Harris Semiconductor

APPNOTE

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Harris Data Acquisition

A Tutorial in Coherent and Windowed Sampling with A/D Converters

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Introduction

The primary application of coherent sampling is sinewave testing of A/D converters. If the proper ratios between f_{IN} and f_S are observed, the need for windowing is eliminated. This greatly increases the spectral resolution of a FFT and creates an ideal environment for critically evaluating the spectral response of the A/D converter. Care must be taken, however, to insure the spectral purity and stability of f_{IN} and f_S in the testing environment. Figure 1 illustrates this procedure.

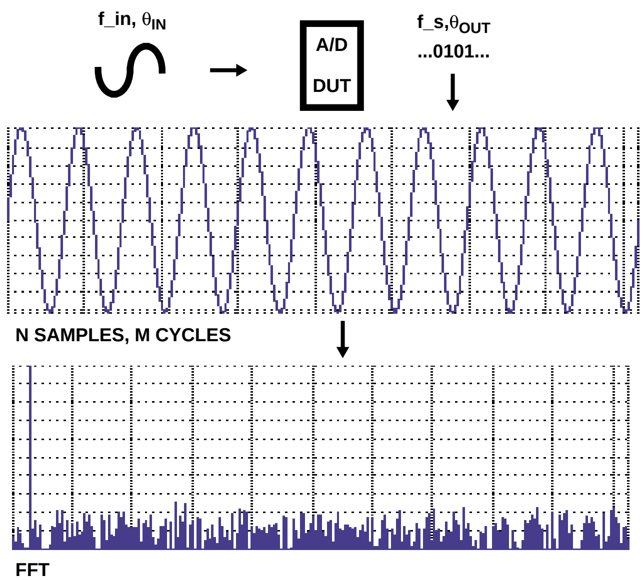

FIGURE 1. TESTING SYSTEM FOR SINEWAVE ANALYSIS

Definition of Coherence

Coherent Sampling of a periodic waveform occurs when an integer number of cycles exist in the sample window. In other words, coherent sampling occurs when the relationship of Equation 1 is rational.

$$\frac{f_{IN}}{f_S} = \frac{M}{N} \qquad \text{(EQ. 1)}$$

Where:

- f_S is the sampling frequency
- f_{IN} is the input frequency
- M is the integer number of cycles in the data record
- N is the integer, factor of 2, number of samples in the record

Ideal Parameters for Coherence

The coherence relationship will work for any arbitrary M and N, but practical values provide better results. A prudent choice for N is a power of 2. The FFT requires the number of samples to be a power of 2 because of its inherent periodicity. The DFT can be performed on an arbitrary sample size, but requires more computation time. M should be odd or prime. By making M odd, we eliminate many common factors with N. A prime M eliminates all common factors with N. Common factors between M and N lead to different harmonics of f_{IN} having the same frequency bin in the FFT after aliasing. The uniqueness of M is absolutely imperative to Harmonic Distortion calculations.

What follows is a mathematical analysis defining a simple rule that evaluates true when two harmonics have equivalent bins in the FFT. Equation 2 represents the location, M_h, in a FFT, of a harmonic h.

$$M_h = \begin{cases} \left| h - N \times INT\left(\dfrac{h + \frac{N}{2}}{N}\right) \right|, & h \neq \dfrac{N}{2} \\ 0, & h = \dfrac{N}{2} \end{cases} \qquad \text{(EQ. 2)}$$

Suppose harmonic 1 and harmonic 2 have the same FFT bin locations, $M_{h1} = M_{h2} = M_h$, then, from Equation 2,

$$M_h = \begin{cases} \left(\left| h_1 - N \times INT\left(\dfrac{h_1 + \frac{N}{2}}{N}\right) \right| = \left| h_2 - N \times INT\left(\dfrac{h_2 + \frac{N}{2}}{N}\right) \right|\right), & h_1 = h_2 \neq \dfrac{N}{2} \\ 0, & h_1 = h_2 = \dfrac{N}{2} \end{cases} \qquad \text{(EQ. 3)}$$

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Consider that h1, h2, and N will always be positive values. Simplifying Equation 3 results in Equation 4.

$$\frac{(h_1 \pm h_2)}{N} = \mathrm{INT}\left(\frac{h_1 + \frac{N}{2}}{N}\right) \pm \mathrm{INT}\left(\frac{h_2 + \frac{N}{2}}{N}\right) \qquad \text{(EQ. 4)}$$

Since the right side of Equation 4 must necessarily be an integer, the left side must also be an integer when the two harmonic locations M_{h1} and M_{h2} are equal.

$$M_{h1} = M_{h2} \Leftrightarrow \frac{(h_1 \pm h_2)}{N} = \mathrm{INT} \qquad \text{(EQ. 5)}$$

Therefore, when Equation 5 is true, the frequency bin of harmonic 1 is equal to the frequency bin of harmonic 2. The FFT provides N/2 frequency bins. To insure the uniqueness of each harmonic of the fundamental frequency bin, Equation 5 must be false for all combinations of h1 and h2 for all multiples of M extending to N.

