Pseudo Velocity Shock Spectrum Rules For Analysis Of Mechanical Shock

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ABSTRACT: I have taken on the job of recording the features and use of the pseudo velocity shock spectrum (PVSS) plotted on four coordinate paper (4CP). Some of the newer rules could be presented as a separate paper, but knowledge of the PVSS on 4CP is so limited that few would understand the application. An integrated document is needed to show how all the concepts fit together. The rules cover the definition, interpretation and accuracy of four coordinate paper, simple shock spectrum shape, drop height and the 2g line, pseudo velocity relation to modal stress, shock severity, destructive frequency range, shock isolation, use with multi degree of freedom systems, low frequency limitation of shaker shock, and relation to the aerospace acceleration SRS concept. I hope that' by showing you the wide applicability of PVSS on 4CP analysis, that I can convince you to use it.

Introduction: Dick Chalmers (Navy Electronics Lab, San Diego, CA) and Howie Gaberson (Navy Facilities Lab in Port Hueneme, CA) worked on shock during the late sixties to define equipment fragility and its measurement. Chalmers' Navy experience in organizing severe ship shocks by induced velocity led us to an independent discovery that induced modal velocity, not acceleration, was proportional to stress. We published that in 1969. Earlier others had discovered and written on the same subject. No one paid any attention. At Chalmers' insistence, in the early 90's, we started pushing the concept again, and we connected it to the pseudo velocity shock spectrum plotted on four coordinate paper (PVSS on 4CP), a 1950's concept. Matlab came along and made the PVSS calculation and 4CP plotting easy. It turns out that PVSS indicates multi degree of freedom system modal velocity through a participation factor. Dick died in 1998 but his results are certainly in this paper. PVSS on 4CP was used at least in the late 50's, and Eubanks and Juskie [23] employed it for installed equipment fragility in their 50-page 1963 Shock and Vibe Paper. Civil, nuclear defense, and Army Conventional Weapons defense, have adopted the convention. Howie has recently been assembling the rules and reasons that explain the use of PVSS on 4CP for measuring the destructive potential of violent shock motions. This paper attempts to assemble them in one convenient document.

Shock Spectrum Definitions: The shock spectrum is a plot of an analysis of a motion (transient motions due to explosions, earthquakes, package drops, railroad car bumping, vehicle collisions, etc.) that calculates the maximum response of many different frequency damped single degree of freedom systems (SDOFs) exposed to the motion. The response can be: positive, negative, or maximum of the two. It can be calculated for during, or residual (after), the shock motion, overall or maximum of the maximum is most common. The SDOFs can be damped or undamped. It can be plotted in terms of relative or absolute: acceleration, velocity, or displacement. The most important plot is on four coordinate paper, (4CP) in terms of pseudo velocity.

PVSS4CP (PSEUDO VELOCITY SHOCK SPECTRUM PLOTTED ON FOUR COORDINATE PAPER) IS A SPECIFIC PRESENTATION OF THE RELATIVE DISPLACEMENT SHOCK SPECTRUM THAT IS EXTREMELY HELPFUL FOR UNDERSTANDING SHOCK. PSEUDO VELOCITY EXACTLY MEANS PEAK RELATIVE DISPLACEMENT, Z, MULTIPLIED BY THE NATURAL FREQUENCY IN RADIANS, $\left(\sqrt{k/m}\right)$.

Many papers were published wasting time calculating eloquent acceleration shock spectra (called SRS) of the classical pulses, (i.e., half sine, haversine, trapezoid, saw tooth). Examples of these articles are [1, 2, 3, 4]. I think these are unimportant. The acronym SRS has come to mean a log log plot of the absolute acceleration shock spectrum and is used extensively by the aerospace community. The structural community and the Navy use the PVSS 4CP.

Shock Spectrum Equation: Fig. 1 is the SDOFs model to explain the shock spectrum where:

y is the shock motion applied to the bogey or heavy wheeled foundation.
x is the absolute displacement of the SDOF mass
z is the relative displacement, x - y.

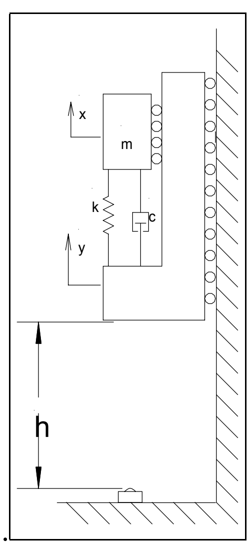
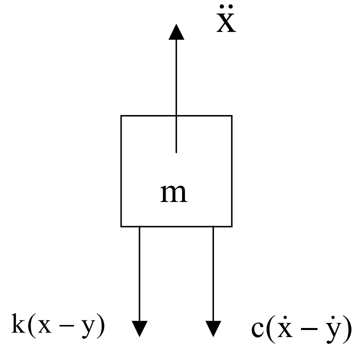


Figure 1. The shock table wheeled bogey with a single degree of freedom system (SDOFs) attached.



The free body diagram of the mass is in Fig. 2.

Figure 2. The free body diagram of the mass with forces.

Applying F = ma on the FBD of Fig. 2 gives us Eq. (1).

$$-c(\dot{x}-\dot{y})-k(x-y)=m\ddot{x} \qquad (1)$$

Using relative coordinates, defined as: z = x - y, gives (Eq. (2)):

$$\begin{aligned} &-c\dot{z}-kz=m(\ddot{z}+\ddot{y}), \quad or, \\ &m\ddot{z}+c\dot{z}+kz=-m\ddot{y} \end{aligned} \qquad (2)$$

Dividing by "m," and substituting the definitions and symbols of Eq. (3a) give Eq. (3).

$$\omega_n=\sqrt{\frac{k}{m}}, \quad \zeta=\frac{c}{c_c}, \text{ and } \quad c_c=2\sqrt{km} \qquad (3a)$$

$$\ddot{z}+2\zeta\omega_n\dot{z}+\omega_n^2 z=-\ddot{y} \qquad (3)$$

Equation (3) is the shock spectrum equation, and the shock spectrum is our tool for understanding shock. In Eq. (3), $\ddot{y}$ is the shock. O'Hara [5] gives the solution explicitly with initial conditions as follows (Eq. (4)):

$$z=z_0e^{-\zeta\omega t}\left(\cos\omega_d\ t+\frac{\zeta}{\eta}\sin\omega_d\ t\right)+\frac{\dot{z}_0e^{-\zeta\omega t}}{\omega_d}\sin\omega_d t-\frac{1}{\omega_d}\int_0^t\ddot{y}(\tau)e^{-\zeta\omega(t-\tau)}\sin\omega_d(t-\tau)d\tau \qquad (4)$$

Where: $z_0, \dot{z}_0 =$ initial values of $z, \dot{z}$

$\omega_d =$ damped natural frequency, $\eta\omega$

$\eta=\sqrt{1-\zeta^2}$

$\tau =$ integration time variable

Shock Spectrum Calculation

Equation (4) is applied from point to point giving a list of z's. The maximum value of z multiplied by the frequency in radians is the pseudo velocity, ωz_{max}, for that frequency. If you think of applying that equation to the whole shock, (as though you knew how to write an equation for the shock) from time equals zero, to after the shock is over, the initial terms will be zero and we have z and a function of time given by Eq. (5).

$$z = -\frac{1}{\omega_d}\int_0^t \ddot{y}(\tau)e^{-\zeta\omega(t-\tau)}\sin\omega_d(t-\tau)d\tau \tag{5}$$

The PVSS, is the maximum value of this for each frequency multiplied by ω

$$PV = \omega z_{\max} = \omega\left[-\frac{1}{\omega_d}\int_0^t \ddot{y}(\tau)e^{-\zeta\omega(t-\tau)}\sin\omega_d(t-\tau)d\tau\right]_{\max} \tag{5a}$$

The undamped equations are Eqs (6), (6a), and (6b).

$$\ddot{z} + \omega_n^{\,2} z = -\ddot{y} \tag{6}$$

$$z = -\frac{1}{\omega}\int_0^t \ddot{y}(\tau)\sin\omega(t-\tau)d\tau \tag{6a}$$

$$PV = \omega z_{\max} = \left[-\int_0^t \ddot{y}(\tau)\sin\omega(t-\tau)d\tau\right]_{\max} \tag{6b}$$

I had to lead you to Eq. (6b), because I want you to believe it. We're coming back to Eq. (6b) when we do multi degree of freedom systems (MDOFS), and shock isolation.

ZERO MEAN SIMPLE SHOCK: The shock in Figures 3, is a zero mean simple shock. Zero mean acceleration means shock begins and ends with zero velocity. This means the motion analyzed includes the drop, as in the case of a drop table shock machine shock. The integral of the acceleration is zero if it has a zero mean. By simple shock I mean one of the common pulses: half sine, initial peak saw tooth, terminal peak saw tooth, trapezoidal, haversine

PVSS-4CP Example, 1 ms, 800 g Half Since: As an example Fig. 3 shows a drop table shock machine 800 g, 1 ms, half sine shock motion and its integrals; this is the motion, y, in Fig. 1. (I saw this 800 g, 1 ms, half sine listed for non operational shock capability on the package of a 60 gig Hammer USB Hard Drive.) Fig. 4 shows its PVSS on 4CP for 5% damping.

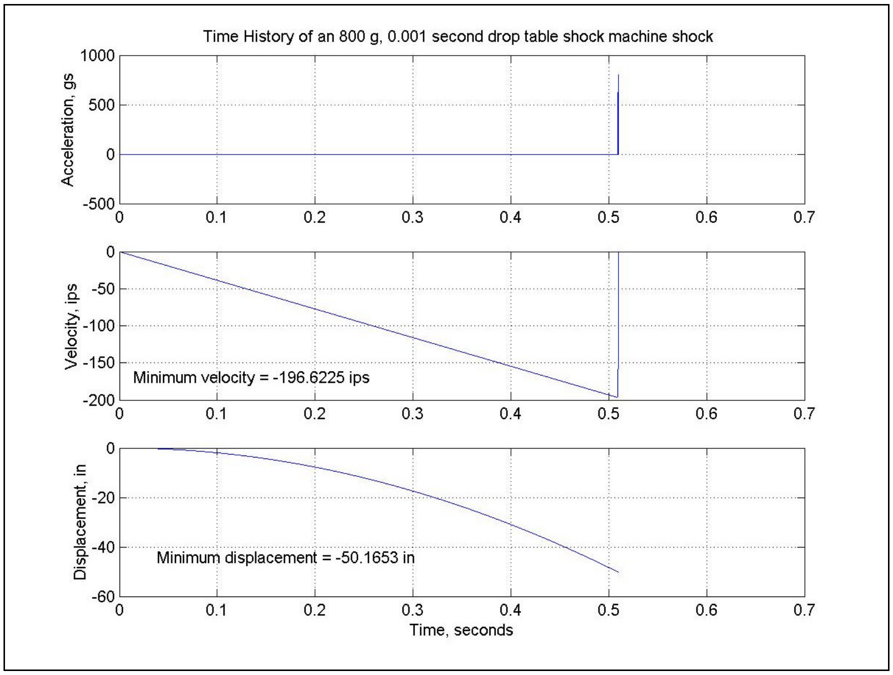


Figure 3. Time history of acceleration, velocity, and displacement of a drop table shock machine half sine shock.

