ATTENUATION OF BENDING SHOCK WAVES IN A BEAM

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November 21, 2008

Distance attenuation curves for pyrotechnic-induced shock pulses in launch vehicles are given in Reference 1. These curves were obtained from measured data. The purpose of this tutorial is to develop analytical attenuation curves for the case of bending wave propagation in a beam. The results are obtained via finite element method, using References 2 and 3.

The sample beam has the following characteristics:

Parameter	Value
Boundary Conditions	Free-Free
Material	Aluminum
Cross-Section	Rectangular
Thickness	0.25 inch
Width	2 inch
Length	200 inch
Damping for Each Mode	%1 for case 1 %5 for case 2
Number of Elements	150
Sample Rate	800K samples/sec

The beam has two rigid-body modes each with a frequency of zero due to the free-free boundary condition. Otherwise, the fundamental frequency is 1.26 Hz.

The excitation source is an initial lateral displacement of 0.1 inch at one of the free ends of the beam. All other points of the beam have zero initial displacement. Furthermore, the initial velocity is zero at all points. This set of initial conditions would nearly impossible to produce with real hardware, but this approach was determined to be very useful for shock propagation analysis, as determined by trial-and-error with a variety of excitation methods.

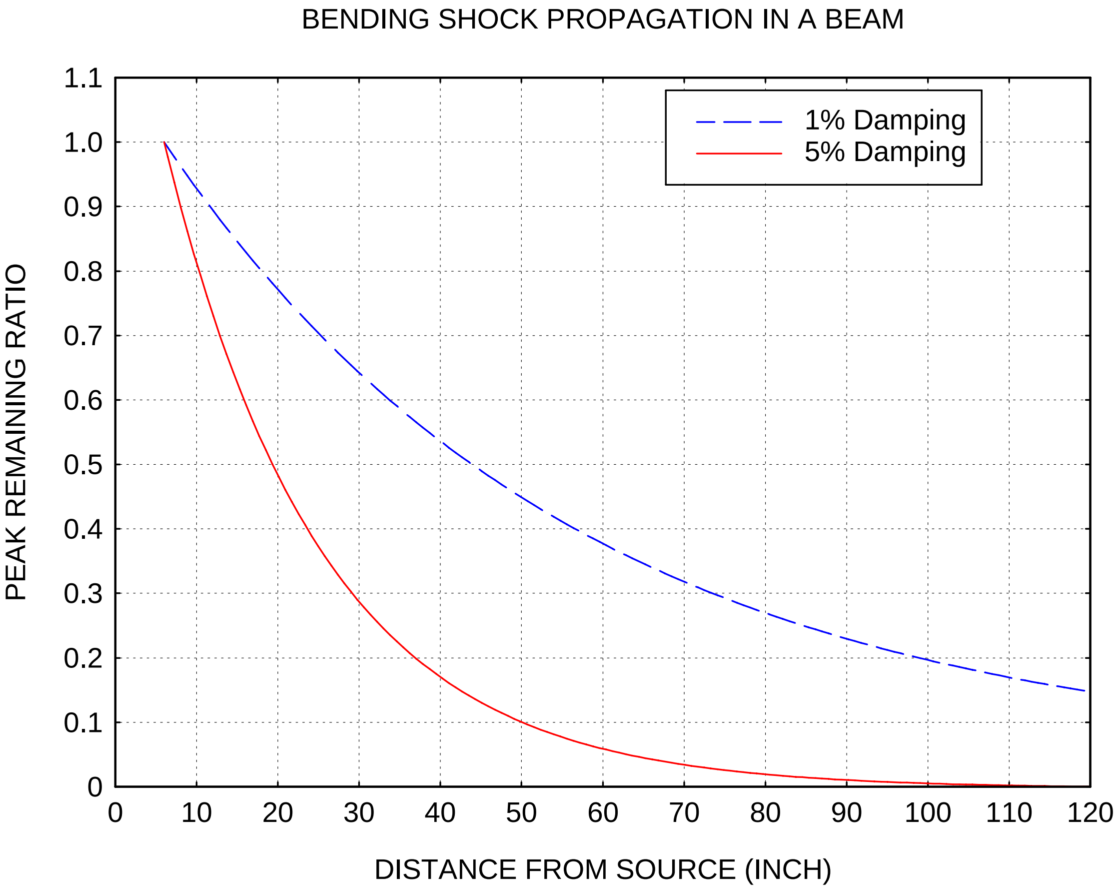

Figure 1.

