Introduction

Consider a single-degree-of-freedom system $S$ constructed from two rigidly-connected subsystems $A$ and $B$.

The combined system $S$ is

![Figure 1](image1.png)

Figure 1.

Subsystem $A$ is

![Figure 2](image2.png)

Figure 2.

Subsystem $B$ is

![Figure 3](image3.png)

Figure 3.

The combined system and the subsystems are assumed to have modal damping which implies a dashpot.
Variables

<table>
<thead>
<tr>
<th>$m_i$</th>
<th>mass</th>
<th>$F_i$</th>
<th>applied force</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_i$</td>
<td>stiffness</td>
<td>$H_i$</td>
<td>receptance function (displacement/force)</td>
</tr>
<tr>
<td>$\xi_i$</td>
<td>modal damping</td>
<td>$x_i$</td>
<td>Displacement</td>
</tr>
<tr>
<td>$\omega_i$</td>
<td>natural frequency</td>
<td>$\omega$</td>
<td>excitation frequency</td>
</tr>
</tbody>
</table>

The subscript indicates the system or subsystem.

Receptance Coupling

The receptance function for subsystem A from Reference 1 is

$$H_a(\omega) = \frac{1}{m_a} \left[ \frac{1}{\omega_a^2 - \omega^2 + j(2\xi_a \omega \omega_a)} \right]$$

(1)

The receptance function for subsystem B is

$$H_b(\omega) = \frac{1}{m_b} \left[ \frac{1}{\omega_b^2 - \omega^2 + j(2\xi_b \omega \omega_b)} \right]$$

(2)

The joint receptance $H_j(\omega)$ is

$$H_j(\omega) = \frac{H_a H_b}{H_a + H_b}$$

(3)

$$H_j(\omega) = \frac{1}{m_a m_b} \left[ \frac{1}{\omega_a^2 - \omega^2 + j(2\xi_a \omega \omega_a)} \right] + \frac{1}{m_b} \left[ \frac{1}{\omega_b^2 - \omega^2 + j(2\xi_b \omega \omega_b)} \right]$$

(4)
\[ H_j(\omega) = \frac{1}{m_b\left[\frac{1}{\omega_a^2 - \omega^2 + j(2\xi_a\omega_0a)}\right] + m_a\left[\frac{1}{\omega_b^2 - \omega^2 + j(2\xi_b\omega_0b)}\right]} \] (5)

\[ H_j(\omega) = \frac{1}{m_b\left[\omega_b^2 - \omega^2 + j(2\xi_b\omega_0b)\right] + m_a\left[\omega_a^2 - \omega^2 + j(2\xi_a\omega_0a)\right]} \] (6)

\[ H_j(\omega) = \frac{1}{m_a\omega_a^2 + m_b\omega_b^2 - (m_a + m_b)\omega^2 + j2\omega(m_a\xi_a\omega_a + m_b\xi_b\omega_b)} \] (7)

The system natural frequency \( \omega_s \) is

\[ \omega_s = \sqrt{\frac{k_a + k_b}{m_a + m_b}} \] (8)

\[ f_s = \frac{\omega_s}{(2\pi)} \] (9)

The coupled system damping from Reference 2 is

\[ \xi_s = \frac{\xi_a m_a\omega_a + \xi_b m_b\omega_b}{\sqrt{(m_a\omega_a^2 + m_b\omega_b^2)(m_a + m_b)}} \] (10)
An example of rigid-coupling is given in Appendix A.

Elastic coupling is given in Appendix B.

References


APPENDIX A

Rigid Coupling Example

Consider the system in Figure 1 and the subsystems in Figures 2 and 3. Assign the following variables.

<table>
<thead>
<tr>
<th>m_a</th>
<th>4 lbm</th>
</tr>
</thead>
<tbody>
<tr>
<td>k_a</td>
<td>1000 lbf/in</td>
</tr>
<tr>
<td>\xi_a</td>
<td>0.05</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>m_b</th>
<th>2 lbm</th>
</tr>
</thead>
<tbody>
<tr>
<td>k_b</td>
<td>8000 lbf/in</td>
</tr>
<tr>
<td>\xi_b</td>
<td>0.05</td>
</tr>
</tbody>
</table>

The system natural frequency per equation (9) is

\[ f_n = 121.1 \text{ Hz} \] (A-1)

The system damping per equation (10) is

\[ \xi_s = 0.04 \] (A-2)

The receptance function is shown in Figure A-1.
Figure A-1.

The magnitude unit is (in/lbf).
An FRF curve-fit was performed on the real and imaginary components of the system receptance functions. The curve-fit yields the expected results.

\[ fn = 121.1 \text{ Hz} \]

\[ \text{damping ratio} = 0.041 \]
APPENDIX B

Elastic Coupling

Consider a single-degree-of-freedom system \( S \) constructed from two subsystems \( A \) and \( B \) connected by a spring.

The combined system \( S \) is

![Diagram of coupled system](image)

Figure B-1.

Subsystem \( A \) is

![Diagram of subsystem A](image)

Figure B-2.

Subsystem \( B \) is

![Diagram of subsystem B](image)

Figure B-3.
The receptance function for subsystem A is

\[
H_a(\omega) = \frac{1}{m_a} \left[ \frac{1}{\omega_a^2 - \omega^2 + j(2\xi_a\omega\omega_a)} \right]
\]  

(B-1)

The receptance function for subsystem A plus the coupling spring is

\[
H_{ac}(\omega) = \frac{1}{m_a} \left[ \frac{1}{\omega_a^2 - \omega^2 + j(2\xi_a\omega\omega_a)} \right] + \frac{1}{k_c}
\]  

(B-2)

The receptance function for system B is

\[
H_b(\omega) = \frac{1}{m_b} \left[ \frac{1}{\omega_b^2 - \omega^2 + j(2\xi_b\omega\omega_b)} \right]
\]  

(B-3)

The joint receptance \(H_{s,b}(\omega)\) at the interface between the coupling spring and subsystem b is

\[
H_{s,b}(\omega) = \frac{H_{ac}H_b}{H_{ac} + H_b}
\]  

(B-4)

**Elastic Coupling Example**

Consider the system in Figure B-1 and the subsystems in Figures B-2 and B-3. Assign the following variables.

| \(m_a\) | 4 lbm |
| \(k_a\) | 1000 lbf/in |
| \(\xi_a\) | 0.05 |

| \(m_b\) | 2 lbm |
| \(k_b\) | 8000 lbf/in |
| \(\xi_b\) | 0.05 |

| \(k_c\) | 2000 lbf/in |
Figure B-4.

The curve in Figure B-4 was calculated using equation (B-4).

The magnitude unit is (in/lbf).

The system modal parameters are

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Mode 1</th>
<th>Mode 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Natural Frequency (Hz)</td>
<td>79</td>
<td>224</td>
</tr>
<tr>
<td>Damping Ratio</td>
<td>0.034</td>
<td>0.044</td>
</tr>
</tbody>
</table>

The modal parameters were extracted from the FRF using the curve-fit shown in Figure B-5.

The results were confirmed in a separate two-degree-of-freedom modal analysis.
Figure B-5.