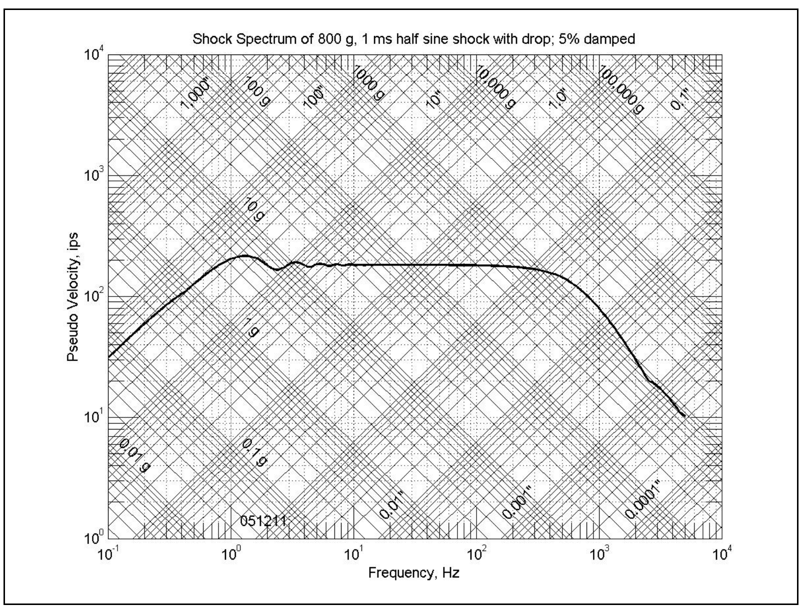


Figure 4. PVSS on 4CP for the half sine shock of Figure 3. Notice the high frequency asymptote is on the constant 800 g line, that the velocity plateau is at a little under 196 ips, and that the low frequency asymptote is on a constant displacement line of about 50 inches

Figure 4, our PVSS on 4CP, for that hard drive non operational shock, shows a lot of information. We'll talk more about this later, but for now you see a peak 800 g constant acceleration line sloping down and to the right for the high frequencies, you see a mid frequency range plateau at just under the velocity change that took place during the impact, (196 ips) and you see a low frequency constant displacement asymptote at the constant maximum displacement of the shock, the 50-inch drop, sloping down and to the left.

Four Coordinate Paper, 4CP is Sine Wave Paper. Every Point Represents a Specific Sine Wave With a Frequency and a Peak Displacement, Velocity, and Acceleration: To explain this 4CP, think of a sine wave vibration, which has a frequency and a peak deflection, a peak velocity, and a peak acceleration. The four are related; knowing any two, the others pop out. Frequency is in Hz. The deflection is in inches, the velocity is in inches per second, ips, and the acceleration is in g's. Four coordinate paper (4CP) is a log log vibration sine wave nomogram displaying the sine wave relationship with four sets of lines, log spaced: vertical for frequency, horizontal for velocity, down and to the right for acceleration, and down and to the left for deflection.

Zero Mean Simple Shock General Shape

WHEN A ZERO MEAN SHOCK PVSS IS PLOTTED ON 4CP IT HAS A HILL SHAPE: THE LEFT UPWARD SLOPE IS A PEAK DISPLACEMENT ASYMPTOTE. THE RIGHT DOWNWARD SLOPE IS THE PEAK ACCELERATION ASYMPTOTE. THE TOP IS A PLATEAU AT THE VELOCITY CHANGE DURING IMPACT.

THE LOGIC FOR PLOTTING PVSS ON 4CP

When we use four coordinate paper for plotting pseudo velocity shock spectra, every point on the plot represents four values. For that frequency the relative displacement, z, and pseudo velocity, ωz, are exact. (Displacement is exactly calculated, and PV is just ωz.) The indicated acceleration (which has to $\omega^2 z_{max}$) is the absolute acceleration at the instant of maximum relative displacement, regardless of the damping. This can be explained as follows. The shock spectrum calculating equation is

$$\ddot{z} + 2\zeta\omega\dot{z} + \omega^2 z = -\ddot{y} \qquad (3)$$

From our definition of the relative coordinate, z, we have

$$\begin{aligned} &z = x - y, \quad and \quad \ddot{z} = \ddot{x} - \ddot{y}, \\ &thus \\ &-\ddot{y} = \ddot{z} - \ddot{x} \end{aligned} \qquad (3b)$$

Substituting (3b) into (3a) we have

$$2\zeta\omega\dot{z} + \omega^2 z = -\ddot{x} \qquad (3c)$$

When the damping is zero, we have Eq (3d), and this is the indicated acceleration on the 4CP. For the undamped case, the indicated acceleration is exact.

$$\omega^2 z_{max} = -\ddot{x} \quad (3d)$$

When the damping is not zero, consider the following. The shock spectrum calculates the maximum value of z. At an instant of maximum z, its derivative, $\dot{z}$, has to be zero. Thus at any instant of maximum z, Eq (3d) still holds. Thus the indicated acceleration on the 4CP for damped spectra, it is indeed the exact absolute acceleration of the mass at the instant that z is equal to z_{max}. But this is not necessarily that maximum acceleration of the mass at that frequency. So the acceleration values on the damped PVSS are only approximate for max acceleration of the mass. It's probably close if damping is small and because the acceleration asymptote is exact at high frequencies.

Similarly and importantly, if you compute an acceleration shock spectrum, the SRS, the pseudo velocity you would get from dividing by ω, that is $\ddot{x}_{max}/\omega$ is not the same as the pseudo velocity ωz_{max}; they don't occur at the same instant. This is a problem and maybe the only way it can be evaluated is to calculate some example cases.

Understanding the PVSS Plateau When PVSS is Plotted On 4CP: All PVSS have a plateau; and it is the region where the shock is most severe so you have to understand it. Sometimes it's very short and sometimes long. Collision shocks don't begin and end with zero velocity, and are almost all plateau.

To explain why the plateau occurs, think with me in the following way. Think of an instantaneous shock. Go back and look at Figure 1. The bogey, is way heavier than the mass, like the table on a drop table shock machine. It is released and falls from a height, h, and hits a shock programmer (pad or whatever) that brings it to rest or zero velocity with one of the traditional simple shock impacts (i.e., half sine, sawtooth, trapezoid, haversine) that has a peak acceleration, $\ddot{y}_{max}$. Both the bogey and the mass fall substantially together and attain a peak velocity of $\dot{y}_i = -\sqrt{2gh}$. Just after the impact, the bogey velocity, $\dot{y}$, suddenly becomes zero, but $\dot{x}$, the mass velocity, hasn't yet changed. Since $\dot{z} = \dot{x} - \dot{y}$, and, $\dot{y}$ has just become zero, $\dot{z}_0 = \dot{x}_o$, and we have the initial velocity case for that undamped homogeneous solution, Eq. (4a), with $\dot{z}_0 = \sqrt{2gh}$, and $z_0 = 0$. We take Eq (3), with no damping, and no shock acceleration, which gives us Eq (4a).

$$z = z_0 \cos\omega t + \frac{\dot{z}_0}{\omega}\sin\omega t \quad (4b)$$

Again: the bogey and the mass fall together, and the shock is over before the spring does any compressing. The bogey suddenly comes to rest and then the mass starts vibrating. This is undamped initial value free vibration of Eq (4b). Just before impact, the mass and the bogy have the same velocity, or

$$\dot{x} = \dot{y} = -\sqrt{2gh} \quad (4c)$$

After impact, $z_0 = 0, \quad \dot{y} = 0$, but still, $\dot{x} = -\sqrt{2gh}$. Since $\dot{z} = \dot{x} - \dot{y}$, $\dot{z}_0 = \dot{x}_0$, in the initial velocity case, with $\dot{z}_0 = \dot{x}_o = \sqrt{2gh}$. so

$$z = \frac{\dot{z}_0}{\omega}\sin\omega t = \frac{-\sqrt{2gh}}{\omega}\sin\omega t, \tag{4d}$$

and max pseudo velocity is.

$$\omega z = \sqrt{2gh} \tag{4e}$$

Now, as simple as that is, that's how/why we get a plateau. All SDOF, with half periods much longer than the impact duration, end up vibrating with the same peak velocity, the impact velocity, no matter what their natural frequency. In this undamped sinusoidal motion, the relative velocity and the pseudo velocity have the same maximum values; they both all continue to vibrate forever with this peak velocity, the impact velocity. The maximum pseudo velocity is the impact velocity, so all SDOFS with periods much longer than the shock, will have the same maximum pseudo velocity. This is why we see the plateau; the shock spectrum of a simple shock will have a constant PV plateau for quite a wide frequency interval.

UNDAMPED PVSS'S OF SIMPLE DROP TABLE SHOCKS HAVE A FLAT CONSTANT PSEUDO VELOCITY PLATEAU AT THE VELOCITY CHANGE THAT TOOK PLACE DURING THE SHOCK.

The High Frequency Asymptote is the Constant Acceleration Line at the Peak Acceleration: There are limits to the frequencies at which this plateau can continue. In the very high frequency region, think of the mass as very light and the spring very stiff; so stiff that the mass exactly follows the input motion. The acceleration of the mass is equal to the acceleration of the foundation. In this region the maximum relative deflection, z, is given by the maximum force in the spring over its stiffness, k. The maximum force is the ma force, $m\ddot{x}$, and $\ddot{x}_{\max} = \ddot{y}_{\max}$. Thus the maximum spring stretch is:

$$z_{\max} = \frac{F_{\max}}{k} = \frac{m\ddot{x}_{\max}}{k} = \frac{m}{k}\ddot{y}_{\max} = \frac{1}{\omega_n^2}\ddot{y}_{\max}. \tag{10}$$

So for the high frequency region the pseudo velocity:

$$PV = \omega z_{\max} = \frac{\ddot{y}_{\max}}{\omega_n} \tag{10a}$$

The very high frequency pseudo velocity asymptote is the peak acceleration divided by the natural frequency, and this is the 4CP constant acceleration line at the peak acceleration. I have calculated and plotted all of the simple shocks [6]. I've found that on the RHS of the PVSS on 4CP, near the intersection of the acceleration asymptote and the plateau, the PVSS starts sloping downward at a higher acceleration than the asymptote but does not exceed twice a_{max}.

THE HIGH FREQUENCY LIMIT OF THE PLATEAU OF THE UNDAMPED PVSS OF THE SIMPLE SHOCKS OF THE HIGH PV REGION IS SET BY THE MAXIMUM ACCELERATION OF THE SHOCK.

The Low Frequency Asymptote of a Zero Mean Shock is a Constant Displacement Line at the Peak Displacement: Now on the low frequency end of the plateau, imagine the following: the mass is heavy and the spring is extremely soft, so the mass won't even start to move until the bogey has fallen, come to rest, and the impact is over. Then it notices it has deflected an amount "h," and it starts vibrating with amplitude "h" forever. The deflection cannot exceed the drop height. Thus, on the left side of the PVSS on 4CP, z = h and the PV will be:

$$\omega z = \omega h$$

And that's a line sloping down and to the left at a constant deflection, "h."

Notice: The Low Frequency Limit of the Plateau of the PVSS on 4CP of a Zero Mean Shock is Set by the Maximum Deflection of the Shock: I want to remind you of Figures 3 and 4, the example 800 g half sine shock. Please notice that there is no net velocity change; it starts at zero velocity and ends at zero velocity; however, there was a sudden 100 ips velocity change during the impact. No net velocity change means the acceleration time trace has a zero integral, or in fact a zero mean or average value.

