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A Methodology for Defining Shock Tests Based on Shock Response Spectra and Temporal Moments *

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Defining acceptable tolerances for a shock test has always been a problem due in large part to the use of Shock Response Spectra (SRS) as the sole description of the shock. While SRS do contain a wealth of information if one knows what to look for, it is commonly accepted that different agencies can generate vastly different time domain test inputs whose SRS all satisfy the test requirements within a stated tolerance band. At an even more basic level, the laboratory test specifications often fail to resemble the field environment even though the SRS appear to be similar. A concise means of bounding the time domain inputs would be of great benefit in reducing the variation in the resulting shock tests. This paper describes a methodology that uses temporal moments to improve the repeatability of shock test specifications.

INTRODUCTION

Shock Response Spectra (SRS) have historically been used to define shock test requirements. The only criteria used to insure that a transient shock test specification is acceptable has been to check to see whether the resulting SRS is within a specified tolerance of the corresponding test requirement SRS.

The nonlinear relationship between SRS and the underlying acceleration history permit a wide range of acceleration histories to satisfy the test requirement. The introduction of test tolerances, which are needed to allow the testing laboratories some latitude to account for the physical realities of testing, tend to make the problem even worse.

Figures 1-5 show an example of the potential range of acceleration histories whose SRS would satisfy a typical test requirement based solely on matching the specified SRS test requirement. Figure 1 shows the acceleration history for the launch ignition shock for a single missile flight. Figure 2 shows the Shock Response Spectra (SRS) for the test requirement derived from the corresponding Maximum Expected Flight Environment (MEFE) and three different test specifications intended to simulate this environment in the laboratory. Figures 3-5 show the acceleration histories for these test specifications. Two of these test specifications were synthesized using a sum of decayed sinusoids technique [Smallwood & Nord, 1974]. The third test specification was derived using a transient random technique {Cap, 1994].

GOALS OF PROPOSED IMPROVEMENT

The objective of any improvement in the current process would be aimed at achieving a better agreement between the test specifications generated by different laboratories and between laboratory test specifications and the underlying field data.

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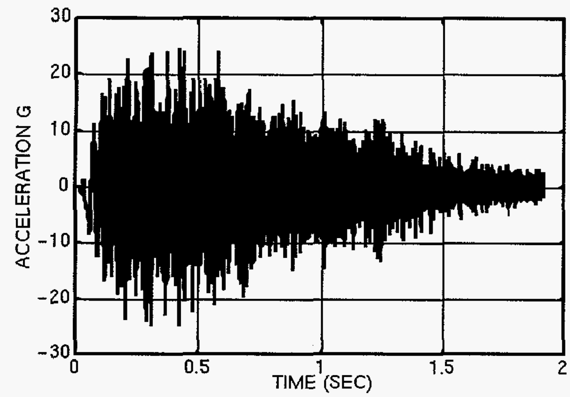

Figure 1: Acceleration History for Response to Launch Ignition Environment (Single Flight)

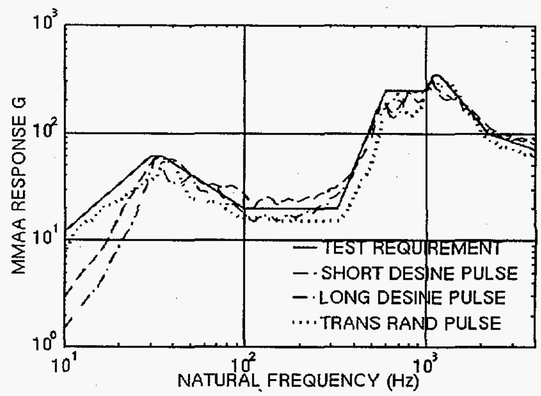

Figure 2: Comparison of SRS for Test Requirement and Three Different Test Specifications

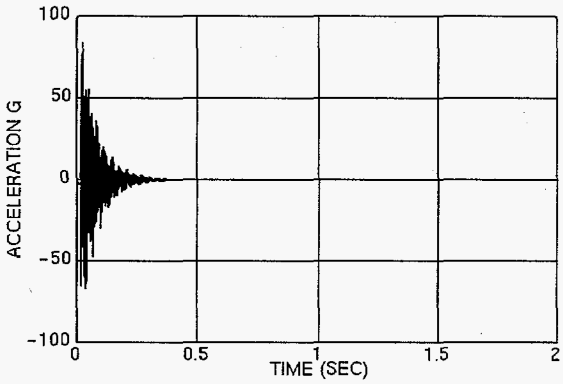

Figure 3: Acceleration History for Short Duration Decayed Sinusoid Test Specification

