

Extending Steinberg's Fatigue Analysis of Electronics Equipment Methodology to a Full Relative Displacement vs. Cycles Curve

Revision C

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Vibration fatigue calculations are "ballpark" calculations given uncertainties in S-N curves, stress concentration factors, natural frequency, damping and other variables. Furthermore, damage increases exponentially with an increase in stress, which magnifies the effect of any error in the stress calculation or in the reference S-N curve. Perhaps the best that can be expected is to calculate the accumulated fatigue to the correct "order-of-magnitude."

Introduction

Electronic components in vehicles are subjected to shock and vibration environments. The components must be designed and tested accordingly. Dave S. Steinberg's *Vibration Analysis for Electronic Equipment* is a widely used reference in the aerospace and automotive industries.

Steinberg's text gives practical empirical formulas for determining the fatigue limits for electronics piece parts mounted on circuit boards. The concern is the bending stress experienced by solder joints and lead wires.

The fatigue limits are given in terms of the maximum allowable 3-sigma relative displacement of the circuit boards for the case of 20 million stress reversal cycles at the circuit board's natural frequency. The vibration is assumed to be steady-state with a Gaussian distribution.

Note that classical fatigue methods use stress as the response metric of interest. But Steinberg's approach works in an approximate, empirical sense because the bending stress is proportional to strain, which is in turn proportional to relative displacement.

The user then calculates the expected 3-sigma relative displacement for the component of interest and then compares this displacement to the Steinberg limit value.

There are several limitations to Steinberg's fatigue limit method:

1. An electronic component's service life may be well below or well above 20 million cycles.

2. A component may undergo nonstationary or non-Gaussian random vibration such that its expected 3-sigma relative displacement does not adequately characterize its response to its service environments.
3. The component's circuit board will likely behave as a multi-degree-of-freedom system, with higher modes contributing non-negligible bending stress, and in such a manner that the stress reversal cycle rate is greater than that of the fundamental frequency alone.

These obstacles can be overcome by developing a “relative displacement vs. cycles” curve, similar to an S-N curve.

Fortunately, Steinberg has provided the pieces for constructing this RD-N curve, with “some assembly required.” Note that RD is relative displacement.

The analysis can then be completed using the rainflow cycle counting for the relative displacement response and Miner's accumulated fatigue equation.

Steinberg's Fatigue Limit Equation

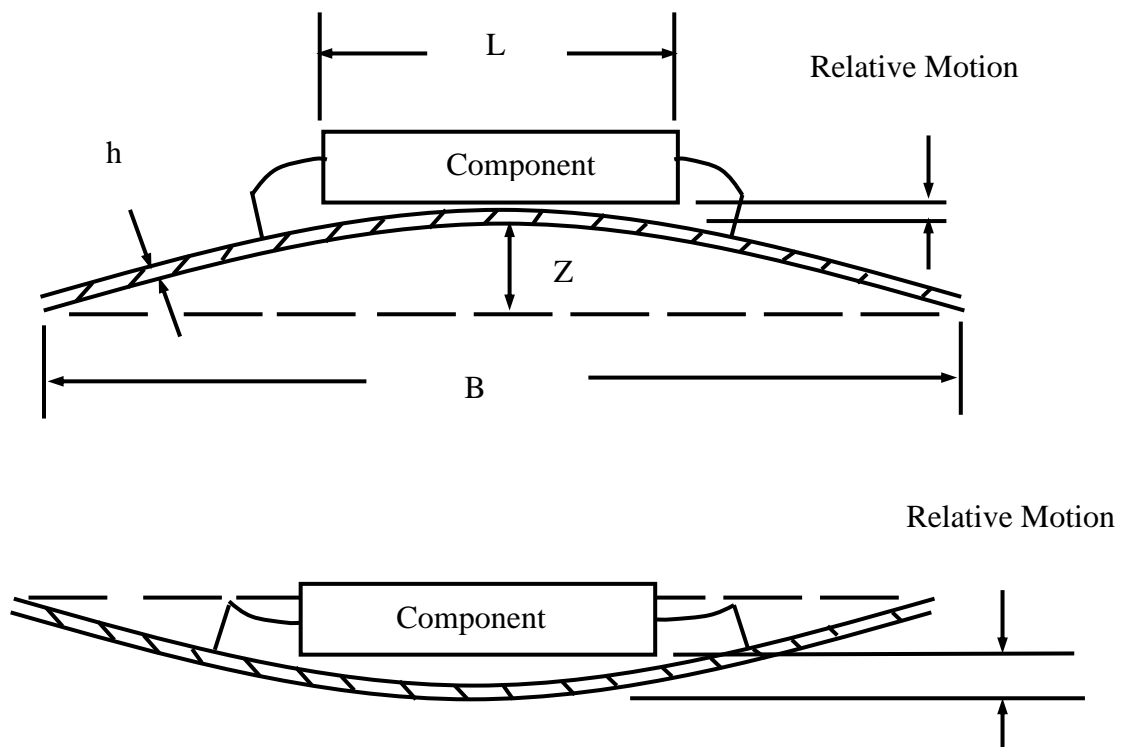


Figure 1. Component and Lead Wires undergoing Bending Motion

Let Z be the single-amplitude displacement at the center of the board that will give a fatigue life of about 20 million stress reversals in a random-vibration environment, based upon the 3σ circuit board relative displacement.

Steinberg's empirical formula for $Z_{3\sigma \text{ limit}}$ is

$$Z_{3\sigma \text{ limit}} = \frac{0.00022 B}{C h r \sqrt{L}} \quad \text{inches} \quad (1)$$

where

B = length of the circuit board edge parallel to the component, inches

L = length of the electronic component, inches

h = circuit board thickness, inches

r = relative position factor for the component mounted on the board
(Table 1)

C = Constant for different types of electronic components (Table 2)
 $0.75 \leq C \leq 2.25$

Equation (1) is taken from Reference 1.

Table 1. Relative Position Factors for Component on Circuit Board	
r	Component Location (Board supported on all sides)
1	When component is at center of PCB (half point X and Y).
0.707	When component is at half point X and quarter point Y.
0.50	When component is at quarter point X and quarter point Y.

