TWO-STAGE ISOLATION FOR HARMONIC BASE EXCITATION
Revision C

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## Introduction

Consider a base plate mass $m_1$ and an avionics mass $m_2$ modeled as two-degree-of-freedom. Evaluate the benefits and drawbacks of this two-stage isolation scheme.

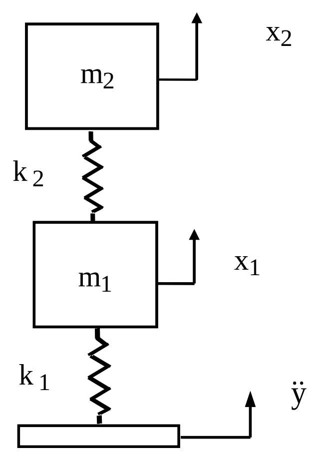


Figure 1.

The system also has damping, but it is modeled as modal damping.

A free-body diagram of mass 1 is given in Figure 2. A free-body diagram of mass 2 is given in Figure 3.

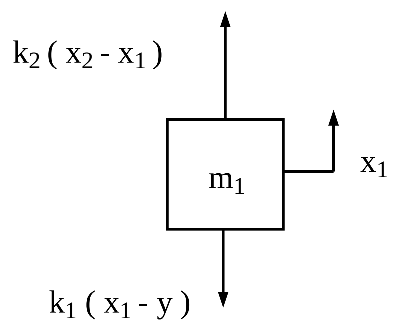


Figure 2.

Determine the equation of motion for mass 1.

$$\Sigma F = m_1 \ddot{x}_1 \tag{1}$$

$$m_1 \ddot{x}_1 = k_2(x_2 - x_1) - k_1(x_1 - y) \tag{2}$$

$$m_1 \ddot{x}_1 + k_1 x_1 - k_2(x_2 - x_1) = k_1 y \tag{3}$$

$$m_1 \ddot{x}_1 + k_1 x_1 + k_2(x_1 - x_2) = k_1 y \tag{4}$$

$$m_1 \ddot{x}_1 + (k_1 + k_2)x_1 - k_2 x_2 = k_1 y \tag{5}$$

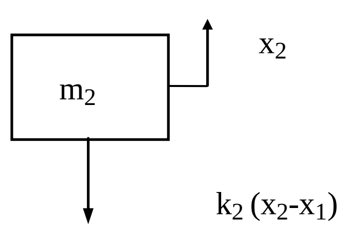


Figure 3.

Derive the equation of motion for mass 2.

$$\Sigma F = m_2 \ddot{x}_2 \tag{6}$$

$$m_2 \ddot{x}_2 = -k_2 \left( x_2 - x_1 \right) \tag{7}$$

$$m_2 \ddot{x}_2 + k_2 \left( x_2 - x_1 \right) = 0 \tag{8}$$

$$m_2 \ddot{x}_2 + k_2 x_2 - k_2 x_1 = 0 \tag{9}$$

Assemble the equations in matrix form.

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} k_1 y \\ 0 \end{bmatrix} \tag{10}$$

Define a relative displacement z such that

$$x_1 = z_1 + y \tag{11}$$

$$x_2 = z_2 + y \tag{12}$$

Substitute equations (11) and (12) into (10).

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{bmatrix} \ddot{z}_1 + \ddot{y} \\ \ddot{z}_2 + \ddot{y} \end{bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{bmatrix} z_1 + y \\ z_2 + y \end{bmatrix} = \begin{bmatrix} k_1 y \\ 0 \end{bmatrix} \tag{13}$$

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{bmatrix} \ddot{z}_1 \\ \ddot{z}_2 \end{bmatrix} + \begin{bmatrix} m_1 \ddot{y} \\ m_2 \ddot{y} \end{bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{bmatrix} y \\ y \end{bmatrix} = \begin{bmatrix} k_1 y \\ 0 \end{bmatrix} \tag{14}$$

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{bmatrix} \ddot{z}_1 \\ \ddot{z}_2 \end{bmatrix} + \begin{bmatrix} m_1 \ddot{y} \\ m_2 \ddot{y} \end{bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} k_1 y \\ 0 \end{bmatrix} = \begin{bmatrix} k_1 y \\ 0 \end{bmatrix} \tag{15}$$

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{bmatrix} \ddot{z}_1 \\ \ddot{z}_2 \end{bmatrix}+\begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}\begin{bmatrix} z_1 \\ z_2 \end{bmatrix}=\begin{bmatrix} -m_1\ddot{y} \\ -m_2\ddot{y} \end{bmatrix} \tag{16}$$

Decoupling

Equation (16) is coupled via the stiffness matrix. An intermediate goal is to decouple the equation.

Simplify,

$$M\,\bar{\ddot{z}}+K\,\bar{z}=\bar{F} \tag{17}$$

where

$$M=\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \tag{18}$$

$$K=\begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \tag{19}$$

$$\bar{z}=\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \tag{20}$$

$$\bar{F}=\begin{bmatrix} -m_1\ddot{y} \\ -m_2\ddot{y} \end{bmatrix} \tag{21}$$

Consider the homogeneous form of equation (17).

$$M\,\bar{\ddot{z}}+K\,\bar{z}=\bar{0} \tag{22}$$

Seek a solution of the form

$$\bar{z}=\bar{q}\exp\left(\,j\omega t\right) \tag{23}$$

The q vector is the generalized coordinate vector.

