Using a Random Vibration Test to Cover a Shock Requirement
Revision F

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Note that a subtle software problem affected the results for the SRS fatigue curves in the previous revisions up to D. The error was corrected starting in Revision E. Also a more robust PSD is used beginning in E.

Introduction

Aerospace and military components must be designed and tested to withstand shock and vibration environments.

Some of this testing occurs as qualification, whereby a sample component is tested to levels much higher than those which it would otherwise encounter in the field. This is done to verify the design.[1] Lot acceptance testing is a similar example.

Now consider a launch vehicle component which must withstand random vibration and pyrotechnic shock. The random vibration specification is in the form of a power spectral density (PSD). The shock requirement is a shock response spectrum (SRS).

Pyrotechnic-type SRS tests are often more difficult to configure and control, and are thus more expensive than shaker table PSD tests. Furthermore, some lower and even mid-level SRS specifications may not have the true damage potential to justify shock testing.

A method for assessing the severity of a shock specification based on the work of Hunt, Gaberson, Steinberg, et al, was given in Reference 1. The conclusion was that a shock specification falling below a certain pseudo velocity limit could potentially be omitted from the test program depending on the component material properties. Caveats were also given such as the need for shock testing fragile spacecraft instruments regardless.

Reference 1 also mentioned that the argument to skip shock testing could be strengthened if the random vibration test was rigorous enough to cover the shock requirement. This approach was previously given in Reference 2 for the case of peak response.

The purpose of this paper is to demonstrate a shock and vibration comparison method based on the fatigue damage spectrum. This method should give greater weight to the random vibration test than the pure peak response method of Reference 2.

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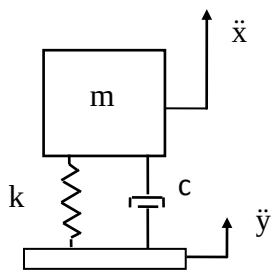
[1] Since the test levels are so conservative, this qualification unit is not actually mounted on a vehicle for field use.

Assumptions



m = mass

c = damping coefficient

k = stiffness

Figure 1. Single-degree-of-freedom System Subject to Base Acceleration

Assume:

1. The component can be modeled as a single-degree-of-freedom (SDOF) system.
2. The system has a linear response.
3. The peak shock and vibration pseudo velocity response levels fall below the threshold in Reference 1.
4. The resulting shock and vibration response stress levels are below the yield point.
5. There are no failure modes due to peak relative displacement, such as misalignment, loss of sway space, mechanical interference, etc.
6. There are no shock-sensitive mechanical switches, relays or reed valves, which might experience chatter or change-of-state during shock.
7. There are no extra-sensitive piece parts such as crystal oscillators, klystrons, traveling wave tubes, magnetrons, etc.
8. The piece parts are Mil-spec quality and have been previously qualified to shock levels similar to those in MIL-STD-202, MIL-STD-883, etc.
9. The natural frequency, amplification factor Q and fatigue exponent b, can be estimated between respective limits.

For electronic components, the effective assumption is that the failure mode would be broken solder joints or lead wires due to circuit board bending.

Rainflow Cycle Counting

SDOF responses must be calculated for each fn and Q of interest, for both the PSD and the for SRS.

A representative time history can be synthesized for the SRS.[2] The Smallwood, ramp invariant, digital recursive filtering relationship is then used for the response calculation, per Reference 3 and Appendix E. The rainflow cycles can then be calculated via Reference 4.

In addition, response PSDs can be calculated for the base input PSD using the textbook SDOF power transmissibility function, as shown in Appendix F. The rainflow cycles are then tabulated via the Dirlik method in References 5 and 6.

Fatigue Damage Spectrum

The fatigue damage spectrum (FDS) is calculated from the response rainflow cycles.

A relative damage index D can then be calculated using

$$D = \sum_{i=1}^{m} A_i^b \, n_i \tag{1}$$

where

A_i	is the *response* amplitude from the rainflow analysis
n_i	is the corresponding number of cycles
b	is the fatigue exponent

Note that the amplitude convention for this paper is: (peak-valley)/2.

The FDS expresses damage as a function of natural frequency with the Q and b values duly noted.

[2] There is currently no method for calculating rainflow cycles directly from an SRS, although this will be investigated in an upcoming paper

Example

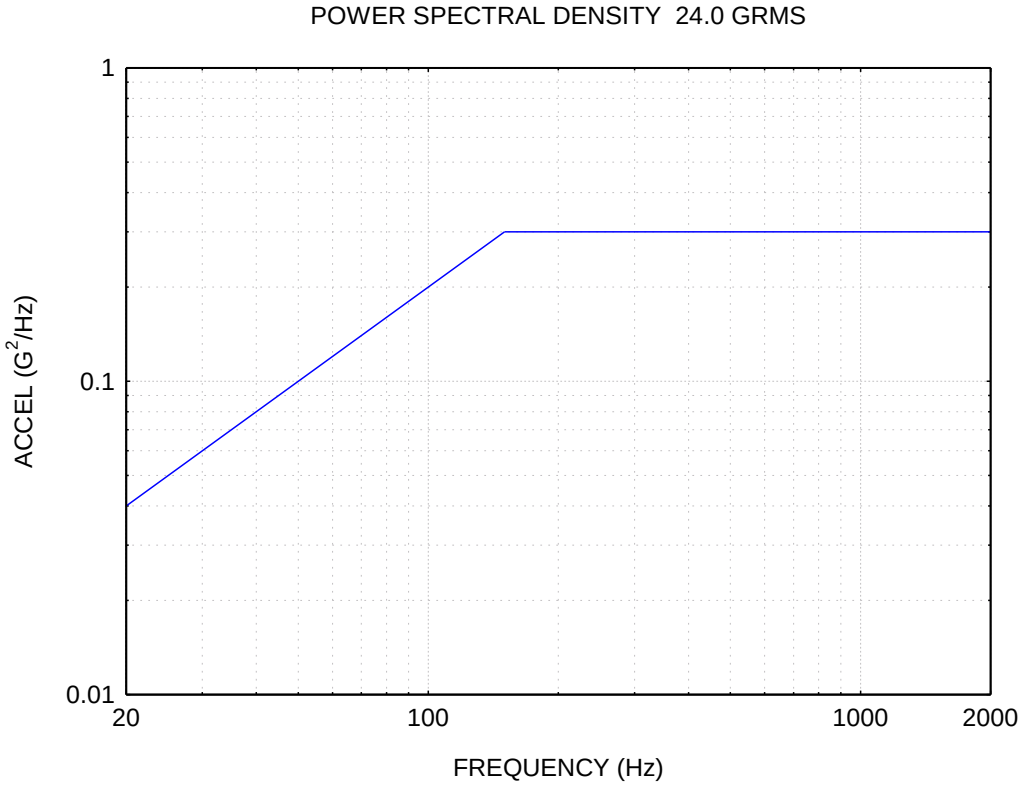


Figure 2.

Power Spectral Density, 24.0 GRMS Overall, 180 sec/axis	
Frequency (Hz)	Accel (G^2/Hz)
20	0.040
150	0.30
2000	0.30

A component must be subjected to the qualification vibration and shock levels given in Figures 2 and 3, respectively.