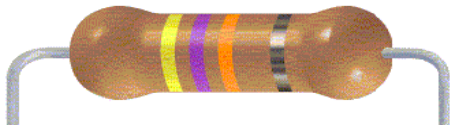
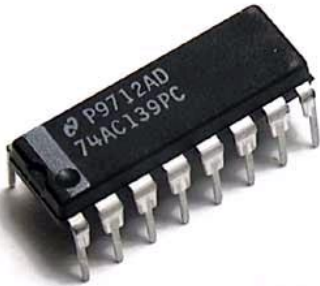
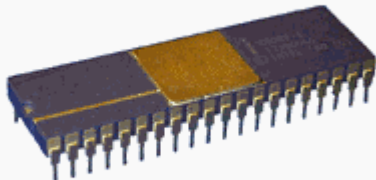
Table 2. Constant for Different Types of Electronic Components		
C	Component	Image
0.75	Axial leaded through hole or surface mounted components, resistors, capacitors, diodes	
1.0	Standard dual inline package (DIP)	
1.26	DIP with side-brazed lead wires	

Table 2. Constant for Different Types of Electronic Components (continued)

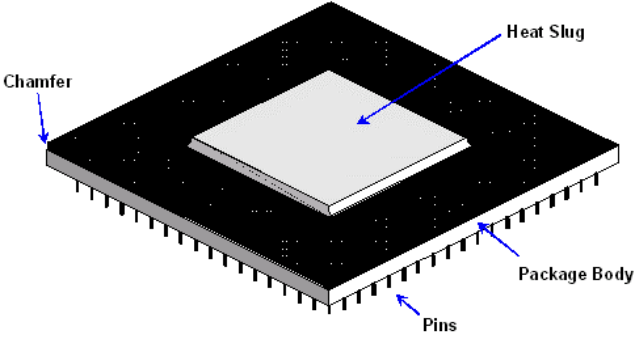


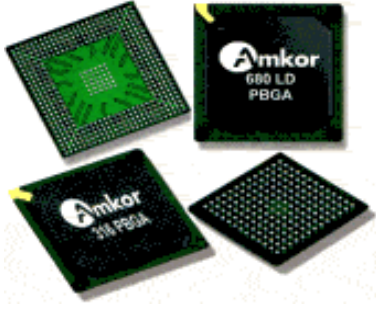
C	Component	Image
1.0	Through-hole Pin grid array (PGA) with many wires extending from the bottom surface of the PGA	
2.25	<p>Surface-mounted leadless ceramic chip carrier (LCCC).</p> <p>A hermetically sealed ceramic package. Instead of metal prongs, LCCCs have metallic semicircles (called castellations) on their edges that solder to the pads.</p>	
1.26	Surface-mounted leaded ceramic chip carriers with thermal compression bonded J wires or gull wing wires.	

Table 2. Constant for Different Types of Electronic Components (continued)		
C	Component	Image
1.75	<p>Surface-mounted ball grid array (BGA).</p> <p>BGA is a surface mount chip carrier that connects to a printed circuit board through a bottom side array of solder balls.</p>	
0.75	<p>Fine-pitch surface mounted axial leads around perimeter of component with four corners bonded to the circuit board to prevent bouncing</p>	—
1.26	<p>Any component with two parallel rows of wires extending from the bottom surface, hybrid, PGA, very large scale integrated (VLSI), application specific integrated circuit (ASIC), very high scale integrated circuit (VHSIC), and multichip module (MCM).</p>	—

Sample Base Input PSD

An RD-N curve will be constructed for a particular case. The resulting curve can then be recalibrated for other cases.

Consider a circuit board which behaves as a single-degree-of-freedom system, with a natural frequency of 500 Hz and $Q=10$. These values are chosen for convenience but are somewhat arbitrary.

The system is subjected to the base input shown in Table 3.

Table 3. Base Input PSD, 8.8 GRMS	
Frequency (Hz)	Accel (G^2/Hz)
20	0.0053
150	0.04
2000	0.04

The duration will be 1260 seconds, but the response results will be extended as a “test to failure.”

Note that accumulated fatigue damage for random vibration response increases approximately linearly with increase in duration. The relationship may have some nonlinearity due to the random occurrence of response peaks above 3-sigma.

Time History Synthesis

The next step is to generate a time history that satisfies the base input PSD in Table 3.

The synthesis is performed using the method in Reference 2. The total 1260-second duration is represented as three consecutive 420-second segments. Separate segments are calculated due to computer processing speed and memory limitations.

The complete time history set is shown in Figure 2.

Each segment essentially has a Gaussian distribution, but the histogram plots are also omitted for brevity. The actual histograms have a slight deviation from the Gaussian ideal due to the effect of the fade in, but this is incidental.

Verification that the complete time history satisfies the PSD specification is shown in Figure 3.

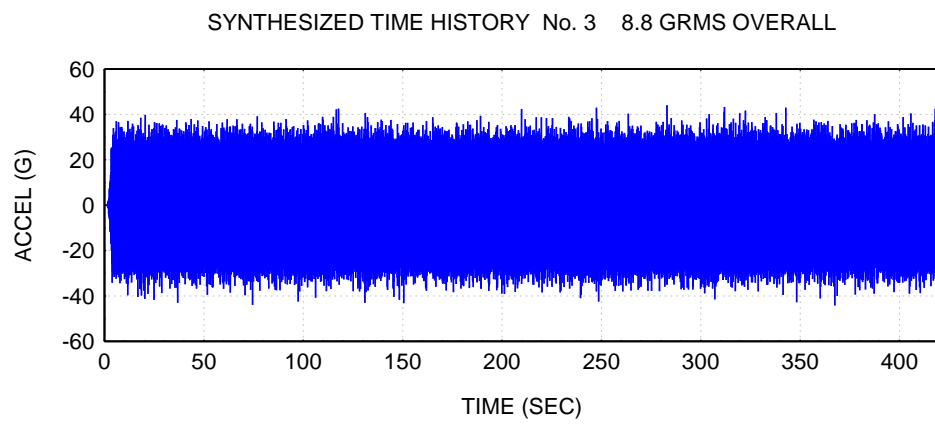
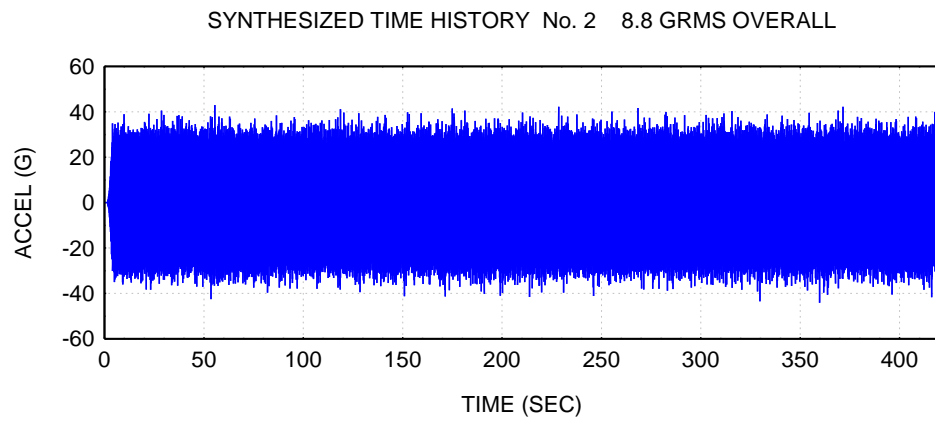
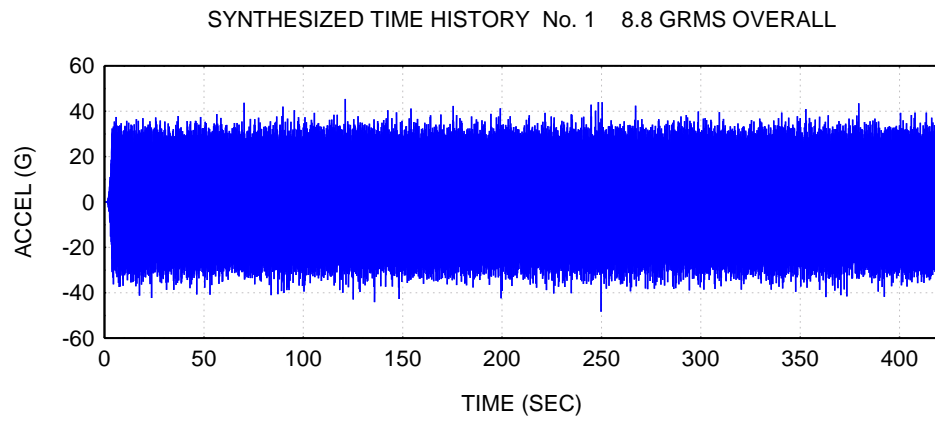


