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Introduction

Random Forcing Function and Response

Consider a turbulent airflow passing over an aircraft wing. The turbulent airflow is a forcing function. Furthermore, the turbulent pressure at a particular location on the wing varies in a random manner with time.

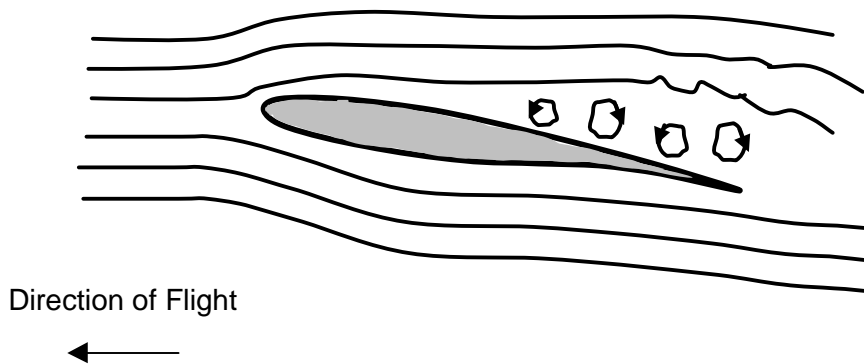


Figure 1.

For simplicity, consider the aircraft wing to be a single-degree-of-freedom system. The wing would vibrate in a sinusoidal manner if it were disturbed from its rest position and then allowed to vibrate freely. The turbulent airflow, however, forces the wing to undergo a random vibration response.

Random Base Excitation

Consider earthquake motion. The ground vibrates in random manner during the transient duration. Buildings, bridges, and other structures must be designed to withstand this excitation.

An automobile traveling down a rough road is also subjected to random base excitation. The excitation may become periodic, however, if the road is a "washboard road."

Common Characteristics

One common characteristic of these examples is that the motion varies randomly with time. Thus, the amplitude cannot be expressed in terms of a "deterministic" mathematical function.

Dave Steinberg wrote in Reference 1:

The most obvious characteristic of random vibration is that it is nonperiodic. A knowledge of the past history of random motion is adequate to predict the probability of occurrence of various acceleration and displacement magnitudes, but it is not sufficient to predict the precise magnitude at a specific instant.

Frequency Content

Pure sinusoidal vibration is composed of a single frequency. On the other hand, random vibration is composed of a multitude of frequencies. In fact, random vibration is composed of a continuous spectrum of frequencies.

Random vibration is somewhat analogous to white light. White light can be passed through a prism to reveal a continuous spectrum of colors. Likewise, random vibration can be passed through a spectrum analyzer to reveal a continuous spectrum of frequencies.

On the other hand, sinusoidal vibration is analogous to a laser beam, where the light wave is composed of a single frequency.

Statistics of a Random Vibration Sample

A sample random vibration time history is shown in Figure 2. This time history was "synthesized," or generated analytically. It has the descriptive statistics shown in Table 1.

Table 1. Random Vibration Descriptive Statistics	
Parameter	Value
Duration	4 sec
Samples	4000
Mean	0.00
Std dev	0.59 G
RMS	0.59 GRMS
Kurtosis	3.04
Maximum	2.30 G
Minimum	-1.96 G

Recall that pure sine vibration has a peak value that is $\sqrt{2}$ times its RMS value.

On the other hand, random vibration has no fixed ratio between its peak and RMS values. The ratio between the absolute peak and RMS values in Table 1 is

$$\frac{2.30 \text{ G peak}}{0.59 \text{ GRMS}} = 3.90 \quad (1)$$

Note that the RMS value is equal to the standard deviation value if the mean is zero. The standard deviation is often represented by sigma, σ .

Thus, the sample in Figure 2 has a peak value of 3.90σ . A different random sample could have a higher or lower peak value in terms of its σ , however.

A typical assumption is that random vibration has a peak value of 3.0σ for design purposes. Again, the example in Figure 2 deviates from this assumption with its peak value of 3.90σ .