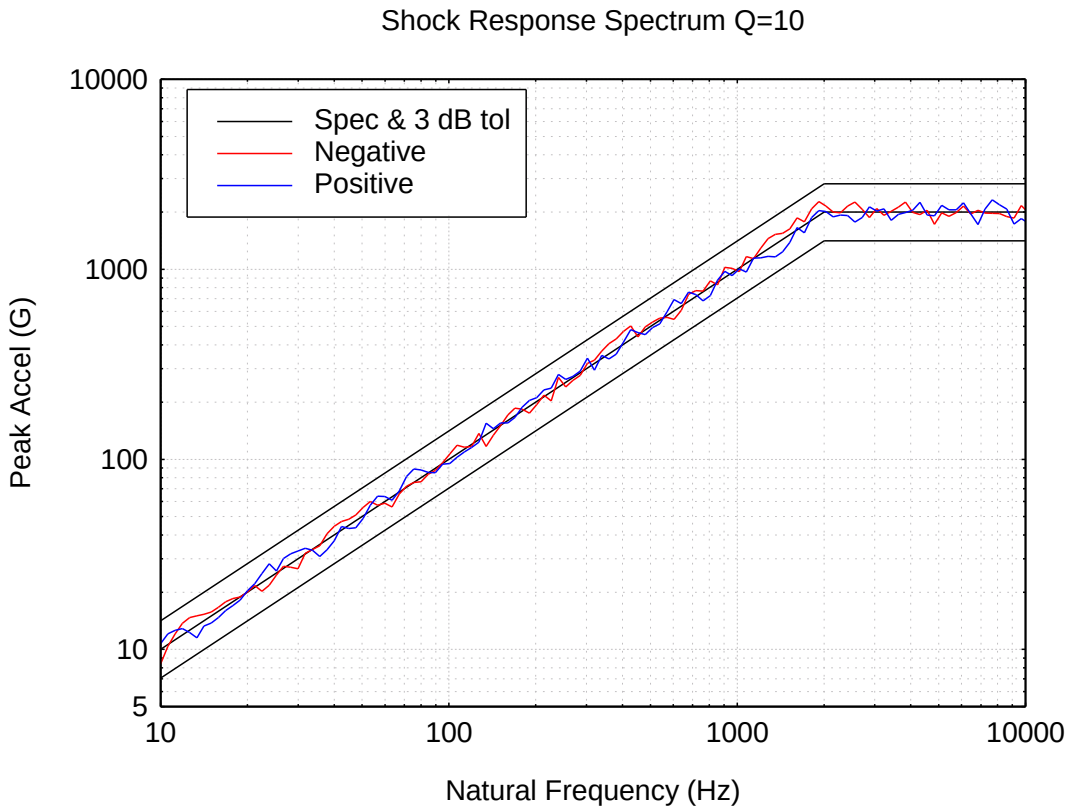


Figure 3.

SRS Q=10, 3 shock/axis	
Natural Frequency (Hz)	Peak Accel (G)
10	10
2000	2000
10000	2000

Note that typical aerospace SRS specifications begin at 100 Hz. The specification for this case began at 10 Hz in order to control the low frequency energy in the time history synthesis.

A sample time history is synthesized to meet the SRS, as shown in Figure 4. The SRS of the synthesized time history is shown together with the specification in Figure 3.

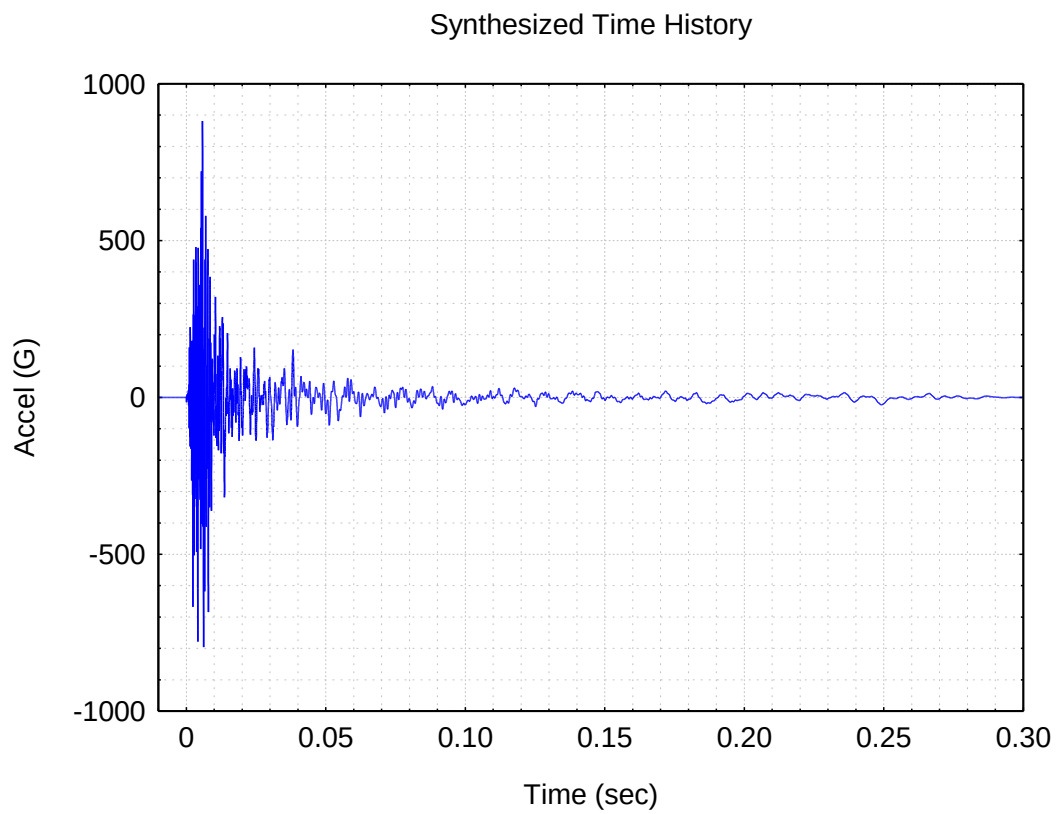


Figure 4.

The initial synthesis is performed using a damped sine function series. The time history is reconstructed as a wavelet series so that the corresponding velocity and displacement time histories each has a zero net value.

Analysis

Fatigue damage spectra were calculated for both the SRS and PSD specifications. This was done for three response metrics: acceleration, pseudo velocity and relative displacement.

The units for acceleration, velocity and displacement were respectively: G, in/sec & in

The natural frequency was an independent variable from 20 to 2000 Hz.

The amplification factor Q was set at either 10 or 30.

The fatigue exponent was set at either 4 or 9.

The resulting spectral plots are shown in Appendices A through D.

Conclusion

The relative difference between the FDS curves for the PSD and SRS were rather insensitive to Q but very sensitive to b.

Q	b	Results
10 & 30	4	The three damage curves from the PSD enveloped the SRS over the frequency domain up to 1900 Hz.
10 & 30	9	The results were mixed, but the SRS curves tended to envelope the corresponding PSD curves.

Note that circuit boards typically have their fundamental bending frequencies from 200 to 800 Hz, based on the author experience as well as Reference 7.

The fatigue exponent range from b=4 to b=9 was very broad for this analysis. Note that Steinberg used b=6.3 for electronic components in Reference 7.

The FDS method in the paper appears useful for comparing shock and random vibration environments. Ideally, the estimates for fn, Q and b could be narrowed for an actual case, compared with the value used in this paper for a hypothetical component.

Postscript

Neither Gaberson nor Steinberg published papers using the FDS method.

But the pseudo velocity FDS offered in this paper was in consideration of Gaberson's principle that dynamic stress is directly proportional to pseudo velocity.

The relative displacement FDS was consistent with Steinberg's empirical formula that circuit board and piece part fatigue is proportional to relative displacement.