The Undamped no Rebound Simple Drop Table Shock Machine Shock Plateau Low Frequency Limit is the 2g Line: On the undamped PVSS on 4CP of a simple no rebound drop table shock machine shock, the shock machine drop height is the constant displacement line going through the intersection of the plateau level and the 2g line. This is because the low frequency, no rebound asymptote is the drop height constant displacement line. The PV everywhere on this line is ωh. Recall that the velocity after a drop, "h" is given by:

$$v^2 = 2gh \tag{11}$$

The undamped velocity plateau PV is at $\omega z = \sqrt{2gh}$. Thus, the LF asymptote intersects the velocity plateau line where $\omega h = \sqrt{2gh}$. Squaring both sides we have the intersection at:

$$\begin{gathered} \omega^2 h^2 = 2gh, \quad or \\ \omega^2 h = 2g \end{gathered} \tag{11a}$$

$\omega^2 h$ is an acceleration. The undamped PV plateau intersects the low frequency simple shock no rebound drop height at an acceleration of 2g's. Flip ahead and notice that I have drawn in the 2g line on Fig. 14b.

No Rebound Must be Stated in the 2g Line Definition: I had to say no rebound because a rebound increases the velocity change during impact, or for a given velocity change a rebound reduces the needed drop height, and will reduce the low frequency asymptote.

Damping Reduces the Plateau Level and Makes it Less Than the Impact Velocity Change. The way I established the plateau was with the undamped homogeneous solution of Eq. (3), the shock spectrum equation for an initial velocity, Eq. (9b). I showed the initial velocity was the impact velocity, or the velocity change at impact. To do the same problem with damping, we need the damped homogenous solution of Eq. (3). In the plateau region, the relative displacement "z" is really an initial velocity problem. From the first two terms of Eq. (4) the homogeneous solution of the shock spectrum equation is:

$$z = \frac{z_0\omega\zeta + \dot{z}_0}{\omega\eta} e^{-\zeta\omega t}\sin\eta\omega t + z_0 e^{-\zeta\omega t}\cos\eta\omega t$$
$$z = z_0 e^{-\zeta\omega t}\left(\cos\eta\omega t + \frac{\zeta}{\eta}\sin\eta\omega t\right) + \frac{\dot{z}_0 e^{-\zeta\omega t}}{\omega\eta}\sin\eta\omega t \qquad (12)$$

At time equal to zero, the initial displacement is 0, and we have an initial velocity so Eq. (1) becomes: (where $\dot{z}_0$ = initial velocity, $=\sqrt{2gh}$)

$$z = \frac{\dot{z}_0 e^{-\zeta\omega t}}{\omega\eta}\sin\eta\omega t \qquad (13)$$

Now with an initial velocity, we'll get a positive maximum and a negative minimum in the first period, and the product of these and the natural frequency will be the positive and negative pseudo velocity plateau shock spectrum values. I want to calculate both because we will ultimately want them. These maxima occur when $\dot{z} = 0$. From differentiating Eq. (13):

$$\dot{z} = \frac{z_0}{\omega\eta}\left[-\varsigma\omega e^{-\varsigma\omega t}\sin\eta\omega t + \eta\omega e^{-\varsigma\omega t}\cos\eta\omega t\right]$$
$$= \frac{z_0}{\eta} e^{-\varsigma\omega t}\left[-\varsigma\sin\eta\omega t + \eta\cos\eta\omega t\right] \qquad (14)$$

Two maxima occur in the first cycle when the bracketed RHS factor in Eq. (14) is zero. From Fig. 1 notice that the larger first value will be negative and the second value positive. I want to calculate the ratio of the maximum and minimum pseudo velocity to the impact velocity for a set of dampings. I will call these R_1 and R_2. To get these we divide Eq. (13) by the impact velocity, $\dot{z}_0$, and multiply it by ω. The R values are given by the two ηωt values from Eq. (14) substituted in Eq. (15).

$$R = \frac{\omega z}{\dot{z}_0} = \frac{e^{-\zeta\omega t}}{\eta} \sin \eta \omega t \quad (15)$$

I wrote a Matlab script to do this and the results are tabulated below:

Damping Table

ζ	R_1	R_2	R_2/R_1
0	1.0000	-1.0000	1.0000
0.0050	0.9922	-0.9767	0.9844
0.0100	0.9845	-0.9541	0.9691
0.0200	0.9695	-0.9104	0.9391
0.0300	0.9548	-0.8689	0.9100
0.0400	0.9406	-0.8294	0.8818
0.0500	0.9267	-0.7918	0.8545
0.1000	0.8626	-0.6290	0.7292
0.1500	0.8062	-0.5005	0.6209
0.2000	0.7561	-0.3982	0.5266
0.2500	0.7115	-0.3162	0.4443
0.3000	0.6715	-0.2500	0.3723
0.3500	0.6355	-0.1965	0.3092
0.4000	0.6029	-0.1530	0.2538
0.4500	0.5733	-0.1177	0.2053
0.5000	0.5463	-0.0891	0.1630

This is disappointing, but true. I cannot teach that the simple shock machine shock PVSS plateau. is at $\sqrt{2gh}$. It's only true for the undamped case. From the table it's down to 93% for 5% damping and in the negative direction at 80%; and for 10% damping it's down to 86% and 63%.

Damping Makes the 2g Line Approximate: The 2g line, a cute concept, is only good for undamped, no rebound simple shocks. It's still handy because it generally roughly shows the LF limit of the plateau, and indicates a general drop height.

Damping in the PVSS on 4CP Shows the Polarity of the Shock: Polarity is the ratio of positive and negative PVSS content in the plateau region of its PVSS. I hope it is obvious that the simple pulse tests have a strong polarity. By this I mean that that the shock is a lot more severe in the direction of the shock than the opposite direction. As an example MIL-STD 810 [20] and the IEC [21] spec both require three hits in the positive and negative directions to account for this, which seems wise to me. Unfortunately, the undamped PVSS of simple shocks shows equal positive and negative amplitudes in the high shock severity plateau region. This is because of the SDOF being undamped, ring with equal positive and negative amplitudes. The damping affects the severe velocity plateau region, but not the asynmptotes. In the 4th column of the Damping Table, I have listed the ratio of the negative to positive plateaus. Since stress is proportional to the plateau levels, simple shock machine shocks cause a reduced stress level in the opposite direction given by the ratio R_2/R_1.

It takes heavy damping to show positive and negative charateristics of the pulse. I assume the simple shocks like the half sine are as “polarized” as a shock can get. For an example I show positive and negative shock spectra of a 200 ips, 100 g half sine with zero and 20% damping to show the polarity of what I consider a grossly polarized shock in Figs. 5a and 5b.

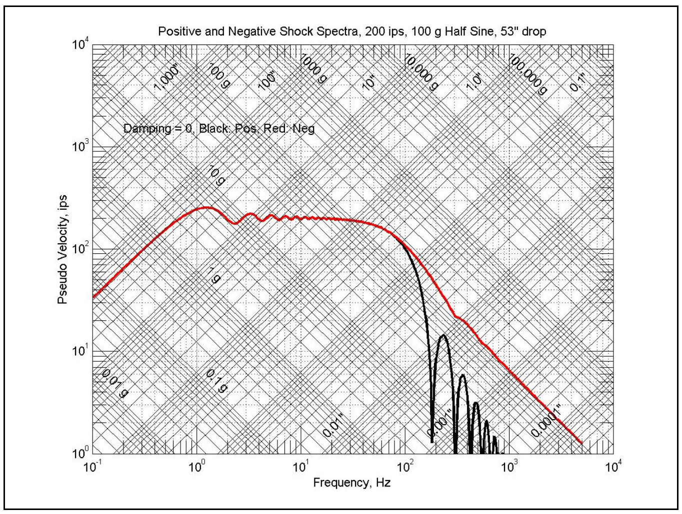


Figure 5a. Positive and negative PVSS for an undamped 200 ips, 100g, half sine. The negative spectrum only exceeds the positive at high frequencies where the PV is low.

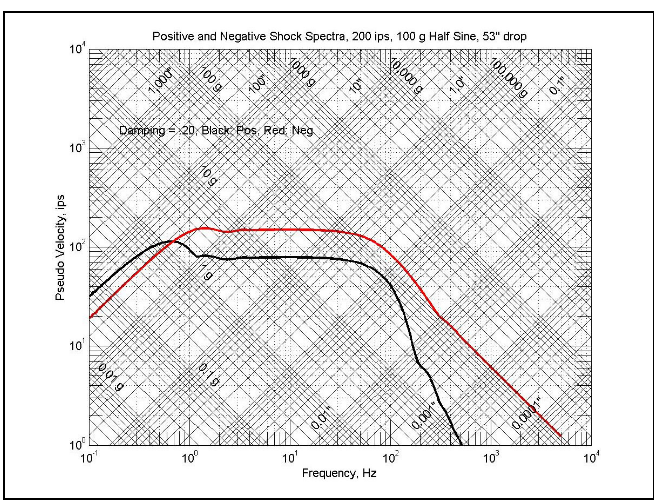


Figure 5b. Positive and negative PVSS for a 20% damped 200 ips, 100 g, half sine. The negative spectrum strongly exceeds (twice) the positive in the high PV plateau.

Modal Velocity is Proportional to Stress, Not G's or Acceleration. THE STRESS IS GIVEN BY $\sigma = k\rho cv$: Chalmers and I published a paper in 1969 [7] in which we proved that modal velocity was proportional to stress in bending vibrations of beams and longitudinal vibrations of rods. The proof uses the partial differential equations for vibrating beams and rods. When vibrating at one of their natural frequencies, one finds that the maximum stress at the maximum stress point in the body, is directly proportional to the maximum modal velocity at the maximum modal velocity point in the body. The equation for the stress during axial or longitudinal, plane wave, vibration of a long rod in any of its modes is given in Eq. (16).

$$\sigma_{max} = \rho c v_{max} \tag{16}$$

Where:

σ_{max} = The maximum stress anywhere in the bar
v_{max} = maximum velocity anywhere in the bar.
$\omega_n = f_n/2\pi$ = frequency in radians/sec; f_n = is frequency in Hz. The sub script n implies the equation only applies at the natural frequencies
c = wave speed = $(E/\rho)^{1/2}$
E = Young's modulus
ρ = mass density; mass per unit volume

In any mode the motion is sinusoidal. At the antinode or peak velocity point, the displacement is given by v/ω and the maximum acceleration is given by $v\omega$; thus the maximum stress is also proportional the acceleration and displacement and is given by:

$$\sigma_{max} = \rho c \omega u_{max} = \rho c \frac{a_{max}}{\omega} \tag{17}$$

But notice, when expressed in terms of the maximum acceleration or displacement, frequency now enters the equation and peak displacement or peak acceleration alone does not indicate high stress. You have to also state the frequency along with the maximum displacement or acceleration of vibration to indicate a severe vibration. This is amazing; any axially vibrating rod, you can know the peak stress, if you measure the peak velocity.

When one analyzes the bending vibrations of beams you get almost the same results. The equation is Eq. (18) below.

$$\sigma = \frac{h}{\eta} \rho c v_{max} \tag{18}$$

The new symbols are given below:

η = radius of gyration = $(I/A)^{1/2}$
I = cross-sectional area moment of inertia about beam neutral axis
A = cross –sectional area
h = distance from the neutral axis to the outer fiber

For a beam vibrating in any one of its modes, stress is proportional to the peak modal velocity and it doesn't matter what the frequency is. Again if you find the position of highest modal velocity, and put that value in Eq. (18) you will get the maximum bending stress at the most highly stressed point on the beam. We could write Eq. (18) as:

$$\sigma_{max} = K_b \rho c v_{max} \text{.....where...} K_b = \frac{h}{\eta} \tag{19}$$

Here "K" is a beam shape factor. Again η is the radius of gyration of the cross section, and "h" is the distance from the neutral axis to the outer fiber. (Typical beam shapes are from 1.2 to 3.)