A modal transient solution was then applied to the finite element method using Matlab script: beam_bending_initial_disp.m, ver 1.1. The modal transient method for systems subjected to initial conditions is given in Reference 3. The response at each node was subjected to a 10 KHz lowpass Butterworth filter, with refiltering for phase correction.

The curves are calculated from the respective peak acceleration values in the time domain and in the lateral axis. Each is a smoothed curve, but with only a slight amount of smoothing.

The curves are normalized so that the peak remaining ratio is unity at 6 inches. This follows the convention in Reference 1. The attenuation curves in Reference 1, however, are not given in terms of a damping ratio.

The results show that damping has a significant effect on the shock wave propagation.

Each of the curves assumes uniform damping. Reality is more complicated given that each mode in a real system would have its own damping ratio.

The curves are characterized by mathematical curve-fitting formulas as follows.

x is the distance from the source (inches)

y is the acceleration peak remaining ratio

The domain is constrained for each curve by

$6 \leq x \leq 120$ inches

The respective peak remaining ratio formulas are

$$y = 0.955 \exp(-0.0195 * (x - 6)) + 0.045 \quad \text{for 1\% damping} \tag{1}$$

$$y = 1.0 \exp(-0.0517 * (x - 6)) \quad \text{for 5\% damping} \tag{2}$$

Sample time histories are given in Appendix A for the 1% damping case.

This study could be expanded further by considering different dimensions, materials, boundary conditions, excitation methods, etc.