References

1. T. Irvine, Shock Severity Limits for Electronic Component, Rev B, Vibrationdata, 2014.

2. H. Caruso and E. Szymkowiak, A Clarification of the Shock/Vibration Equivalence in Mil-Std-180D/E, Journal of Environmental Sciences, 1989.

3. David O. Smallwood, An Improved Recursive Formula for Calculating Shock Response Spectra, Shock and Vibration Bulletin, No. 51, May 1981.

4. ASTM E 1049-85 (2005) Rainflow Counting Method, 1987.

5. Halfpenny & Kim, Rainflow Cycle Counting and Acoustic Fatigue Analysis Techniques for Random Loading, RASD International Conference, Southampton, UK, July 2010.

6. Halfpenny, A Frequency Domain Approach for Fatigue Life Estimation from Finite Element Analysis, nCode International Ltd., Sheffield UK.

7. Dave Steinberg, Vibration Analysis for Electronic Equipment, Second Edition, Wiley-Interscience, New York, 1988.

APPENDIX A

Q=10, b=4

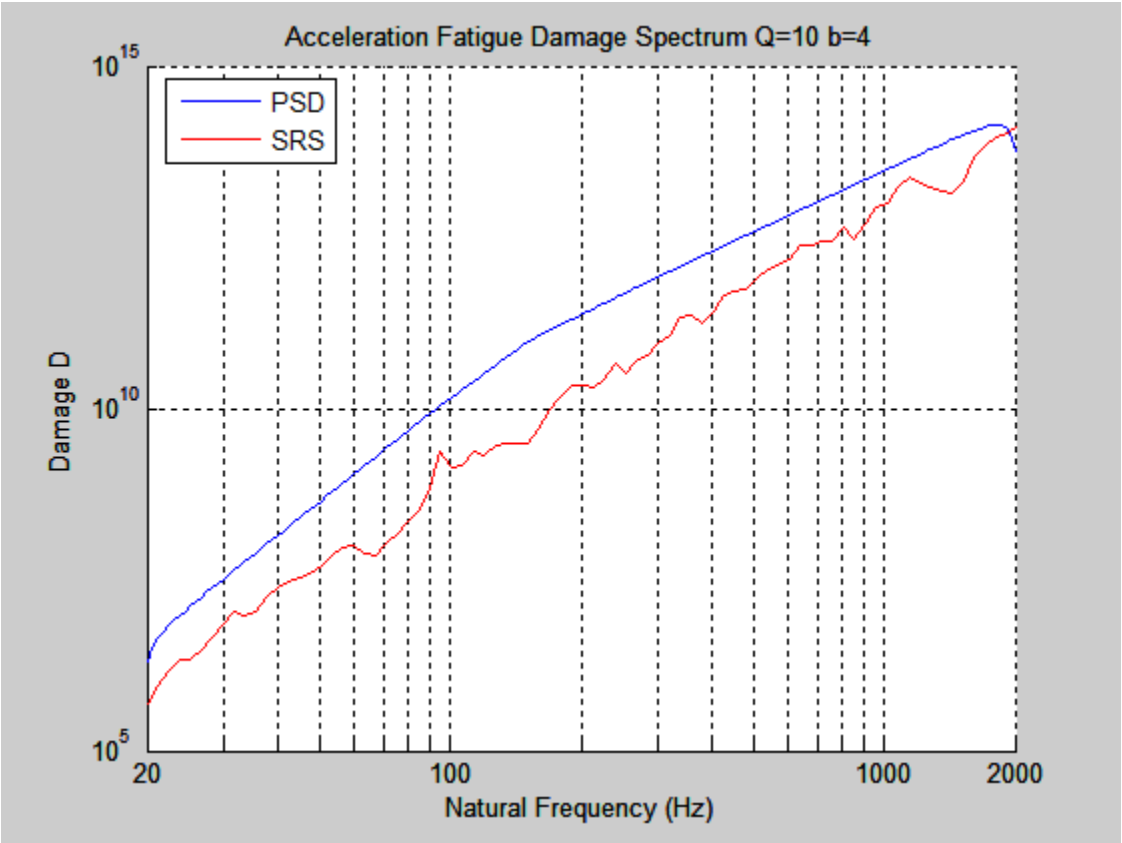


Figure A-1.

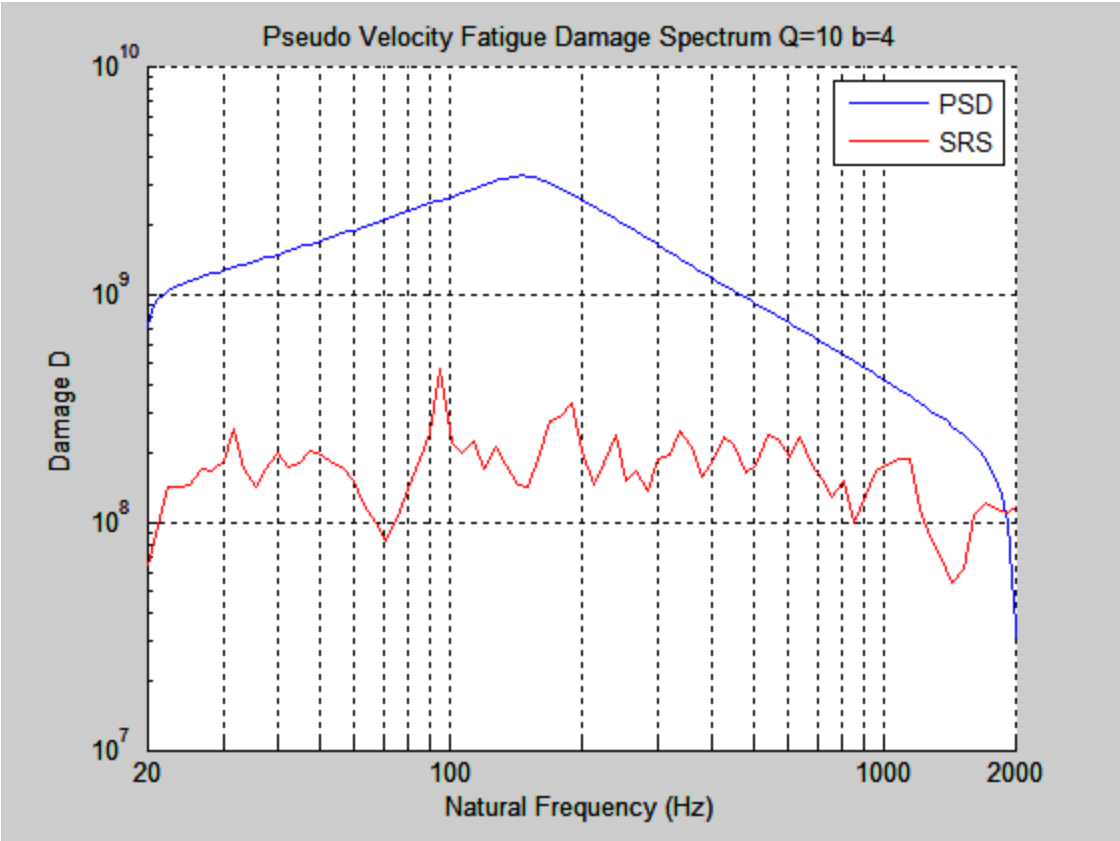

Pseudo Velocity Fatigue Damage Spectrum Q=10 b=4
PSD
SRS
Damage D
10^{10}
10^{9}
10^{8}
10^{7}
20
100
1000
2000
Natural Frequency (Hz)

Figure A-2.