Figure 2.

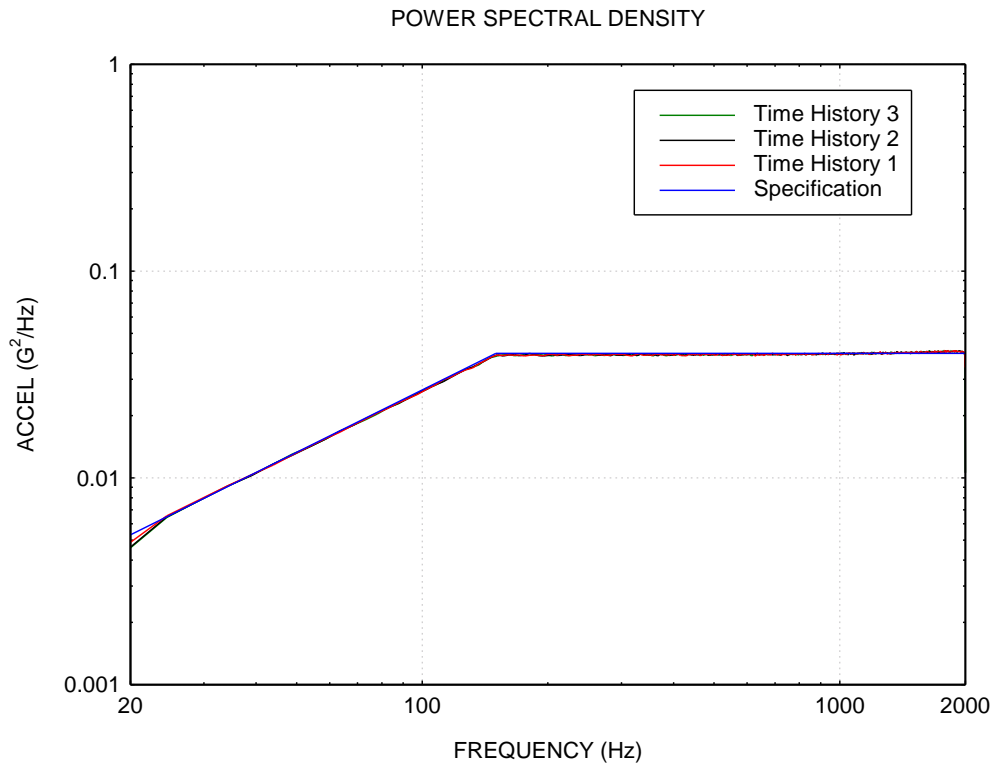


Figure 3.

Each curve has an overall level of 8.8 GRMS. The curves are nearly identical.

SDOF Response

The response analysis is performed using the ramp invariant digital recursive filtering relationship in Reference 3.

The response results are shown in Figure 4. Descriptive statistics are shown in Table 4.