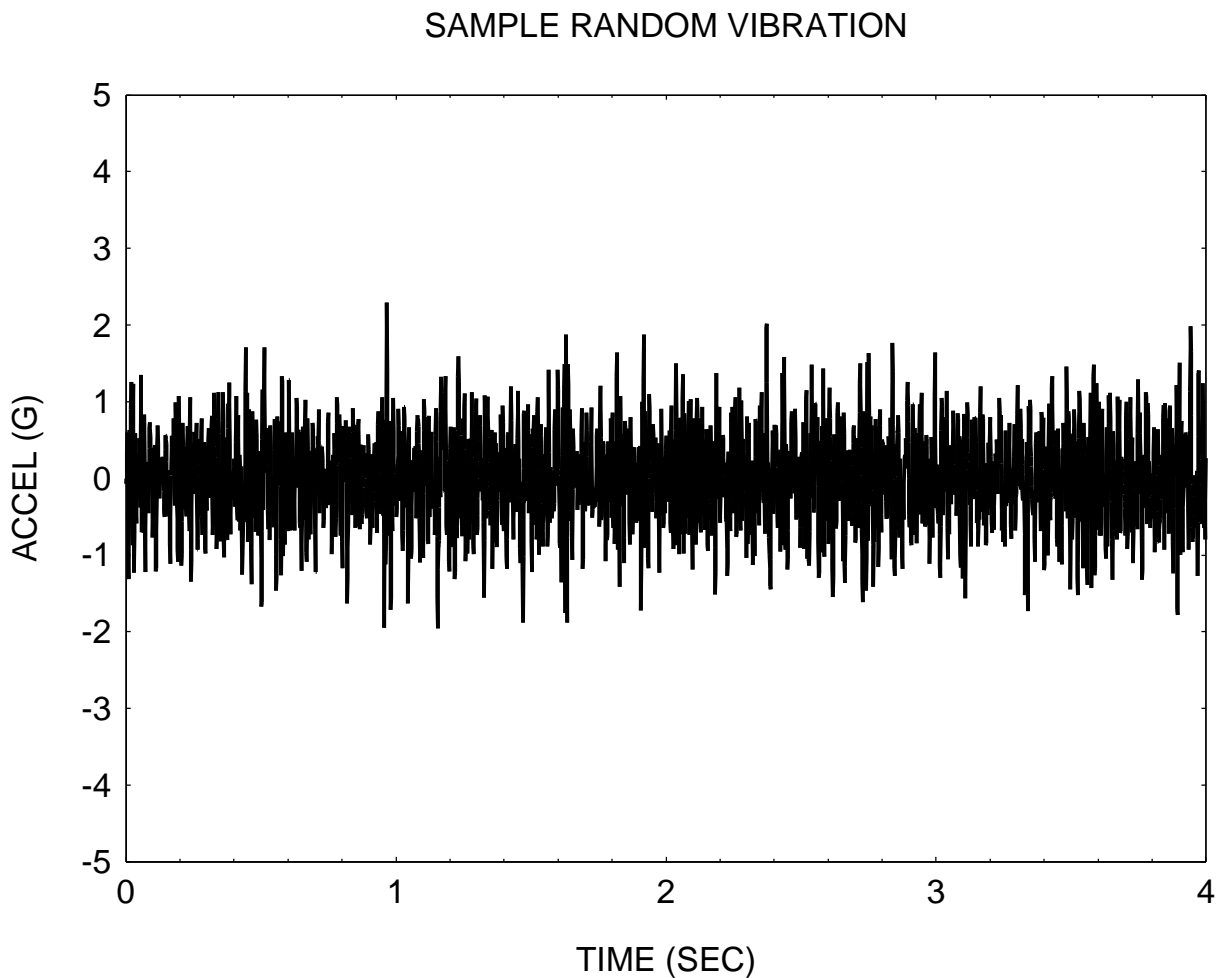


Figure 2.

The time history of Figure 2 is shown again in Figure 3. The amplitude in Figure 3 is scaled in terms of the σ value. According to theory, the amplitude should be within the $\pm 1\sigma$ limits 68.26% of the time.