Sampling at the Nyquist rate of $f_S = 2*f_{IN}$ is a classic problem. As an example, consider N = 4096, $f_{IN} = f_S/2$, M = N/2. Equation 5 evaluates as $((h_1 \pm h_2)/4096)$ = INT. If we substitute h = x*M for h1 and h2 where x represents the harmonic number, the equation simplifies to $((x_1 \pm x_2)/2)$ = INT. Whenever this equation holds true, the two harmonics have the same frequency bin. In this case, every odd harmonic will have the same bin as the fundamental and every even harmonic will have the same bin as DC. Therefore, no information about harmonic distortion or signal to noise ratio can be calculated.

Non-Ideal Parameters for Coherence

Thus far, we have talked about the complexities involved in coherent sampling that usually involve tedious iterative calculations to get the correct sampling ratio. That process ultimately resolves a very accurate solution. When the integer relationship of Equation 1 is not observed, artifacts result in the FFT spectrum.

Figure 2 is a 4096 point sample record from an ideal 10-bit A/D converter. To illustrate how the FFT interprets the 4096 points of data, the same 4096 point data record has been shifted in time by 4096 points and copied onto the graph. The FFT assumes that the 4096 data points represent a periodic waveform that extends to infinity in both directions. Because of this assumed periodicity, the calculation time of the FFT is reduced significantly and a smaller number of samples is required. Observe Figure 3. The waveform has been mirrored in the same way as Figure 1, except the record now contains 1.1 cycles instead of the original 1.0 cycles of Figure 1. It is obvious that if this is to be the signal that we perform the FFT on, the results will be degraded.

The characteristics of non-coherent sampling are obvious for gross errors, but are illusive with smaller error. Figure 4 is the FFT spectrum of a coherently sampled ideal 10-bit A/D converter. Notice there is no significant activity below -80db and the harmonic components are virtually nonexistent. Conversely, adjusting the coherence relationship to reflect a change in M of as little as 0.005, the harmonic components are significantly increased, and a unique condition called spreading occurs. Spreading, sometimes referred to as smearing or leakage, causes a spike centered around the frequency bin of the fundamental. The width of the spike is an indication of the magnitude of non-coherence. A concept crucial to understanding the nature of the problem is interpreting what a 0.005 change in M really means with respect to f_{IN} and f_S. If N and f_S stay constant, and M is increased by 0.005, then the input frequency, f_{IN}, is increased by f_S/N*0.005 causing leakage in the time domain window that leads to non-coherence.