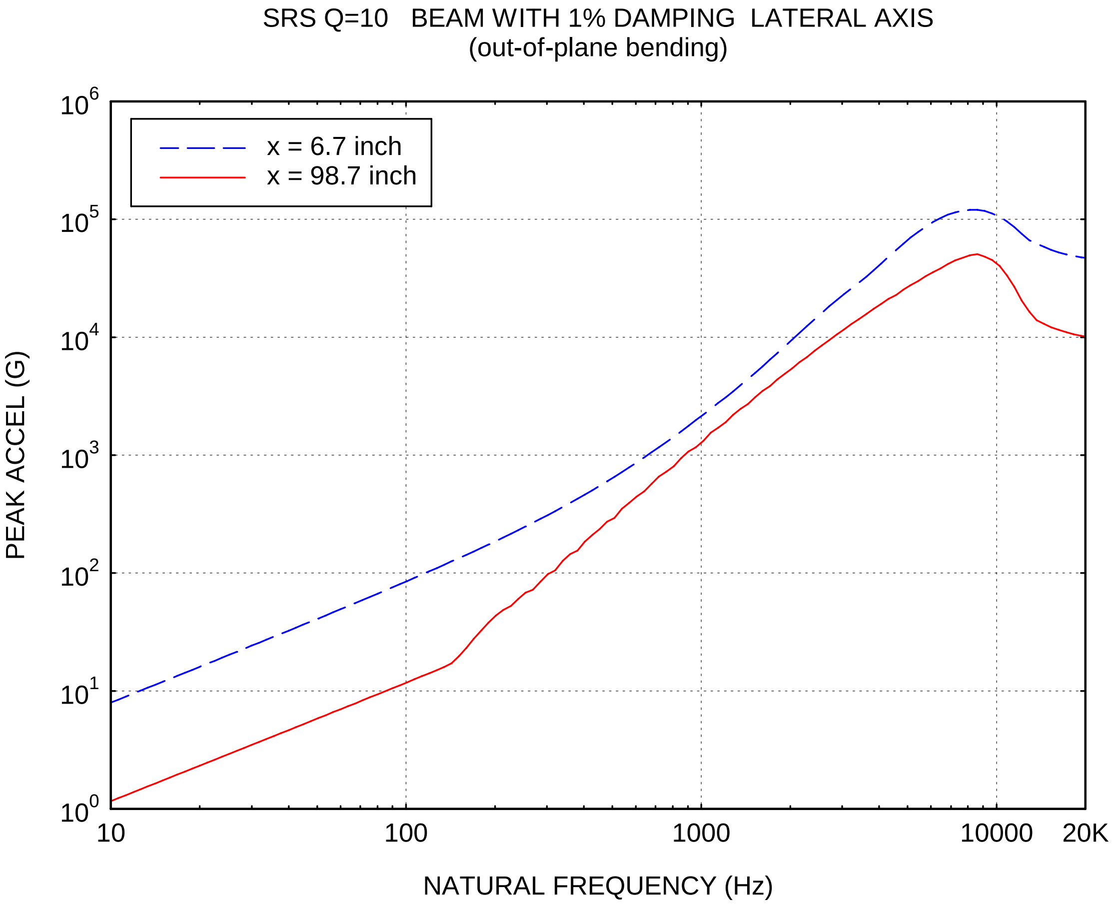

Figure 2.

The SRS comparison shows that the attenuation varies with respect to natural frequency below 10 KHz. The attenuation is 3.9 dB near 1000 Hz. The attenuation is 17 dB at the low frequency, and it is 13.4 dB at the high frequency end.

Note that the Q=10 values assumes that a component with 5% damping would be mounted to the beam. The beam itself has a damping value of 1%.

References

1. W. Kacena, M. McGrath, A. Rader; Aerospace Systems Pyrotechnic Shock Data, Vol. VI, NASA CR 116406, Goddard Space Flight Center, 1970.
2. T. Irvine, Transverse Vibration of a Beam via the Finite Element Method, Rev E, Vibrationdata, 2008.
3. T. Irvine, Free Vibration of a Two-Degree-of-Freedom System Subjected to Initial Velocity and Displacement, Vibrationdata, 2005.