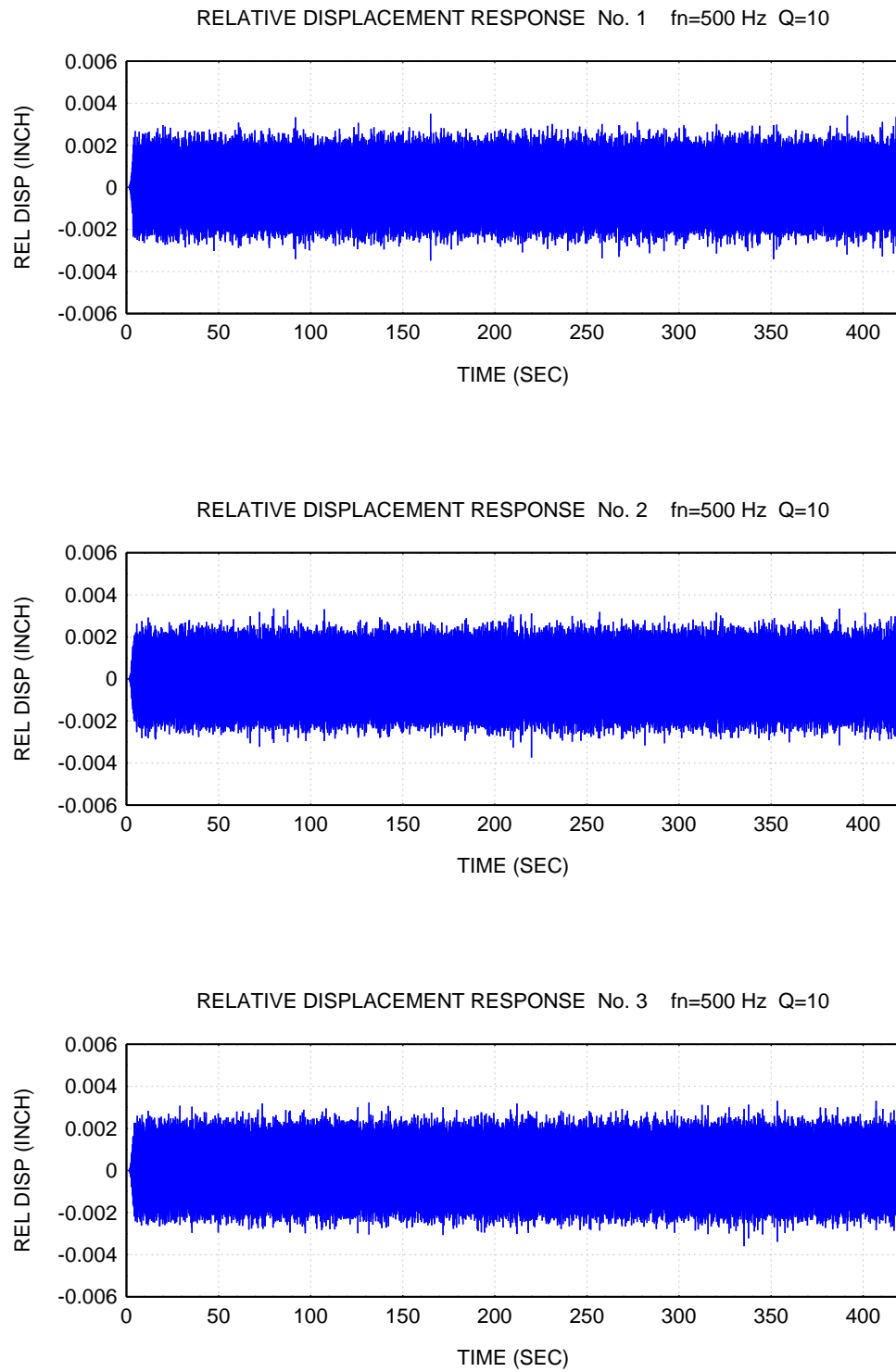


Figure 4.

Table 4. Relative Displacement Response Statistics				
No.	1-sigma (inch)	3-sigma (inch)	Kurtosis	Crest Factor
1	0.00068	0.00204	3.02	5.11
2	0.00068	0.00204	3.03	5.44
3	0.00068	0.00204	3.01	5.25

Note that the crest factor is the ratio of the peak-to-standard deviation, or peak-to-rms assuming zero mean.

Kurtosis is a parameter that describes the shape of a random variable's histogram or its equivalent probability density function (PDF).

Rainflow Cycle Counting

Table 5. Relative Displacement Results from Rainflow Cycle Counting, Bin Format, Unit: inch, 1260-sec Duration									
Range		Cycle Counts	Average Amplitude	Max Amp	Min Amp	Average Mean	Max Mean	Min Valley	Max Peak
Upper Limit	Lower Limit								
0.0060	0.0067	12.0	0.0031	0.0034	0.000	-0.0036	0.0034	0.0060	0.0067
0.0054	0.0060	147.5	0.0028	0.0030	0.000	-0.0032	0.0033	0.0054	0.0060
0.0047	0.0054	1269.5	0.0025	0.0027	0.000	-0.0029	0.0029	0.0047	0.0054
0.0040	0.0047	6287.0	0.0021	0.0023	0.000	-0.0027	0.0026	0.0040	0.0047
0.0034	0.0040	22101.0	0.0018	0.0020	0.000	-0.0023	0.0024	0.0034	0.0040
0.0027	0.0034	59422.5	0.0015	0.0017	0.000	-0.0021	0.0021	0.0027	0.0034
0.0020	0.0027	120350.0	0.0012	0.0013	0.000	-0.0037	0.0035	0.0020	0.0027
0.0013	0.0020	176108.5	0.0008	0.0010	0.000	-0.0034	0.0034	0.0013	0.0020
0.0010	0.0013	91787.5	0.0006	0.0007	0.000	-0.0034	0.0033	0.0010	0.0013
0.0007	0.0010	75836.0	0.0004	0.0005	0.000	-0.0028	0.0030	0.0007	0.0010
0.0003	0.0007	51072.0	0.0003	0.0003	0.000	-0.0028	0.0026	0.0003	0.0007
0.0002	0.0003	22220.0	0.0001	0.0002	0.000	-0.0029	0.0027	0.0002	0.0003
0.0000	0.0002	72289.5	0.0000	0.0001	0.000	-0.0026	0.0028	0.0000	0.0002

Next, a rainflow cycle count was performed on the relative displacement time histories using the method in Reference 4. The combined, binned results are shown in Table 5. Note that: Amplitude = (peak-valley)/2.

The total number of cycles was 698903. This corresponds to a rate of 555 cycles/sec over the 1260 second duration. This rate is about 10% higher than the 500 Hz natural frequency.

The binned results are shown mainly for reference, given that this is a common presentation format in the aerospace industry. The binned results could be inserted into a Miner's cumulative fatigue calculation.

The method in this analysis, however, will use the raw rainflow results consisting of cycle-by-cycle amplitude levels, including half-cycles. This brute-force method is more precise than using binned data.

Also, note that rainflow counting can be performed more quickly using a C++ program rather than a Matlab script, in the author's experience. This is especially true if the time history has millions of data points.

Miner's Accumulated Fatigue

Let n be the number of stress cycles accumulated during the vibration testing at a given level stress level represented by index i .

Let N be the number of cycles to produce a fatigue failure at the stress level limit for the corresponding index.

Miner's cumulative damage index CDI is given by

$$CDI = \sum_{i=1}^m \frac{n_i}{N_i} \quad (2)$$

where m is the total number of cycles or bins depending on the analysis type.

In theory, the part should fail when

$$CDI = 1.0 \quad (3)$$

Miner's index can be modified so that it is referenced to relative displacement rather than stress.

Note that the zero-to-peak form of relative displacement will be used throughout the remainder of this paper.

Derivation of the RD-N Curve

The exponent b is taken as 6.4 for PCB-component lead wires. This number is derived in Reference 1, section 7.3, page 177. It represents generic metal. It is used in Reference 1 for both sine and random vibration.