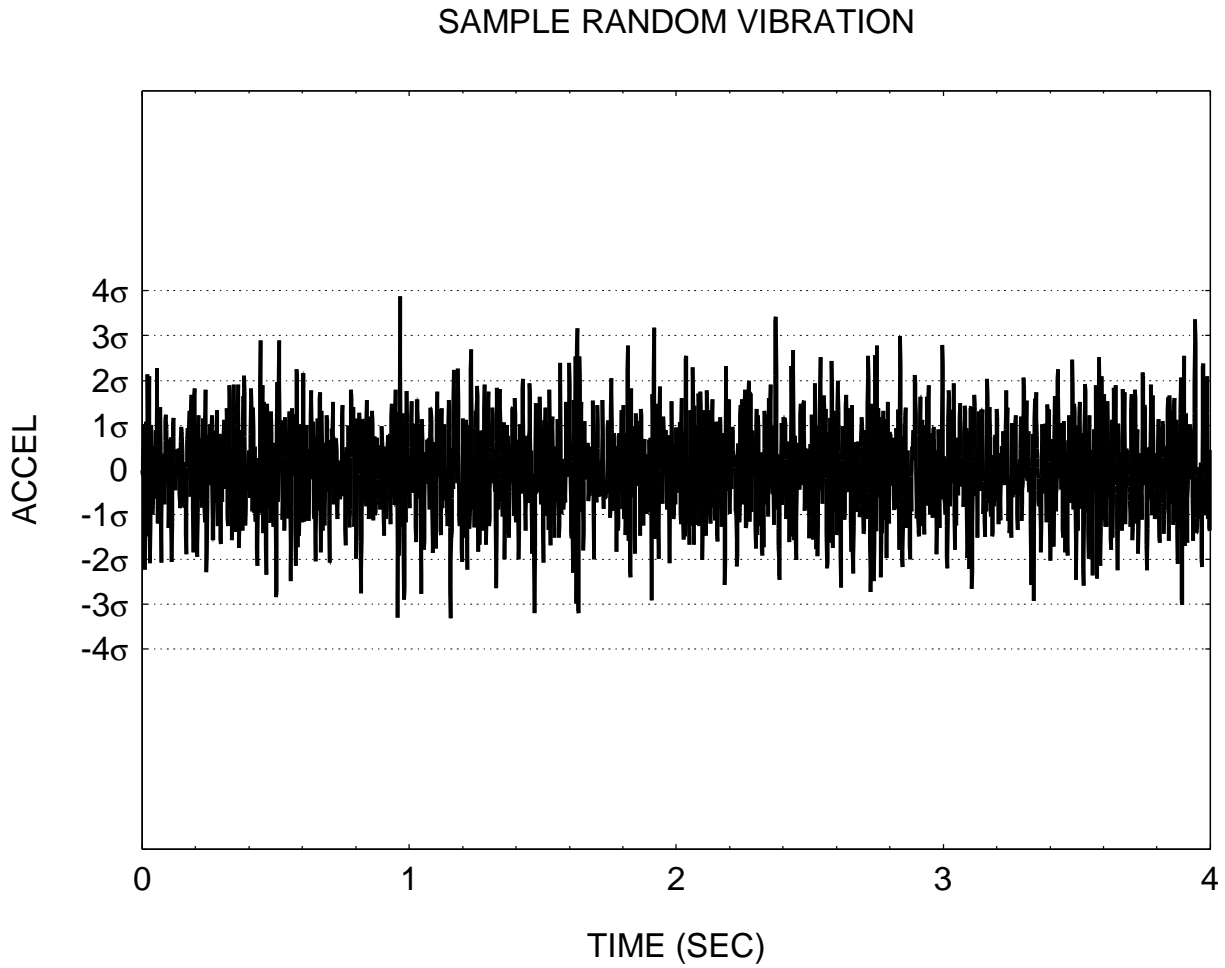


Figure 3.

Histogram

The histogram of the time history in Figure 2 is shown in Figure 4.

Note that the histogram of the random vibration sample has a "bell-shaped" curve. The histogram is an approximate example of a Gaussian or normal distribution.

The histogram shows that the random vibration signal has a tendency to remain near its mean value, which in this case is zero.

In contrast, recall the histogram of a sinusoidal signal has the shape of a bathtub. Sine vibration thus tends to remain at its positive and negative peak values.

For this and other reasons, sine and random are two very different forms of vibration. There really is no "equivalency" between the two forms, although many engineers have tried to derive a relation.

