omitted for brevity.

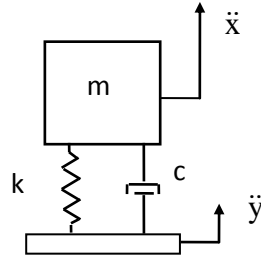


Figure 3. SDOF System, Base Excitation

The variables are

m	mass
c	damping coefficient
k	Stiffness
x	mass displacement
y	base displacement

The double-dot denotes acceleration.

Fatigue Analysis

The rainflow fatigue cycles are then calculated for each response PSD using the Dirlik method from Reference 3. The damage index is then calculated from the cycles using the method shown in Appendix A.

The fatigue exponent is 6.4. The damage results are shown for relative displacement and absolute acceleration in Tables 3 and 4, respectively.

Table 3. Relative Damage from Relative Displacement				
Natural Freq (Hz)	Damage PSD Band 1	Damage PSD Band 2	Total Damage from Each Band	Damage from Original PSD
200	0.000895	2.86E-15	8.95E-04	8.93E-04
400	2.31E-06	4.77E-15	2.31E-06	2.39E-06
600	6.87E-08	1.29E-14	6.87E-08	7.23E-08
800	5.35E-09	8.62E-14	5.35E-09	6.00E-09

Table 4. Relative Damage from Absolute Acceleration				
Natural Freq (Hz)	Damage PSD Band 1	Damage PSD Band 2	Total Damage from Each Band	Damage from Original PSD
200	1.21E+20	1.24E+09	1.21E+20	1.20E+20
400	2.22E+21	6.23E+12	2.22E+21	2.30E+21
600	1.19E+22	2.51E+15	1.19E+22	1.25E+22
800	3.66E+22	6.19E+17	3.66E+22	4.11E+22

The damage units for relative displacement and acceleration are ($\text{in}^{6.4}$) and ($\text{G}^{6.4}$) respectively.

Conclusions

The results show that the total damage from the band splitting approach is approximately equal to that from the original PSD case as long as the natural frequency is, say, one octave less than the split frequency, which was 1036 Hz in this example. Thus, the splitting is valid for natural frequencies up to about 518 Hz.

The splitting still gives reasonable damage results at higher natural frequencies, but the difference increases relative to the original PSD value.

Again, the duration of each of the two bands is the same as that from the original PSD specification.

Damage accumulates linearly with time for stationary vibration. Thus the duration for each of the two bands could be scaled upward to provide total damage equal to the original specification.

The scale factor for the 800 Hz natural frequency case would be 1.12, for a duration of 404 seconds for each band. In practice this would only be necessary for the band containing the natural frequency.

These calculates are a “snapshot” for an SDOF system with a natural frequency between 200 and 800 Hz and with an amplification factor of $Q=10$ and fatigue exponent $b=6.4$. These parameters may be

unknown for a given component and thus should be varied between estimated limits for a rigorous fatigue analysis.

Furthermore, the fatigue analysis can be extended for the case of a multi-degree-of-freedom system.

References

1. Test Methods and Control, Martin Marietta, M-67-45 (Rev 4), Denver, Colorado, January 1989.
2. T. Irvine, An Introduction to the Vibration Response Spectrum, Revision D, Vibrationdata, 2009.
3. T. Irvine, A Fatigue Damage Spectrum Method for Comparing Power Spectral Density Base Input Specifications, Revision A, Vibrationdata, 2014.

APPENDIX A

A relative damage index D can be calculated using

$$D = \sum_{i=1}^m A_i^b n_i \quad (A-1)$$

where

A_i is the *response* amplitude from the rainflow analysis

n_i is the corresponding number of cycles

b is the fatigue exponent

Note that the amplitude convention for this paper is (peak-valley)/2.