The goal is to determine an RD-N curve of the form

$$\log_{10} (N) = -6.4 \log_{10} (RD) + a \quad (4)$$

where

N is the number of cycles

RD relative displacement (inch)

a unknown variable

The unknown variable will be determined by calibration.

$$a = \log_{10} (N) + 6.4 \log_{10} (RD) \quad (5)$$

Let N = 20 million reversal cycles.

$$a = 7.30 + 6.4 \log_{10} (RD) \quad (6)$$

Now assume that the process in the preceding example was such that its 3-sigma relative displacement reached the limit in equation (1) for 20 million cycles. This would require that the duration 1260 second duration be multiplied by 28.6.

$$28.6 = (20 \text{ million cycles-to-failure}) / (698903 \text{ rainflow cycles})$$

Now apply the RD-N equation (4) along with Miner's equation (2) to the rainflow cycle-by-cycle amplitude levels with trial-and-error values for the unknown variable a. Multiply the CDI by the 28.6 scale factor to reach 20 million cycles. Iterate until a value of a is found such that CDI=1.0.

The numerical experiment result is

$$a = -11.20 \quad \text{for a 3-sigma limit of 0.00204 inch}$$

The 3-sigma value matches that in Table 4.

Substitute into equation (4).

$$\log_{10} (N) = -6.4 \log_{10} (RD) - 11.20 \quad \text{for a 3-sigma limit of 0.00204 inch} \quad (7)$$

Equation (7) will be used for the “high cycle fatigue” portion of the RD-N curve. A separate curve will be used for “low cycle fatigue.”

The low cycle portion will be based on another Steinberg equation that the maximum allowable relative displacement for shock is six times the 3-sigma limit value at 20 million cycles for random vibration.

But the next step is to derive an equation for a as a function of 3-sigma limit without resorting to numerical experimentation.

Let $RD_x = RD$ at $N=20$ million.

$$RD_x = 10^{\left[\frac{a - 7.30}{6.4} \right]} \quad (8)$$

$$RD_x = 0.0013 \text{ inch for } a = -11.20 \quad (9)$$

$$a = 7.3 + 6.4 \log_{10} (0.0013) = -11.20 \quad \text{for a 3-sigma limit of 0.00204 inch} \quad (10)$$

The RD_x value is not the same as the $Z_{3\sigma}$ limit .

But RD_x should be directly proportional to $Z_{3\sigma}$ limit .

So postulate that

$$a = 7.3 + 6.4 \log_{10} (0.0026) = -9.24 \quad \text{for a 3-sigma limit of 0.00408 inch} \quad (11)$$

This was verified by experiment where the preceding time histories were doubled and CDI =1.0 was achieved after the rainflow counting.

Thus, the following relation is obtained.

$$a = 7.3 + 6.4 \log_{10} \left[(0.0013) \frac{Z_{3\sigma \text{ limit}}}{0.00204 \text{ inch}} \right] \quad (12)$$

$$a = 7.3 + 6.4 \log_{10} \left[(0.637) Z_{3\sigma \text{ limit}} \right] \quad (13)$$

$$\log_{10} (N) = -6.4 \log_{10} (RD) + 7.3 + 6.4 \log_{10} \left[(0.637) Z_{3\sigma \text{ limit}} \right] \quad (14)$$

$$\log_{10} (N) = 7.3 - 6.4 \left\{ -\log_{10} \left[(0.637) Z_{3\sigma \text{ limit}} \right] + \log_{10} (RD) \right\} \quad (15)$$

$$\log_{10} (N) = 7.3 - 6.4 \log_{10} \left[\frac{RD}{0.637 Z_{3\sigma \text{ limit}}} \right] \quad (16)$$

$$\log_{10} (N) = 6.05 - 6.4 \log_{10} \left[\frac{RD}{Z_{3\sigma \text{ limit}}} \right] \quad (17)$$

The final RD-N equation for high-cycle fatigue is

$$\log_{10} \left[\frac{RD}{Z_{3\sigma \text{ limit}}} \right] = \frac{6.05 - \log_{10} (N)}{6.4} \quad (18)$$

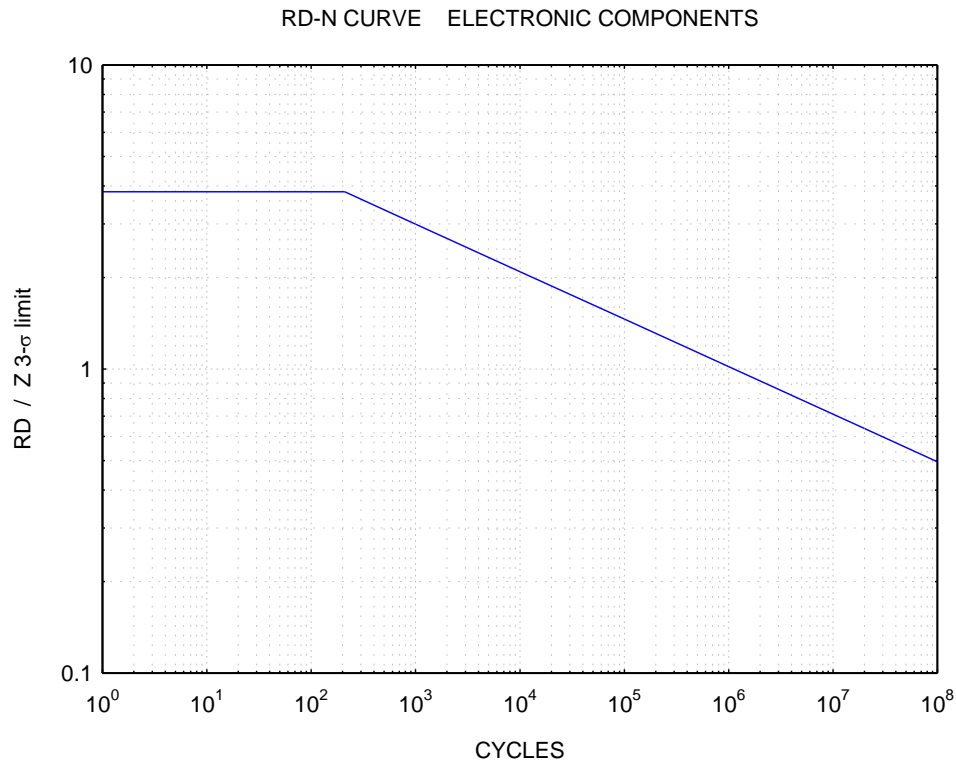


Figure 5.

Equation (18) is plotted in Figure 5 along with the low-cycle fatigue limit.

RD is the zero-to-peak relative displacement.

Again, the low cycle portion is based on another Steinberg equation that the maximum allowable relative displacement ratio for shock is six times the 3-sigma limit value at 20 million cycles for random vibration. The physical explanation is that a “strain hardening effect” occurs at the relative displacement ratio corresponding to the plateau region below 200 cycles.

In reality, transition between the plateau and the downward ramp would be a smooth curve. This is a topic for future research.