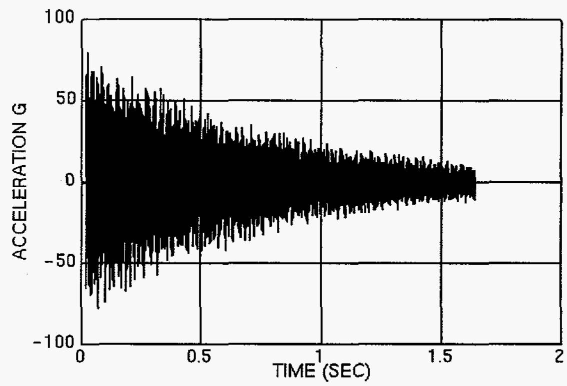

Figure 4: Acceleration History for Long Duration Decayed Sinusoid Test Specification

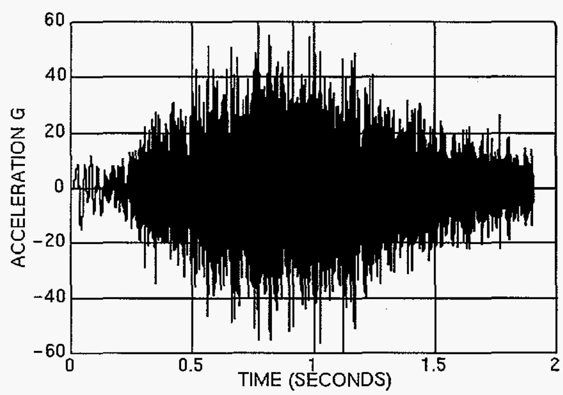

Figure 5: Acceleration History for Transient Random Test Specification

A concise means of constraining the time domain inputs without adding any significant restrictions on the test personnel would be of great benefit in reducing the variation in the resulting shock tests. With this goal in mind, our plan was to develop a time domain test requirement to be used in conjunction with the SRS for defining shock tests. This concept is analogous to the use of tolerances on both the overall g_{rms} and the Acceleration Spectral Density (ASD) profile when specifying a random vibration test.

Temporal moments [Smallwood, 1994] (a review of which has been included in Appendix A) have several qualities which make them ideal for this purpose: 1) temporal moments are easily computed using modern digital computers, 2) since they are scalar quantities, they are compact (the first 3-4 temporal moments should provide enough information for our purposes), and 3) because they are scalar quantities they can be easily scaled and toleranced.

Our experience [Smallwood, 1994] indicates that two pulses with nearly identical temporal moments and similar spectral content will have approximately the same SRS as well. Therefore, the use of temporal moments should not restrict the test engineer's ability to synthesize an acceptable test pulse, but rather aid him in that effort.

The energy (E), rms duration (D), and root energy amplitude (R) are probably adequate for most applications associated with conventional shock testing. These moments are analogous to the velocity, pulse width, and amplitude used to specify a classical haversine or half-sine drop table shock test. The corresponding higher order temporal moments will obviously diverge for profiles whose general shape does not look like the desired field environment. However, the matching of these higher moments would appear to be less important to the overall credibility of the resulting test specification.

OVERVIEW OF PROPOSED PROCESS

The procedure to implement a test defined with both SRS and temporal moments would be accomplished in two phases. Phase I would address the processing of the field data using the following steps:

1) Compute the SRS and temporal moments for the field data.
2) Apply desired margins to the SRS, energy (E), and root energy amplitude (R) (observing units).
3) Develop desired SRS test requirements (typically some sort of straight line envelope).
4) Define testing tolerances for SRS and temporal moments that are compatible with the desired test technique.

Phase II would address the processing of the laboratory test using the following steps:

1) Identify testing technique and develop best effort test specification (input profile).
2) Compute SRS and temporal moments for the test specification.
3) Compare test specification values with test requirements and iterate until the specification meets the requirement +/- tolerances.

Three case studies were analyzed to demonstrate how the technique might be applied. The first two cases looked at shaker shock testing, while the third case study looks at a resonant fixture test.

CASE STUDY #1: CORRELATING TEMPORAL MOMENTS WITH SHAKER SHOCK INPUT PARAMETERS

If temporal moments are to be of any use in defining shaker table shock tests, it will be necessary to relate the temporal moments to the input parameters used to define the shock test. With this idea in mind, the first case study examined the relationship between temporal moments and the parameters used to define two common shaker shock input acceleration histories: 1) sum of decayed sinusoids [Smallwood & Nord, 1974], and 2) wavsyn pulses [Smallwood, 1974].