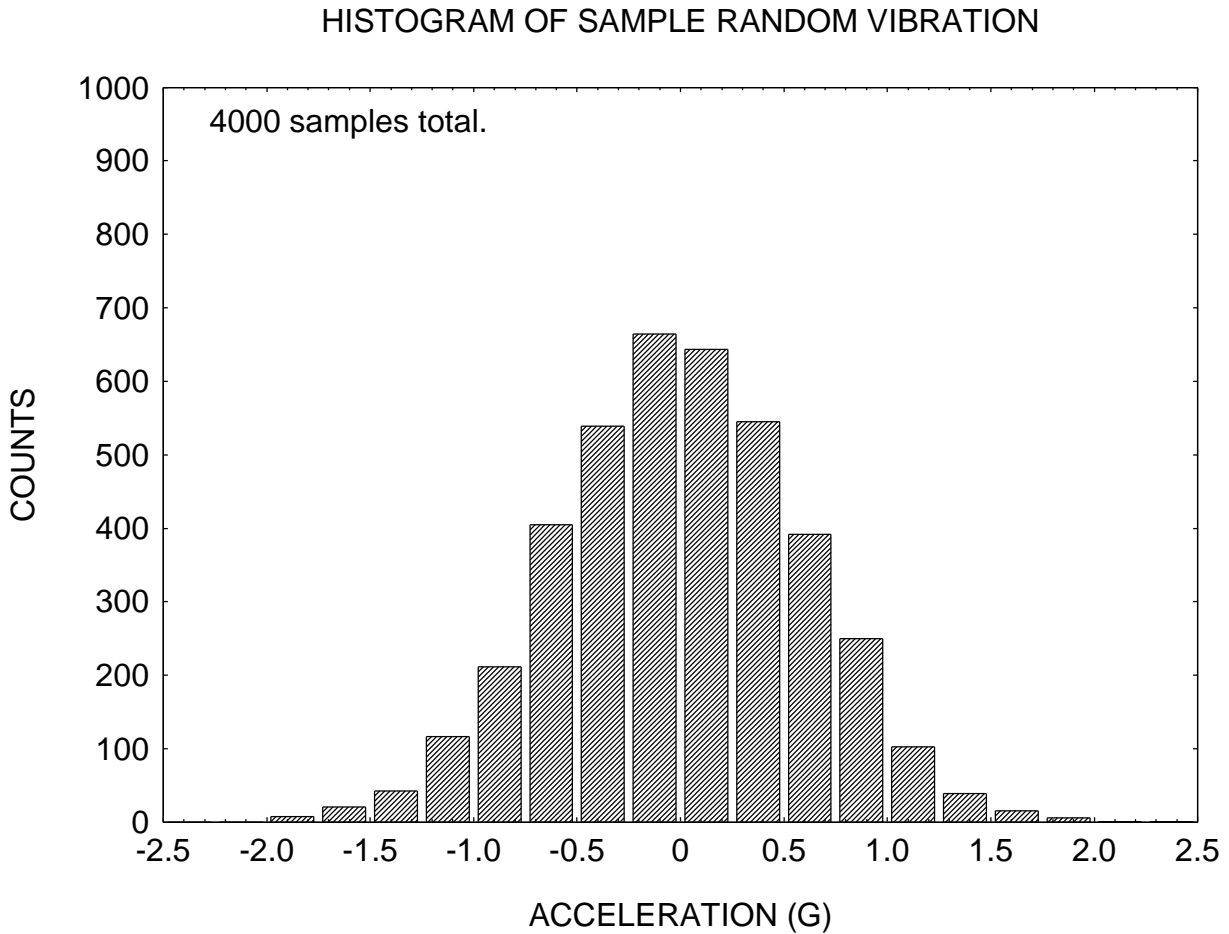


Figure 4.

Probability Density Function

The histogram in Figure 4 can be converted to a "probability density function." This would be done by dividing each bar by the total number of samples, 4000 in this case. The resulting function would be a probability density function. Furthermore, the amplitude along the X-axis could be represented in terms of σ . The resulting probability density function would then approximate a "normal probability density function."

The equation which characterizes the normal probability function is well-known. It is available in References 1 and 2. This equation can be integrated to determine the probability that an amplitude will occur inside or outside certain limits.

