AN INTRODUCTION TO MUSIC THEORY   Revision A

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Historical Background

Pythagoras of Samos was a Greek philosopher and mathematician, who lived from approximately 560 to 480 BC. Pythagoras and his followers believed that all relations could be reduced to numerical relations. This conclusion stemmed from observations in music, mathematics, and astronomy.

Pythagoras studied the sound produced by vibrating strings. He subjected two strings to equal tension. He then divided one string exactly in half. When he plucked each string, he discovered that the shorter string produced a pitch which was one octave higher than the longer string. A one-octave separation occurs when the higher frequency is twice the lower frequency.

German scientist Hermann Helmholtz (1821-1894) made further contributions to music theory. Helmholtz wrote "On the Sensations of Tone" to establish the scientific basis of musical theory.

Natural Frequencies of Strings

A note played on a string has a fundamental frequency, which is its lowest natural frequency. The note also has overtones at consecutive integer multiples of its fundamental frequency. Plucking a string thus excites a number of tones.

Ratios

The theories of Pythagoras and Helmholz depend on the frequency ratios shown in Table 1.

Table 1. Standard Frequency Ratios

Ratio	Name
1:1	Unison
1:2	Octave
1:3	Twelfth
2:3	Fifth
3:4	Fourth
4:5	Major Third
3:5	Major Sixth
5:6	Minor Third
5:8	Minor Sixth

These ratios apply both to a fundamental frequency and its overtones, as well as to relationship between separate keys.

Consonance

Now consider two strings which are plucked simultaneously. The degree of harmony depends on how the respective fundamental frequencies and overtones blend together.

Music notes which blend together in a pleasing manner are called *consonances*. Notes with a displeasing blend are *dissonances.*

Helmholtz gave a more mathematical definition of these terms:

> When two musical tones are sounded at the same time, their united sound is generally disturbed by the beats of the upper partials, so that a greater or less part of the whole mass of sound is broken up into pulses of tone, and the joint effect is rough. This relation is called *Dissonance.* But there are certain determinant ratios between pitch numbers, for which this rule suffers an exception, and either no beats at all are formed, or at least only such as have so little intensity that they produce no unpleasant disturbances of the united sound. These exceptional cases are called *Consonances.*

Helmholtz has defined degrees of consonance as shown in Table 2.

Table 2. Consonances	
Degree	Interval
Absolute	Octave, Twelfth, Double Octave
Perfect	Fifth, Fourth
Medial	major Sixth, major Third
Imperfect	minor Sixth, minor Third

For reference, a glossary of musical terms is given in Appendix A.

Octave

Again, a one-octave separation occurs when the higher frequency is twice the lower frequency. The octave ratio is thus 2:1

A note's first overtone is one octave higher than its fundamental frequency.

Consider a modern piano keyboard, as shown in Appendix A. The beginning key on the left end is an A0 note with a fundamental frequency of 27.5 Hz. A piano key has harmonic overtones at integer multiples of its fundamental frequency. Thus, the A0 key also produces a tone at 55.0 Hz, which is one octave higher than the fundamental frequency. The second overtone is at 82.5 Hz.

The twelfth key to the right of A0 is A1, counting both the black and white keys. The A1 note has a fundamental frequency of 55.0 Hz. The A1 note is thus one octave higher than the A0 note, in terms of their respective fundamental frequencies. In fact, there is a one-octave separation between any two piano keys which are twelve keys apart.

A pleasing, harmonious sound is produced when two notes separated by one octave are played simultaneously on a piano or other musical instrument. Helmholtz calls such a pair an *absolute consonance.* Thus, the A0 and A1 keys are an absolute consonance.

This effect is shown for the A0 note and the A1 note in Table 3.

Table 3.
Comparison of Two Notes and Their Respective Overtones up to 165 Hz.

A0 Frequency (Hz)	A1 Frequency (Hz)
27.5	
55.0	55.0
82.5	
110.0	110.0
137.5	
165.0	165.0

The overtones of the A1 note thus coincide with the evenly numbered overtones of the A0 note. Again, these two notes are separated by one octave.

Hermann Helmholz wrote:

> A note accompanied by its Octave consequently becomes brighter in quality, because the higher upper partial tones on which brightness of quality depends, are partially reinforced by the additional Octave.

<u>Twelfth</u>

A twelfth is two notes which form a frequency ratio of 1:3.

A note's second overtone is a twelfth higher than its fundamental frequency.

Recall the A0 note with its fundamental frequency of 27.5 Hz. Its second overtone is 82.5 Hz, which is three times higher than its fundamental frequency.

Ideally, there would be a key with a fundamental frequency of 82.5 Hz. The nearest is the E2 key which has a fundamental frequency of 82.407 Hz. This frequency approximately meets the goal. Thus, the E2 key is considered as a twelfth higher than A0. A comparison is shown in Table 4.

Table 4. Comparison of Two Notes and Their Respective Overtones up to 250 Hz.	
A0 Frequency (Hz)	E2 Frequency (Hz)
27.5	
55.0	
82.5	82.407
110.0	
137.5	
165.0	164.814
192.5	
220.0	
247.5	247.221

Thus A0 and E2 have three tones very nearly in common in the frequency domain up to 250 Hz.