The form for a decayed sinusoid shock pulse, x(t), is shown in Eq. (1). Table I shows the temporal moments for various individual and combined decayed sinusoid shock pulses.

$$x(t) = X_o e^{-2\zeta\pi ft} \sin(2\pi ft) \tag{1}$$

where X_0 is the amplitude, ζ is the decay rate, f is the frequency, and t is time. The characteristic time constant, τ_{ds}, for a decayed sinusoid is defined in Eq. (2).

$$\tau_{ds} = \frac{1}{2\zeta\pi f} \tag{2}$$

Table I: Temporal Moment Calculations for Decayed Sinusoid Shock Pulses

Case	Decayed Sinusoid Parameters				Temporal Moments			
	f (Hz)	X_0 (g)	ζ	τ_{ds}	E (g^2s)	R (g)	τ (sec)	D (sec)
1	20	10	0.1	0.080	7.9	14.1	0.040	0.039
2	40	10	0.05	0.080	31.8	28.3	0.040	0.040
3	80	10	0.025	0.080	127.2	56.6	0.040	0.040
4	40	10	0.1	0.040	15.8	28.3	0.020	0.020
5	80	10	0.1	0.040	31.5	56.6	0.010	0.010
6	80	20	0.025	0.020	127.2	56.6	0.040	0.040
7 (1)	–	–	–	0.080	169.3	64.9	0.039	0.040

Note (1): Case 7 is a shock pulse made up of the individual decayed sinusoids shown in cases 1-3.

Several important conclusions can be drawn from this case study

1) The results for cases 1-6 show that the centroid, τ, and rms duration, D, are both equal to $0.5\tau_{ds}$ (within numerical round-off error). This result agrees with the theoretical prediction for the temporal moments of an exponential window [Smallwood, 1994].
2) A comparison of the results for cases 1-3 against the results for case 7 show that the centroid, τ, and rms duration, D, are conserved for a combination of decayed sinusoids each having the same values for τ_{ds}.
3) A comparison of the results for cases 1-3 against the results for case 7 show that for a combination of decayed sinusoids the root energy amplitude, R, is equal to the rms of the individual R's. Similarly, the energy for the combination of decayed sinusoids is the sum of the individual energies. It is not known if this will be true if τ_{ds} is not held constant.

These conclusions demonstrate a direct, linear relationship between the parameters used to generate a decayed sinusoid shock pulse and the corresponding temporal moments that we are trying to match. While the third conclusion is interesting, it is not considered to be crucial since the energy and root energy amplitude of the pulse are constrained by the choice for τ_{ds} and the need to match the desired SRS (which is usually done automatically by the shaker shock synthesis algorithms). Therefore, it will only be necessary for the test engineer to correctly define the parameters associated with the characteristic time constant, τ_{ds}, in order to produce the optimum decayed sinusoid shaker shock test pulse.

The form for a wavsyn pulse, x(t), is shown in Eq. (3). Table II shows the temporal moments for individual and combined wavsyn pulses.

$$x(t) = X_0 \sin(2\pi b_m t)\sin(2\pi f_m t) \quad \text{for } 0 < t < T_m$$
$$= 0 \quad \text{for } t > T_m \tag{3}$$

where X_0 is the amplitude, $f_m = N_m b_m$, $T_m = 1/(2b_m)$, and N_m is an odd integer.

Table II: Temporal Moment Calculations for Wavsyn Pulses

Case	Wavsyn Parameters				Temporal Moments			
	f_m (Hz)	X_0 (g)	N_m	T_m	E (g^2s)	R (g)	τ (sec)	D (sec)
1	20	10	7	0.18	4.4	11.8	0.50	0.032
2	40	10	15	0.19	4.7	11.8	0.50	0.034
3	80	10	29	0.19	4.5	11.8	0.50	0.033
4	40	10	7	0.09	2.2	11.8	0.50	0.016
5	80	10	7	0.044	1.1	11.8	0.50	0.008
6	80	20	29	0.18	18.1	23.5	0.50	0.033
7 (1)	–	–	–	0.18	13.7	20.3	0.09	0.033

Note (1): Case 7 is a shock pulse made up of the individual wavsyn pulses shown in cases 1-3.

