KARMAN VORTEX SHEDDING AND THE STROUHAL NUMBER
Revision A

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Variables

D	=	diameter
f_S	=	Strouhal frequency
Re	=	Reynolds number
S	=	Strouhal number
U	=	Free stream velocity
ν	=	Kinematic viscosity

Introduction

Wind has a number of effects upon structures.

For example, a steady wind exerts a quasi-static drag force upon a structure. This effect is readily apparent as trees are bent backward by the oncoming wind pressure.

Wind may also exert a lift force and moment upon a structure, contributing to a self-excited oscillation of the structure. The original Tacoma Narrows Bridge, which had an aerodynamic instability, failed due to self-excited oscillation.

Furthermore, the wind's lift force may generate vortices in the wake behind the structure. Under certain conditions, the vortices may be periodic, forming a "Karman vortex street." This effect is shown in Figure 1 as taken from Reference 1.

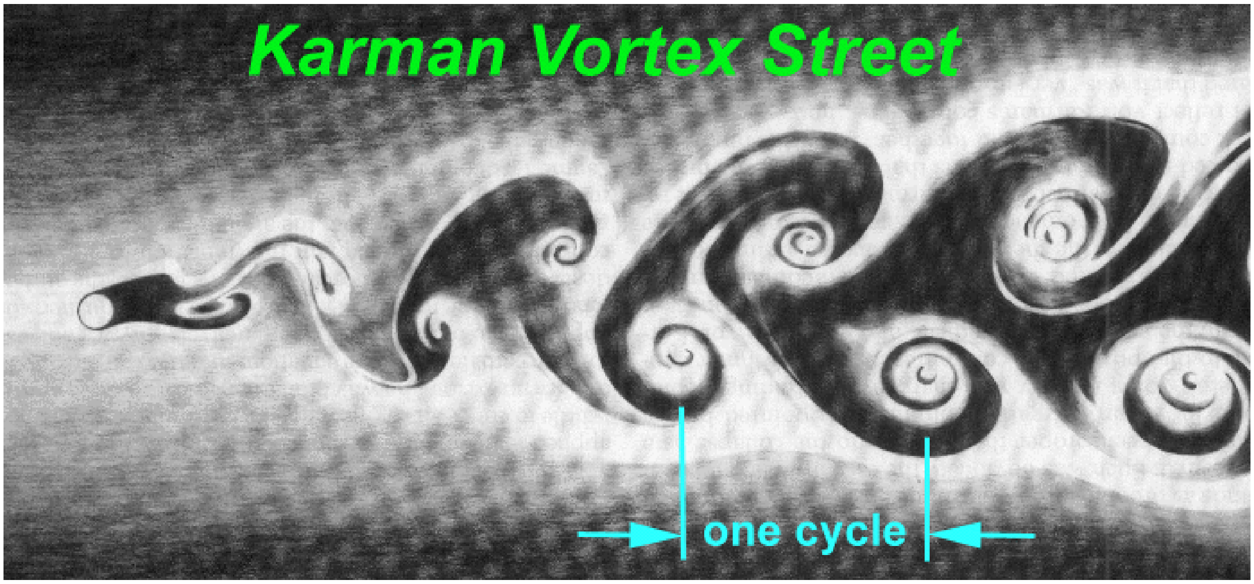


Figure 1. Vortex Shedding around a Spherical Body

This report is concerned with Karman vortex shedding. The specific purpose is to discuss the "Strouhal number" which relates the frequency of Karman vortex shedding to a characteristic dimension of the body and the wind speed.

Circular Cylinder

Two key parameters for analyzing vortex shedding are the Reynolds number and the Strouhal number. An empirical relationship between these numbers for a circular section is given in Table 1.

Table 1.
Strouhal Number vs. Reynolds Number for a Circular Section (Reference 2)

Reynolds Number Re	Strouhal Number S
< 30	0
50	0.13
500	0.20
1000	0.21
10^4	0.20
10^5	0.19
10^6	0.21
10^7	0.23

Note that the Reynolds number Re is defined as

$$Re = UD/\nu \tag{1}$$

Air has the following kinematic viscosity under normal conditions per Reference 3.

$$\nu \cong 1.6\ (10^{-4})\ ft^2/sec$$

The flow regimes across a circular cylinder for various Reynolds numbers are shown in Table 2, taken from Reference 4.

The Strouhal number S is defined as

$$S = f_S D/U \tag{2}$$

where f_S is the frequency of full cycles of vortex shedding in Hertz.

The f_S value shall be called the "Strouhal frequency" rather than the "vortex shedding frequency" in this report because periodic vortex shedding does not occur for all flow regimes, as is apparent in Table 2.

Note that the Strouhal frequency is also referred to as the "lift oscillation frequency" by some sources.

