Joint Receptance for Rigid & Elastically Coupled Subsystems
Revision A

By Tom Irvine
Email: tom@vibrationdata.com

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Introduction

Consider a single-degree-of-freedom system S constructed from two rigidly-connected subsystems A and B.

The combined system S is

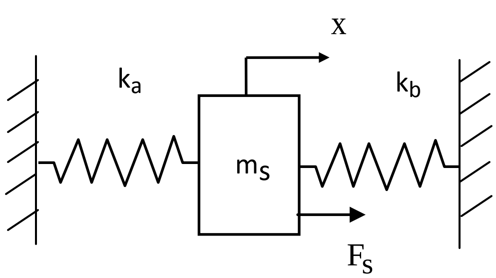

Figure 1.

Subsystem A is

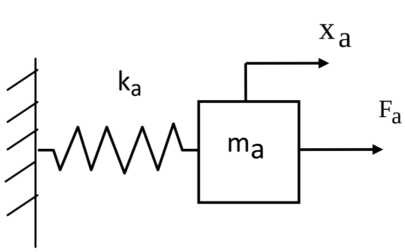

Figure 2.

Subsystem B is

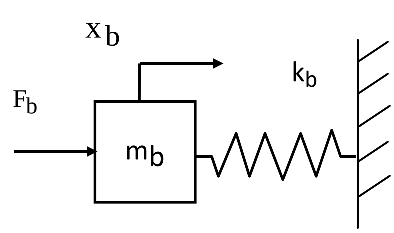

Figure 3.

The combined system and the subsystems are assumed to have modal damping which implies a dashpot.

Variables

m_i	mass		F_i	applied force
k_i	stiffness		H_i	receptance function (displacement/force)
ξ_i	modal damping		x_i	Displacement
ω_i	natural frequency		ω	excitation frequency

The subscript indicates the system or subsytem

Receptance Coupling

The receptance function for subsystem A from Reference 1 is

$$H_a(\omega) = \frac{1}{m_a}\left[\frac{1}{\omega_a^2 - \omega^2 + j\left(2\xi_a \omega\omega_a\right)}\right] \tag{1}$$

The receptance function for subsystem B is

$$H_b(\omega) = \frac{1}{m_b}\left[\frac{1}{\omega_b^2 - \omega^2 + j\left(2\xi_b \omega\omega_b\right)}\right] \tag{2}$$

The joint receptance $H_j(\omega)$ is

$$H_j(\omega) = \frac{H_a H_b}{H_a + H_b} \tag{3}$$

$$H_j(\omega) = \frac{\dfrac{1}{m_a m_b}\left[\dfrac{1}{\omega_a^2 - \omega^2 + j\left(2\xi_a \omega\omega_a\right)}\right]\left[\dfrac{1}{\omega_b^2 - \omega^2 + j\left(2\xi_b \omega\omega_b\right)}\right]}{\dfrac{1}{m_a}\left[\dfrac{1}{\omega_a^2 - \omega^2 + j\left(2\xi_a \omega\omega_a\right)}\right] + \dfrac{1}{m_b}\left[\dfrac{1}{\omega_b^2 - \omega^2 + j\left(2\xi_b \omega\omega_b\right)}\right]} \tag{4}$$

$$H_j(\omega)=\frac{\left[\dfrac{1}{\omega_a^2-\omega^2+j\left(2\xi_a\omega\omega_a\right)}\right]\left[\dfrac{1}{\omega_b^2-\omega^2+j\left(2\xi_b\omega\omega_b\right)}\right]}{m_b\left[\dfrac{1}{\omega_a^2-\omega^2+j\left(2\xi_a\omega\omega_a\right)}\right]+m_a\left[\dfrac{1}{\omega_b^2-\omega^2+j\left(2\xi_b\omega\omega_b\right)}\right]} \tag{5}$$

$$H_j(\omega)=\frac{1}{m_b\left[\omega_b^2-\omega^2+j\left(2\xi_b\omega\omega_b\right)\right]+m_a\left[\omega_a^2-\omega^2+j\left(2\xi_a\omega\omega_a\right)\right]} \tag{6}$$

$$H_j(\omega)=\frac{1}{m_a\omega_a^2+m_b\omega_b^2-\left(m_a+m_b\right)\omega^2+j2\omega\left(m_a\xi_a\omega_a+m_b\xi_b\omega_b\right)} \tag{7}$$

The system natural frequency ω_s is

$$\omega_s=\sqrt{\frac{k_a+k_b}{m_a+m_b}} \tag{8}$$

$$f_s=\omega_s/(2\pi) \tag{9}$$

The coupled system damping from Reference 2 is

$$\xi_s=\frac{\xi_a\, m_a\omega_a+\xi_b\, m_b\omega_b}{\sqrt{\left(m_a\omega_a{}^2+m_b\omega_b{}^2\right)\left(m_a+m_b\right)}} \tag{10}$$

An example of rigid-coupling is given in Appendix A.