Normal Probability Values

Consider a random vibration time history $x(t)$. Again, the amplitude $x(t)$ cannot be calculated for a given time.

Nevertheless, the probability that $x(t)$ is inside or outside of certain limits can be expressed in terms of statistical theory.

The probability values for the amplitude are given in Tables 2a and 2b for selected levels in terms of the standard deviation or σ value.

Statement	Probability Ratio	Percent Probability
$-\sigma < x < +\sigma$	0.6827	68.27%
$-2\sigma < x < +2\sigma$	0.9545	95.45%
$-3\sigma < x < +3\sigma$	0.9973	99.73%

Statement	Probability Ratio	Percent Probability
$ x > \sigma$	0.3173	31.73%
$ x > 2\sigma$	0.0455	4.55%
$ x > 3\sigma$	0.0027	0.27%

The probability tables can be used as follows. Suppose that a random vibration time history has a total duration of 60 seconds. For what amount of time will the amplitude exceed 1σ in terms of absolute value?

$$(60 \text{ sec})(0.3173) = 18.22 \text{ sec} \quad (2)$$

Suppose the same time history is digitized such that it consists of 200,000 points. How many points will exceed 2σ in terms of absolute value?

$$(200,000)(0.0455) = 9100 \text{ samples} \quad (3)$$

Types of Random Vibration

Random vibration can be broadband or narrow band. It can be stationary or non-stationary. In addition, white noise and pink noise are two special cases of random vibration.

Sometimes, measured data has a "sine-on-random" characteristic.

Computer hard disk drives are an example of device where both sine and random vibration environments are a concern, as discussed in Appendix A.

Furthermore, note that a great deal of analysis effort is spent searching vibration data for particular sinusoids which may be hidden inside a random signal.

Automobile Example

Consider that a vendor is developing a GPS navigation system for rental cars. The GPS system interfaces with a computer mounted underneath the dashboard. The computer has a small display screen with superimposed road maps so that the driver can navigate to his or her destination.

The navigation computer must be designed to withstand vibration. The engine, transmission, and aerodynamic effects are each sources of vibration. Nevertheless, the dominant source is vibration transmitted from the road, through the tires and suspension, and into the vehicle chassis.

Appropriate vibration test levels must thus be derived. The vendor begins by searching for reference data in MIL-STD-810E, SAE standards, and other sources. Although some useful data is available, the vendor reaches the inevitable conclusion that he must take field measurements to establish the maximum expected vibration level for his specific product.

The vendor thus mounts accelerometers inside a test car adjacent to the navigation system mounting location. A data acquisition system is used to monitor the data. A test driver then drives the car through a variety of roads, including city streets and highways.

What type of vibration is expected? The data will probably show that vibration is random. An exception might occur if a particular natural frequency is excited. This would be resonant excitation. The natural frequency would have a sinusoidal time history response. Even so, the amplitude of the sinusoid might vary in a somewhat random manner. For simplicity, consider that the response will be random vibration.

The next question is whether the random vibration "characteristics" vary with speed, road conditions, total vehicle weight, or other variables. The amount of variability will affect the amount of data which must be collected and analyzed.

A possible outcome is that the character of the random vibration might be reasonably consistent as the vehicle travels at 60 mph down a particular interstate highway. On the other hand, the character could vary considerably as the vehicle experiences stop-and-go driving over a city street filled with potholes, railroad tracks, and construction detours.

Stationary and Nonstationary Definitions

For simplicity, consider that random vibration can be "characterized" in terms of its statistical properties, such as mean value, standard deviation, and kurtosis.

The random vibration is "stationary" if these statistical properties remain constant with time. Otherwise, it is "nonstationary."

Stationary vibration is an idealized concept. All vibration is ultimately nonstationary. Nevertheless, certain types of random vibration may be regarded as reasonably stationary.

For example, the vehicle in the previous example experiences stationary vibration as it travels at a constant speed of 60 mph down a flat interstate highway. Or at least the vibration is stationary as long as the vehicle remains under those conditions.

The vibration during the stop-and-go city driving is clearly non-stationary, however. The vehicle experiences a series of transient vibration events as it crosses railroad tracks, encounters potholes and other obstacles. Transient vibration is nonstationary.