Hunt [8] gives a more scientific derivation and also did it for thin rectangular plates, tapered rods and wedges. He felt strongly that it extended to all elastic structure, and for practical situations the shape factor stays under two. He speaks of the maximum value of K being half an order of magnitude or 5.

There Are Absolute Limits to Modal Velocities That Structure Can Tolerate Modal Velocities Above 100 IPS Can be Severe. It is Doubtful That Anyone Ever Sees 700 IPS in Structural Modes: Some example severe velocities values are given in Table I. These are peak velocities to attain the indicated stress, not counting any stress concentrations, nonuniformities, or other configurations. For long term and random vibration, fatigue limits as well as the stress concentrations, and the actual configuration would make the values much lower. Stress velocity relations are used in statistical energy analysis [9].

Table I. Severe Velocities

Material	E (psi)	σ (psi)	ρg (lb/in^3)	v_{max} (ips) rod σ/(ρc)	v_{max} Beam Rectangular	v_{max} Plate
Douglas fir	1.92×10^6	6,450	0.021	633	366	316
Aluminum 6061-T6	10.0×10^6	35,000	0.098	695	402	347
Magnesium AZ80A-T5	6.5×10^6	38,000	0.065	1015	586	507
Steel Structural	29×10^6	33,000 100,000	0.283	226 685	130 394	113 342

Chalmers and I wrote it in 1969. [7] Hunt [8] knew this in 1960, Ungar [10] wrote about it in 1962, Crandall commented on it in 1962 [11], Lyon [9] finally seemed to be the first to use it in his 1975 book. I doubt it is yet being used in machine design, materials, or vibration texts. These are absolute limits and there is no getting around them.

Why Pseudo Velocity and not Absolute or Relative Velocities are Best For Shock Spectra: The relative velocity and the absolute velocities are real velocities. PV is a pseudo velocity. When we solve the transient excitation vibration problem for the lumped mass MDOF system, and when we work it out for a continuous beam with all its modes, we find that the induced modal velocity is determined by the PVSS equation.

Additionally, PV has the important low frequency asymptote of the peak shock displacement that is nice to know. PV happens to come out just about equal to relative velocity in the important high plateau region, and is about equal to relative velocity there. The relative velocity shock spectrum does not show the nice maximum acceleration asymptote either.

The Relative Velocity Spectrum has a Low Frequency Asymptote Equal to the Peak Shock Velocity. For the undamped simple shock situation in the plateau region, since the mass is left vibrating sinusoidaly, the maximum PV and relative velocity are identical. So in the undamped plateau for simple shocks they both have the same value, but at low frequencies there is major difference. Again think of the situation with a very heavy mass on a very soft spring. The mass doesn't even start to move until the shock is over. The peak relative velocity has to be the peak shock velocity and this becomes the low frequency asymptote for a relative velocity shock spectrum. I can’t ever remember seeing anyone use the relative velocity shock spectrum. I haven’t tried to explain how it behaves in the high frequency region, but in Reference [12] we show many calculated spectra that show it drops off to below the constant acceleration asymptote. Figure 6, shows an example; notice the maximum velocity low frequency asymptote and the useless high frequency asymptote. Also notice that both spectra are almost the same in the severe high PV plateau. The relative velocity shock spectrum doesn't have any nice features at all, and that’s why it doesn’t seem to be used.

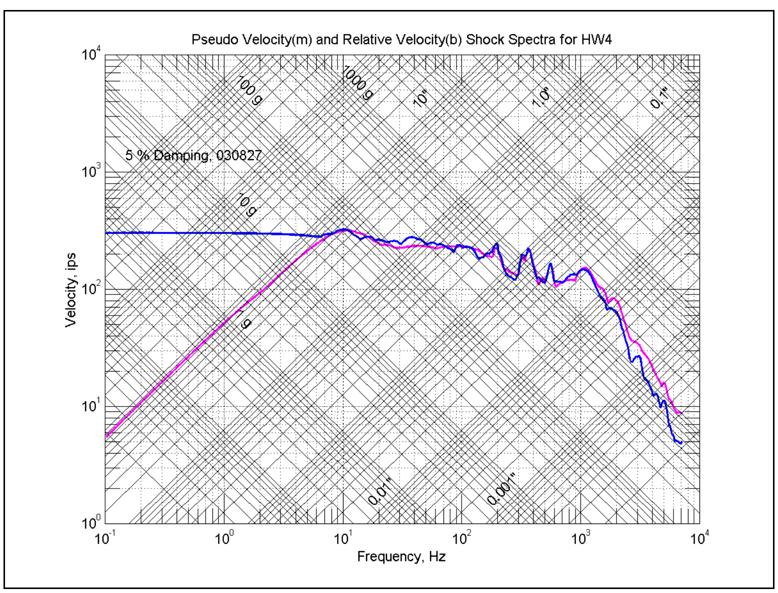


Figure 6. This a superposition of the PVSS and the relative velocity shock spectrum for a 5% damped explosive shock.

Lumped Mass Multi Degree of Freedom (MDOF) System Response is Proportional to Peak Pseudo Velocity: Scavuzzo and Pusey [13] present normal mode analysis of a lumped mass MDOF system excited by a shock in matrix terms as Eq. (20)

$$[m]\{\ddot{z}\}+[k]\{z\}=[m]\{1\}\ddot{y} \tag{20}$$

They developed a modal solution of the motion of each mass as an element of the vector {z}. The motion of each of the masses, z_b, is the sum of the motion in each mode where Z_a is a^{th} modal vector, and q_a is time (history) response of the a^{th} mode.

$$\{z\}=\sum_{a=1}^{N}\{Z_a\}q_a \tag{21}$$

$$\{\ddot{z}\}=\sum_{a=1}^{N}\{Z_a\}\ddot{q}_a \tag{22}$$

After finding the mode shapes, we substitute these into Eq. (20) and obtain the time response of each mode by solving Eq. (23).

$$\ddot{q}_b+\omega^2 q_b=-P_b\ddot{y} \tag{23}$$

This is our old friend the undamped SDOFs shock spectrum Eq. (6), except that the shock acceleration is multiplied by P_b, the participation factor for that mode. If $\omega_b q_b$ is the modal pseudo velocity of the b^{th} mode, we see that the modal pseudo velocity for mode "b," is the product of the participation factor times the PVSS value at the mode "b" modal frequency.

$$\omega_b q_b = P_b PVSS(\omega_b) \tag{24}$$

Thus, P_b times the undamped PVSS determines the peak modal pseudo velocity in each mode.

The Modal Velocity of Undamped Continuous Systems and Hence the Stress is Proportional to the PVSS at the Modal Frequency: The shock excitation of a simply supported beam illustrates the multi degree of freedom elastic systems shock response problems. You start with the beam vibration partial differential equation [14] given by Eq (25).

$$\frac{\partial^2 y}{\partial t^2}+\frac{EI}{\rho A}\frac{\partial^4 y}{\partial x^4}=0 \tag{25}$$

You solve this for the simply supported end conditions and find that the simply supported beam free vibration solution given by Eq. (25a).

$$y = \left(A\sin\omega_n t + B\cos\omega_n t\right)\sin\frac{n\pi x}{l} \quad n = 1,2,3,\ldots \tag{25a}$$

This says the beam can indeed undergo free vibrations, but only in modes where n is a positive integer. The natural frequencies are given by:

$$\omega_n = \frac{n^2\pi^2}{l^2}\sqrt{\frac{EI}{A\rho}} \tag{26}$$

Where:

I = cross-sectional area moment of inertia about beam neutral axis
A = cross –sectional area
$\omega_n = f_n/2\pi$ = frequency in radians/sec; f_n = is frequency in Hz. The sub script n implies the equation only applies at the natural frequencies
c = wave speed = $(E/\rho)^{1/2}$
E = Young's modulus
ρ = mass density; mass per unit volume
l = beam length

Now we write Eq. (25a) as a shape function and a time function defining the shape function as:

$$\phi_n(x) = \sin\frac{n\pi x}{l} \tag{27}$$

The time function is Eq. (28).

$$\begin{aligned} q_n &= A\sin\omega_n t + B\cos\omega_n t \\ \dot{q}_n &= \omega_n A\cos\omega_n t - \omega_n B\sin\omega_n t \end{aligned} \tag{28}$$

Using Eqs (27) and (28) we can write Eq. (25a) as:

$$y = \phi_n q_n \tag{29}$$

Now we find the response to a base excited shock motion, $\ddot{z}(t)$, will be the sum of the motions in each of it's modes:

$$y(x,t) = \sum_{n=1}^{\infty}\phi_n(x)\,q_n(t) \tag{30}$$

The trick is to say that y is the motion relative to the supports, and z is the motion of the supports (the shock). Making that substitution into Eq (25), after a page and a half of manipulating, we find that the time function for each mode has to satisfy:

$$\ddot{q}_n + \omega_n^2 q_n = -\frac{4}{n\pi}\ddot{z}\,, \quad for \quad n = 1,3,5,7,9,..... \tag{31}$$

This is great. Except for the coefficient in front of $\ddot{z}$, (call is a participation factor, P_n) this is the forced SDOFs equation used to calculate the PVSS, the shock spectrum. A shock applied rigorously to a simply supported beam leads to the same equation used to calculate the PVSS.

$$\omega_n q_{n,\max} = \frac{4}{n\pi} PVSS(\omega_n) \tag{32}$$

I assure you if we do the same thing for a plate or a shell, we'll get the same type of result.

NOW THIS IS ABSOLUTE PROOF THAT THE MAXIMUM MODAL VELOCITY OF A BEAM EXPOSED TO SHOCK IS GIVEN BY A PARTICIPATION FACTOR TIMES THE SHOCK PVSS VALUE AT THE MODAL FREQUENCY. MAXIMUM MODAL VELOCITY IS DIRECTLY PROPORTIONAL TO MAXIMUM STRESS.

An Important MDOF Lesson is That Elastic Systems only Accept Shock Energy at Their Modal Frequencies: In both lumped mass and the continuous elastic cases: these elastic systems (our equipment) only accept shock energy at their modal frequencies. To damage equipment, the shock PVSS plateau has to be high at these modal frequencies. And it's important to point out that equipment has a lowest modal frequency; no highest.

Shock Isolation is Accomplished by Blocking High PV Shock Content at Equipment Modal Frequencies. This is Done With a Damped Elastic Foundation or Raft Which Reduces The PVSS in the High Frequency Region: From the MDOF analyses of both lumped mass and the simply supported beam example we found that linear structure only accepts shock transient energy at it modal or natural frequencies. It only undergoes dynamic elastic deflections at its modal frequencies. ALL EQUIPMENT HAS A LOWEST NATURAL FREQUENCY. If we can prevent high PV shock content at the low mode frequency and above from entering the equipment, we can protect the equipment. We can with isolators; we mount the equipment on a damped spring so that the equipment becomes the mass. Consider the severe shock motion shown in the PVSS of Fig. 7a. This has severe PV content above 200 ips from 4.5 to 400 Hz.