APPENDIX A

Time History Plots, 1% Damping Case

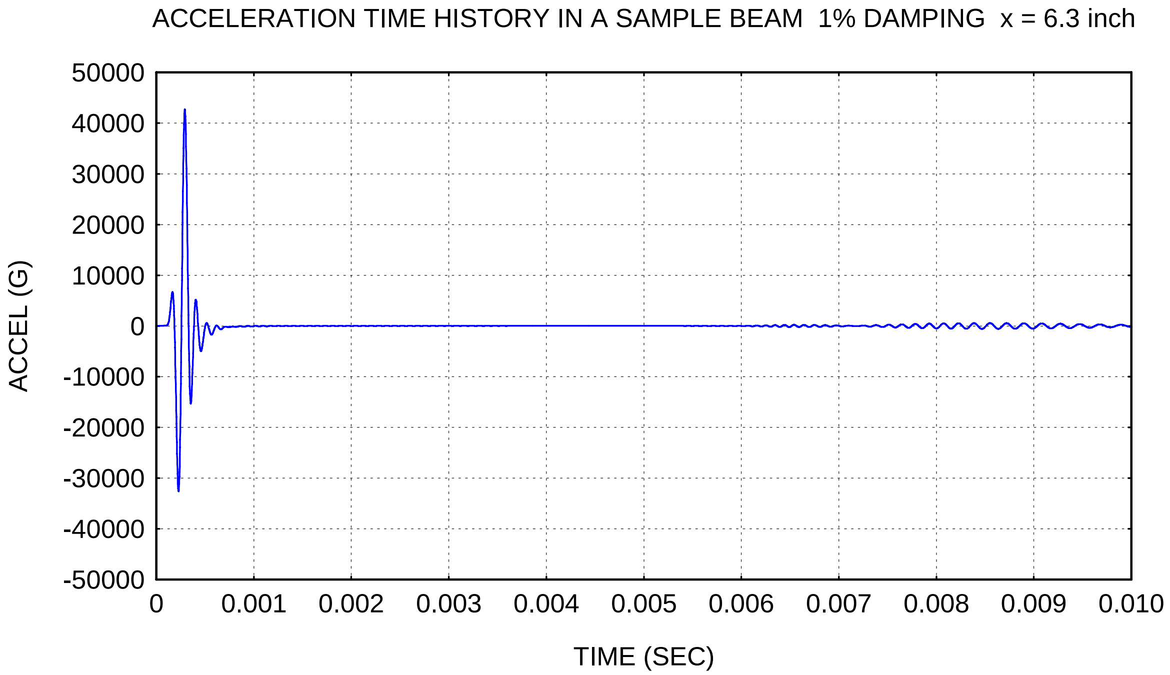

Figure A-1.

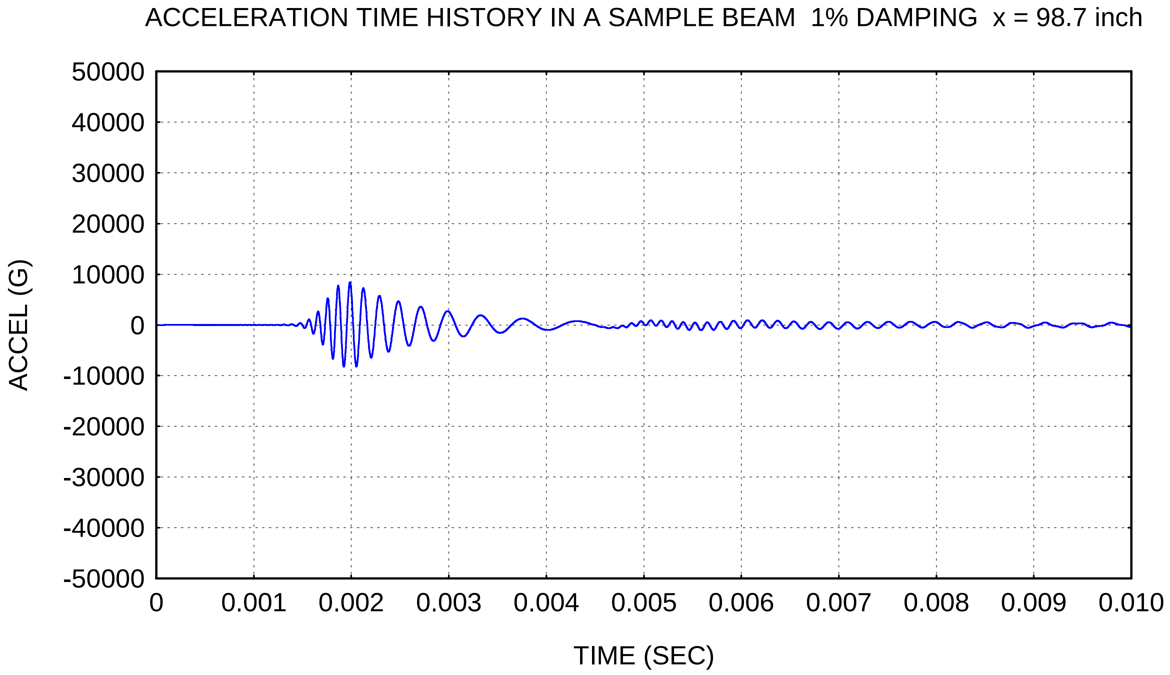

Figure A-2.