Rocket Vehicle Example

A rocket vehicle clearly experiences nonstationary vibration during its powered flight. A sample acceleration time history is shown in Figure 5.

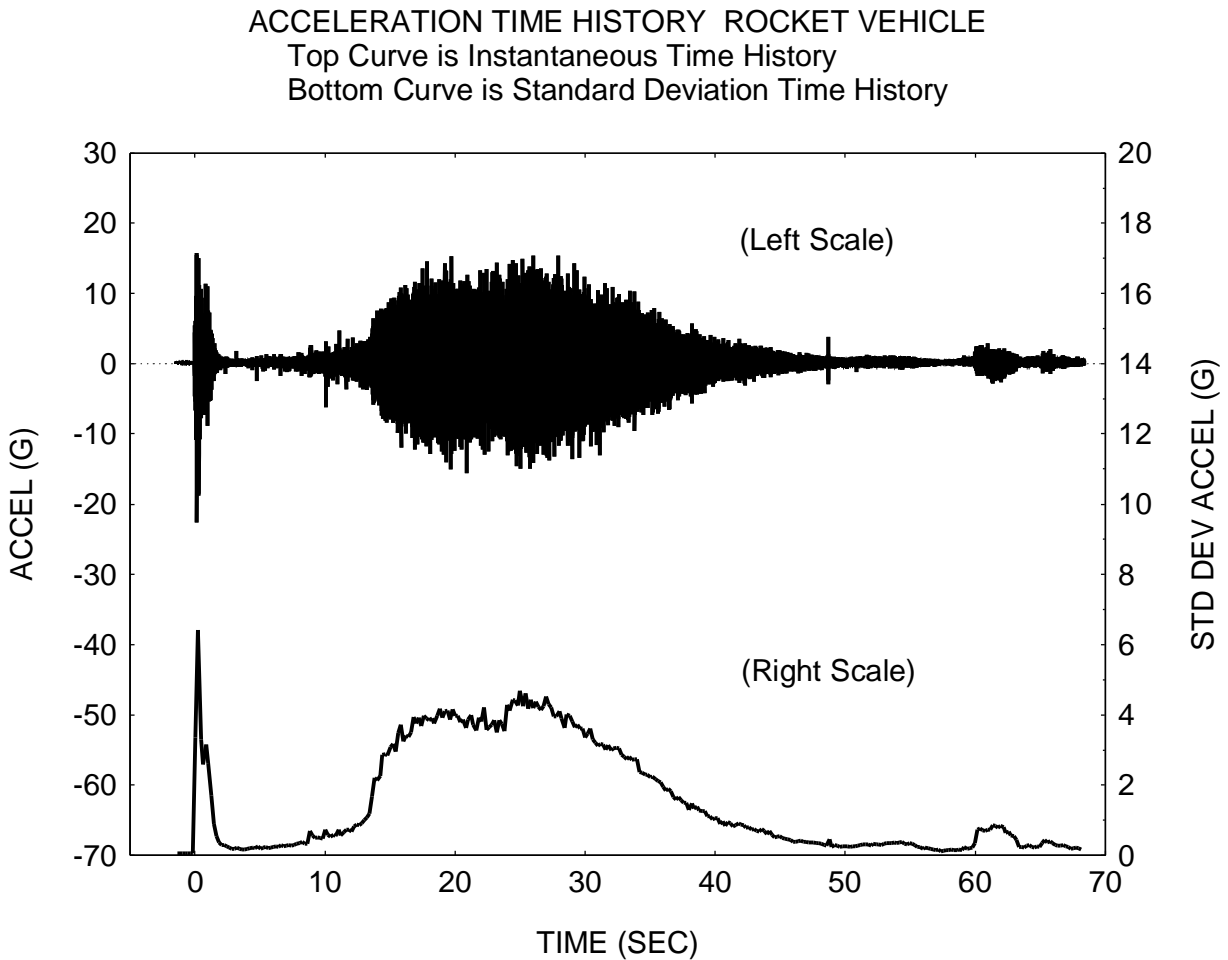


Figure 5.

Note that Figure 5 is a double-Y plot which contains two time history curves. The upper curve is an instantaneous time history, as measured directly by the accelerometer. The bottom curve is a standard deviation time history. For this example, a standard deviation value was calculated for every 0.2 second interval.