Note that

$$\dot{\bar{z}} = j\omega\,\bar{q}\exp(j\omega t) \tag{24}$$

$$\ddot{\bar{z}} = -\omega^2\,\bar{q}\exp(j\omega t) \tag{25}$$

Substitute equations (23) through (25) into equation (22).

$$-\omega^2 M\,\bar{q}\exp(j\omega t) + K\bar{q}\exp(j\omega t) = \bar{0} \tag{26}$$

$$\left\{-\omega^2 M\,\bar{q} + K\bar{q}\right\}\exp(j\omega t) = \bar{0} \tag{27}$$

$$-\omega_n{}^2 M\,\bar{q} + K\bar{q} = \bar{0} \tag{28}$$

$$\left\{-\omega^2 M + K\right\}\bar{q} = \bar{0} \tag{29}$$

$$\left\{K - \omega^2 M\right\}\bar{q} = \bar{0} \tag{30}$$

Equation (30) is an example of a generalized eigenvalue problem. The eigenvalues can be found by setting the determinant equal to zero.

$$\det\left\{K - \omega^2 M\right\} = 0 \tag{31}$$

$$\det\left\{\begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} - \omega^2\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\right\} = 0 \tag{32}$$

$$\det\begin{bmatrix} (k_1 + k_2) - \omega^2 m_1 & -k_2 \\ -k_2 & k_2 - \omega^2 m_2 \end{bmatrix} = 0 \tag{33}$$

$$\left[(k_1 + k_2) - \omega^2 m_1\right]\left[k_2 - \omega^2 m_2\right] + k_2(k_1 + k_2) - k_2{}^2 = 0 \tag{34}$$

$$\omega^4 m_1 m_2 - \omega^2\left[m_1 k_2 + m_2(k_1 + k_2)\right] + k_1 k_2 = 0 \tag{35}$$

The eigenvalues are the roots of the polynomial.

$$\omega_1^2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \tag{36}$$

$$\omega_2^2 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \tag{37}$$

where

$$a = m_1 m_2 \tag{38}$$

$$b = -\left[m_1 k_2 + m_2\left(k_1 + k_2\right)\right] \tag{39}$$

$$c = k_1\ k_2 \tag{40}$$

$$\omega_1^2 = \frac{\left[m_1 k_2 + m_2\left(k_1 + k_2\right)\right] - \sqrt{(m_1 k_2 + m_2 (k_1 + k_2))^2 - 4 m_1 m_2 k_1 k_2}}{2 m_1 m_2} \tag{41}$$

$$\omega_2^2 = \frac{\left[m_1 k_2 + m_2\left(k_1 + k_2\right)\right] + \sqrt{(m_1 k_2 + m_2 (k_1 + k_2))^2 - 4 m_1 m_2 k_1 k_2}}{2 m_1 m_2} \tag{42}$$

The eigenvectors are found via the following equations.

$$\left\{ K - \omega_1{}^2 \, M \right\} \bar{q}_1 = \bar{0} \tag{43}$$

$$\left\{ K - \omega_2{}^2 \, M \right\} \bar{q}_2 = \bar{0} \tag{44}$$

where

$$\bar{q}_1 = \begin{bmatrix} q_{11} \\ q_{12} \end{bmatrix} \tag{45}$$

$$\bar{q}_2 = \begin{bmatrix} q_{21} \\ q_{22} \end{bmatrix} \tag{46}$$

An eigenvector matrix Q can be formed. The eigenvectors are inserted in column format.

$$Q = [\ \bar{q}_1 \quad | \quad \bar{q}_2\ ] \tag{47}$$

$$Q = \begin{bmatrix} q_{11} & q_{21} \\ q_{12} & q_{22} \end{bmatrix} \tag{48}$$

The eigenvectors represent orthogonal mode shapes.

Each eigenvector can be multiplied by an arbitrary scale factor. A mass-normalized eigenvector matrix $\hat{Q}$ can be obtained such that the following orthogonality relations are obtained.

$$\hat{Q}^T M \hat{Q} = I \tag{49}$$

and

$$\hat{Q}^T K \hat{Q} = \Omega \tag{50}$$

where

superscript T represents transpose

I is the identity matrix

$\Omega$ is a diagonal matrix of eigenvalues

Note that

$$\hat{Q} = \begin{bmatrix} \hat{q}_{11} & \hat{q}_{12} \\ \hat{q}_{21} & \hat{q}_{22} \end{bmatrix} \tag{51}$$

$$\hat{Q}^T = \begin{bmatrix} \hat{q}_{11} & \hat{q}_{21} \\ \hat{q}_{12} & \hat{q}_{22} \end{bmatrix} \tag{52}$$

Rigorous proof of the orthogonality relationships is beyond the scope of this tutorial. Further discussion is given in References 5 and 6.

Nevertheless, the orthogonality relationships are demonstrated by an example in this tutorial.

Now define a modal coordinate $\eta(t)$ such that

$$\bar{z} = \hat{Q}\ \bar{\eta} \tag{53}$$

$$z_1 = \hat{q}_{11}\,\eta_1 + \hat{q}_{12}\,\eta_2 \tag{54}$$

$$z_2 = \hat{q}_{21}\,\eta_1 + \hat{q}_{22}\,\eta_2 \tag{55}$$

Recall

$$x_1 = z_1 + y \tag{56}$$

$$x_2 = z_2 + y \tag{57}$$

The displacement terms are

$$x_1 = y + \hat{q}_{11}\,\eta_1 + \hat{q}_{12}\,\eta_2 \tag{58}$$

$$x_2 = y + \hat{q}_{21}\,\eta_1 + \hat{q}_{22}\,\eta_2 \tag{59}$$

The velocity terms are

$$\dot{x}_1 = \dot{y} + \hat{q}_{11}\,\dot{\eta}_1 + \hat{q}_{12}\,\dot{\eta}_2 \tag{60}$$

$$\dot{x}_2 = \dot{y} + \hat{q}_{21}\,\dot{\eta}_1 + \hat{q}_{22}\,\dot{\eta}_2 \tag{61}$$

The acceleration terms are

$$\ddot{x}_1 = \ddot{y} + \hat{q}_{11}\,\ddot{\eta}_1 + \hat{q}_{12}\,\ddot{\eta}_2 \tag{62}$$

$$\ddot{x}_2 = \ddot{y} + \hat{q}_{21}\,\ddot{\eta}_1 + \hat{q}_{22}\,\ddot{\eta}_2 \tag{63}$$