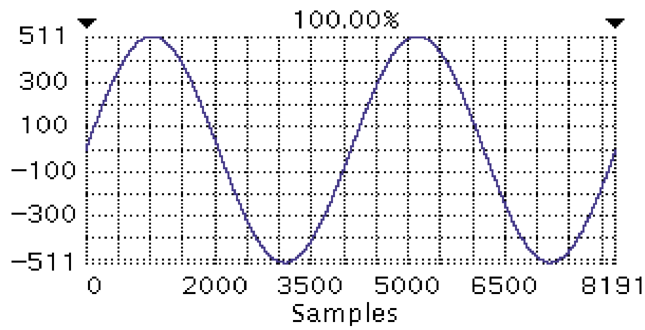


FIGURE 2. TRANSIENT RESPONSE OF A COHERENTLY SAMPLED DATASET AS SEEN BY FFT

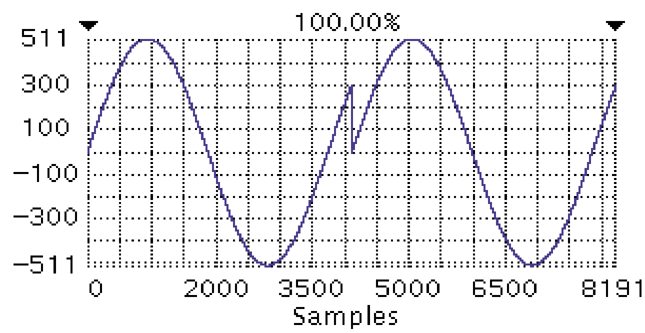


FIGURE 3. TRANSIENT RESPONSE OF A NON-COHERENTLY SAMPLED DATASET AS SEEN BY FFT

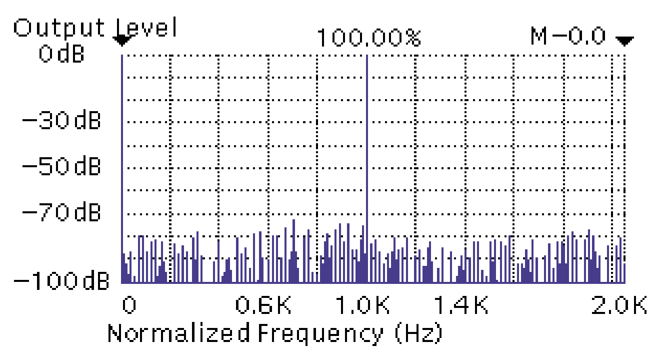


FIGURE 4. FFT SPECTRUM OF A COHERENTLY SAMPLED WAVEFORM

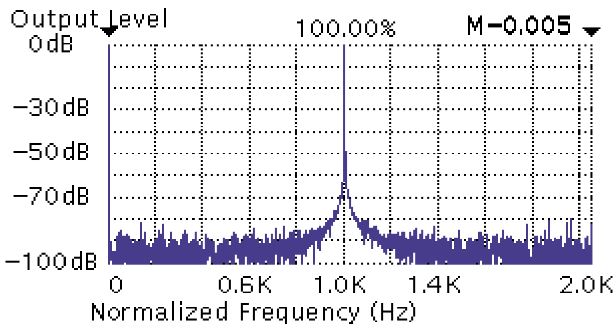


FIGURE 5. FFT SPECTRUM OF A NON-COHERENTLY SAMPLED WAVEFORM

An important specification for A/D testing is ENOB or Effective Number of Bits. Figure 6 shows the effect on ENOB performance of a shift in M from M-0.5 to M+0.5. Since the data is based on an Ideal converter, we expect to be able to achieve 10-bit accuracy. Indeed, there is a region where 10 bits is achievable, but it is very slim. The acuity of this region highlights why coherence must be accurately observed. Although, Figure 6 is somewhat misleading. Assuming the input frequency was 10MHz, the range of Figure 6 would represent an input frequency range from 9995117Hz to 10004883Hz. In most instances, high frequency equipment will perform within 1Hz of the programmed value resulting in a range of ±0.5Hz or a variability in M of ±0.00005 for our high-frequency example. Further narrowing in on the ideal range of variability for M, Figure 7 shows the change in ENOB for a change in M of ±0.0005. The assumed accuracy of our high-frequency source puts it within the 9.95 - 10.0 bit range for an Ideal signal.

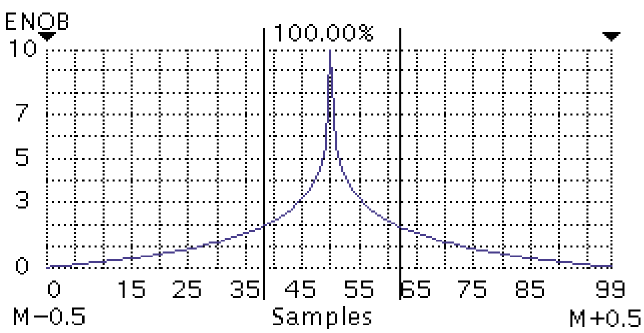


FIGURE 6. ENOB vs M FOR M-0.5 TO M+0.5

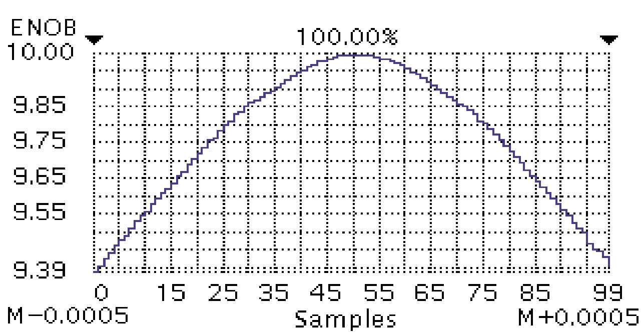


FIGURE 7. ENOB vs M FOR M-0.0005 TO M+0.0005

Unwrapping

A coherently sampled sine wave can be reassembled using a concept called unwrapping. Figure 8 shows a sinewave with M = 11 sampled N = 4096 times. Figure 9 is the same waveform after unwrapping is applied. If a waveform has been coherently sampled, the unwrapped waveform should look like one cycle sampled N times.