There are several reasons for computing a standard deviation time history.

1. An instantaneous time history may have well over 100,000 data points. Plotting this complete set may be impractical depending on the available computer hardware and graphing software. On the other hand, the standard deviation calculation can reduce this set to a 100 or so points. Thus, the standard deviation time history is a "data reduction tool."
2. The standard deviation time history can be used to check whether a signal is stationary. The example in Figure 5 is clearly nonstationary judging from the instantaneous time history. Nevertheless, there may be other cases where the distinction is subtle.
3. The standard deviation time history can be used to clarify sinusoidal signals which are masked by random vibration. The standard deviation of a sine function is 0.707 times the peak value. On the other hand, the standard deviation of random vibration may be 0.30 times the peak value. Thus, the standard deviation calculation favors sine vibration.

Qualitative Description of the Rocket Example

The acceleration time history in Figure 5 is measured data from a ground-launched rocket vehicle. The rocket experience motor ignition vibration and launch acoustics effects during the first few two seconds. The source of the acoustic excitation is the turbulent, high-speed exhaust flow.

Thereafter, the vibration level decays to a relatively benign level for several seconds. During this phase, the rocket is traveling at a subsonic speed. The main vibration source at this time is the motor burn.

At 14 seconds, the vibration level abruptly increases. The vehicle is accelerating through the transonic velocity. Shock waves form around the vehicle. Furthermore, the vehicle passes through its "maximum dynamic pressure" condition. Thus, aerodynamic buffeting effects become the dominant vibration source. The aerodynamic effects continue until about the 40 second mark.

The motor burn ends near the 60 second mark. Note that the vehicle has attitude control thrusters. The thrusters use bursts of nitrogen gas to correct the orientation of the vehicle. The thrusters were the main source of vibration from 60 to 65 seconds.

Random Vibration Testing

Electronic and mechanical components may be mounted on an electromagnetic shaker table for the purpose of vibration testing. The design of an electromagnetic shaker is somewhat similar to an audio loudspeaker.

There are three common types of vibration testing:

1. Verification that the component can withstand simulated vibration environments
2. Stimulation of latent parts and workmanship defects
3. Identification of natural frequencies, transmissibility functions, and other parameters

The shaker provides a base input to the test item. A control computer applies a random vibration signal to the shaker via a power amplifier. The shaker applies mechanical vibration to the test item.

The shaker vibration is monitored by accelerometers. The accelerometers provide a feedback signal to the control computer, as shown in Figure 6.