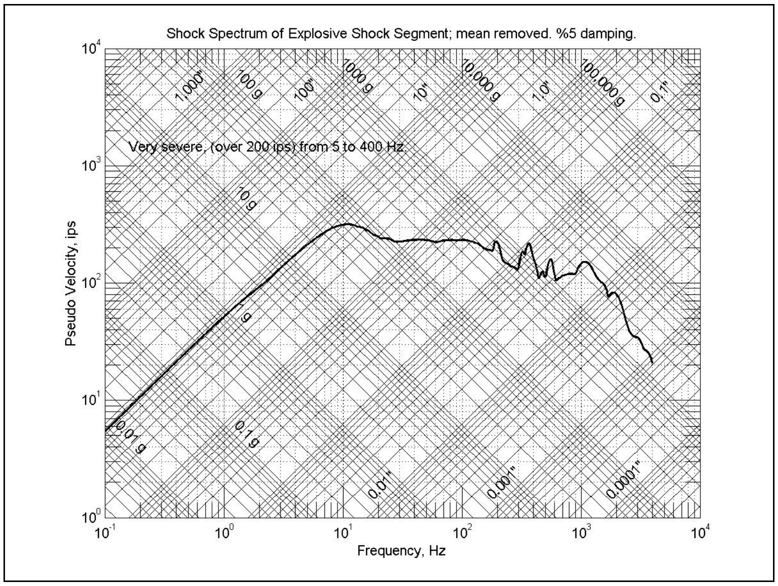


Figure 7a. Five percent damped PVSS of an explosive shock motion.

Say our equipment had a low mode frequency of 20 Hz; this shock has over 200 ips PV content at this frequency and just above, and would probably fail the equipment. We'll try isolating at 4 Hz with a 15% damped isolator. We mount the equipment on an isolator so that the equipment-isolator combination, behaves like a 15% damped SDOFs with a natural frequency of 4 Hz. At this low a frequency the equipment behaves like a mass and has no dynamic elastic deflections. Figure 7b is the time history of the explosive shock of Fig. 7a.

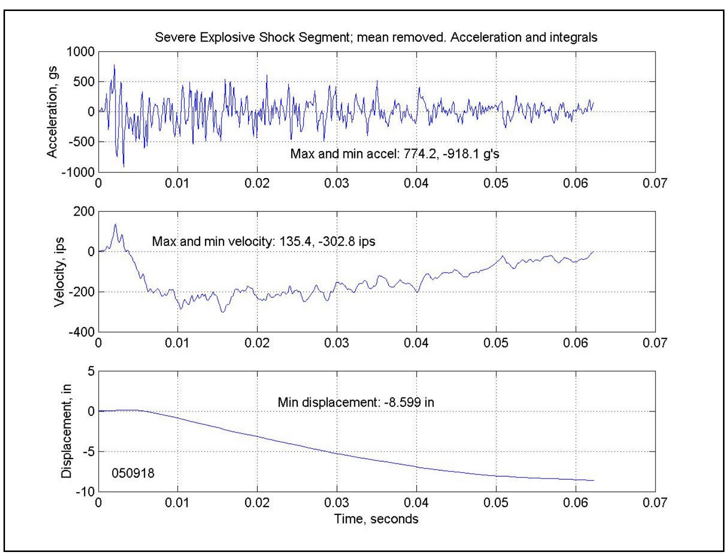


Figure 7b. Time history of the explosive shock to be isolated.

I have modified my SS (shock spectrum) program to calculate a list of absolute mass accelerations for any frequency and damping of an SDOFs exposed to a shock. Then I calculate a shock spectrum of this motion to see what the isolation has done. To see what an isolator can do for us, consider a 15% damped 4 Hz SDOFs. Figure 8a is the resulting shock motion of the SDOFs mass. Now in Fig. 8b I show the PVSS's for both motions and we can see what has been accomplished. Notice the severe PV frequency range of each PVSS. The isolated PVSS has low content at 20 Hz and above.

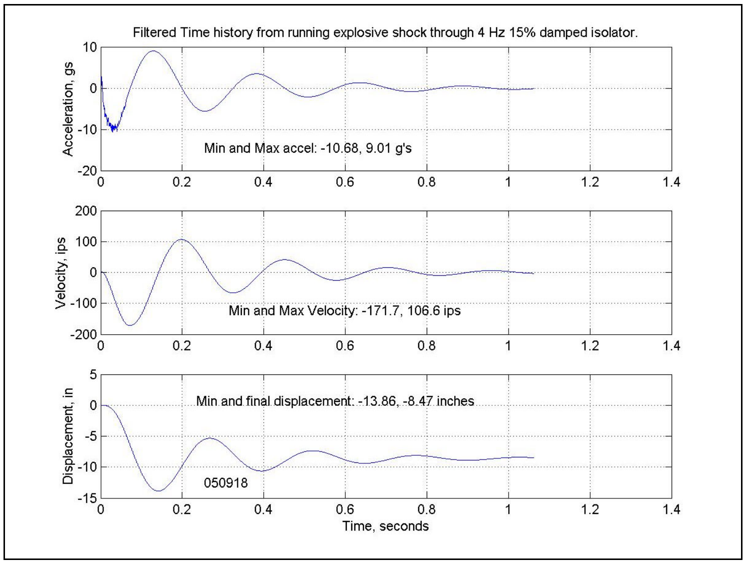


Figure 8a. The motion of a 4 Hz, 15% damped SDOF mass exposed to the shock of Figure 7b.

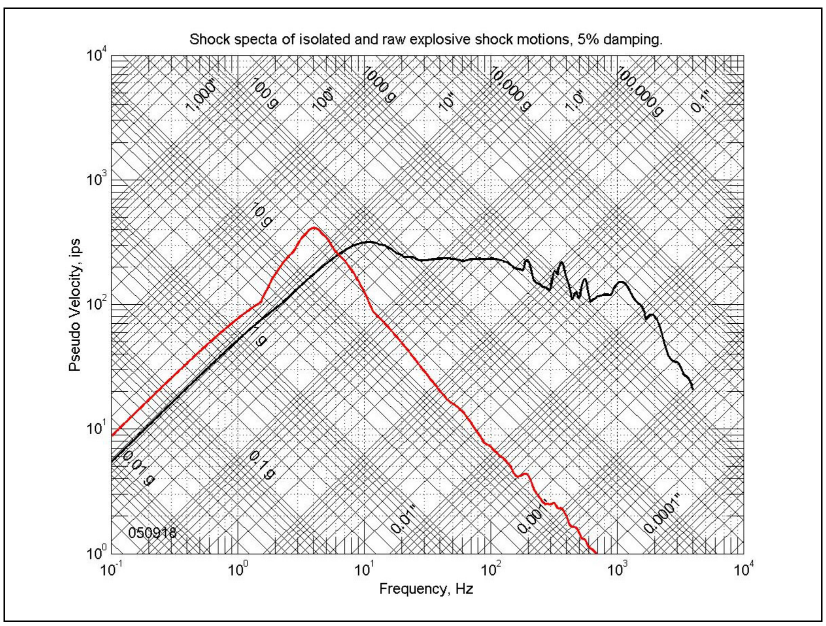


Figure 8b. This shows the PVSS of the Figure 7b motion, and the PVSS of the Figure 8a motion. The isolation is successful.

Pseudo Velocity is the Square Root of One Half the Energy Per Unit Mass Stored in the SDOFs. As Such the PVSS 4CP Shows the Frequencies and the Energy Density the Shock is Able to Deliver to an SDOF. This is One Reason it Works so Well: The PVSS shock spectrum algorithm finds the peak relative displacement for a base excited SDOF. That peak "z" is closely related to the maximum energy stored in the elastic member during the transient event. The energy "U" stored in the spring at any instant is $kz^2/2$. Thus energy per unit mass would be:

$$\frac{U}{m} = \frac{1}{2}\frac{k}{m}z^2 = \frac{1}{2}(\omega z)^2 \quad and \quad \omega^2 = \frac{k}{m}$$
$$PV = \omega z = \sqrt{\frac{1}{2}\frac{U}{m}} \qquad (33)$$

The reasons why PV is such a good damage indicator are a little difficult, but this energy argument is extremely important. Additionally, peak modal velocity in elastic structure is proportional to peak stress, and not acceleration.

Integrating Shock Acceleration to Velocity and Displacement Provides Useful Information. You Must Interpolate the Data to Have at Least 10 Samples Per Period of the Highest Frequency Present: An important part of shock analysis is integrating the acceleration time history to velocity and displacement (Fig 9).

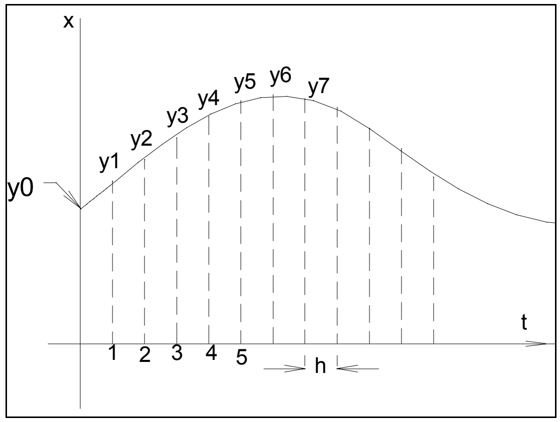


Figure 9. Function of time or acceleration time curve to integrate to velocity.

Let's discuss the trapezoidal or "straight-line-between-data-points" approximate discrete integration. Our sampling rate has to be high enough so that a straight line drawn between the points is a good enough approximation to the "true" curve. In Matlab, this usually means to interpolate a "signal-analyzer-sampled-signal" by 4. This provides ten samples per period of the highest frequency present. (I have heard people say to interpolate it by 6.)

Imagine y(t) is the function we are going to approximately integrate. The area under the curve between y1 and y2, a time h apart, dA, using this straight line approximation, is:

$$\mathrm{dA} = \frac{\mathrm{y1} + \mathrm{y2}}{2}\mathrm{h} \qquad (34)$$

And between y2 and y3 is similarly:

$$\mathrm{dA} = \frac{\mathrm{y2} + \mathrm{y3}}{2}\mathrm{h} \qquad (34b)$$

The total area between y1 and y6 would be:

$$\begin{aligned}\sum \Delta A &= \frac{y1+y2}{2}h + \frac{y2+y3}{2}h + \frac{y3+y4}{2}h + \frac{y4+y5}{2}h + \frac{y5+y6}{2}h \\ &= \left(\frac{y1}{2} + y2 + y3 + y4 + y5 + \frac{y6}{2}\right)h\end{aligned} \qquad (34c)$$

If the beginning and ending y values are zero or small, the fact that they are halved wouldn't matter, and we could say that the integral of y(x) when we have an equally spaced set of discrete values of y_i are given by:

$$\int_{t_1}^{t_2} y dt = h \sum y_i \qquad (34d)$$

“h” is one over the sample rate f_s. That’s how we can say the sum of the data is its integral.

By Examining Shock Acceleration Integration as a Straight Line Between the Points, and Assuming the Initial and Final Values are Zero, One Finds the Integral Equal to the Sum of the Values Divided by the Sampling Rate: Since my shock spectrum calculating algorithm [15] approximates the acceleration as a straight line between the points, I do the same thing for the time histories. Using Matlab I interpolate the data so that it is digitized to 10 sample per highest frequency present; this allows me to integrate successfully as well. I have a little Matlab script to accomplish this plot the three time histories one on top of the other as indicated in Fig. 10, which shows an as received El Centro Earthquake time history.