Several important conclusions can be drawn from this case study

1) The results for cases 1-6 show the rms duration, D, to be equal to ≈0.18 times the overall wavsyn pulse duration, T_m. (the jitter seen in the values in Table II is due to the relatively small number of half cycles used in the study). This is consistent with the theoretical prediction for the temporal moments of a half-sine window [Smallwood, 1994].
2) A comparison of the results for cases 1-3 against the results for case 7 show that the rms duration, D, is conserved for a combination of wavsyn pulses each having the same values for T_m.
3) A comparison of the results for cases 1-3 against the results for case 7 show that for a combination of wavsyn pulses the root energy amplitude, R, is equal to the rms of the individual R's. Similarly, the energy for the combination of wavsyn pulses is the sum of the individual energies.

These conclusions demonstrate a direct, linear relationship between the parameters used to generate a wavsyn shock pulse and the corresponding temporal moments that we are trying to match. While the third conclusion is interesting, it is not considered to be crucial since the energy and root energy amplitude of the pulse are constrained by the choice for T_m and the need to match the desired SRS (which is usually done automatically by the shaker shock synthesis algorithms). Therefore, it will only be necessary for the test engineer to correctly define the parameters associated with the characteristic time constant, T_m in order to produce the optimum wavsyn shaker shock test.

CASE STUDY #2: OPTIMIZATION OF SHAKER SHOCK TEST SPECIFICATIONS

This case study considered the temporal moments for a set of three best effort shock pulses synthesized using sums of decayed sinusoids and transient random inputs. The objective was to determine just how close one could get to the desired values for the temporal moments. The SRS and acceleration histories used in this example are shown in Figures 2-5. Table III presents temporal moments for the underlying MEFE environment and the three test specifications. The MEFE temporal moments were derived from the temporal moments of the individual flights using the same techniques that were applied to obtain the MEFE SRS profile (i.e., assume a lognormal distribution with a standard deviation of 3 dB).

Table III: Temporal Moments for MEFE Ignition Shock Environment and Best Effort Shaker Shock Test Specifications

	E (g^2sec)	E_{Error}	D (sec)	R (g)	R_{Error}
MEFE Envelope	275	–	0.39	26.5	–
Transients Random	290	0.2 dB	0.36	28.6	0.7 dB
Short Decayed sinusoid	45	-7.9 dB	0.05	36.5	2.8 dB
Long Decayed sinusoid	353	1.1 dB	0.34	31.5	1.5 dB

The transient random input was designed specifically to match the temporal moments of the MEFE profile so it is not surprising that its temporal moments are closely matched to those of the underlying field environment. The decayed sinusoid inputs were designed to match only the MEFE SRS. Since the long duration decayed sinusoid pulse has approximately the correct rms duration, it also provides a fairly close match for the other field environment temporal moments. However, the short duration decayed sinusoid is a poor match for all of the temporal moments.

Based on experience with decayed sinusoid testing (and drawing on parallels from drop table testing), a possible set of tolerances for a shaker shock test might look as follows:

1) +/-3 dB on the SRS.
2) +/-1 dB on the energy (E) and the root energy amplitude (R) in each frequency analysis band.
3) +/- 10% on the rms duration.

It is believed that by constraining the range of permitted values for the temporal moments, it might be possible to relax the tolerances on the SRS to as much as +/- 6 dB, thereby making it easier to accommodate local exceedances associated with fixture resonances and other such anomalies without sacrificing the overall fidelity of the intended test levels. However, such an approach will not produce the desired results unless band limited temporal moments are also employed.

CASE STUDY #3: RESONANT FIXTURE TEST SPECIFICATIONS

Resonant fixture testing is inherently less precise than shaker shock testing owing to several factors. The primary reasons for this are the limited degree of tailoring possible with the fixtures themselves and the somewhat variable excitation sources - which are often based on the rapid release of energy. However, it is the very fact that resonant fixture exhibits a high degree of variability (with a corresponding high variability in the amplitude and frequency content of the resulting SRS) that makes the use of temporal moments so potentially valuable.

This problem is illustrated in Figures 6-9 using the example of a resonant fixture test conducted at Sandia in order to simulate a pyrotechnic stage separation shock environment. Figure 6 shows the acceleration history for one of the higher responses measured within a zone during the pyroshock event. Figure 7 shows the SRS for the acceleration history shown in Figure 6, the SRS envelope of the field responses for that entire zone, and the resulting SRS test requirement. Figure 7 shows that the SRS test requirement is fairly close in magnitude to the SRS of the individual response location. Therefore, one would expect that the temporal moments for a test specification whose SRS matched the test requirement SRS would not be significantly higher than the temporal moments of any acceleration history within that zone.