Sine and Random Damage Equivalence

Continue with the previous example for the case where there were 20 million cycles and CDI=1.0. This is also the special case where the 3-sigma relative displacement equals Steinberg's $Z_{3\sigma}$ limit.

Note that

$$\left[\frac{RD}{Z_{3\sigma} \text{ limit}} \right] = 0.64 \quad \text{at 20 million cycles} \quad (19)$$

Again, RD and $Z_{3\sigma}$ limit are different parameters which have a common length dimension.

RD is the zero-to-peak relative displacement, which varies cycle-by-cycle.

$Z_{3\sigma}$ limit is the 3-sigma relative displacement limit from Steinberg. Note that 1.11% of the absolute response peaks are greater than 3-sigma for the case where the response peaks have a Rayleigh distribution.

Any individual point along the RD-N curve must be considered in terms of an “equivalent sine” oscillation.

Again the relative displacement ratio at 20 million cycles is 0.64.

$$(0.64)(3\text{-sigma}) = 1.9\text{-sigma} \quad (20)$$

This suggests that “damage equivalence” between sine and random vibration occurs when the response sine amplitude (zero-to-peak) is approximately equal to the random vibration 2-sigma amplitude. This relationship was previously derived in Reference 5. The base input sine amplitude would then be the response sine amplitude divided by the Q factor.

Please also refer to the example in Appendix A which gives experimental verification of the equivalent sine for a case where CDI < 1 and the total cycle number is < 20 million.

Examples

Examples for particular piece parts are given in Appendices A through C.

Conclusion

A methodology for developing RD-N curves for electronic components was presented in this paper. The method is an extrapolation of the empirical data and equations given in Steinberg's text.

The method is particularly useful for the case where a component must undergo nonstationary vibration, or perhaps a series of successive piecewise stationary base input PSDs.

The resulting RD-N curve should be applicable to nearly any type of vibration, including random, sine, sine sweep, sine-or-random, shock, etc.

It is also useful for the case where a circuit board behaves as a multi-degree-of-freedom system.

This paper also showed in a very roundabout way that "damage equivalence" between sine and random vibration occurs when the sine amplitude (zero-to-peak) is approximately equal to the random vibration 2-sigma amplitude.

This remains a "work-in-progress." Further investigation and research is needed.

References

1. Dave S. Steinberg, Vibration Analysis for Electronic Equipment, Second Edition, Wiley-Interscience, New York, 1988.
2. T. Irvine, A Method for Power Spectral Density Synthesis, Rev B, Vibrationdata, 2000.
3. David O. Smallwood, An Improved Recursive Formula for Calculating Shock Response Spectra, Shock and Vibration Bulletin, No. 51, May 1981.
4. ASTM E 1049-85 (2005) Rainflow Counting Method, 1987.
5. T. Irvine, Sine and Random Vibration Equivalent Damage, Revision A, Vibrationdata, 2004.
6. T. Irvine, Plate Bending Frequencies via the Finite Element Method with Rectangular Elements, Revision A, 2011.
7. T. Irvine, Modal Transient Analysis of a Multi-degree-of-freedom System with Enforced Motion, Revision C, Vibrationdata, 2011.

APPENDIX A

Single-degree-of-freedom, Stationary Vibration Example

Consider a Ball Grid Array (BGA) mounted at the center of an electronics board. The parameters are given in Table A-1.

Assume that the circuit board behaves as a single-degree-of-freedom system with a natural frequency of 400 Hz and $Q=10$.

Table A-1. Example Parameters		
B	=	6.0 inch
		Circuit board length
L	=	1.0 inch
		Part length
h	=	0.093 inch
		Circuit board thickness
r	=	1.0
		Center of circuit board from Table 1
C	=	1.75
		BGA component from Table 2
fn	=	400 Hz
		Circuit board natural frequency
Q	=	10
		Amplification factor

Calculate the relative displacement limit.

$$Z_{3\sigma \text{ limit}} = \frac{0.00022 B}{\text{Chr} \sqrt{L}} \quad \text{inches} \quad (\text{A-1})$$

$$Z_{3\sigma \text{ limit}} = \frac{0.00022 [6.0]}{[1.75][0.093][1.0] \sqrt{1.0}} \quad \text{inches} \quad (\text{A-2})$$

$$Z_{3\sigma \text{ limit}} = 0.0081 \quad \text{inches} \quad (\text{A-3})$$

Now consider that the circuit board is subjected to the base input in Table 1 plus 12 dB. The input level is thus 35.2 GRMS overall.

The synthesized time histories from Figure 4 are raised by 12 dB are used as a base input.

The relative displacement response is calculated via Reference 3. The levels are shown in Table A-2.

Table A-2. Relative Displacement Response Statistics				
No.	1-sigma (inch)	3-sigma (inch)	Kurtosis	Crest Factor
1	0.0038	0.0114	3.04	4.81
2	0.0038	0.0114	3.04	5.25
3	0.0038	0.0114	3.02	5.05

The rainflow cycle counting is performed using Reference 4.

The CDI is calculated using the curve in Figure 5.

The result is: CDI = 0.242 for 1260 seconds and 12 dB margin

The number of rainflow cycles was 574,680. The rainflow cycle rate was 456 Hz.

Equivalent Sine

Consider a 400 Hz sine function over 1260 seconds. The total number of cycles is 504,000.

Derive the sine amplitude so the resulting CDI is the same as that for the random vibration case.

Now divide the actual cycle number by the CDI.

$$(504,000 \text{ cycles}) / (0.242) = 2,082,644 \text{ cycles} \quad (\text{A-4})$$

Thus 2,082,644 cycles would have been allowed for the yet-to-be-determined relative displacement amplitude.

Note that from the RD-N curve

$$\left[\frac{\text{RD}}{\text{Z}_{3\sigma} \text{ limit}} \right] = 0.908 \quad \text{at } 2,082,644 \text{ cycles} \quad (\text{A-5})$$

$$\text{RD} = 0.908 \text{ Z}_{3\sigma} \text{ limit} \quad (\text{A-6})$$

Again, for this example,

$$\text{Z}_{3\sigma} \text{ limit} = 0.0081 \text{ inches} \quad (\text{A-7})$$

Now calculate the equivalent sine amplitude with respect to this limit.

$$\text{RD} = 0.908 (0.0081 \text{ inches}) \quad (\text{A-8})$$

$$\text{RD} = 0.0074 \text{ inches} \quad (\text{A-9})$$

Now divide the equivalent sine amplitude by the random vibration relative displacement response standard deviation from Table A-2.

$$(0.0074 \text{ inches} / 0.0038 \text{ inches}) = 1.9 \quad (\text{A-10})$$

Thus, an “equivalent sine” would be approximately 2-sigma in terms of the respective sine and random relative displacement response amplitudes.