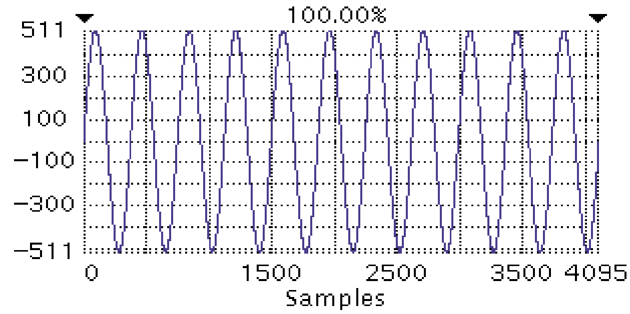


FIGURE 8. COHERENTLY SAMPLED WAVEFORM FOR M = 11

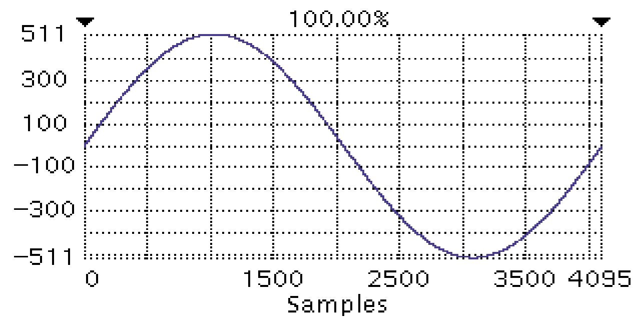


FIGURE 9. COHERENTLY SAMPLED WAVEFORM AFTER UNWRAP FOR M = 11

Windowing

Leakage is not a problem in all cases. It does not affect transient data as long as the transient occurrence is fully contained within the sample window. Leakage only occurs when the FFT is used to extrapolate frequency information from the sampled waveform. The actual source of leakage is not the signal itself but the window used in acquisition. The amount of leakage depends on the window shape and how the signal fits into the window.

Consider the coherently sampled waveform of Figure 12. The window of acquisition is rectangular and precisely set so that an integer number of cycles are captured. Therefore, leakage does not occur, the noise floor is nearly ideal for a 10 bit device, and harmonic distortion is nonexistent. In many cases the signal or sampling variables can not be precisely controlled. This makes it difficult to obtain exactly an integer number of cycles. But, leakage can be avoided or controlled by modifying the window to fit the data or to modify the data to a better form.

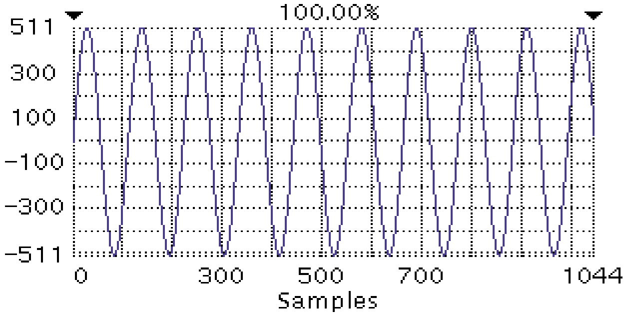


FIGURE 10. 9.5 CYCLE SINE WAVE

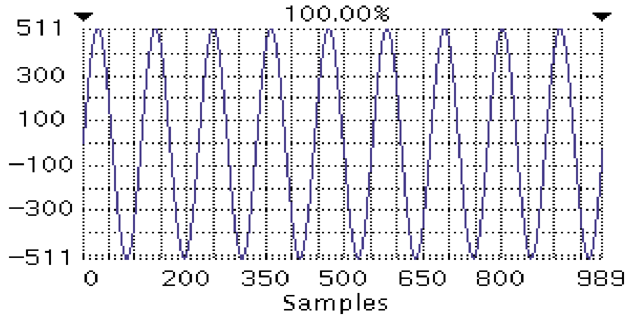


FIGURE 11. 9.0 CYCLE SINE WAVE

Resampling and Interpolative Resampling

For example, if 9.5 cycles of a waveform are captured, the data window can be shortened to disregard the extra 0.5 cycle. Assuming the original data set is 1045 samples long, as in Figure 10, after discarding the extra 0.5 cycle, the data set is reduced to 990 samples as illustrated by Figure 11. A 990 point DFT can now be performed on the modified data to produce accurate results free of leakage. This presents a problem to FFT algorithms that are limited to power of 2 sample sets. There is, however, a solution. The new sample set can be resampled to fit a given sample size or interpolation techniques can be applied to fit the waveform into the appropriately sized sample set. The 9.0 cycle waveform of Figure 11 was resampled to provide 1024 samples. Figure 15 shows the affects of the linear resampling in the frequency domain. Figure 14 is a similar example except the data has been resampled using linear interpolation. Linear resampling preserves the levels of the original data and avoids leakage, but does produce a more discontinuous waveform leading to harmonic distortion. Linear interpolative resampling circumvents this problem but does not entirely preserve the original data. Figure 12 is provided as a reference.