Substitute equation (53) into the equation of motion, equation (17).

$$M\hat{Q}\ \bar{\ddot{\eta}} + K\hat{Q}\ \bar{\eta} = \bar{F} \tag{64}$$

Premultiply by the transpose of the normalized eigenvector matrix.

$$\hat{Q}^T M \hat{Q}\, \ddot{\bar{\eta}} + \hat{Q}^T K \hat{Q}\, \bar{\eta} = \hat{Q}^T \bar{F} \tag{65}$$

The orthogonality relationships yield

$$I\, \ddot{\bar{\eta}} + \Omega\, \bar{\eta} = \hat{Q}^T \bar{F} \tag{66}$$

For the sample problem, equation (66) becomes

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \ddot{\eta}_1 \\ \ddot{\eta}_2 \end{bmatrix} + \begin{bmatrix} \omega_1^2 & 0 \\ 0 & \omega_2^2 \end{bmatrix} \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix} = \begin{bmatrix} \hat{q}_{11} & \hat{q}_{21} \\ \hat{q}_{12} & \hat{q}_{22} \end{bmatrix} \begin{bmatrix} -m_1 \ddot{y} \\ -m_2 \ddot{y} \end{bmatrix} \tag{67}$$

Note that the two equations are decoupled in terms of the modal coordinate.

Now assume modal damping by adding an uncoupled damping matrix.

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \ddot{\eta}_1 \\ \ddot{\eta}_2 \end{bmatrix} + \begin{bmatrix} 2\xi_1 \omega_1 & 0 \\ 0 & 2\xi_2 \omega_2 \end{bmatrix} \begin{bmatrix} \dot{\eta}_1 \\ \dot{\eta}_2 \end{bmatrix} + \begin{bmatrix} \omega_1^2 & 0 \\ 0 & \omega_2^2 \end{bmatrix} \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix} = \begin{bmatrix} \hat{q}_{11} & \hat{q}_{21} \\ \hat{q}_{12} & \hat{q}_{22} \end{bmatrix} \begin{bmatrix} -m_1 \ddot{y} \\ -m_2 \ddot{y} \end{bmatrix} \tag{68}$$

Equation (68) yields two equations

$$\ddot{\eta}_1 + 2\xi_1 \omega_1 + \omega_1^2\, \eta_1 = -\left[\hat{q}_{11} m_1 + \hat{q}_{21} m_2\right] \ddot{y} \tag{69}$$

$$\ddot{\eta}_2 + 2\xi_2 \omega_2 + \omega_2^2\, \eta_2 = -\left[\hat{q}_{12} m_1 + \hat{q}_{22} m_2\right] \ddot{y} \tag{70}$$

Now assume a harmonic base input.

$$\ddot{y} = A \exp(j\omega t) \tag{71}$$

Assume a harmonic modal displacement.

$$\eta_i = \psi_i \exp(j\omega t) \tag{72}$$

$$\dot{\eta}_i = j\, \omega_i\, \psi_i \exp(j\omega t) \tag{73}$$

$$\ddot{\eta}_i = -\omega_i^2 \psi_i \exp(j\omega t) \tag{74}$$

By substitution,

$$\left\{ -\omega^2 + j\,2\xi_1 \omega_1 \omega + \omega_1^2 \right\} \psi_1 \exp(j\omega t) = -\left[ \hat{q}_{11} m_1 + \hat{q}_{21} m_2 \right] A \exp(j\omega t) \tag{75}$$

$$\left\{ -\omega^2 + j\,2\xi_2 \omega_2 \omega + \omega_2^2 \right\} \psi_2 \exp(j\omega t) = -\left[ \hat{q}_{12} m_1 + \hat{q}_{22} m_2 \right] A \exp(j\omega t) \tag{76}$$

$$\left\{ \left[ \omega_1^2 - \omega^2 \right] + j\,2\xi_1 \omega_1 \omega \right\} \psi_1 \exp(j\omega t) = -\left[ \hat{q}_{11} m_1 + \hat{q}_{21} m_2 \right] A \exp(j\omega t) \tag{77}$$

$$\left\{ \left[ \omega_2^2 - \omega^2 \right] + j\,2\xi_2 \omega_2 \omega \right\} \psi_2 \exp(j\omega t) = -\left[ \hat{q}_{12} m_1 + \hat{q}_{22} m_2 \right] A \exp(j\omega t) \tag{78}$$

$$\eta_1 = \psi_1 \exp(j\omega t) = \frac{-\left[ \hat{q}_{11} m_1 + \hat{q}_{21} m_2 \right]}{\left\{ \left[ \omega_1^2 - \omega^2 \right] + j\,2\xi_1 \omega_1 \omega \right\}} A \exp(j\omega t) \tag{79}$$

$$\eta_2 = \psi_2 \exp(j\omega t) = \frac{-\left[ \hat{q}_{12} m_1 + \hat{q}_{22} m_2 \right]}{\left\{ \left[ \omega_2^2 - \omega^2 \right] + j\,2\xi_2 \omega_2 \omega \right\}} A \exp(j\omega t) \tag{80}$$

The modal velocity is

$$\dot{\eta}_1 = \frac{-j\omega \left[ \hat{q}_{11} m_1 + \hat{q}_{21} m_2 \right]}{\left\{ \left[ \omega_1^2 - \omega^2 \right] + j\,2\xi_1 \omega_1 \omega \right\}} A \exp(j\omega t) \tag{81}$$

$$\dot{\eta}_2 = \frac{-j\omega \left[ \hat{q}_{12} m_1 + \hat{q}_{22} m_2 \right]}{\left\{ \left[ \omega_2^2 - \omega^2 \right] + j\,2\xi_2 \omega_2 \omega \right\}} A \exp(j\omega t) \tag{82}$$