Time-to Failure

Again, damage is approximately linearly proportional to duration. .

Thus, by extrapolation: CDI = 1.0 for 5204 seconds and 12 dB margin

The time-to-failure is thus about 87 minutes.

APPENDIX B

Single-degree-of-freedom, Nonstationary Vibration

Reconsider the Ball Grid Array mounted at the center of an electronics board from Appendix A.

Change the board natural frequency to 450 Hz. All other material and geometry parameters remain the same.

Again,

$$Z_{3\sigma \text{ limit}} = 0.0081 \quad \text{inches} \quad (\text{B-1})$$

The part is subjected to the flight accelerometer data shown as in Figure B-1. The corresponding Waterfall FFT is given in Figure B-2.

This is a sine sweep driven by a “resonant burn” effect in the solid rocket motor combustion chamber. It is nonstationary. The duration is about 20 seconds.

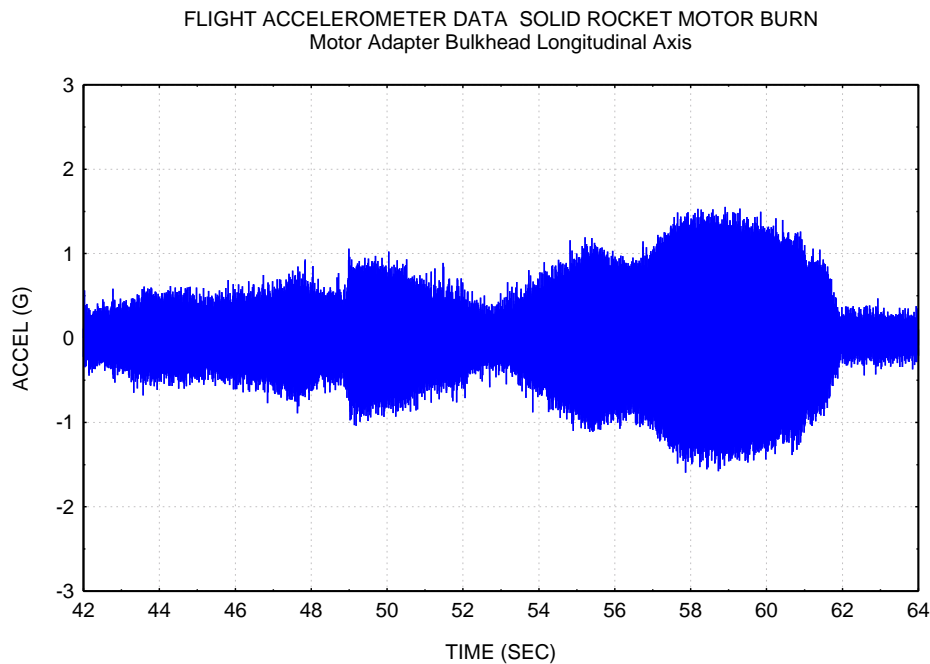


Figure B-1.

Waterfall FFT Solid Rocket Motor Oscillation

Flight Accelerometer Data Motor Adapter Bulkhead Longitudinal Axis

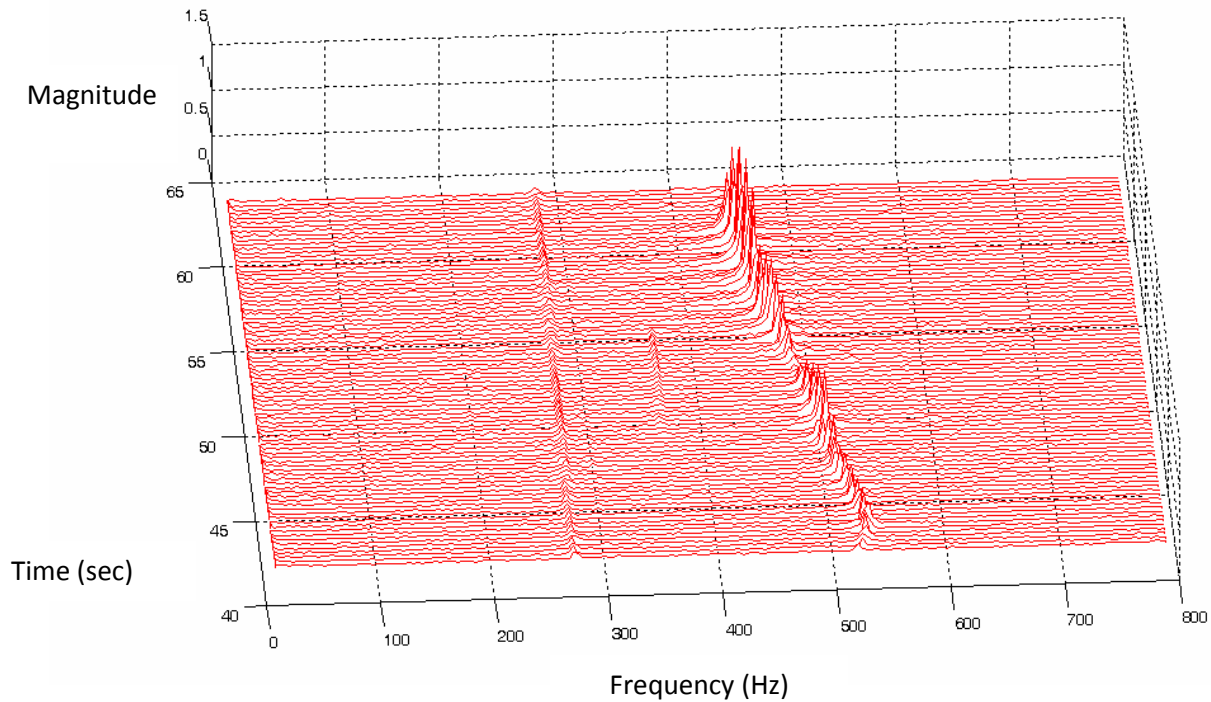


Figure B-2.

The overall response relative displacement is 0.00018 inches for the zero margin case.

The rainflow cycle counting is performed using Reference 4.

The CDI is calculated using the curve in Figure 6.

The result is:

$$\text{CDI} = 3.64\text{e-}11 \quad \text{for 20 seconds and zero margin}$$

$$\text{CDI} = 3.03\text{e-}09 \quad \text{for 20 seconds and 6 dB margin}$$

The CDI is negligibly low for each margin.