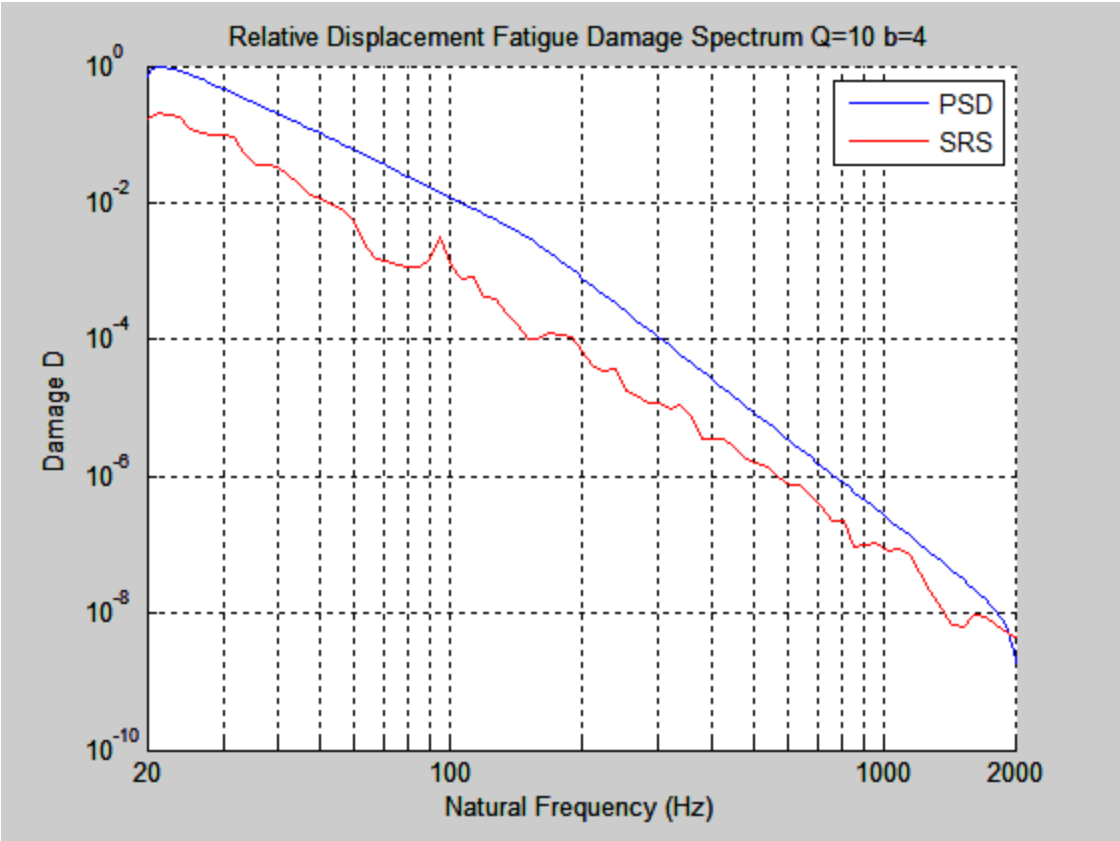

Relative Displacement Fatigue Damage Spectrum Q=10 b=4
PSD
SRS
Damage D
10^{0}
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
20
100
1000
2000
Natural Frequency (Hz)

Figure A-3.

APPENDIX B

Q=10, b=9

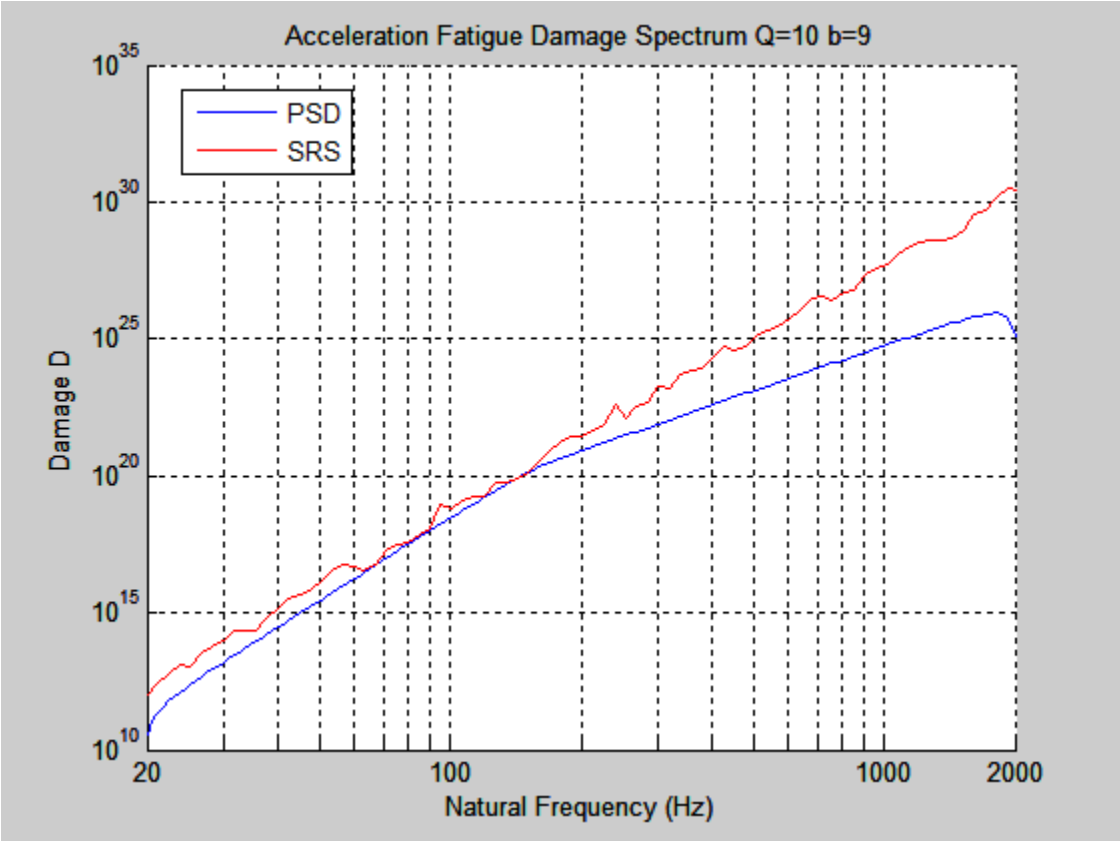


Figure B-1.