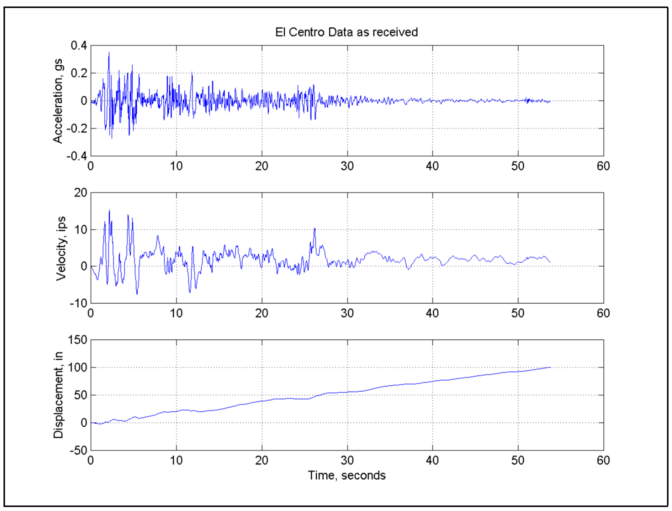


Figure 10a. El Centro acceleration time history as received and two integrals.

In Fig. 10a we see the peak velocity and acceleration, the largest velocity changes, which often tells you something about the plateau. Since the velocity does not end at zero, the PVSS-4CP will not have a low frequency asymptote.

Shock time history editing is difficult. The topic includes high and low pass filtering and discrete wavelet filtering. The pyro shock appendix of [28] presents some concepts.

Editing by Removing the Mean. If the Mean Value of the Shock is Zero, it Has no Velocity Change, and Thus Will Have a Displacement Asymptote: From the section on integrating, we see that the sum of the data values divided by the sampling rate is its integral. When we remove the mean from the data, we make this sum zero, which means the integral of the shock acceleration is zero and that there was no velocity change. If we take the initial velocity to be zero, the final velocity is zero. This is true for a large group of important shocks, but judgment is needed. These shocks have a peak displacement asymptote equal to the maximum deflection during the shock, an important quantity to know. Figure 10b, shows El Centro with the mean removed.

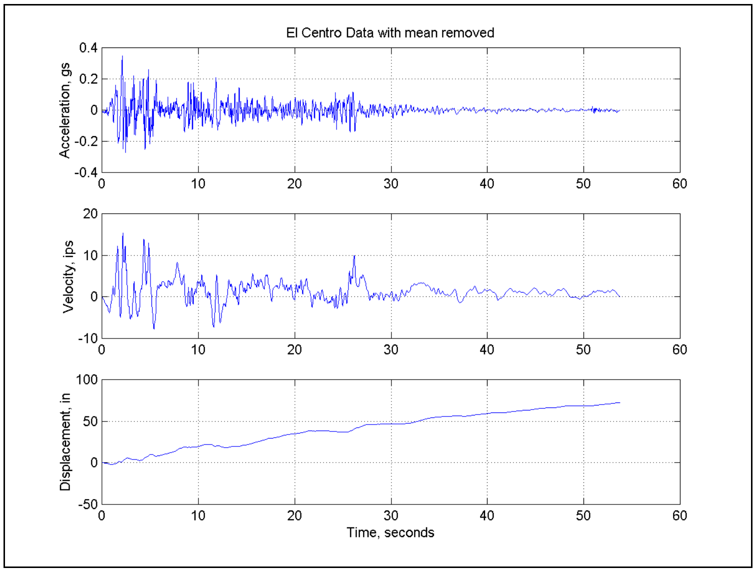


Figure 10b. El Centro with the accelkeration mean reoved. Note that the velocity ends at zero and now the peak displacement is about 70 inches.

Detrending Shock Data Makes the Initial and Final Value of the Displacement Zero. This is Often a Good Editing Technique: Detrending, or removing any linear trend from the data, causes the displacement to end at zero. There may be times when you have reason to believe that the final displacement was indeed zero, or you have a reason to display the data with the final displacement zero. In that case detrend the data. Figure 10c shows El Centro with the acceleration detrended.

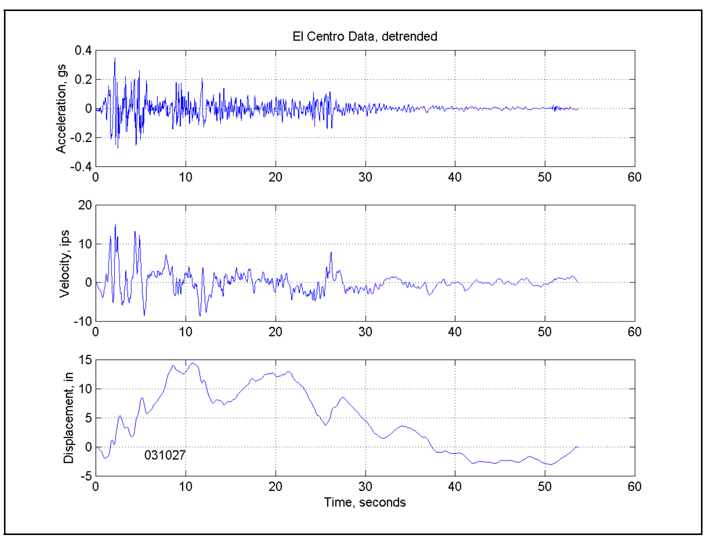


Figure 10c. The El Centro earthquake acceleration detrended and its integrals. Now notice that final displacement is zero, and that peak displacement is a little under 15

inches. An expert told me that he believes this to be a better representation of what actually happened.

Shock Analysis Methods Were Tested Against Six Equally Severe Different Shock Tests. Methods Tested Were Acceleration Time History, Acceleration Shock Spectra, Fourier Transform Magnitude and PVSS-4CP. The Best Analysis is the Damped PVSS-4CP. It Shows the Five Failure Causing Environments Similar, and the Sixth Environment Weak: A series of tests was conducted to evaluate several transient shock motion analysis methods to determine the best indicator of shock severity [16, 17, 18, 18a]. Six squirrel cage blowers were exposed to six different shocks that could be incrementally increased until the blower failed. The acceleration time histories of the six different strongest mechanical shocks applied to six identical blowers were collected; five of the shocks were just sufficient to cause the failure of each blower. A sixth shock did not fail the blower. The five failure causing shocks are equally severe. The acceleration time histories, the Fourier transform magnitudes, the damped and undamped overall acceleration and pseudo velocity shock spectra were compared to see which data analysis method would show the equally severe shocks most similar. Acceleration shock spectra could not identify the weaker shock. The 5 or 20% damped overall pseudo velocity shock spectra look the best. They showed the weaker LW72 shock weaker than the rest. It is my opinion, based on this evidence and theoretical proofs that stress is proportional to modal velocity, that if one if forced to compare the severity of drastically different shock time histories, one should compare their damped overall pseudo velocity shock spectra. The appropriate damping level was not determined; the range of five to twenty per cent was adequate.

The six analyzed shock are shown in Fig. 11, and are described as:

HS54, a 54-inch drop half sine
TP60, a 60-inch drop- terminal peak
PB24, the 24-inch drop onto a hard phenolic block
LW72, a 72-inch hammer drop on the Navy Lightweight shock machine
MW36, a 36-inch blow on the Navy Medium Weight shock machine
HW4, a 4th shot on the Navy Floating Shock Platform

By examining Figure 11, I hope you'll conclude that G’s as any kind of a concept of shock severity is useless, even in the face of 50 years of tradition. This then necessarily argues that all design methods using g’s are probably wrong and must be critically re-examined.

In the region beyond 100 Hz, the Fourier transform magnitudes cover a band of velocities and accelerations of three orders of magnitude. The plots were somewhat puzzling scribbling in the high frequency range. They were of no value especially in showing LW72 the weakest shock. The acceleration shock spectra, undamped or damped could not show LW72 the weaker shock as well.

Looking at the undamped PVSS of LW72 (which did not fail the blower) it had higher amplitudes than PB24 and HS54 (which did fail the blower.), so the undamped pseudo velocity spectrum is unsatisfactory from this higher ordinates point of view. However, with 5% damping the LW72 amplitudes are convincingly below those of the five severe shocks. A small amount of damping reveals the curve to be less severe. The extreme 20% damping the LW72 shock appear even less severe.

On the PVSS 4CP, damping brings the curves closer together. Damping greatly reduces the levels composed of jagged peaks. High narrow peaks on the Fourier transform and on an undamped shock spectrum are due to ringing in the response at the frequency.

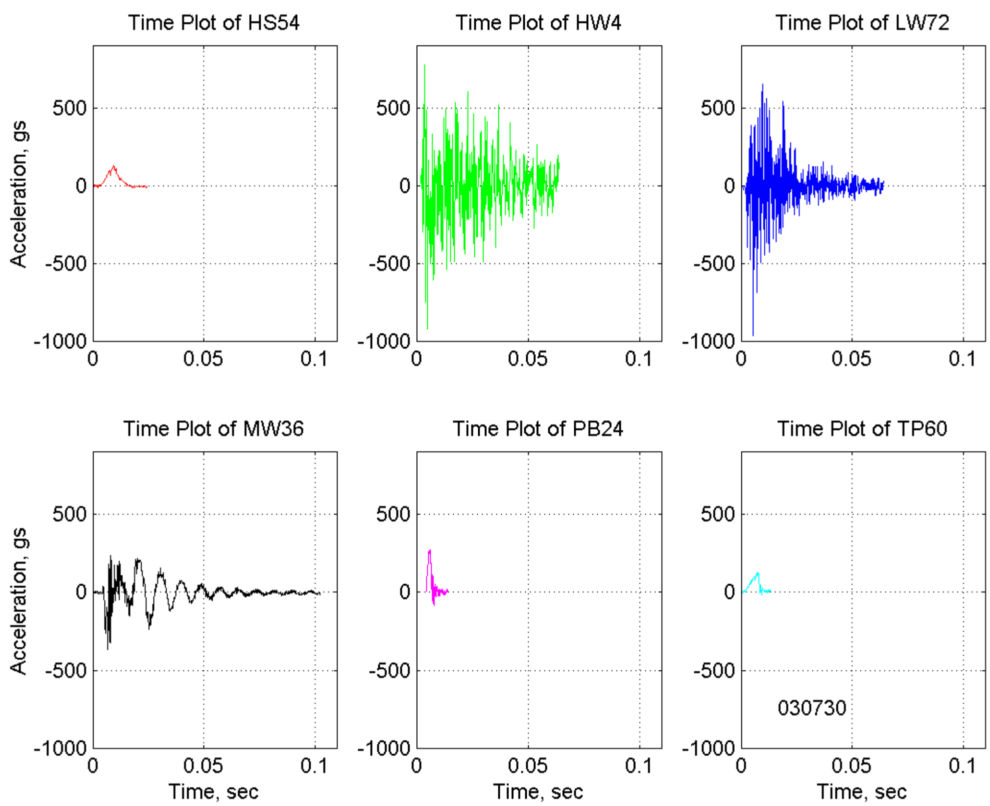


Figure 11. Here are the time histories of all six shocks. All but LW72 failed the blower. This should cure you from trying to estimate the severity of a shock in term of its g's.