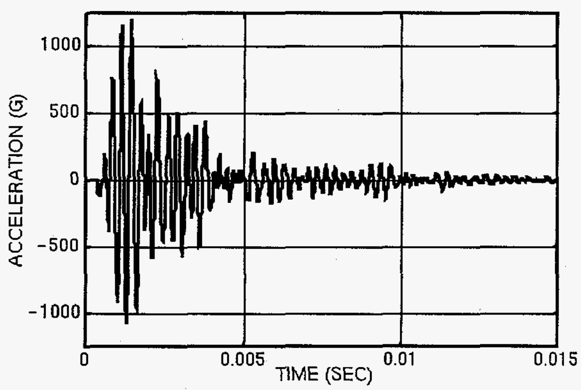

Figure 6: Acceleration History for Severe Field Environment

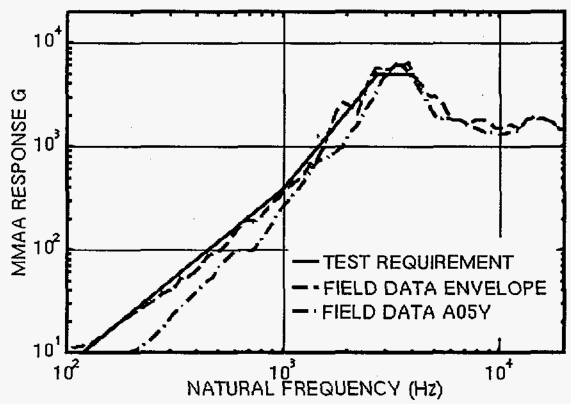

Figure 7: Comparison of SRS for Field Environment and Test Requirement

Figure 8 shows the acceleration history for a typical test specification (Test #19). Figure 9 shows the SRS for the test requirement (with the corresponding +/-6 dB tolerance bands) along with two separate resonant fixture inputs used to simulate the required SRS (Tests 12 and 19).

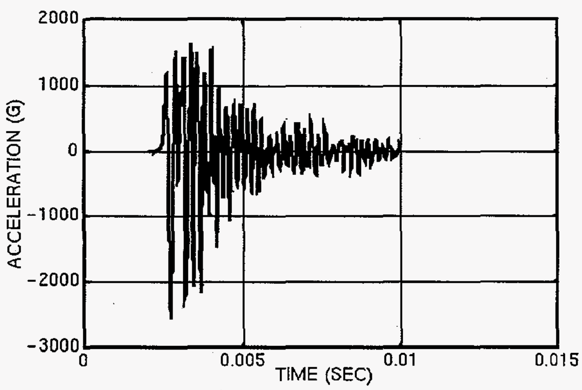

Figure 8: Acceleration History for a Typical Test Specification

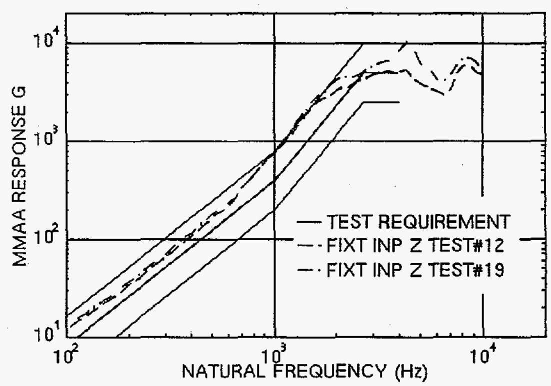

Figure 9: Comparison of SRS for Test Requirement and Two Best Effort Test Specifications

Table IV shows the temporal moments for the field environment and the two test specifications. The temporal moments for the field environment were generated by taking the most severe values from all of the accelerometer responses within the zone (i.e., maximum E and R, minimum D).

Table IV: Temporal Moments for Pyroshock Environment and Best Effort Resonant Fixture Test Specifications

	E (g^2sec)	E_{Error}	D (sec)	R (g)	R_{Error}
Field Response	809	–	0.002	617	–
Comp Test #12	1453	2.5 dB	0.001	1204	5.8 dB
Comp Test #19	2384	4.3 dB	0.001	1492	7.7 dB

The SRS for both test specifications were within the typical +/-6 dB tolerance bands and therefore were considered to be satisfactory inputs at the time of the tests. However, while the energy (E) values for these test specifications are pretty good, the rms duration is only half as long as desired and as a result the root energy amplitude is much higher than the desired value. It is possible that these differences are due to the presence of high frequency energy in the test pulses that was not present in the field environment (this would tend to shorten the rms duration and increase the root energy amplitude).