But assume the component had been subject to a random vibration test, but not a sine sweep test, before the flight.

A CDI could be calculated for the random vibration test in order to determine whether it covered the flight sine sweep in terms of fatigue.

APPENDIX C

Multi-degree-of-freedom, Stationary Vibration Example

Consider a Ball Grid Array (BGA) mounted on electronics board. The parameters are given in Table C-1.

Table C-1. Example Parameters		
B	= 3.0 x 6.0 inch	Circuit board length & width
L	= 0.5 inch	Part length
h	= 0.093 inch	Circuit board thickness
r	= 1.0	Center of Board from Table 1
C	= 1.75	BGA component from Table 2
Q	= 10	Amplification factor for all modes

The board boundary conditions are shown in Figure C-1.

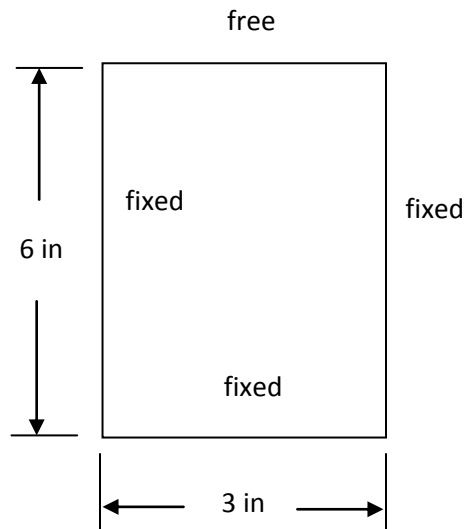


Figure C-1.

Calculate the relative displacement limit.

Set B = 3.0 inch, the width, for conservatism. Note that the BGA in this example is square.

$$Z_{3\sigma \text{ limit}} = \frac{0.00022 B}{\text{Chr} \sqrt{L}} \quad \text{inches} \quad (\text{C-1})$$

$$Z_{3\sigma \text{ limit}} = \frac{0.00022 [3.0]}{[1.75][0.093][1.0]\sqrt{0.5}} \quad \text{inches} \quad (\text{C-2})$$

$$Z_{3\sigma \text{ limit}} = 0.0057 \quad \text{inches} \quad (\text{C-3})$$

Also assume that the total board mass is 0.31 lbm with a uniform distribution.

The finite element method is used to calculate the circuit board response via Reference 5.

The first four natural frequencies of the board are

n	fn(Hz)
1	818
2	937
3	1210
4	1667

The relative displacement transmissibility for the center of the board is shown in Figure C-2.

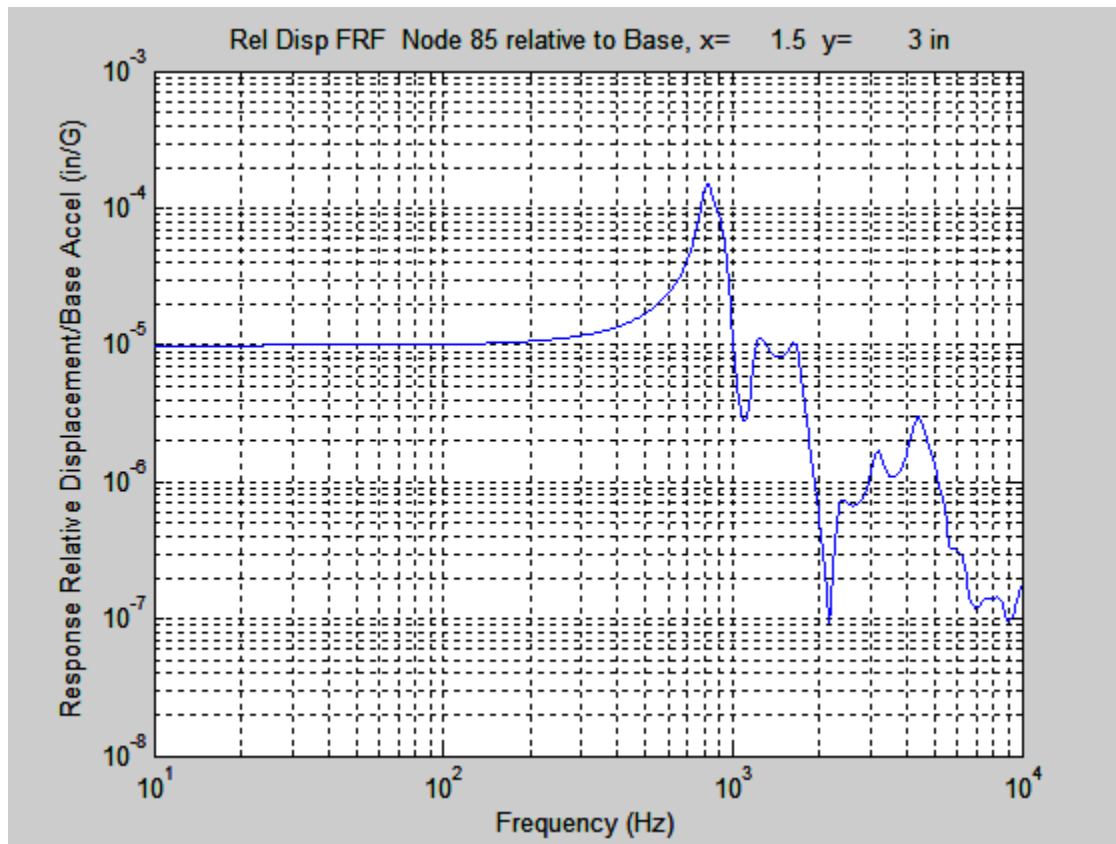


Figure C-2.

Note that the largest peak represents the combined response of the 818 and 937 Hz modes

Now consider that the circuit board is subjected to the base input in Table 1 plus 12 dB. Determine the time-to-failure.

The first (top) synthesized time history from Figure 4 is used as a base input. Only the first 150 seconds of this record is used due to the author's computer memory limitations. The full duration will be used in the next revision of this paper. Pushing large time histories through multi-degree-of-freedom finite element models for modal transient analysis is a challenge.

The response for the 150-second duration is calculated via Reference 6.

The rainflow cycle counting is performed using Reference 4.

The CDI is calculated using the curve in Figure 5.

The result is: $CDI = 0.000338$ for 150 seconds and 12 dB margin

Again, damage is approximately linearly proportional to duration.

Failure would occur at about 123 hours for the 12 dB margin case.

Note the relative displacement tends to be higher for lower natural frequency components. In this case, the first two natural frequencies were 818 and 937 Hz, which were sufficiently high for a relatively long life at the elevated level.