The modal acceleration is

$$\ddot{\eta}_1 = \frac{\omega^2 \left[\hat{q}_{11} m_1 + \hat{q}_{21} m_2\right]}{\left\{\left[\omega_1{}^2 - \omega^2\right] + j\, 2 \xi_1 \omega_1 \omega\right\}} \; A \exp(j\omega t) \tag{83}$$

$$\ddot{\eta}_2 = \frac{\omega^2 \left[\hat{q}_{12} m_1 + \hat{q}_{22} m_2\right]}{\left\{\left[\omega_2{}^2 - \omega^2\right] + j\, 2 \xi_2 \omega_2 \omega\right\}} \; A \exp(j\omega t) \tag{84}$$

Recall

$$\ddot{x}_1 = \ddot{y} + \hat{q}_{11} \ddot{\eta}_1 + \hat{q}_{12} \ddot{\eta}_2 \tag{85}$$

$$\ddot{x}_2 = \ddot{y} + \hat{q}_{21} \ddot{\eta}_1 + \hat{q}_{22} \ddot{\eta}_2 \tag{86}$$

$$\ddot{x}_1(t) = \left\{ 1 + \frac{\omega^2 \hat{q}_{11} \left[\hat{q}_{11} m_1 + \hat{q}_{21} m_2\right]}{\left\{\left[\omega_1{}^2 - \omega^2\right] + j\, 2 \xi_1 \omega_1 \omega\right\}} + \frac{\omega^2 \hat{q}_{12} \left[\hat{q}_{12} m_1 + \hat{q}_{22} m_2\right]}{\left\{\left[\omega_2{}^2 - \omega^2\right] + j\, 2 \xi_2 \omega_2 \omega\right\}} \right\} A \exp(j\omega t) \tag{87}$$

$$\ddot{x}_2(t) = \left\{ 1 + \frac{\omega^2 \hat{q}_{21} \left[\hat{q}_{11} m_1 + \hat{q}_{21} m_2\right]}{\left\{\left[\omega_1{}^2 - \omega^2\right] + j\, 2 \xi_1 \omega_1 \omega\right\}} + \frac{\omega^2 \hat{q}_{22} \left[\hat{q}_{12} m_1 + \hat{q}_{22} m_2\right]}{\left\{\left[\omega_2{}^2 - \omega^2\right] + j\, 2 \xi_2 \omega_2 \omega\right\}} \right\} A \exp(j\omega t) \tag{88}$$

The Fourier transform equation is

$$\hat{X}_i(f) = \int_{-\infty}^{\infty} \ddot{x}_i(t) \exp[-\, j\omega t]\, dt \tag{89}$$

Take the Fourier transform of each side of equations (87) and (88).

$$\hat{X}_1(f)/A = \left\{1 + \frac{\omega^2 \hat{q}_{11}\left[\hat{q}_{11} m_1 + \hat{q}_{21} m_2\right]}{\left\{\left[\omega_1^2 - \omega^2\right] + j\,2\xi_1\,\omega_1\,\omega\right\}} + \frac{\omega^2 \hat{q}_{12}\left[\hat{q}_{12} m_1 + \hat{q}_{22} m_2\right]}{\left\{\left[\omega_2^2 - \omega^2\right] + j\,2\xi_2\,\omega_2\,\omega\right\}}\right\} \tag{90}$$

$$\hat{X}_2(f)/A = \left\{1 + \frac{\omega^2 \hat{q}_{21}\left[\hat{q}_{11} m_1 + \hat{q}_{21} m_2\right]}{\left\{\left[\omega_1^2 - \omega^2\right] + j\,2\xi_1\,\omega_1\,\omega\right\}} + \frac{\omega^2 \hat{q}_{22}\left[\hat{q}_{12} m_1 + \hat{q}_{22} m_2\right]}{\left\{\left[\omega_2^2 - \omega^2\right] + j\,2\xi_2\,\omega_2\,\omega\right\}}\right\} \tag{91}$$

References


1. T. Irvine, An Introduction to the Shock Response Spectrum Revision P, Vibrationdata, 2002.

2. T. Irvine, Response of a Single-degree-of-freedom System Subjected to a Classical Pulse Base Excitation, Revision A, Vibrationdata, 1999.

3. R. Cook, Finite Element Modeling for Stress Analysis, Wiley, New York, 1995.

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5. Bathe, Finite Element Procedures in Engineering Analysis, Prentice-Hall, New Jersey, 1982.

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7. L. Meirovitch, Analytical Methods in Vibrations, Macmillan, New York, 1967.

APPENDIX A

EXAMPLE 1

Normal Modes Analysis

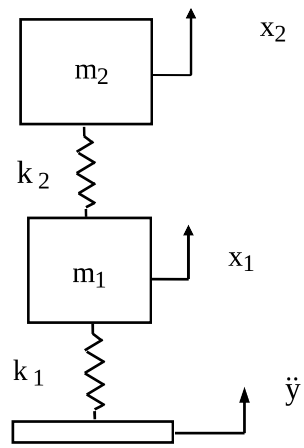



Figure A-1.

A 5-lbm avionics component ($m_2$) is mounted on a 2-lbm base plate ($m_1$). Each spring stiffness is 4.6e+04 lbf/in. Analyze the energy transmitted to the avionics mass with and without the base plate stage.

| Table A-1. Parameters | |
|---|---|
| Variable | Value |
| $m_1$ | 2 lbm |
| $m_2$ | 5 lbm |
| $k_1$ | 4.6e+04 lbf/in |
| $k_2$ | 4.6e+04 lbf/in |

Furthermore, assume that each mode has a damping value of 5%.