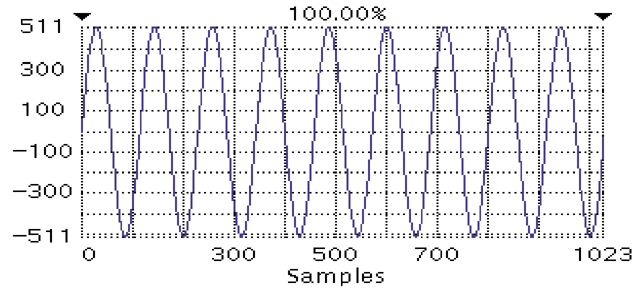


FIGURE 12. COHERENT 9.0 CYCLE SINEWAVE

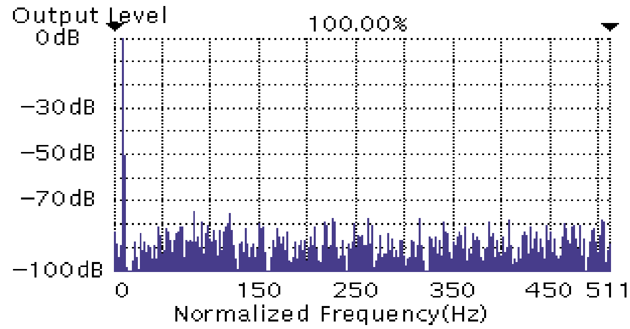


FIGURE 13. FFT OF COHERENT 9.0 CYCLE SINEWAVE

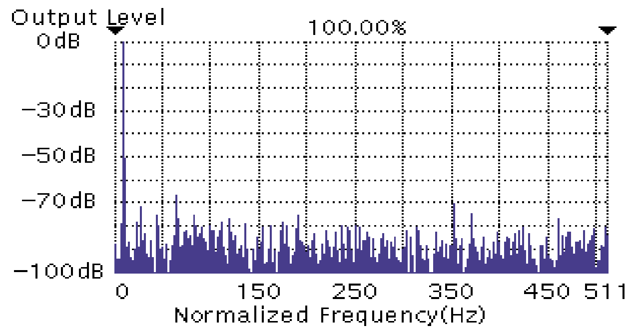


FIGURE 14. FFT OF FIGURE 11 AFTER INTERPOLATION

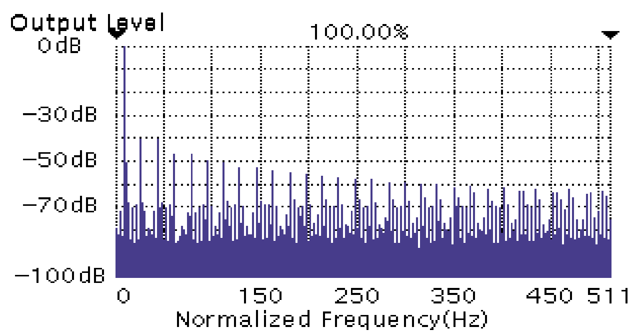


FIGURE 15. FFT OF FIGURE 11 AFTER RESAMPLING

Different Window Shapes

Since almost periodic data does not have a definable period, the above techniques are not applicable. Windowing the data can however force the data to begin and end at the same or nearly the same level. The technique of mathematical windowing is accomplished by multiplying the sampled waveform by an appropriate function. This prevents discontinuity at the window edge. Eliminating the discontinuity does not always eliminate leakage, but it does help to reduce it.

There are several functions that taper at the window edges. They are shown in Table 1. The first column is the actual window which should be applied to the sampled signal. The second column defines the shape equation for each window. As a basis for comparison, the third column contains the normalized frequency domain magnitude for each window. The fourth column lists the peak magnitude in the frequency domain as compared to that of the rectangular window. The decreased major lobe magnitude is due to the addressed area (energy) of each window as compared to the rectangular window. Adjusting the amplitude of the window will accommodate this difference. The fifth column lists the amplitude of the highest side-lobe in decibels referenced to the major lobe peak.

The 3dB bandwidth of the major lobe is given in the sixth column. These bandwidth values are normalized to Beta, the reciprocal of the window's time duration. The last column of parameters lists the theoretical rate of decay (roll-off) of the side lobes.

In choosing a windowing function, the bandwidth and side lobe levels should be considered. In general, the lower the side lobes, the less leakage in the frequency domain of the windowed data. However, lowering the side lobes also results in more energy being concentrated in widening the major lobe. Figures 17 and 18 illustrate these qualities. The Extended Cosine Bell has a very narrow major lobe and very high side lobes whereas the Parzen window has very low side lobes but a wide major lobe. Figure 16 is provided as a reference. Table 1 lists the windows in order of decreasing side lobe level and as a result they are listed in order of increasing bandwidth. The exception is the Hamming because of its non-zero edges.