Elastic coupling is given in Appendix B.

References

1. T. Irvine, An Introduction to Frequency Response Functions, Vibrationdata, 2000.
2. T. Irvine, Notes on Damping in FRF Substructuring, Revision A, Vibrationdata, 2014.

APPENDIX A

Rigid Coupling Example

Consider the system in Figure 1 and the subsystems in Figures 2 and 3. Assign the following variables.

m_a	4 lbm
k_a	1000 lbf/in
ξ_a	0.05

m_b	2 lbm
k_b	8000 lbf/in
ξ_b	0.05

The system natural frequency per equation (9) is

$$\text{fn}= \quad 121.1 \text{ Hz} \tag{A-1}$$

The system damping per equation (10) is

$$\xi_s = 0.04 \tag{A-2}$$

The receptance function is shown in Figure A-1.

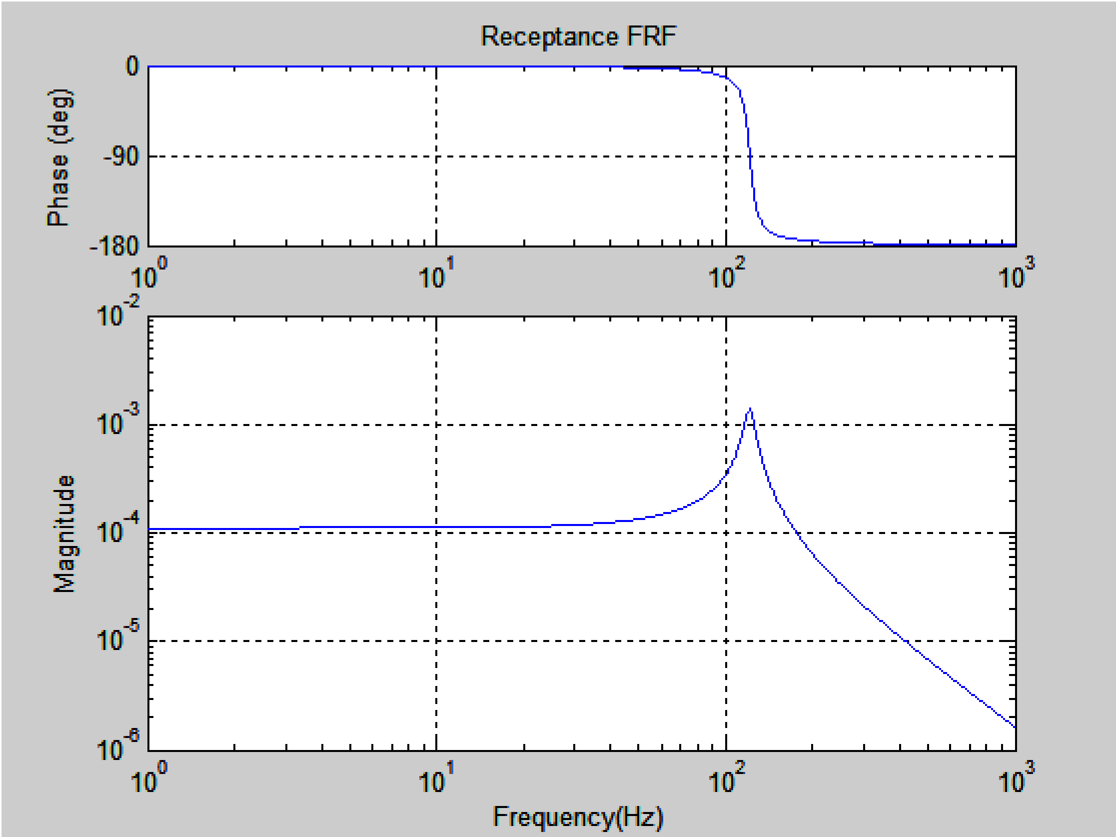

Figure A-1.

The magnitude unit is (in/lbf).