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{bmatrix} \ddot{z}_1 \\ \ddot{z}_2 \end{bmatrix}+\begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}\begin{bmatrix} z_1 \\ z_2 \end{bmatrix}=\begin{bmatrix} -m_1\ddot{y} \\ -m_2\ddot{y} \end{bmatrix} \tag{A-1}$$

Solve for the acceleration response time histories. The homogeneous, undamped problem is

$$\begin{bmatrix} 2/386 & 0 \\ 0 & 5/386 \end{bmatrix}\begin{bmatrix} \ddot{z}_1 \\ \ddot{z}_2 \end{bmatrix}+\begin{bmatrix} 9.2e+04 & -4.6e+04 \\ -4.6e+04 & 4.6e+04 \end{bmatrix}\begin{bmatrix} z_1 \\ z_2 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{A-2}$$

The natural frequencies are

$$f_1 = 201.3 \text{ Hz} \tag{A-3}$$

$$f_2 = 706.5 \text{ Hz} \tag{A-4}$$

FRF Analysis

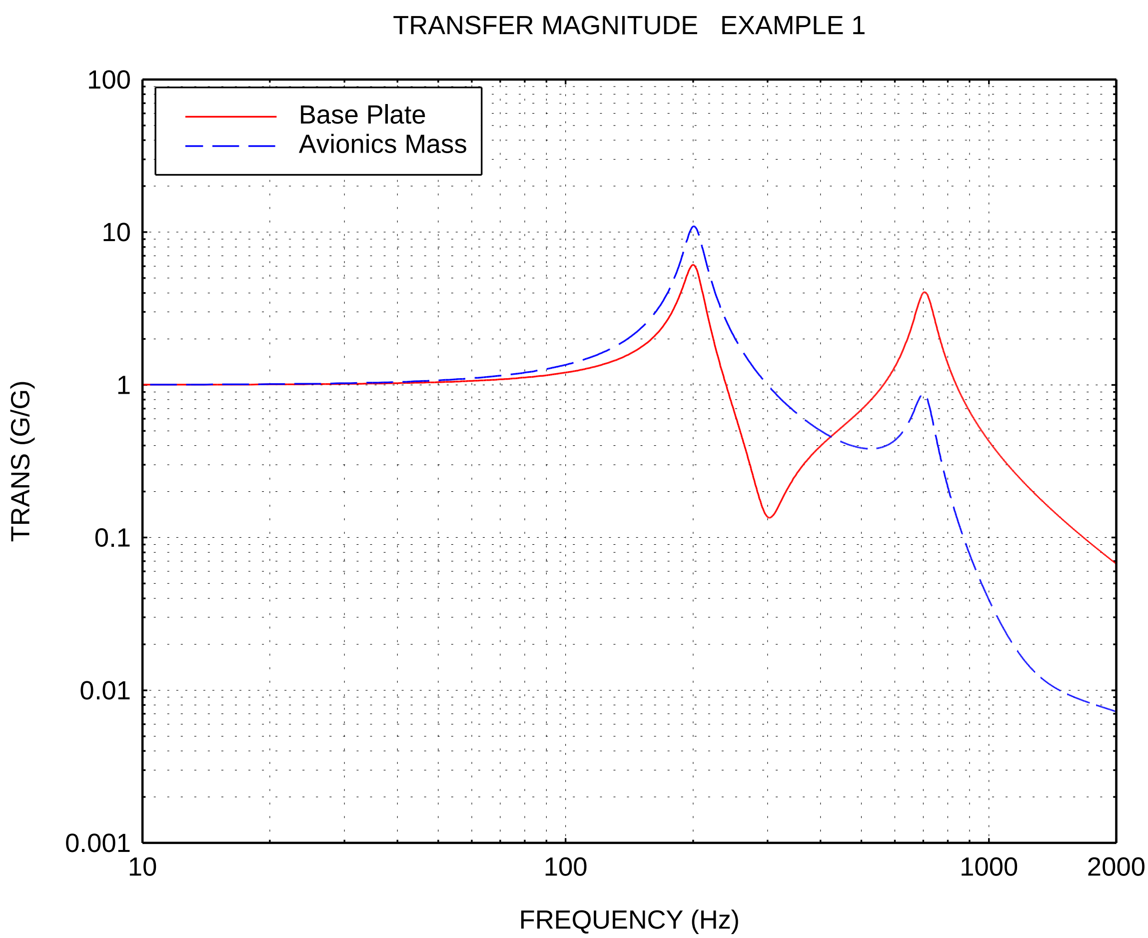



Figure A-2.