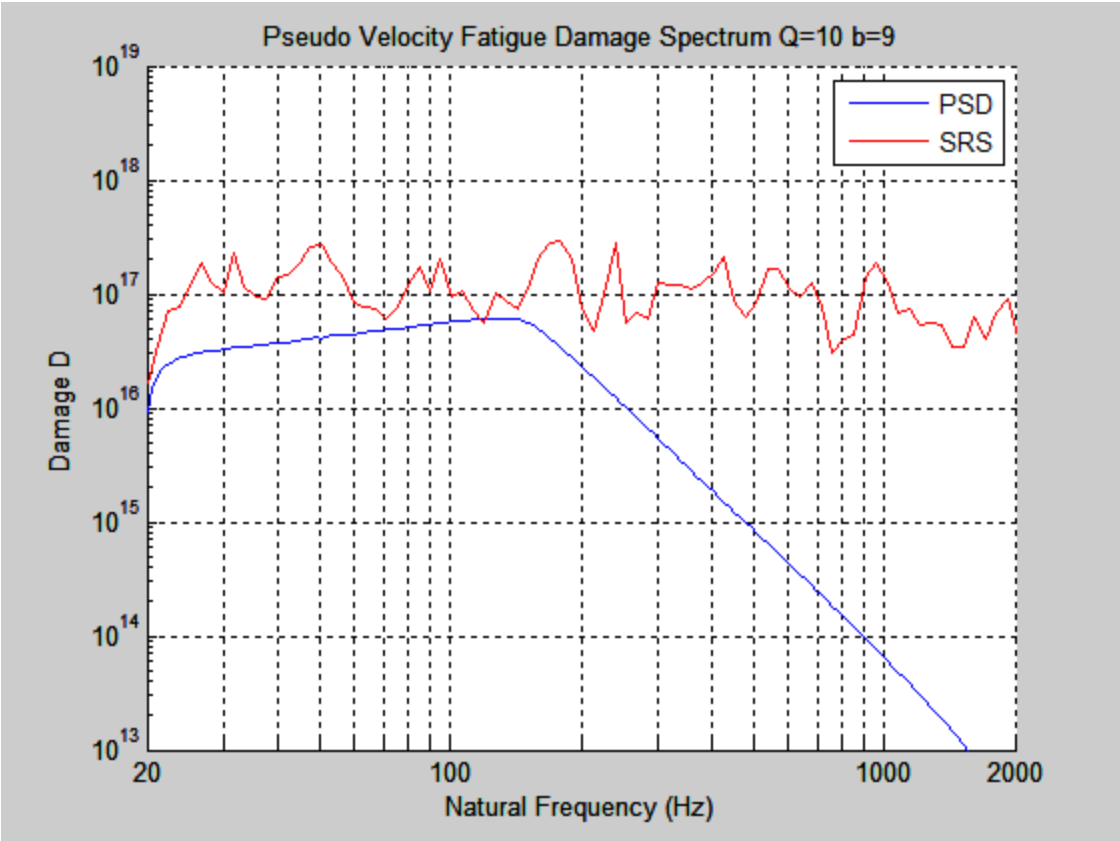

Pseudo Velocity Fatigue Damage Spectrum Q=10 b=9
PSD
SRS
Damage D
10^{19}
10^{18}
10^{17}
10^{16}
10^{15}
10^{14}
10^{13}
20
100
1000
2000
Natural Frequency (Hz)

Figure B-2.

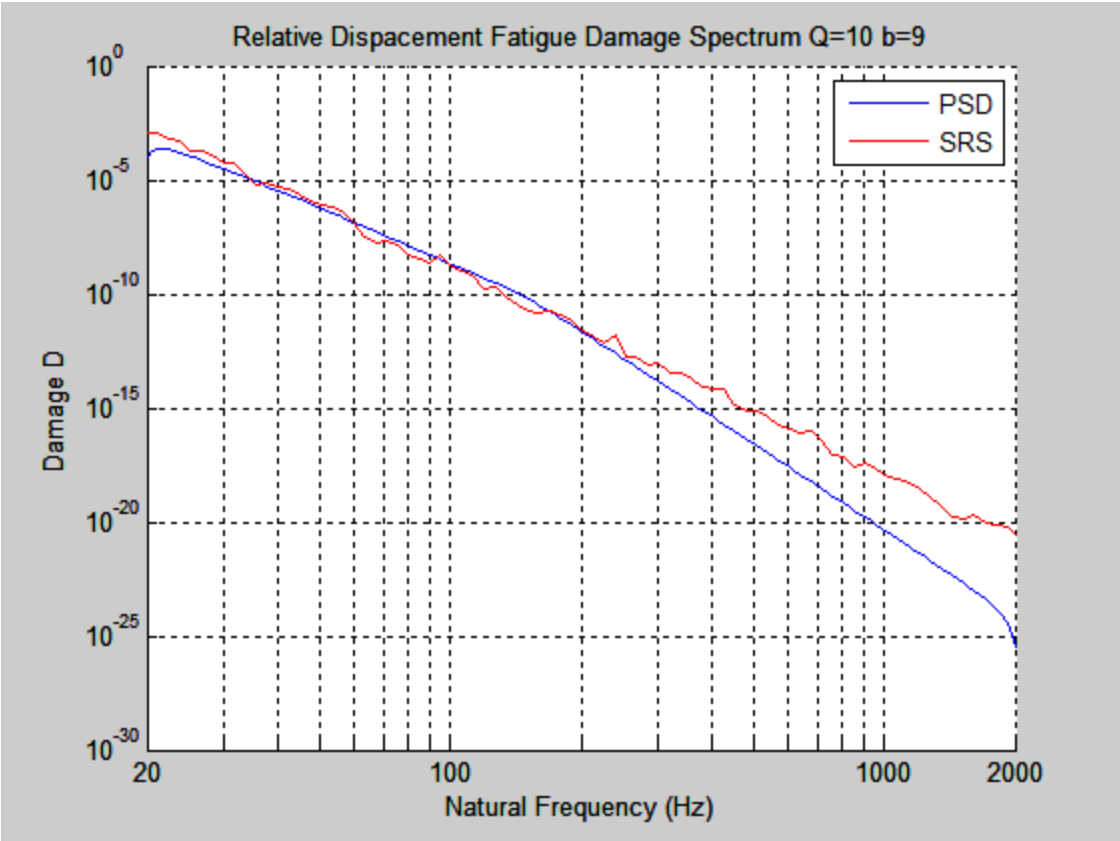

Relative Dispacement Fatigue Damage Spectrum Q=10 b=9
PSD
SRS
Damage D
10^{0}
10^{-5}
10^{-10}
10^{-15}
10^{-20}
10^{-25}
10^{-30}
20
100
1000
2000
Natural Frequency (Hz)

Figure B-3.