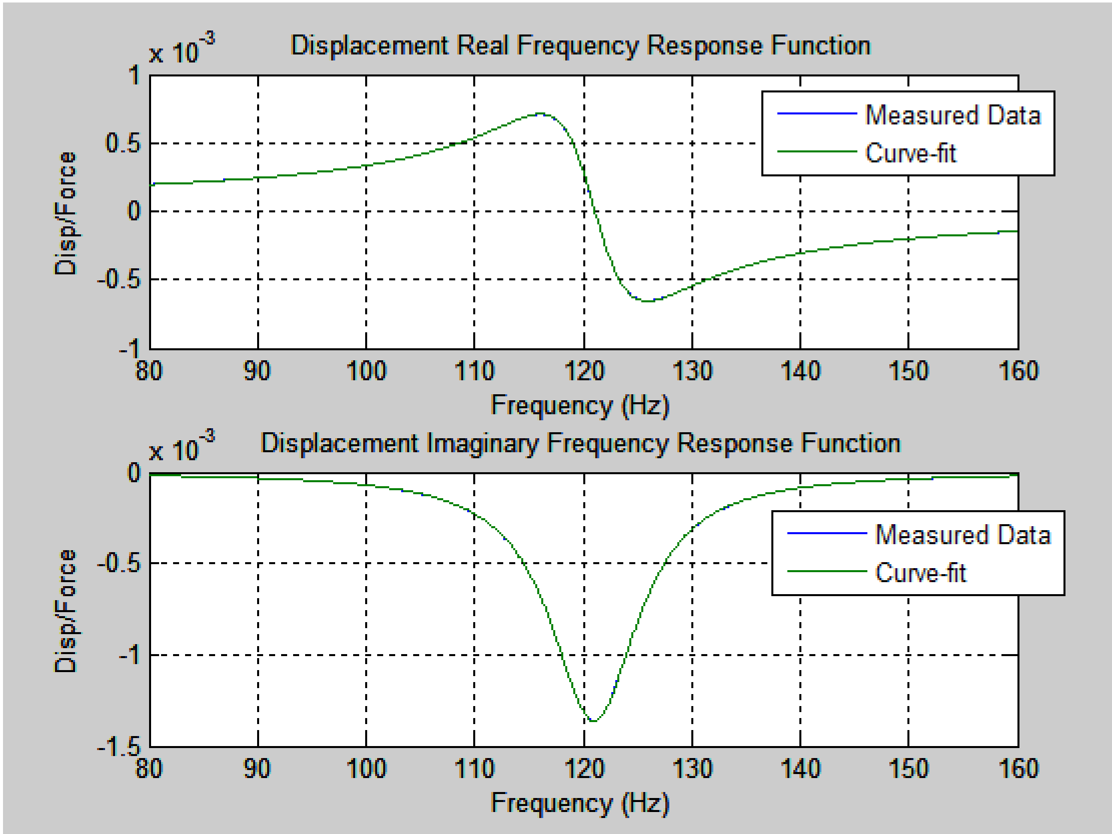

Figure A-2.

An FRF curve-fit was performed on the real and imaginary components of the system receptance functions. The curve-fit yields the expected results.

fn = 121.1 Hz

damping ratio = 0.041

APPENDIX B

Elastic Coupling

Consider a single-degree-of-freedom system S constructed from two subsystems A and B connected by a spring.

The combined system S is

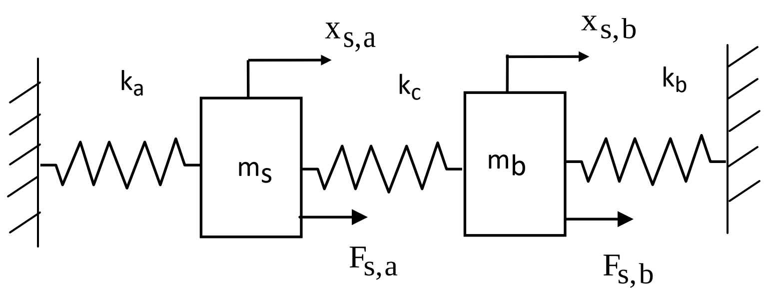

Figure B-1.

Subsystem A is

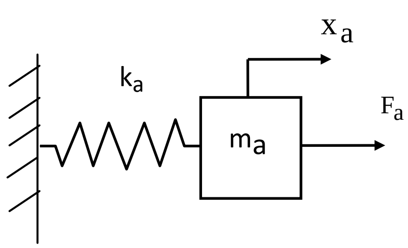

Figure B-2.