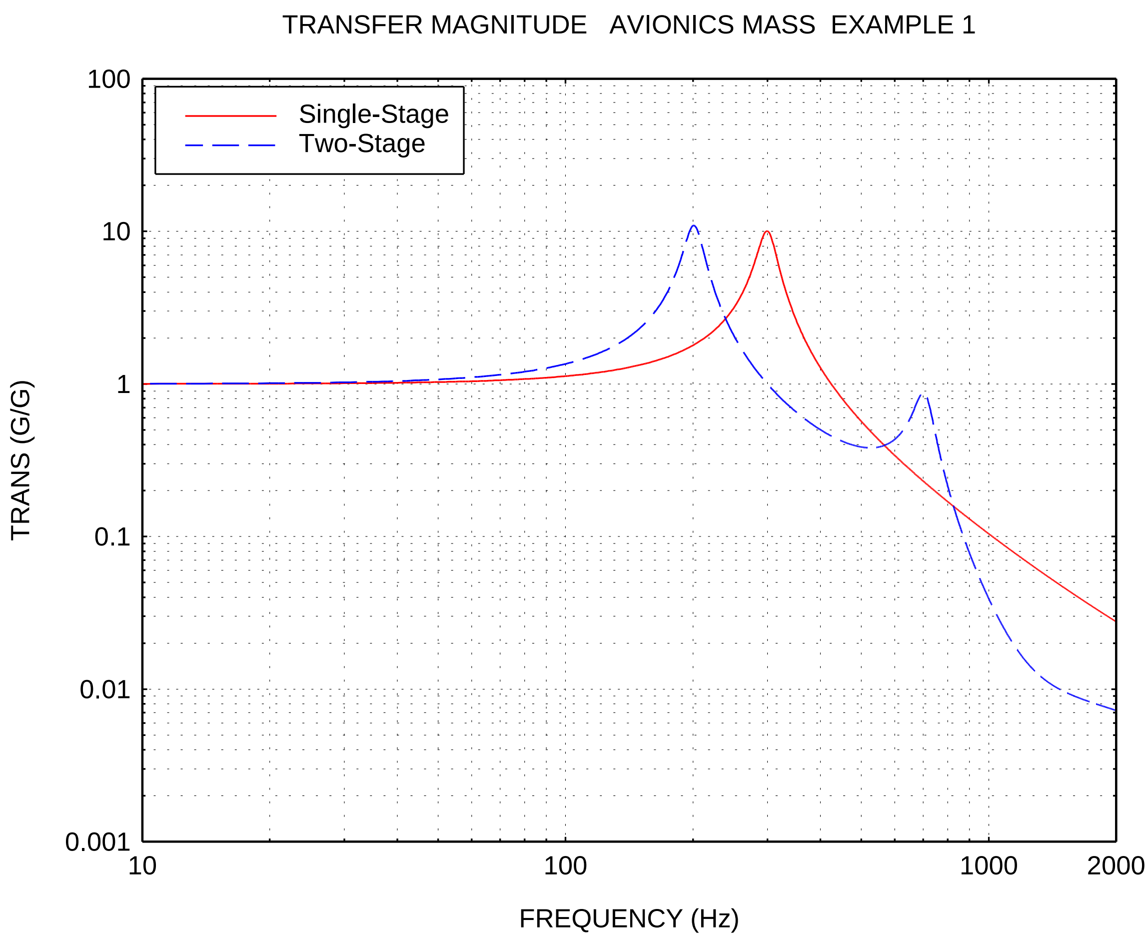



Figure A-3.

The Single-Stage curve represents the avionics mass and its spring by themselves.

The results are mixed. The optimum design depends on the base excitation frequency.

APPENDIX B

EXAMPLE 2

Repeat the example from Appendix A but with the base plate and avionics mass both at 5 lbm.

| Table B-1. Parameters | |
|---|---|
| Variable | Value |
| $m_1$ | 5 lbm |
| $m_2$ | 5 lbm |
| $k_1$ | 4.6e+04 lbf/in |
| $k_2$ | 4.6e+04 lbf/in |

The natural frequencies are

$$f_1 = 185.4 \text{ Hz} \quad \text{(B-1)}$$

$$f_2 = 485.3 \text{ Hz} \quad \text{(B-2)}$$

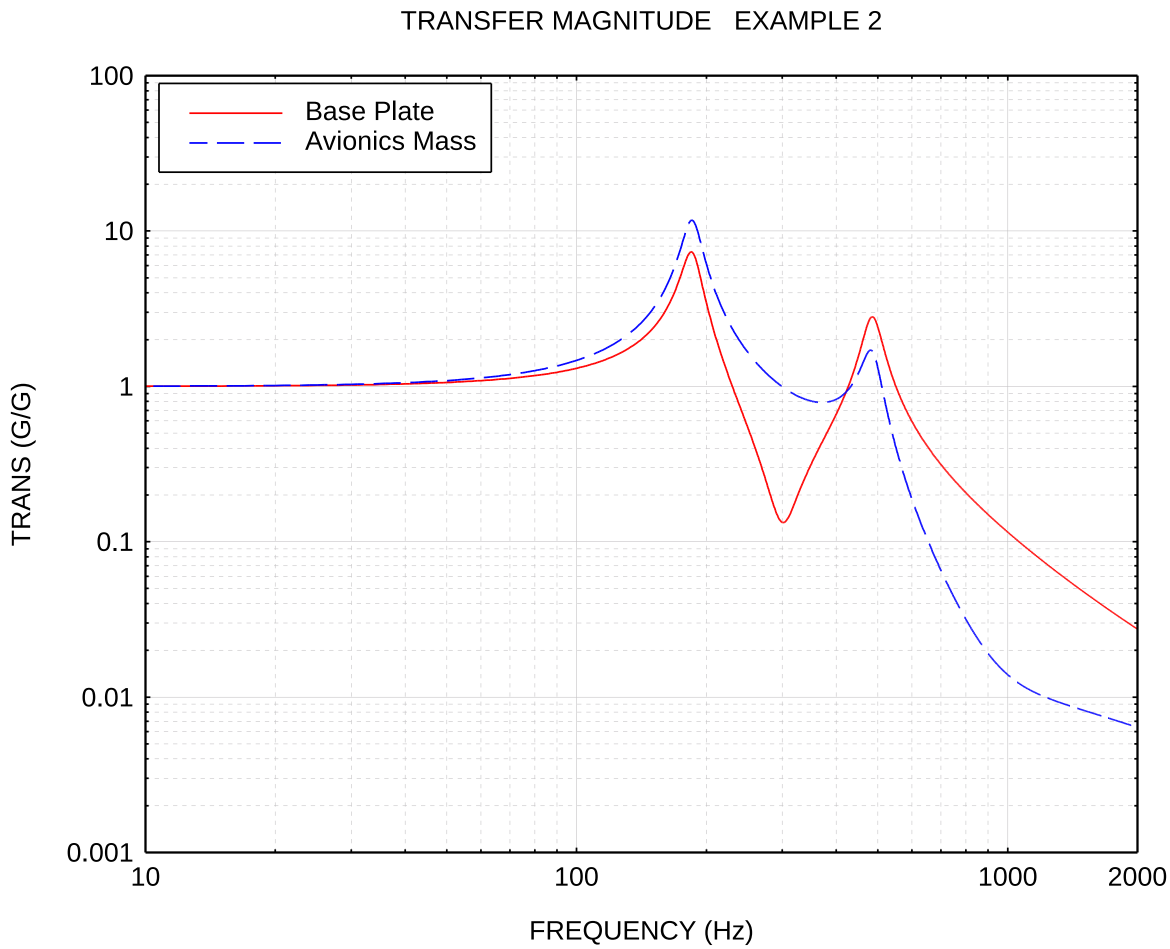

TRANSFER MAGNITUDE EXAMPLE 2
Base Plate
Avionics Mass
100
10
1
0.1
0.01
0.001
TRANS (G/G)
10
100
1000
2000
FREQUENCY (Hz)

Figure B-2.