APPENDIX C

Q=30, b=4

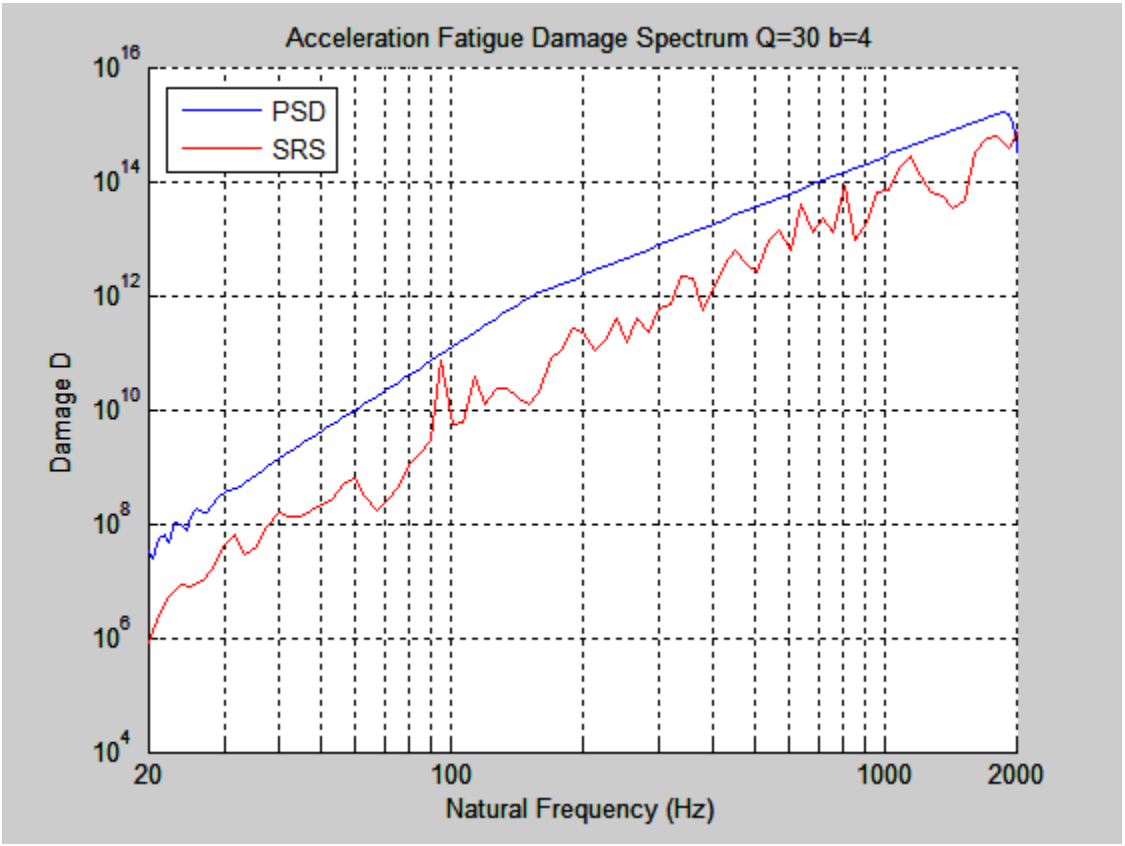


Figure C-1.

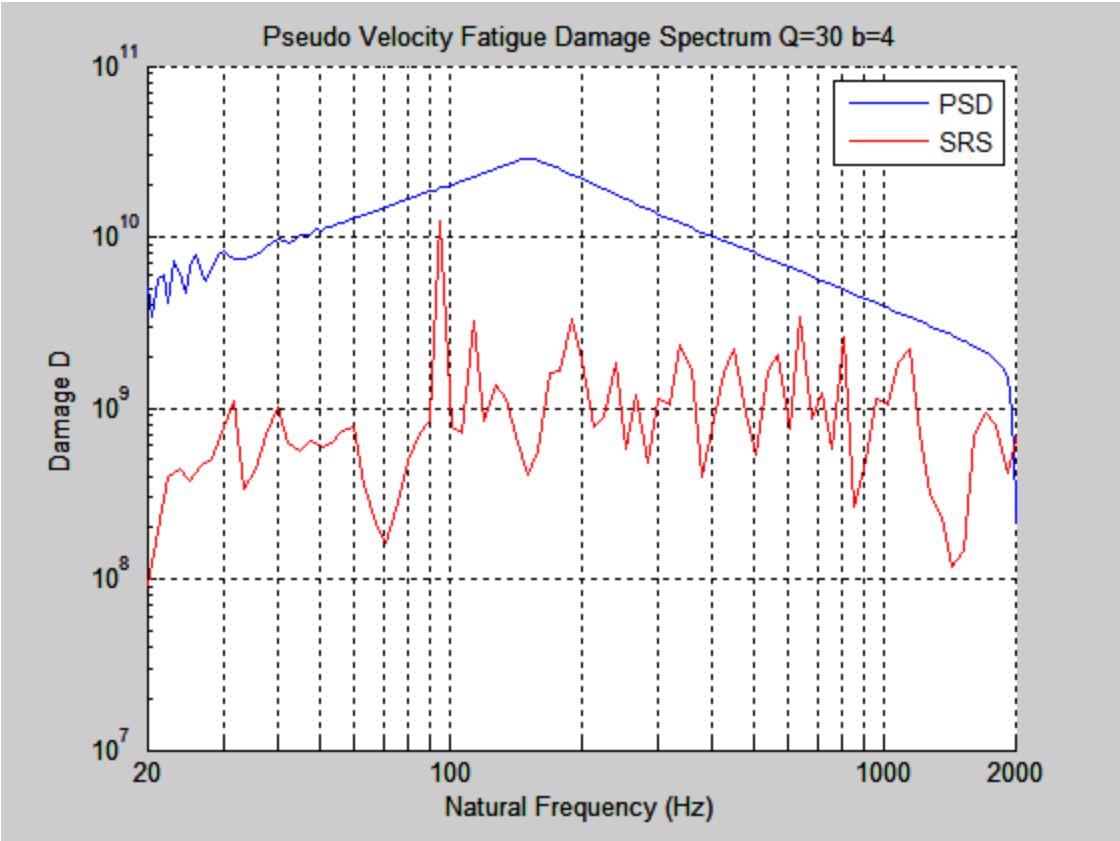

Pseudo Velocity Fatigue Damage Spectrum Q=30 b=4
PSD
SRS
Damage D
10^{11}
10^{10}
10^{9}
10^{8}
10^{7}
20
100
1000
2000
Natural Frequency (Hz)

Figure C-2.

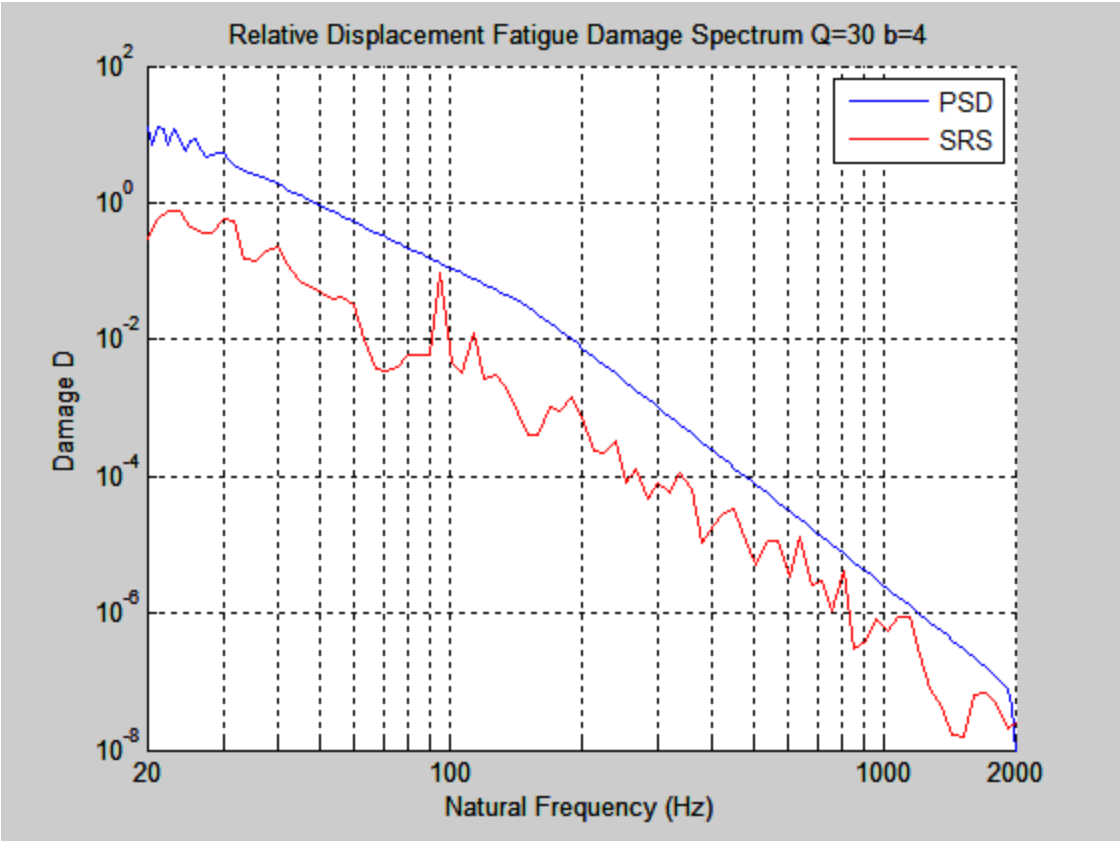

Relative Displacement Fatigue Damage Spectrum Q=30 b=4
PSD
SRS
Damage D
10^{2}
10^{0}
10^{-2}
10^{-4}
10^{-6}
10^{-8}
20
100
1000
2000
Natural Frequency (Hz)

Figure C-3.