The Strouhal frequency is thus

$$f_S = S\ U/D \tag{3}$$

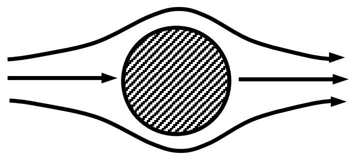
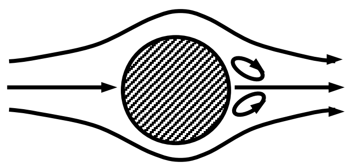
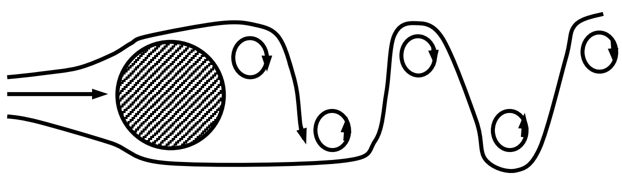
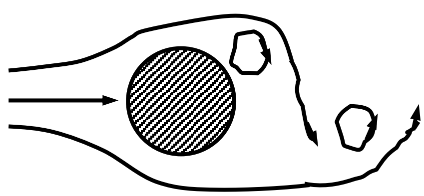
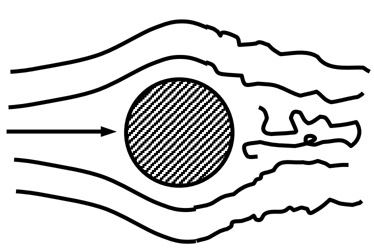
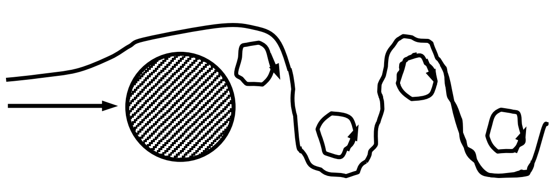
<u>Examples of Circular Cylinders</u>

Bishop notes in Reference 5 that electrical transmission lines and tall steel chimneys may oscillate due to vortex shedding. He gives three methods for designing or modifying chimneys to solve this problem:

1. Guy wires
2. Fire-brick lining
3. Helical spoilers or strakes

Bishop further notes that vortex shedding may also occur in water. For example, the periscope of a moving submarine may oscillate thus yielding a blurred image.

Table 2. Regimes of Fluid Flow across Circular Cylinders

	**$Re < 5$**	**Regime of unseparated flow**
	**$5 \le Re < 40$**	**A fixed pair of vortices in wake**
	**$40 \le Re < 90$**	**Vortex street is laminar**
	$90 \le Re < 150$	**Vortex street is laminar**
	**$150 \le Re < 300$**	**Transition range to turbulence in vortex**
	$300 \le Re < 3(10^5)$	**Vortex street is fully turbulent**
	**$3(10^5) \le Re < 3.5(10^6)$**	**Laminar boundary layer has undergone turbulent transition and wake is narrower and disorganized**
	**$3.5(10^6) \le Re$**	**Reestablishment of turbulent vortex street**

Rectangular Section

The Strouhal number for a non-circular section is still given by equation (2), except that D represents a characteristic dimension rather than the diameter. The Strouhal number for a rectangular section is shown in Figure 2, taken from Reference 6.