Egg Fragility Tested to 19.6 IPS Plateau Example: As an example of the use of PVSS-4CP and perhaps a lower limit of equipment fragility, I carefully dropped a fresh refrigerated large egg from a short beam on gear driven tripod onto a 2-inch thick aluminum block. The egg was held with the long axis horizontal in a plastic sandwich bag with a 1-inch hole in the bottom exposing the shell surface. The bag was tied with a string to the tripod beam. I adjusted the height with the tripod crank and measured the drop distance with inside calipers. To drop the egg I cut the string with scissors. At one-half inch drop the egg was not damaged; from the 1-inch drop, the egg cracked. The undamped shock spectrum of the shock the shock the egg survived has to have a low frequency asymptote of one-half inch. This has to intersect the 2g line at a pseudo velocity of:

g = 386.087; h=.5; v=sqrt(2*g*h)
v = 19.6491 ips

From those values, we can sketch the egg fragility (the most severe shock the egg is known to have survived) on 4CP as shown in Fig. 12.

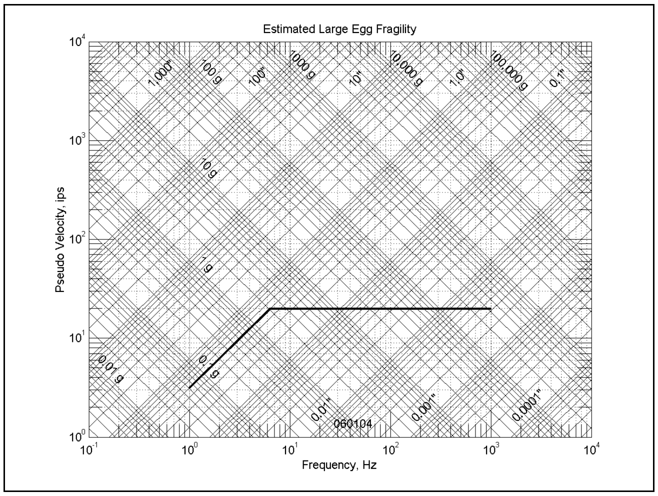


Figure 12. A estimated PVSS of a shock that an egg survived.

I have inserted this to give an idea of the lower limit of what we expect equipment to survive.

You Must add the Drop Acceleration to Simple Drop Table Shocks to See the Low Frequency Plateau Limit: Consider a half sine shock without including the shock machine table drop. I illustrate with a half sine shock that has a velocity change of 100 ips and peak "g" level of 200g's. Let's plot this shock along with its two integrals, and then calculate the shock spectrum for this unrealistic half sine shock. Integrating the half sine gives the duration to be 2.034 ms. Figure 13a shows the acceleration shock and its two integrals.

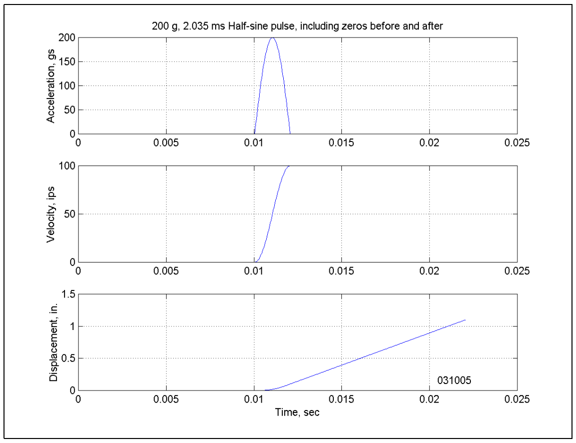


Figure 13a. Acceleration, velocity, and displacement of a half sine acceleration shock preceded and followed by zeros. The shock has a peak “g” level of 200g's, and a velocity change of 100 ips; and continues at this velocity forever.

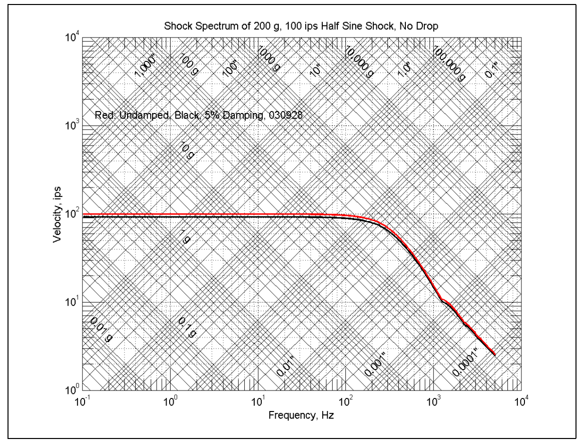


Figure 13b. The PVSS of a 200g, 100 ips half sine aceleration shock. In the lower right corner the spectra are asymptotic to 200g’s. From a little under 200 Hz down to 0.1 Hz the spectra show a constant velocity of 100 ips. That probably can not be.

Figure 13b shows the PVSS on 4CP as a half sine is usually analyzed. This is decptive and not true. The velocity change of 100 ips shows up as it should and the acceleration level is assympotic to 200g’s, as expected. But it does not have the low frequency asynptote. It has to be limited by the peak drop height which will be the intersection of the undamped plateau with the 2g line.

Now we consider a realistic half sine shock with the 1 g drop indicated in the acceleration time history. We drop it through a distance, such that a shock programmer delivering our half sine, just brings it to rest. The velocity begins and ends at zero, thus the shock will have a zero mean. The shock machine table falls with a 1 g acceleration for a time t_{drop}. The area (g x t_{drop}) equals the velocity change, and since $v^2 = 2gh$, the drop height can be computed. The time history plot and the integrals are shown in Fig. 14a.

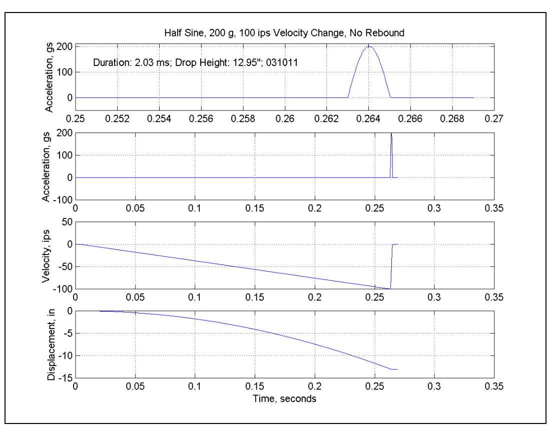


Figure 14a. A 200g, 100 ips half sine shock preceded by a 12.95-inch drop and its integrals.

This is a shock that could occur, and its PVSS on 4CP is shown on Fig. 14b.

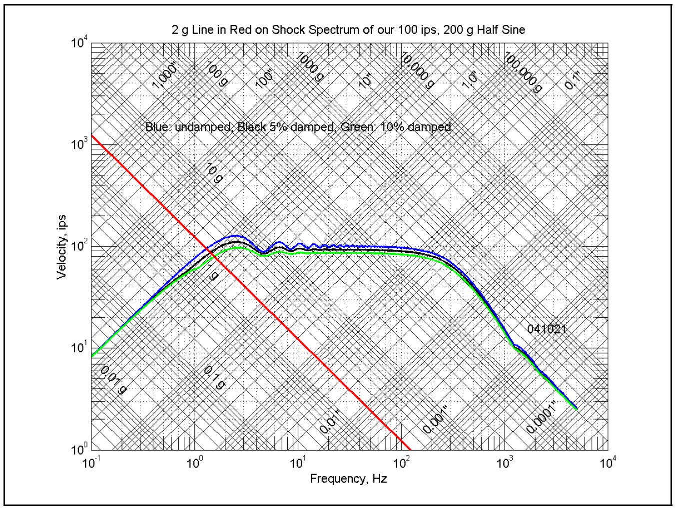


Figure 14b. Shock spectrum of a 100 ips, 200g half sine with the correct drop included.

Its shock spectra for 0, 5%, and 10% damping are shown. Notice now we see the 13-inch drop hieght assymptote at the low frequencies. We also see the 100 ips velocity change plateau (and the damping effect) and the 200g peak assymptote clearly. Finally, this figure shows the frequency range over which the shock is severe. It is most severe where its plateau is at 100 ips, from about 1.5 to 200 Hz.

DO NOT BE DECEPTIVE ABOUT THE SHOCK LOW FREQUENCY CAPABILITY. IF YOUR PVSS SHOCK SPECTRUM ROUTINE CANNOT ACCEPT THE LONG STRING OF 1 G VALUES FOR THE DROP, THEN DRAW THE KNOWN LOW FREQUENCY ASYMPTOTE IN AT THE KNOWN DROP HEIGHT. IF YOU KNOW IT DROPPED 53 INCHES, DRAW THAT CONSTANT DISPLACEMENT LINE IN ON THE PVSS 4CP PLOT WITH A NOTE THAT THE DEFLECTION ASYMPTOTE HAS BEEN DRAWN VICE CALCULATED.

All Drop Table Shock Machine Simple Shocks are Equally Severe and Have the Same PVSS on 4CP: One might expect this if we compare simple drop table shocks with the same velocity change on impact, which means the same drop height, and the same peak acceleration. This was observed many years ago by Gertel [19] and Vigness [25] showed it in his non-dimensional shock spectrum plots, although he did not come out and state it. I pointed this out in [6]. Two plots from that paper are given in Figs. 15a, and 15b.

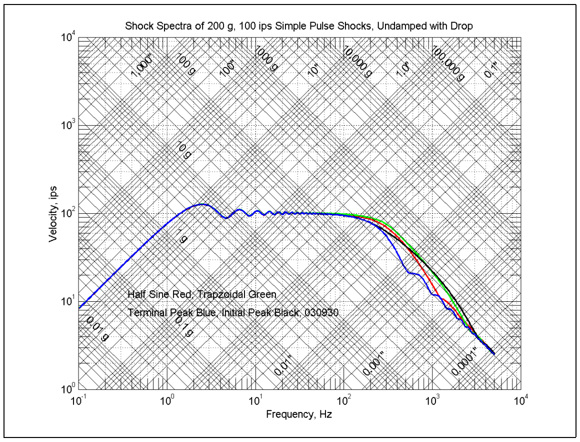


Figure 15a. Composite undamped shock spectrum plots of the half sine, the trapezoid, the initial peak and the terminal peak saw tooth shocks.

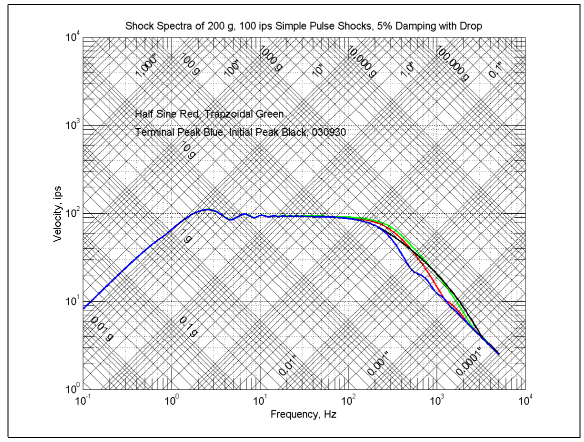


Figure 15b. Composite 5% damped shock spectrum plots of the half sine, the trapezoid, the initial peak and the terminal peak saw tooth shocks.

All of the simple pulses developed on a drop table shock machine by a programmer that results in zero velocity when the pulse is over will have a velocity change of the square root of 2gh. They will all have the same drop height or maximum displacement, hence the same low frequency asymptote. Since they all have the same velocity change, they all have the same plateau region. Since I adjusted the pulses to have the same peak acceleration, they all must have the same high frequency asymptote. The only way their shock spectra can differ are at the two corners, and this can be seen in Figs. 15a and 15b. I had trouble getting the acceleration asymptotes to appear in the trapezoid and the initial peak saw tooth. These have abrupt rise times that cause an initial doubling of the peak accelerations. I had to decrease the rise time abruptness by using a half cosine ramp rise of 30% in the trapezoid and 20% in the initial peak wave form, but this is allowed in the specification of attainable test shocks [20, 21].