APPENDIX D

Q=30, b=9

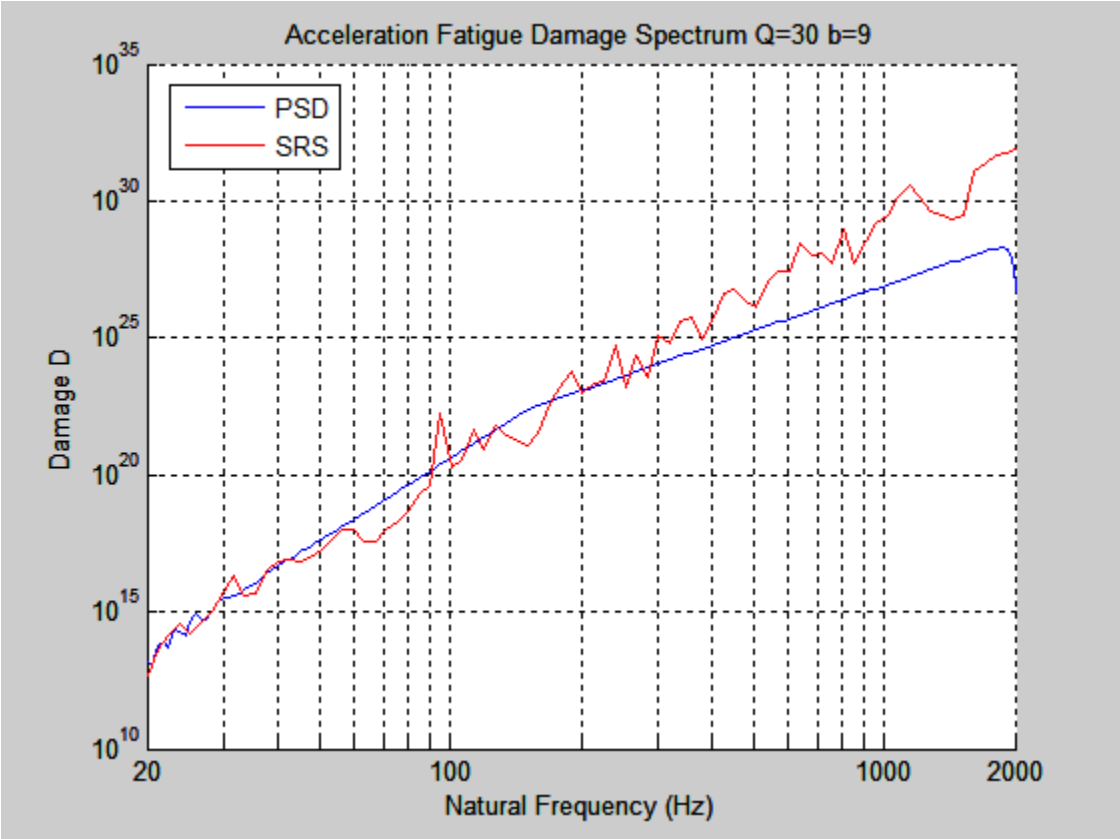


Figure D-1.

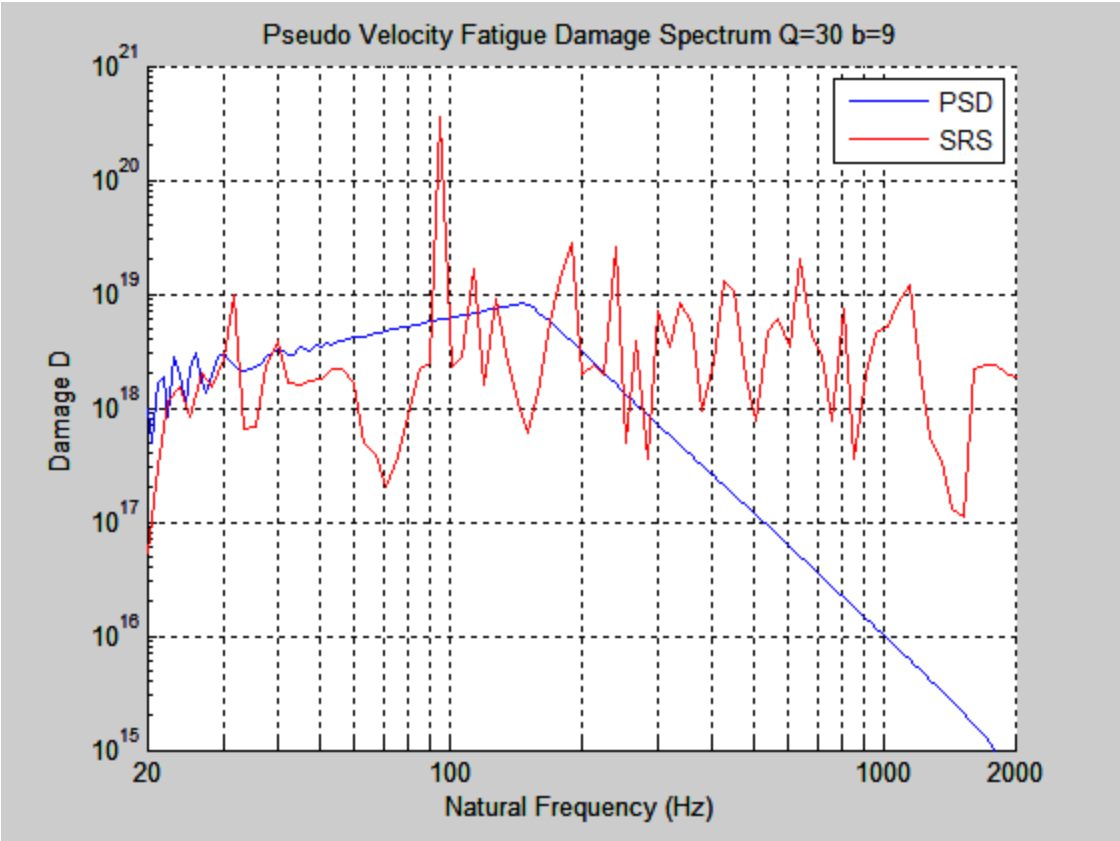

Pseudo Velocity Fatigue Damage Spectrum Q=30 b=9
PSD
SRS
Damage D
10^{21}
10^{20}
10^{19}
10^{18}
10^{17}
10^{16}
10^{15}
20
100
1000
2000
Natural Frequency (Hz)

Figure D-2.