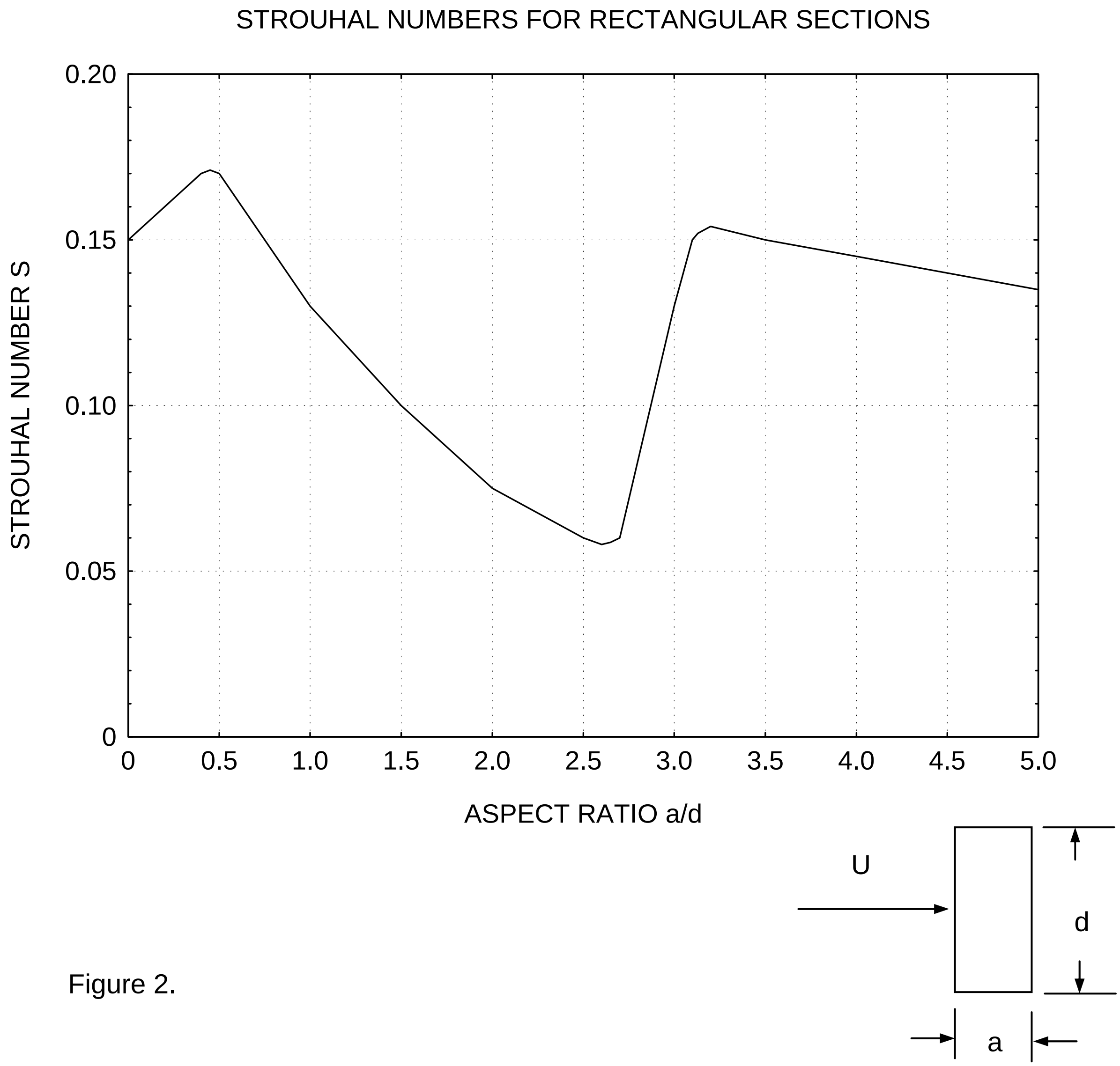


Figure 2.

Tacoma Narrows Bridge

The original Tacoma Narrows Bridge collapsed in 1940. It experienced severe torsional oscillations driven by a 42 mile per hour wind.

The fundamental weakness of the Tacoma Narrows Bridge was its extreme flexibility, both vertically and in torsion. This weakness was due to the shallowness of the stiffening girders and the narrowness of the roadway, relative to its span length.

Engineers still debate the exact cause of its collapse, however. Three theories are:

1. Random turbulence
2. Periodic vortex shedding
3. Aerodynamic instability (negative damping)

A cross-section of the bridge is shown in Figure 3.

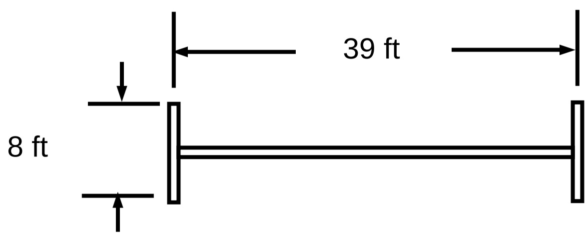


Figure 3. Cross-section of Tacoma Narrows Bridge Span

The Strouhal number for the bridge cross-section is S = 0.11 according to Reference 7. Furthermore, the characteristic dimension is the girder height 8 ft. The Strouhal frequency for a 42 mph (61.6 ft/sec) wind is thus

$$f_S = (0.11)\,(61.1 \text{ ft/sec}) \,/\, (8 \text{ ft}) \quad (4)$$

$$f_S = 0.84 \text{ Hz} \quad (5)$$

The Strouhal frequency is rounded to 1 Hz in Reference 7. On the other hand, the bridge's torsional oscillation was observed to be 0.2 Hz. The Strouhal frequency was thus at least two octaves greater than the torsional mode frequency.

Billah and Scanlan thus argue strongly in Reference 7 that the Karman vortex shedding could not have driven the torsional mode oscillation. They argue instead that an aerodynamic instability resulted in a self-excited oscillation, which caused the failure. Nearly seventy years after the failure, the matter is not completely settled.

Nevertheless, Bishop writes in Reference 5 that vortex shedding was part of the self-excited oscillation.

Billah and Scanlan concede in Reference 7 that a flutter-like, natural vortex shedding may have accompanied the self-excited oscillation, but they argue that this vortex shedding differed from the Karman vortex street as shown in Figure 1.