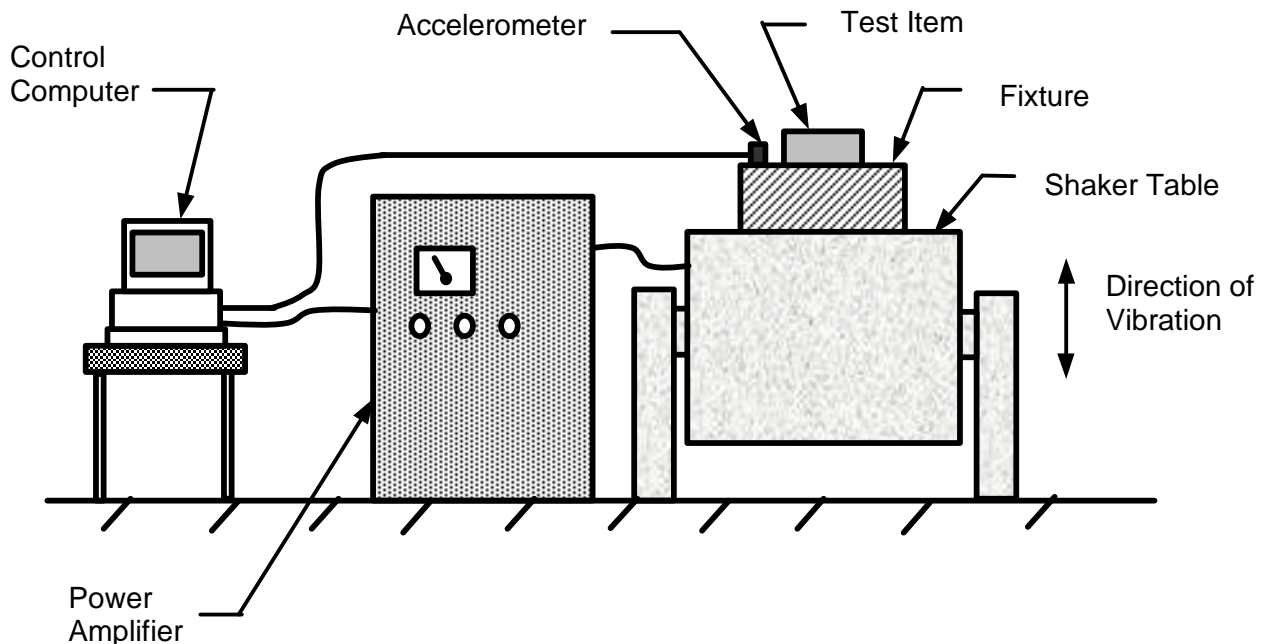


Figure 6. Typical Vibration Test Set-up

Environmental Stress Screening

One type of vibration test is environmental stress screening.

Environmental stress screening involves subjecting circuit boards to random vibration per NAVMAT P-9492, or some related standard. In addition, thermal cycling may be performed.

The purpose of this screening is to detect bad parts and workmanship. Specifically, this is done at an early point in the production process where repair would be the least expensive.

Environmental stress screening is designed to precipitate "infant mortality failures."

Some purists argue that environmental stress screening is a screen of the manufacturing process rather than a test of the component.

Power Spectral Density Function

Random vibration can be represented in the frequency domain in terms of a Fourier transform or power spectral density function. These functions are described in References 3 and 4.

References

1. D. Steinberg, "Vibration Analysis for Electronic Equipment," Wiley-Interscience, New York, 1988.
2. T. Irvine, Integration of the Normal Distribution Curve, Vibrationdata Publications, 1999.
3. T. Irvine, Power Spectral Density Units: [G^2 / Hz], Vibrationdata Publications, 2000.
4. T. Irvine, An Introduction to Spectral Functions, Vibrationdata Publications, 1998.

APPENDIX A

Hard Disk Drive Vibration

A hard disk is made from metal or some other rigid material. The disk is called a platter. It is coated with a magnetic material that is used to store data as transitions of magnetic polarity. Each polarity corresponds to a “1” or a “0.”

One or more platters are mounted on a single spindle shaft. The drive platters are divided into cylinders.

Each drive may have a spin rate somewhere between 3600 rpm and 10,000 rpm. This corresponds to the frequency domain between 60 Hz and 167 Hz. Future designs may have even higher spin rates.

A read/write head is mounted on the actuator arm. There is typically one head on each side of every platter.

The heads move in unison back and forth across the platter according to a control algorithm. The algorithm compensates for the flexibility of the actuator arm, platter vibration, and other disturbances.

Source Energy

Rotating imbalance, misalignment, and other defects may produce sinusoidal vibration at the spin rate or at integer multiples thereof.

Movement of the head assembly produces additional vibration. For example, random seek movement may cause rotational oscillations with a broadband random frequency content. This oscillation causes in-plane rotation of the drive and wrapper assembly.

Error and Failure Modes

Hard drive units have a number of error and failure modes related to vibration.

Impact

A particular concern is that the head or arm might impact against the surface of one or more disks, thereby creating voids in the recording film. This damage could cause data errors.

There are several means by which this impact could occur. For example, the vibration could come from an external source that propagates into the arm, possibly exciting a head/arm natural frequency. Another scenario is that excessive disk vibration causes the disk to impact against the head or arm.

Stack Shift

Stack shift is also a concern. This occurs when individual disks shift from their initial center of rotation. This shift could occur if the shock and vibration forces overcome the initial clamping forces. Stack shift may produce a sinusoidal position error signal with a frequency equal to two times (2X) the spin frequency.

Servo Control Algorithm

Vibration may interfere with the control algorithm. This is a particular concern during write operations. The algorithm is designed to prevent existing data from being inadvertently overwritten. Excessive vibration may cause a delay in the writing process. This condition is called latency.

Control algorithms vary from one hard drive model to the next. Most algorithms, however tend to be sensitive in the frequency domain from 500 Hz to 600 Hz.