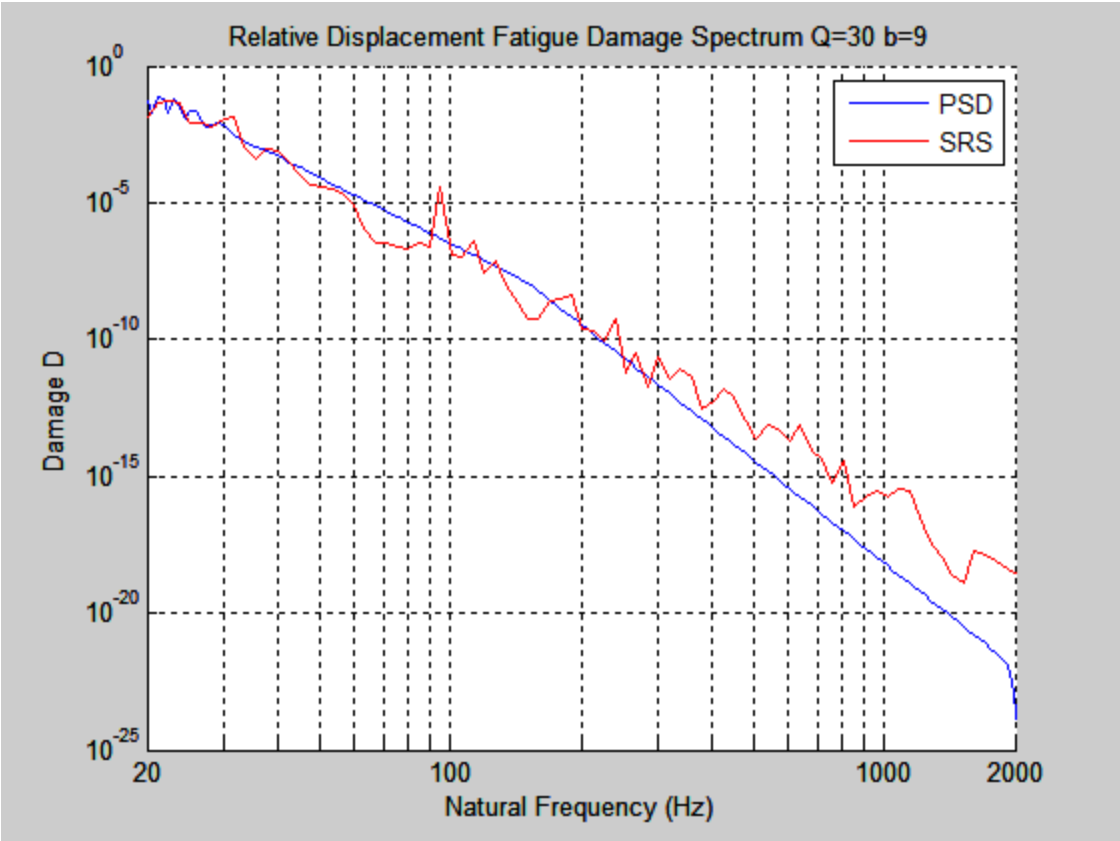

Relative Displacement Fatigue Damage Spectrum Q=30 b=9
PSD
SRS
Damage D
10^{0}
10^{-5}
10^{-10}
10^{-15}
10^{-20}
10^{-25}
20
100
1000
2000
Natural Frequency (Hz)

Figure D-3.

APPENDIX E

Smallwood Ramp Invariant Digital Recursive Filtering Relationship

Variables

$\ddot{y}_i$	=	Base acceleration
T	=	Time step
fn	=	Natural frequency
ξ	=	Viscous damping ratio

The angular natural frequency ω_n is

$$\omega_n = 2\pi f_n \tag{E-1}$$

The damped natural frequency ω_d is

$$\omega_d = \omega_n \sqrt{1-\xi^2} \tag{E-2}$$

Absolute Acceleration

The acceleration response $\ddot{x}_i$ is

$$\begin{aligned}
\ddot{x}_i = \quad & +2\exp[-\xi\omega_n T]\cos[\omega_d T]\ddot{x}_{i-1} \\
& -\exp[-2\xi\omega_n T]\ddot{x}_{i-2} \\
& +\left\{1-\left(\frac{1}{\omega_d T}\right)\exp(-\xi\omega_n T)\sin(\omega_d T)\right\}\ddot{y}_i \\
& +\left\{2\exp(-\xi\omega_n T)\left(-\cos(\omega_d T)+\left(\frac{1}{\omega_d T}\right)\sin(\omega_d T)\right)\right\}\ddot{y}_{i-1} \\
& +\left\{\exp(-2\xi\omega_n T)-\left(\frac{1}{\omega_d T}\right)\exp(-\xi\omega_n T)\sin(\omega_d T)\right\}\ddot{y}_{i-2}
\end{aligned} \tag{E-3}$$

Relative Displacement

The relative displacement u_i is

$$
\begin{aligned}
u_i = \; & \\
& + 2\exp\left[-\xi\omega_n T\right]\cos\left[\omega_d T\right] u_{i-1} \\
& - \exp\left[-2\xi\omega_n T\right] u_{i-2} \\
& - \frac{1}{\omega_n^3 T}\left\{ 2\xi\left[\exp\left(-\xi\omega_n T\right)\cos\left(\omega_d T\right) - 1\right] + \exp\left(-\xi\omega_n T\right)\left[\frac{\omega_n}{\omega_d}\left[2\xi^2 - 1\right]\sin\left(\omega_d T\right)\right] + \omega_n T \right\} \ddot{y}_i \\
& - \frac{1}{\omega_n^3 T}\left\{ -2\omega_n T\exp\left(-\xi\omega_n T\right)\cos\left(\omega_d T\right) + 2\xi\left[1 - \exp\left(-2\xi\omega_n T\right)\right] - 2\frac{\omega_n}{\omega_d}\left[2\xi^2 - 1\right]\exp\left(-\xi\omega_n T\right)\sin\left(\omega_d T\right) \right\} \ddot{y}_{i-1} \\
& - \frac{1}{\omega_n^3 T}\left\{ \left(2\xi + \omega_n T\right)\exp\left(-2\xi\omega_n T\right) + \exp\left(-\xi\omega_n T\right)\left\{ \frac{\omega_n}{\omega_d}\left[2\xi^2 - 1\right]\sin\left(\omega_d T\right) - 2\xi\cos\left(\omega_d T\right) \right\} \right\} \ddot{y}_{i-2}
\end{aligned}
\tag{E-4}
$$

Pseudo Velocity

Pseudo velocity is calculated by differentiating the relative displacement.

APPENDIX F

SDOF Response to a PSD Base Input

Variables

Z_{PSD}	=	Relative displacement PSD
PV_{PSD}	=	Response pseudo velocity PSD
$\ddot{x}_{PSD}$	=	Response acceleration PSD
$\ddot{Y}_{APSD}$	=	Base input acceleration PSD
f	=	Excitation frequency
fn	=	Natural frequency
ξ	=	Viscous damping ratio

Note that the amplification factor Q is related to the damping ratio as

$$Q = \frac{1}{2\xi} \tag{F-1}$$

Relative Displacement

The relative displacement $Z_{PSD}(f)$ is

$$Z_{PSD}(f) = \left(\frac{1}{2\pi}\right)^4 \left[\frac{1}{(f_n^2 - f^2)^2 + (2\xi f f_n)^2}\right] \ddot{Y}_{APSD}(f) \tag{F-2}$$

Pseudo Velocity

The pseudo velocity $PV_{PSD}(f)$ is

$$PV_{PSD}(f) = (2\pi f)^2 \; Z_{PSD}(f) \tag{F-3}$$

Acceleration

The response acceleration $\ddot{x}_{PSD}$ is

$$\ddot{x}_{PSD} = \left[\frac{1+(2\xi\rho)^2}{(1-\rho^2)^2+(2\xi\rho)^2} \right] \ddot{Y}_{APSD}(f), \quad \rho = f / f_n \tag{F-4}$$