Shock Spectra From Explosive Events Have a Similarity to Simple Pulse Tests, and can be Approximated: The explosive shock spectrum plot shown on shock isolation section, Fig. 7a has a peak acceleration of about 2,000g's. Its plateau is at about 210 ips, and its peak displacement is at 9 inches. The point here is that a simple pulse with a peak "g" level of 2,000g's, and a velocity change of 210 ips, would have a shock spectrum matching the severe and high frequency regions. That high "g" level would be hard to obtain on any large drop table shock machine. To attain the velocity change with only a 9-inch drop would require a bungee or spring assist, but it could be done, if a "g" level of 13.8gs during the drop could be arranged. This would give the 9-inch displacement low frequency asymptote. In this sense I make the statement that the simple pulse tests are similar to explosive shock spectra and can be used to test for them.

Rebound in the Drop Table Shock Machine Half Sine Shock Reduces the Low Frequency Content [6]: In the case of the half sine shock, there is often a coefficient of restitution to deal with because the shock can be formed by impact with a rubber like pad. In examining past data, I have found values from 0.3 to 0.5. With a rebound the required drop height is reduced because the velocity change is the sum of the falling velocity and the rebound velocity. In the case of a 0.33 coefficient of restitution, to keep the same velocity change, the required drop height is reduced from about 13 inches to 7.32 inches. The PVSS is shown in Figure 16b. The displacement asymptote is now at 7.3 inches, and the low frequency severe velocity range is decreased or the lowest severe velocity increased from about 1.5 Hz to about 3 Hz.

Shaker Generated Shocks Have a Severely Limited Low Frequency Asymptote and can Have Almost a Negligible Plateau. If You Know the Peak Displacement of the Shaker, Then You Know the Displacement Asymptote, Draw it in on the PVSS 4CP: Half sine shock tests are also conducted on an electrically driven shaker and these have a limited displacement capability [6]. Therefore, the PVSS on 4CP of a shaker generated shock will reflect this with a drastically reduced low frequency capability of shaker generated shock.

Lang [22] considers a host of pre- and post-pulses that allow the shaker armature to start from its center position, perform the half sine shock, and return the armature to its center position. Let's consider a half sine with rectangular pre and post pulses with a magnitude of θ times the maximum acceleration of the half sine to zero the mean. The area of the pre and post pulses must equal the velocity change from Eq. (2), which yields the pre and post pulse duration, t_p in Eq. (35). The time history and its integrals are given in Fig. 16a.

$$2\theta \ddot{x}_m t_p = \frac{\ddot{x}_m}{\pi f_d} = \frac{2\ddot{x}_m t_d}{\pi} \qquad t_p = \frac{t_d}{\pi\theta} \tag{35}$$

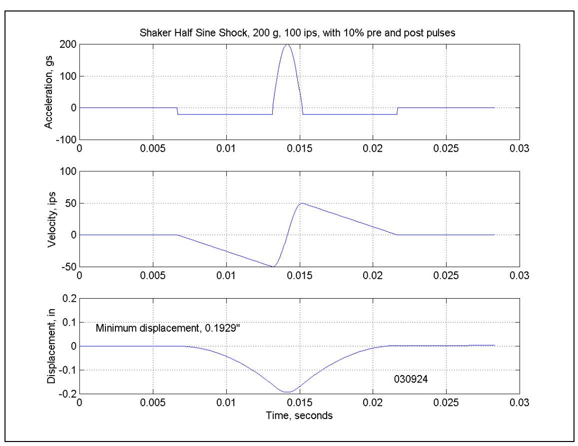


Figure 16a. Acceleration time history with 10% rectangular pre and post pulses to accomplish the 200g, 100 ips half sine shock on a shaker.

From the shaker owner's point of view, this is not good. The shaker armature motion is all in one direction, however the peak displacement is only about 0.2 inch, and this limits the low frequency severe portion of its shock spectrum. The undamped and 5% damped shock spectra are shown in the curves in Fig. 16b. The shock now is only severe from about 60 to 200 Hz. Centering the shaker armature so it is returnd to zero, further reduces the displacement to about 0.11 inch as well as the displacement assymptote.

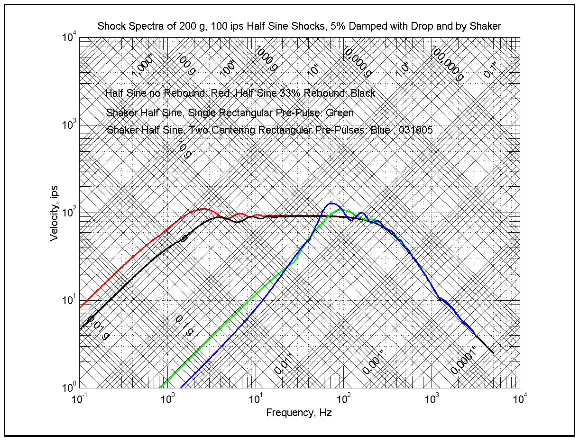


Figure 16b. Shock spectrum comparison of shaker and drop table 200g, 100 ips, half sine shocks for the 5% damped case.

Comparing shaker shocks with the drop table shocks, one notes a reduced high velocity severe region. Shaker simulated half sines would be inadequate for machinery and equipment with lower modal frequencies. This is including the shocks synthesizing a shock spectrum with a collection of oscillatory motions. The beauty of shaker shock is that the direction of the shock or its polarity can be reversed.

Damping Strongly Affects Multi Cycle Shocks: Multi cycle shocks are more severely affected by damping than simple shocks. To illustrate I have taken a specific synthesized earthquake shock [26] that I assume is performed by a hydraulic shaker. The acceleration time history and its integrals are shown in Fig. 17a.

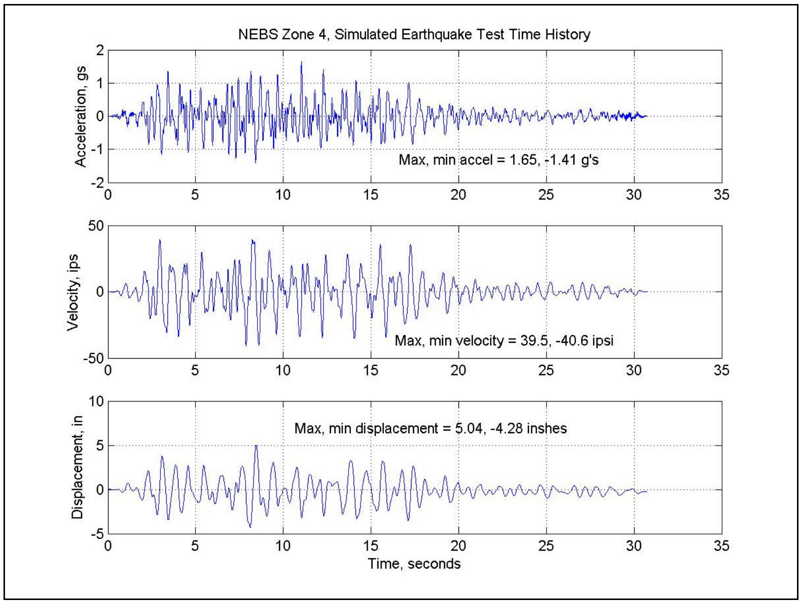


Figure 17a. NEBS Zone 4 Synthesized Earthquake Shock [26]

I think what we should notice here is that the peak velocity change is of the order of 80 ips, and the peak displacement is 5 inches. Notice its undamped PVSS shown in Fig. 17b goes to 800 ips.

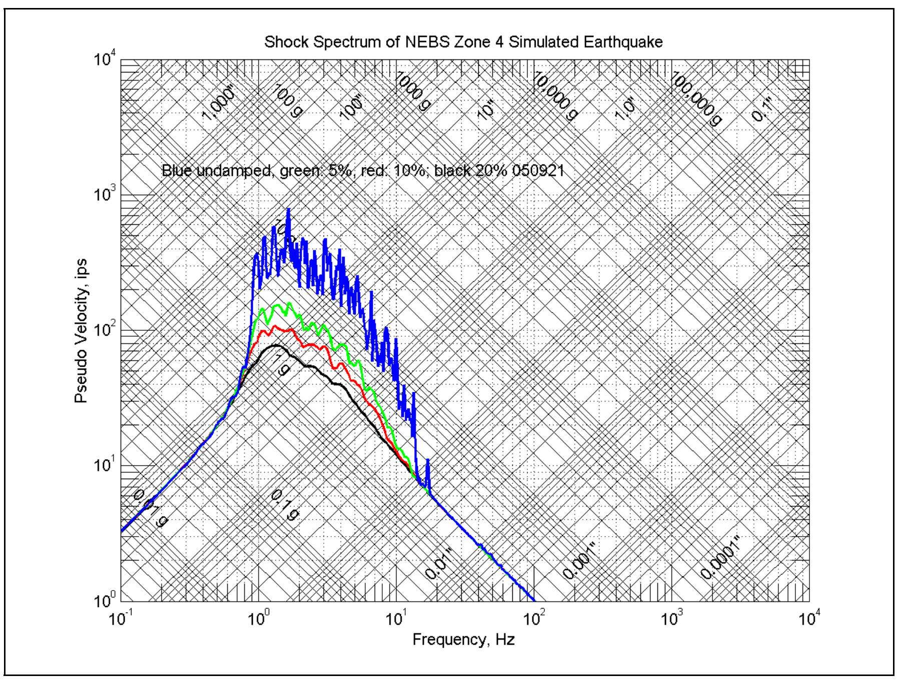


Figure 17b. PVSS of NEBS Zone for Multi Cycle Synthesized Shock

Note first that the reduction in maximum PV from undamped to 5% damped is from 800 to 150 ips. In a simple shock the plateau would only be reduced to 93% of the undamped value. Notice also that the peak "g" level and the peak displacement don't seem to cut off the plateau as in simple shocks. The shock also has a narrow plateau, which gets narrower with increasing damping. Depending on the damping, the plateau width runs from about 0.8 Hz to a little under 4 Hz.

The Extensive Literature Presenting Theoretical Calculations of Acceleration Shock Spectra is Unimportant: I could certainly give you ten well known references to burn, but I better not. I'm a little bit bitter because I wasted time trying to understand some of them. Theoretical acceleration shock spectra of simple pulses have been in vogue for a long time [24]. I assume there were many doctoral theses calculating them. Those writings may be classically interesting, especially to the authors who wrote them, but they have nothing to do with shock severity. Shock severity is why we study shock. The nuclear power design people and the earthquake community have successfully ignored that literature since the 40's. You can ignore it as well.

PVSS-4CP Previous USAGE History: My experience might have begun with Eubanks and Juskie's 1963 paper [23]. I have not read their references. It was well known in 1960 in the ASME publication on shock [24]. The nuclear defense engineers, and the earthquake community have never strayed from PVSS-4CP, although they do like to draw their 4CP with period vice frequency as the abscissa. Lalanne's exhaustive recent shock monograph has one example 4CP plot in the book.

FINAL COMMENTS

A sizable fraction of the rules presented here are original with me, but you can't call it research or scientific scholarly work. This is the dog work that teachers have to do to prepare a decent lecture. I hope that is what I have provided. Please tell me if you can think of better explanations for the rules I have tried to prove or justify, and certainly help me correct my mistakes.

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