Fifth

A fifth is two notes which form a frequency ratio of 2:3.

A note's second overtone is a fifth higher than its first overtone .

Recall the A0 note with its fundamental frequency of 27.5 Hz. A fifth higher would be 41.25 Hz. Such a note does not exist in an exact sense. On the other hand, the E1 note has a frequency of 41.203 Hz, which is approximately equal to the exact fifth. Thus, E1 is considered as a fifth higher than A0. A comparison is shown in Table 5.

Table 5. Comparison of Two Notes and Their Respective Overtones up to 165 Hz.	
A0 Frequency (Hz)	E1 Frequency (Hz)
27.5	
	41.203
55.0	
82.5	82.406
110.0	
	123.609
137.5	
165.0	164.812

Thus, A0 and E1 have two overtones very nearly in common in the frequency domain up to 165 Hz.

Reference

1. Hermann Helmholtz, On the Sensations of Tone, Dover, New York, 1954.

APPENDIX A

Glossary

Consonance – a simultaneous combination of sounds conventionally regarded as pleasing.

Dissonance - a simultaneous combination of sounds conventionally regarded as lacking harmony.

Harmony – a combination of musical considered to be pleasing.

Harmonic – a tone in the harmonic series of overtones produced by a fundamental tone.

Harmonic Series – a series of tones consisting of a fundamental tone and the overtones produced by it, whose frequencies are at integral multiples of the fundamental frequency.

Interval – the difference in pitch between two musical tones

Octave – the interval of eight diatonic degrees between two tones, one of which has twice the frequency as the other.

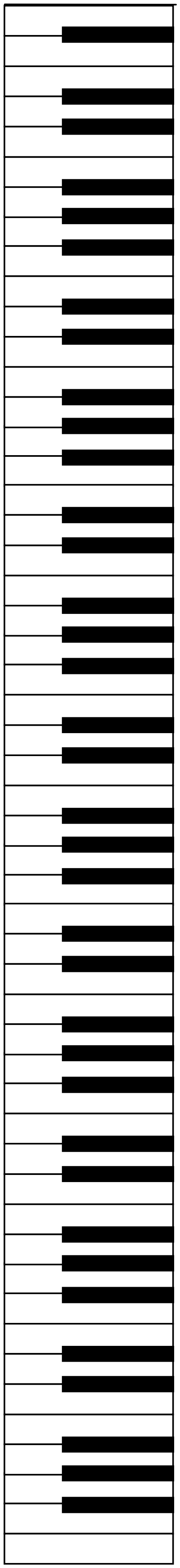
Overtone – a harmonic.

Partial – a harmonic.

Pitch – the frequency of a tone.

Reference: American Heritage Dictionary, Houghton Mifflin Company, Boston, 1982.

APPENDIX B



Middle C (C4)

A0	27.5	A0#	29.135
B0	30.868		
C1	32.703	C1#	34.648
D1	36.708	D1#	38.891
E1	41.203		
F1	43.654	F1#	46.249
G1	48.999	G1#	51.913
A1	55.000	A1#	58.270
B1	61.735		
C2	65.406	C2#	69.296
D2	73.416	D2#	77.782
E2	82.407		
F2	87.307	F2#	92.499
G2	97.999	G2#	103.83
A2	110.00	A2#	116.54
B2	123.47		
C3	130.81	C3#	138.59
D3	146.83	D3#	155.56
E3	164.81		
F3	174.61	F3#	185.00
G3	196.00	G3#	207.65
A3	220.00	A3#	233.08
B3	246.94		
C4	261.63	C4#	277.18
D4	293.66	D4#	311.13
E4	329.63		
F4	349.23	F4#	369.99
G4	392.00	G4#	415.30
A4	440.00	A4#	466.16
B4	493.88		
C5	523.25	C5#	554.37
D5	587.33	D5#	622.25
E5	659.25		
F5	698.46	F5#	739.99
G5	783.99	G5#	830.61
A5	880.00	A5#	932.33
B5	987.77		
C6	1046.5	C6#	1108.7
D6	1174.7	D6#	1244.5
E6	1318.5		
F6	1396.9	F6#	1480.0
G6	1568.0	G6#	1661.2
A6	1760.0	A6#	1864.7
B6	1979.5		
C7	2093.0	C7#	2217.5
D7	2349.3	D7#	2489.0
E7	2637.0		
F7	2793.8	F7#	2960.0
G7	3136.0	G7#	3322.4
A7	3520.0	A7#	3729.3
B7	3951.1		
C8	4186.0		

PIANO KEYBOARD

The number beside each key is the fundamental frequency in units of cycles per seconds, or Hertz.

OCTAVES

For example, the A4 key has a frequency of 440 Hz.

Note that A5 has a frequency of 880 Hz. The A5 key is thus one octave higher than A4 since it has twice the frequency.

OVERTONES

An overtone is a higher natural frequency for a given string. The overtones are “harmonic” if each occurs at an integer multiple of the fundamental frequency.