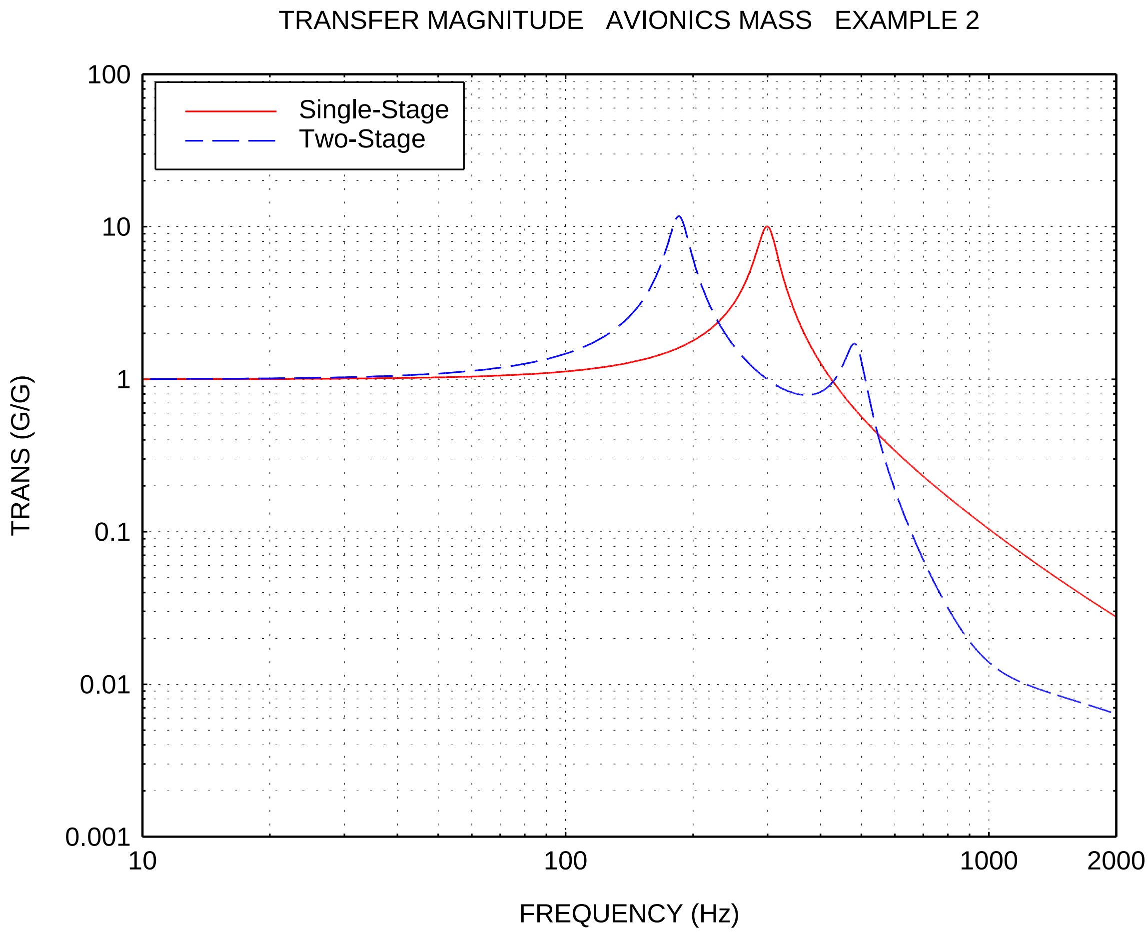



Figure B-3.

The Single-Stage curve represents the avionics mass and its spring by themselves.

Again, the results are mixed. The optimum design depends on the base excitation frequency.

APPENDIX C

EXAMPLE 3

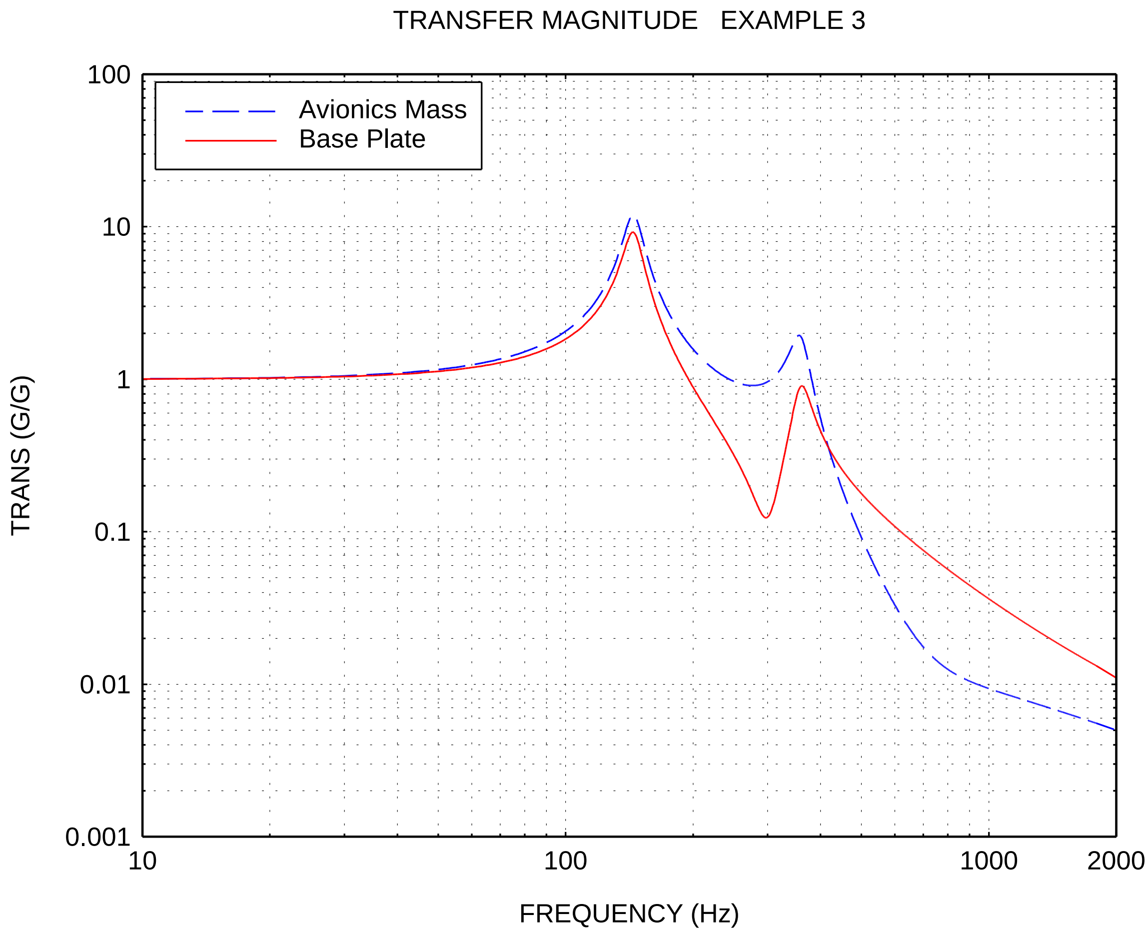
Repeat the example from Appendix A but with the base plate at 15 lbm.