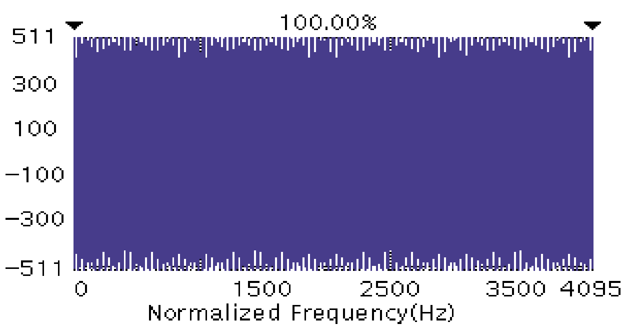


FIGURE 16A. RECTANGULAR WINDOWED SINE WAVE

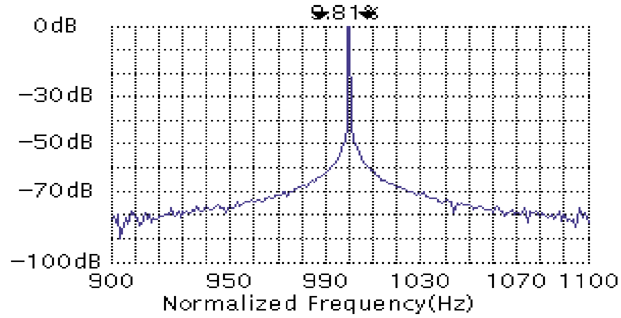


FIGURE 16B. FFT SPECTRUM OF RECTANGULAR WINDOWED SINE WAVE

In terms of spectral separation, the greater the window's bandwidth, the less selectivity it provides for equal amplitude and adjacent frequencies. The wide bandwidth causes them to blur together. Alternatively, lower side lobe levels increases selectivity between adjacent components of unequal amplitudes since the lower magnitude components are no longer buried in the leakage skirts. Usually, it takes a lot of trial and error before the correct window function is selected. Figures 19 through 22 provide some insight into the trial and error pitfalls involved in selecting the correct window shape. In Figure 21, the large side lobes of the Extended Cosine Bell window overshadow the original signals apparent in Figure 19. Conversely, the wide major lobes of the Parzen Window absorb one and another as is evident by Figure 22. A compromise can be arrived at by using the Hanning shape. The FFT spectrum of Figure 20 has reduced leakage to a minimum while continuing to preserve spectral separation of the signals.