With regards to potential tolerancing schemes for resonant fixture testing, it is important to remember that such a test is often used to simulate pyrotechnic environments. In such a test the predominate feature of the test requirement SRS is the knee frequency (with a relatively flat profile to the SRS for higher frequencies and a steep roll-off in the SRS for lower frequencies). Therefore, it would seem logical to define the primary analysis band to be centered about the knee frequency (such as +/- 1 octave). The bands on either side of the primary band would most likely have significantly lower values for the energy (E) and root energy amplitude (R) associated with them.

A possible set of tolerances for a resonant fixture test might look as follows (assume a 2500 Hz knee frequency for this example):

1) Three frequency analysis bands: a) 0-1250 Hz, b) 1250-5000 Hz, c) > 5000 Hz.
2) +/-6 dB on the SRS in all analysis bands.
3) +/-2 dB on the energy (E) and the root energy amplitude (R) in the primary frequency analysis band. The energy and root energy amplitude in the fringe bands should be less than some yet to be determined fraction of the energy in the primary frequency band.
4) +/- 20% on the rms duration.

It is interesting to note that the test specifications in this example would be considered overtests using these criteria.

As was the case for the shaker shock tests, it is believed that use of temporal moments should provide information that will be useful in tailoring the test during the setup phase, and will also make it easier for the test engineer to know whether or not a given test input is acceptable or whether it will require additional tailoring by providing a quantitative set of scalar measurements with which to judge the test pulse.

DISCUSSION ON BAND LIMITED TEMPORAL MOMENTS

The use of band limited temporal moments give the test engineer greater latitude in tailoring the input shock pulse. With regards to the use of band limited temporal moments in defining shaker shock tests, there should not be any major constraints with using as many as 5-6 frequency bands for a typical 20-2000 Hz shaker shock test input.

However, given the relatively short duration inputs typically used in resonant fixture testing, the frequency bands used in defining the temporal moments will have to be kept very broad in order to maintain a minimum value for the product of the frequency bandwidth and the rms duration (this is analogous to the minimum BT product used in random vibration testing to define the minimum acceptable variance error).

FUTURE WORK

In order for this technique to become practical, two fundamental databases must be developed.

1) Collect representative database of field environments and establish appropriate temporal moments parameters (i.e., which moments are needed, how many frequency bands are needed, etc.).
2) Monitor laboratory tests to establish viable tolerances.

CONCLUSIONS

The direct relationship between the parameters used to define a shaker shock and the temporal moments of the resulting acceleration history ensures that temporal moments can be used to assist the test engineer in quantifying the relationship between a laboratory shock test pulse and the underlying field environment. While the relationships between the temporal moments and the related test setup parameters are somewhat more indirect for resonant fixture testing, the ability of quantify the differences between laboratory test pulses and the underlying field environment is just as useful. Therefore, temporal moments can be used to improve the realism of the test specifications without making the test any more difficult to develop and implement.

REFERENCES

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APPENDIX A REVIEW OF TEMPORAL MOMENTS

The ith temporal moment, $m_i(a)$, of a time history, x(t), about a time location, a, is defined by the formula in Eq. (A-1).

$$m_i(a) = \int_{-\infty}^{+\infty} (t-a)^i x^2(t)dt \qquad \text{(A-1)}$$

The resulting moments. $m_i(a)$, are converted into more physically meaningful terms as shown in Table V.

Table V: Physical Interpretations of Temporal Moments

Energy (E)	$E = m_0$
centroid (τ)	$\tau = m_1/m_0$
RMS Duration (D)	$D^2 = m_2(\tau)/E$
Root Energy Amplitude (R)	$R^2 = E/D$
Skewness (S_τ)	$S_\tau^3 = m_3(\tau)/E$

The Energy, centroid, and rms Duration are fairly intuitive quantities. The Root Energy Amplitude is very useful because it has the same units as the original acceleration history and can therefore be used as a measure of the "peak" amplitude. The Skewness is a measure of the pulses symmetry (a positive Skewness denotes a sharp rise time followed by a gradual decay, a negative Skewness denotes just the opposite, while a zero skewness denotes a symmetric pulse).

The temporal moments can be computed for the entire frequency range of interest or for several frequency bands if deemed necessary.