Conclusion

Vortex shedding exerts an alternating lift force on the structure. The force is perpendicular to the wind flow.

Periodic vortex shedding is a concern because the Strouhal frequency could occur at, or near, a natural frequency of the structure. If so, the structure would undergo resonant vibration. It could thus experience a failure due to either yielding or fatigue.

As an aside, an alternate relationship between the Strouhal and Reynolds number for a circular cross-section is given in Appendix A.

References

1. James Koughan, "The Collapse of the Tacoma Narrows Bridge, Evaluation of Competing Theories of its Demise, and the Effects of the Disaster of Succeeding Bridge Designs," The University of Texas at Austin, 1996.
2. R. Blevins, Fluid Dynamics Handbook, Van Norstrand Reinhold, New York, 1984.
3. I. Shames, Mechanics of Fluids, McGraw-Hill, New York, 1982.
4. C. Harris, editor; Shock and Vibration Handbook, 3rd edition; A. Davenport and M. Novak, "Chapter 29 Part II: Structures Induced by Wind," McGraw-Hill, New York, 1988.
5. R. Bishop, Vibration, Cambridge University Press, London, 1979.
6. H. Bachmann, et al., Vibration Problems in Structures, Birkhauser Verlag, Berlin, 1995.
7. K. Billah and R. Scanlan, "Resonance, Tacoma Narrows Bridge Failure, and Undergraduate Physics, Textbooks;" American Journal of Physics, 1991.
8. Transmission Line Reference Book, Wind-Induced Conductor Motion, Electric Power Research Institute, Palo Alto, California, 1979.

APPENDIX A

STROUHAL NUMBER vs. REYNOLDS NUMBER FOR A CYLINDER

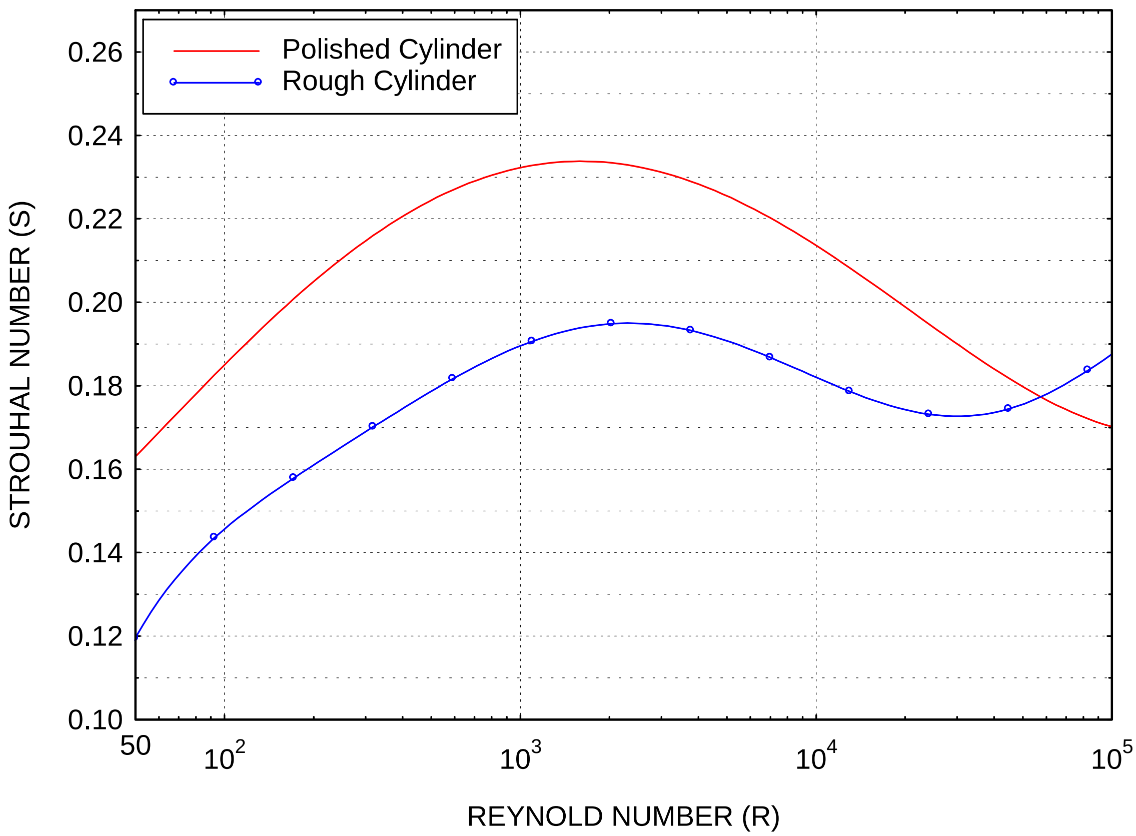


Figure A-1.

The curves are taken from Reference 8.