Subsystem B is

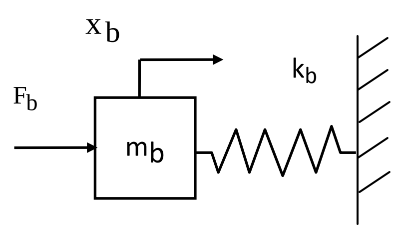

Figure B-3.

The receptance function for subsystem A is

$$H_a(\omega) = \frac{1}{m_a}\left[\frac{1}{\omega_a^2 - \omega^2 + j\left(2\xi_a \omega\omega_a\right)}\right] \tag{B-1}$$

The receptance function for subsystem A plus the coupling spring is

$$H_{ac}(\omega) = \frac{1}{m_a}\left[\frac{1}{\omega_a^2 - \omega^2 + j\left(2\xi_a \omega\omega_a\right)}\right] + \frac{1}{k_c} \tag{B-2}$$

The receptance function for system B is

$$H_b(\omega) = \frac{1}{m_b}\left[\frac{1}{\omega_b^2 - \omega^2 + j\left(2\xi_b \omega\omega_b\right)}\right] \tag{B-3}$$

The joint receptance $H_{s,b}(\omega)$ at the interface between the coupling spring and subsystem b is

$$H_{s,b}(\omega) = \frac{H_{ac}H_b}{H_{ac} + H_b} \tag{B-4}$$

<u>Elastic Coupling Example</u>

Consider the system in Figure B-1 and the subsystems in Figures B-2 and B-3. Assign the following variables.

m_a	4 lbm
k_a	1000 lbf/in
ξ_a	0.05

m_b	2 lbm
k_b	8000 lbf/in
ξ_b	0.05

k_c	2000 lbf/in

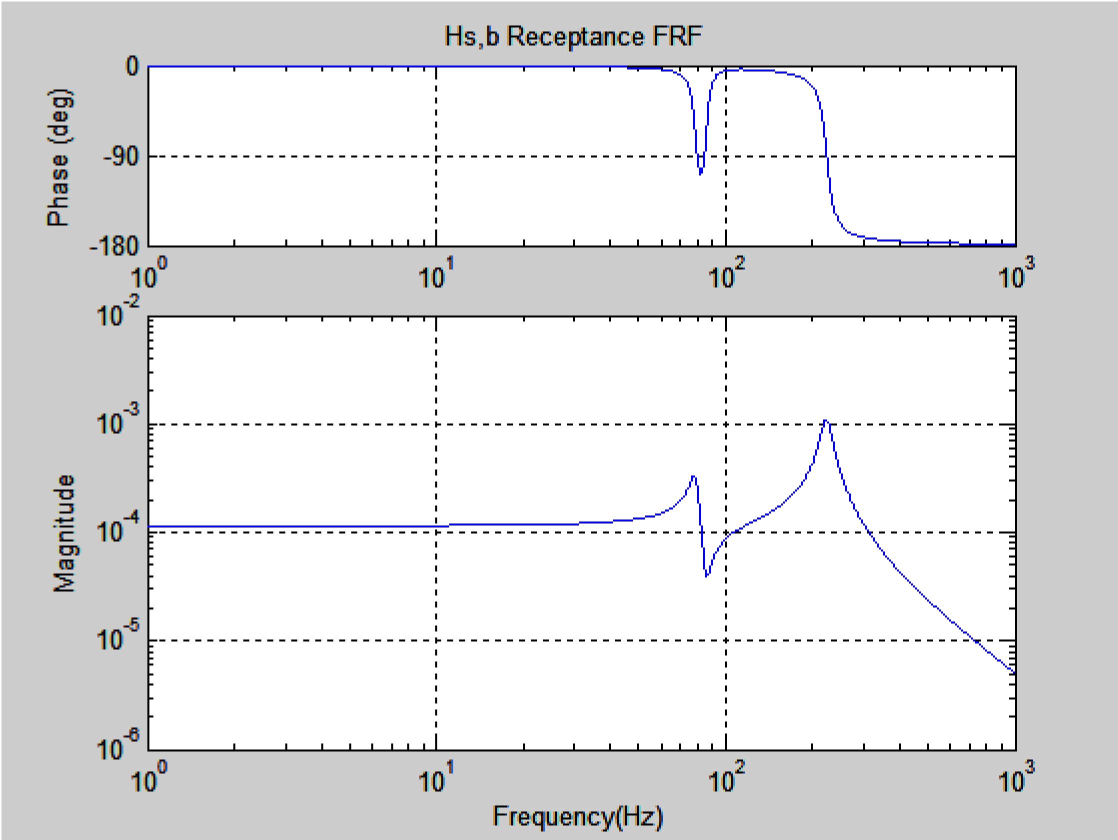

Figure B-4.