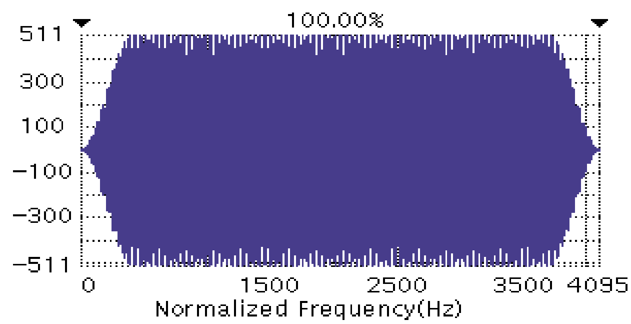


FIGURE 17A. EXTENDED COSINE BELL WINDOWED SINE WAVE

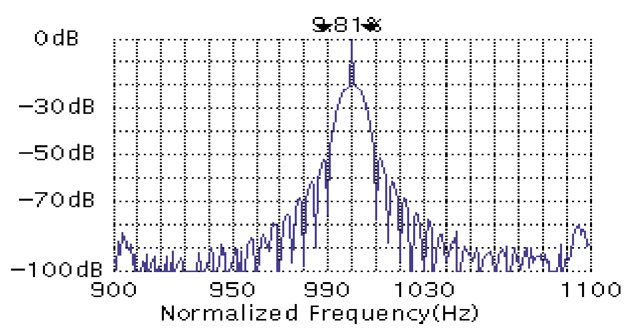


FIGURE 17B. FFT OF EXTENDED COSINE BELL WINDOWED SINE WAVE

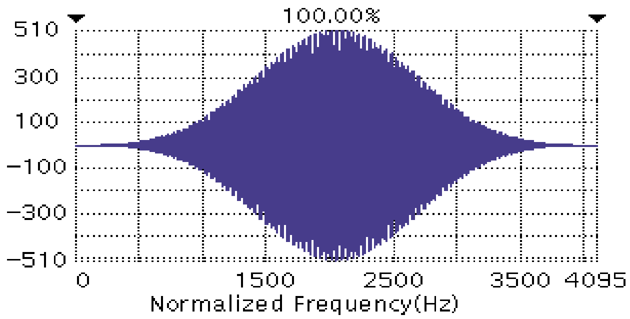


FIGURE 18A. PARZEN WINDOWED SINEWAVE

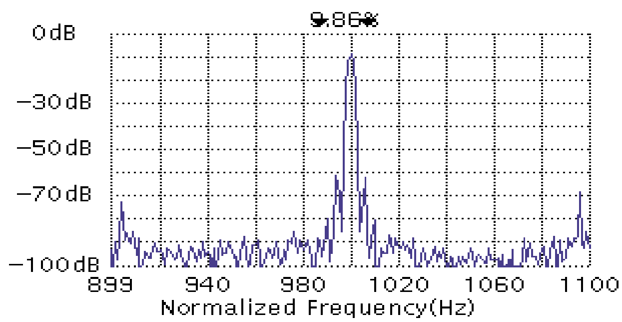


FIGURE 18B. FFT SPECTRUM OF PARZEN WINDOWED SINEWAVE

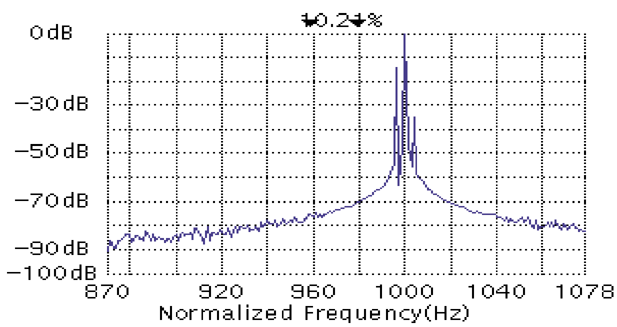


FIGURE 19. FFT OF A 3-SIGNAL NON-COHERENT WAVEFORM

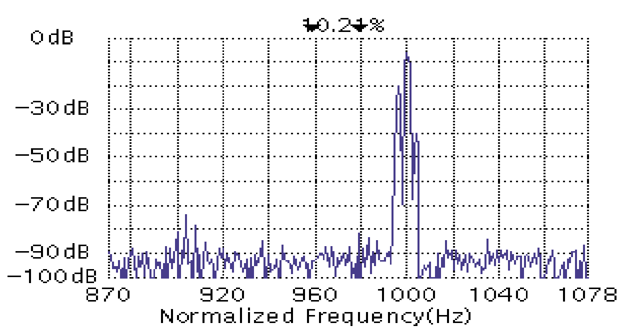


FIGURE 20. FFT OF WAVEFORM OF FIGURE 18 WINDOWED WITH A HANNING SHAPE

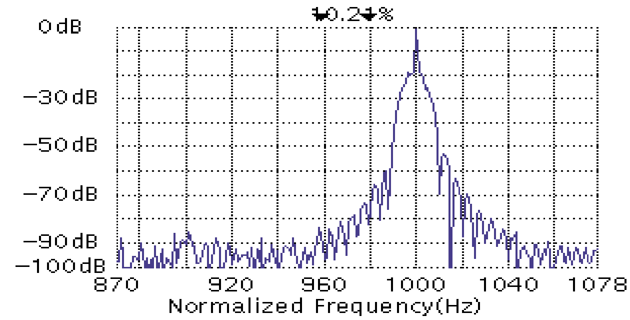


FIGURE 21. FFT OF WAVEFORM OF FIGURE 18 WINDOWED WITH AN EXTENDED COSINE BELL SHAPE

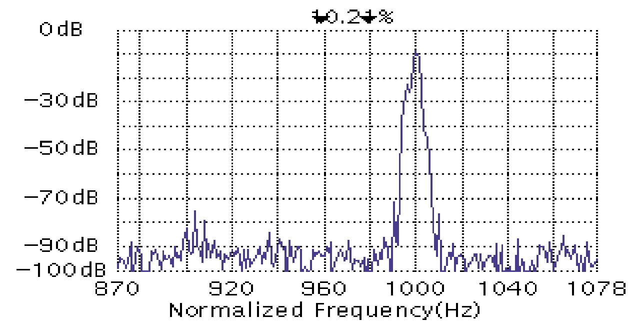


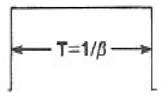
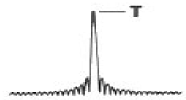
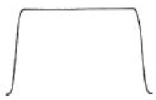
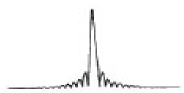
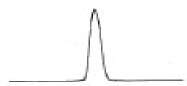
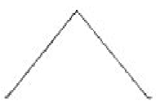
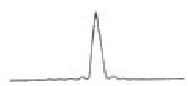
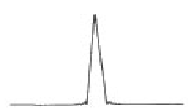
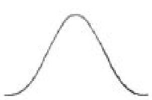
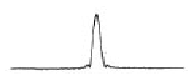
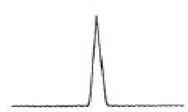
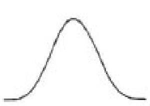
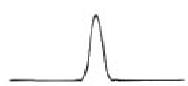
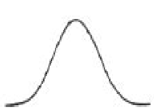
FIGURE 22. FFT OF WAVEFORM OF FIGURE 18 WINDOWED WITH A PARZEN SHAPE

Conclusion

Coherent testing of A/D converters provides an ideal environment for evaluating the spectral response. The rules for coherent sampling are simple. M must be prime or odd. N must be a factor of 2. The sampling and input frequency and phase must be stable and predictable. There are two methods for evaluating coherence. Unwrapping the waveform will show non-coherent anomalies in the time domain. The algorithm for unwrapping is provided below. Another indication of noncoherence is leakage skirting, or spreading, in the FFT spectrum. In the case of a single tone, sine wave curve fitting can be used to calculate signal to noise ratio. When the rules of coherent testing are not observed, windowing may be applied to try and resolve spectral components.