| Table C-1. Parameters | |
|---|---|
| Variable | Value |
| $m_1$ | 15 lbm |
| $m_2$ | 5 lbm |
| $k_1$ | 4.6e+04 lbf/in |
| $k_2$ | 4.6e+04 lbf/in |

The natural frequencies are

$$f_1 = 144.6 \text{ Hz} \quad \text{(C-1)}$$

$$f_2 = 359.2 \text{ Hz} \quad \text{(C-2)}$$



TRANSFER MAGNITUDE   EXAMPLE 3
Avionics Mass
Base Plate
TRANS (G/G)
100
10
1
0.1
0.01
0.001
10
100
1000
2000
FREQUENCY (Hz)


Figure C-1.

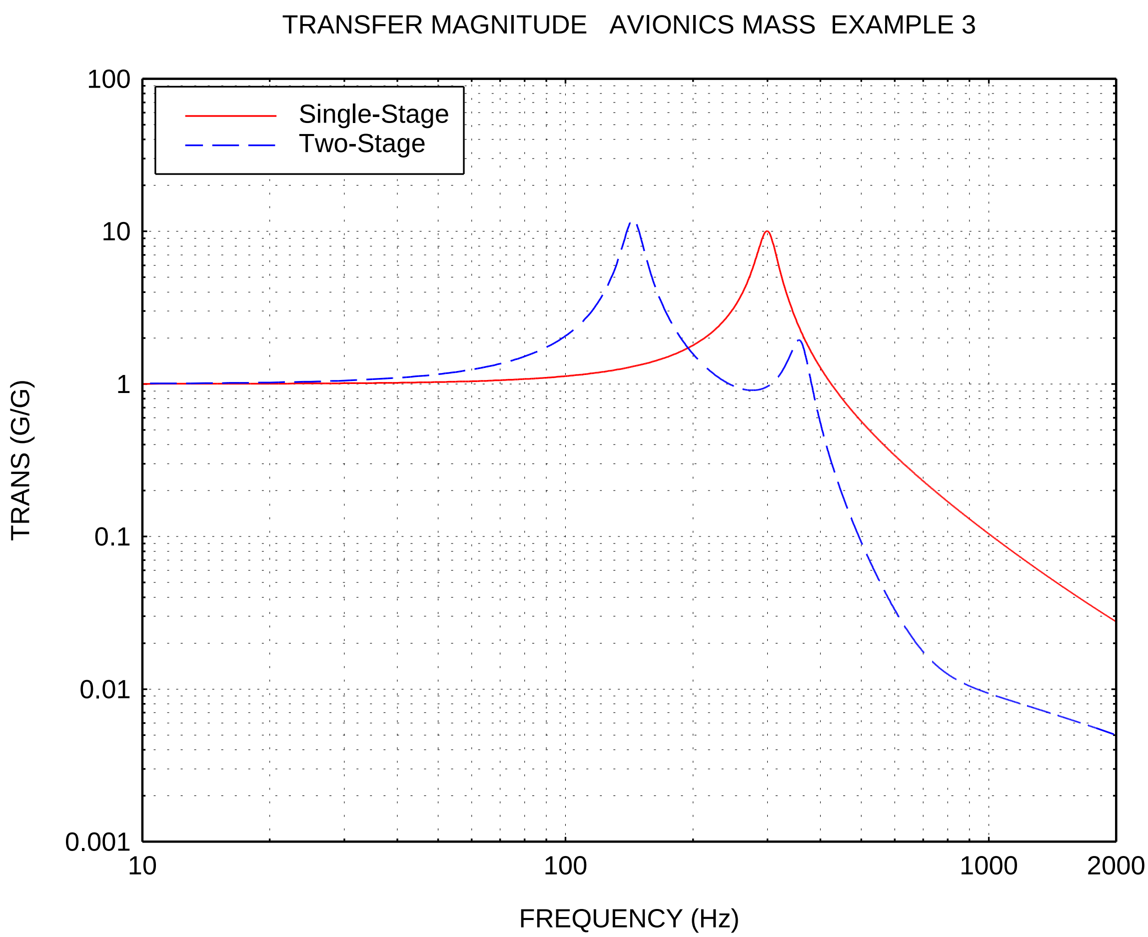



Figure C-2.

The Single-Stage curve represents the avionics mass and its spring by themselves.

The Two-Stage design provides greater attenuation above an excitation frequency of 200 Hz.