The curve in Figure B-4 was calculated using equation (B-4).

The magnitude unit is (in/lbf).

The system modal parameters are

Parameter	Mode 1	Mode 2
Natural Frequency (Hz)	79	224
Damping Ratio	0.034	0.044

The modal parameters were extacted from the FRF using the curve-fit shown in Figure B-5.

The results were confirmed in a separate two-degree-of-freedom modal analysis.

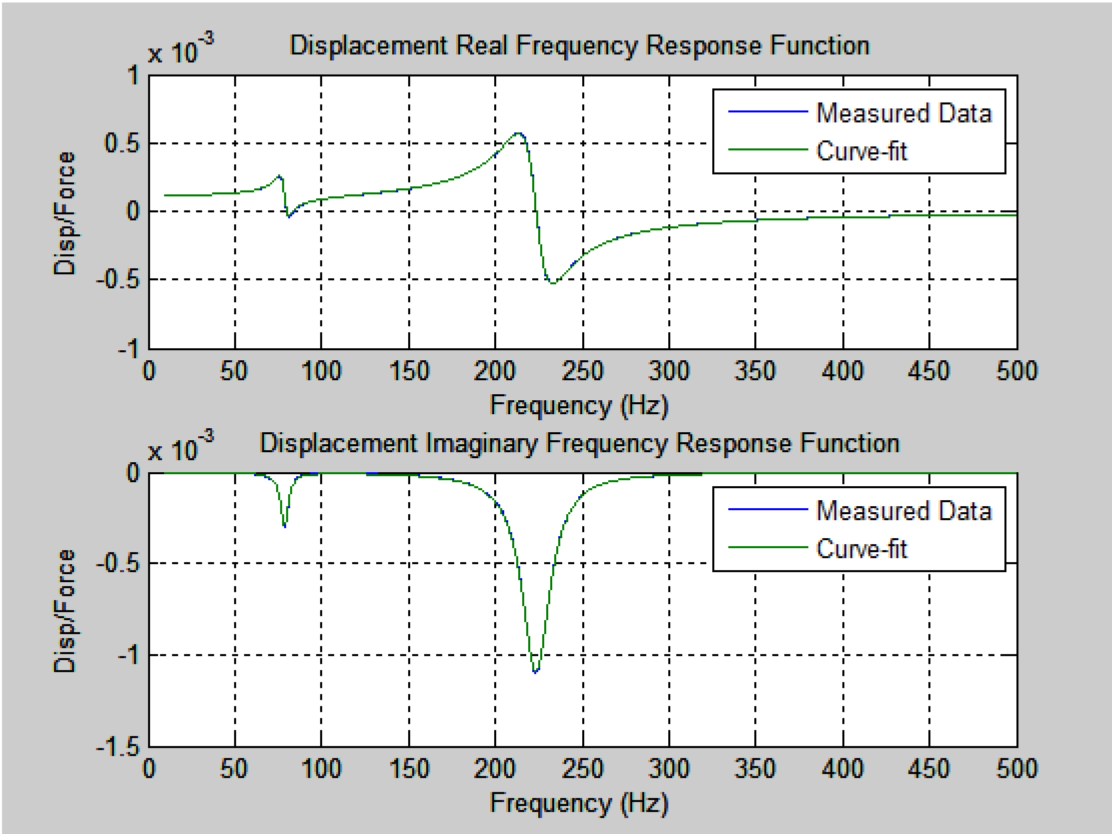
x 10^-3
Displacement Real Frequency Response Function
1
0.5
0
-0.5
-1
Disp/Force
0 50 100 150 200 250 300 350 400 450 500
Frequency (Hz)
Measured Data
Curve-fit
x 10^-3
Displacement Imaginary Frequency Response Function
0
-0.5
-1
-1.5
Disp/Force
0 50 100 150 200 250 300 350 400 450 500
Frequency (Hz)
Measured Data
Curve-fit

Figure B-5.