When should windowing be used, and when should it not? If windowing is needed, which windowing function should be used? The answer to these questions depend upon what you are looking for. If a waveform has adjacent components of nearly equal magnitude, you may want to leave the data in the rectangular window. The increased major lobe width of another window shape may cause the two adjacent components to leak into each other and appear as one. On the other hand, if there is a small component near a large component, a low side-lobe window will decrease leakage around the large component and make the small component easier to distinguish. Ultimately, selecting the window is a compromise between needed side-lobe reduction and a tolerable increase in major lobe width.

TABLE 1. WINDOWING FUNCTIONS

UNITY AMPLITUDE WINDOW	SHAPE EQUATION	FREQUENCY DOMAIN MAGNITUDE	MAJOR LOBE HEIGHT	HIGHEST SIDE LOBE (dB)	BAND-WIDTH (3dB)	THEORETICAL ROLL-OFF
Rectangle 	A = 1 for t = 0 to T		T	-13.2	0.86β	6
Extended Cosine Bell 	A = 0.5(1-cos(2π5t/T)) for t = 0 to T/10 and t = 9T/10 to T A = 1 for t = T/10 to 9T/10		0.9T	-13.5	0.95β	18 Beyond 5B
Half Cycle Sine 	A = sin(2π0.5t/T) for t = 0 to T		0.64T	-22.4	1.15β	12
Triangle 	A = 2t/T for t = 0 to T/2 A = -2t/T +2 for t = T/2 to T		0.5T	-26.7	1.27β	12
Cosine^2(Hanning) 	A = 0.5(1-cos(2πt/T)) for t = 0 to T		0.5T	-31.6	1.39β	18
Half Cycle Sine^3 	A = sin^3(2π0.5t/T) for t = 0 to T		0.42T	-39.5	1.61β	24
Hamming 	A = 0.08 + 0.46(1-cos(2πt/T)) for t = 0 to T		0.54T	-41.9	1.26β	6 Beyond 5B
Cosine^4 	A = (0.5(1-cos(2πt/T)))^2 for t = 0 to T		0.36T	-46.9	1.79β	30
Parzen 	A = 1-6(2t/T-1)^2 + 6|2t/T-1|^3 for t = T/4 to 3T/4 A = 2(1-|2t/T-1|)^3 for t = 0 to T/4 and t = 3T/4 to T		0.37T	-53.2	1.81β	24

Algorithms

The coherence algorithm accepts known values for each of the coherence parameters and evaluates the closest value to the initial guess so that all values are integer related. N must be a power of 2 greater than 4. The error is not more than ±(N/2) for f_s and ±(f_s/N) for f_in.

The unwrap algorithm accepts two arrays(tsample and unwrap) the number of cycles captured(M), and the array length(N). The tsample array is the sampled waveform. The unwrap array is the unwrapped waveform. It should look like one cycle of the waveform but sampled N times. The variable M is the number of cycles in the record.

The alias algorithm accepts N and fbin as variables and computes the correct FFT bin assignment of fbin.

References

[1] R.W. Ramirez, *The FFT Fundamentals and Concepts*, 1986, pgs. 140-141.

[2] M.. Mahoney, T*utorial DSP-Based Testing of Analog and Mixed-Signal Circuits*, 1987, pgs. 45-58.


```
void coherence(f_in,f_s,N,M)
    double *f_in,*f_s;
    int N,*M;
    {
    int K;

            K = (*f_s + N/2)/N;
           *M = (int)(*f_in)/(int)K/2*2+1;
          *f_s = K*N;
         *f_in = K*(*M);

    }
    /* Sample Call */

    /* coherence(&f_in,f_s,N,M); */

    }/* End of Coherence Algorithm */

void unwrap_algorithm(tsample,unwrap,size_cap,M)
    int tsample[],unwrap[],M,size_cap;
    {
    int i,j;
    for (i=0; i<size_cap;i++)
        {
        j = M*i % size_cap;
        unwrap[j] = tsample[i];
        }

    /* Sample Call */
    /* unwrap_algorithm(tsample,unwrap,size_cap,f_bin);
    */

    }/* End of Unwrap Algorithm */

void alias_algorithm(fbin,N)
    int *fbin,N;
    {
    *fbin=fabs((float) ( *fbin - N *((*fbin+N/2)/N) ) );
    if (*fbin == N/2) *fbin = 0;

    /* Sample Call */
    /* alias_algorithm(&fbin,M); */

    }/* End of Alias Algorithm */
```

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