BINGHAM COSINE TAPER WINDOW COMPENSATION FACTOR

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A Fourier transform may have a leakage error, whereby energy is smeared across adjacent frequency bands.

The leakage error can be reduced by subjecting the time history to a window, such a Bingham window. The Bingham window is given in equation (1).

$$w(t)=\begin{cases} 0.5\left(1-\cos\left(10\pi t/T\right)\right), & 0\le t\le T/10 \\ 1, & T/10\le t\le 9T/10 \\ 0.5\left(1+\cos\left(10\pi\left(t-(9T/10)\right)/T\right)\right), & 9T/10\le t\le T \end{cases} \tag{1}$$

The Bingham window forces the signal to start and stop at zero amplitude as shown in Figures 1 and 2.

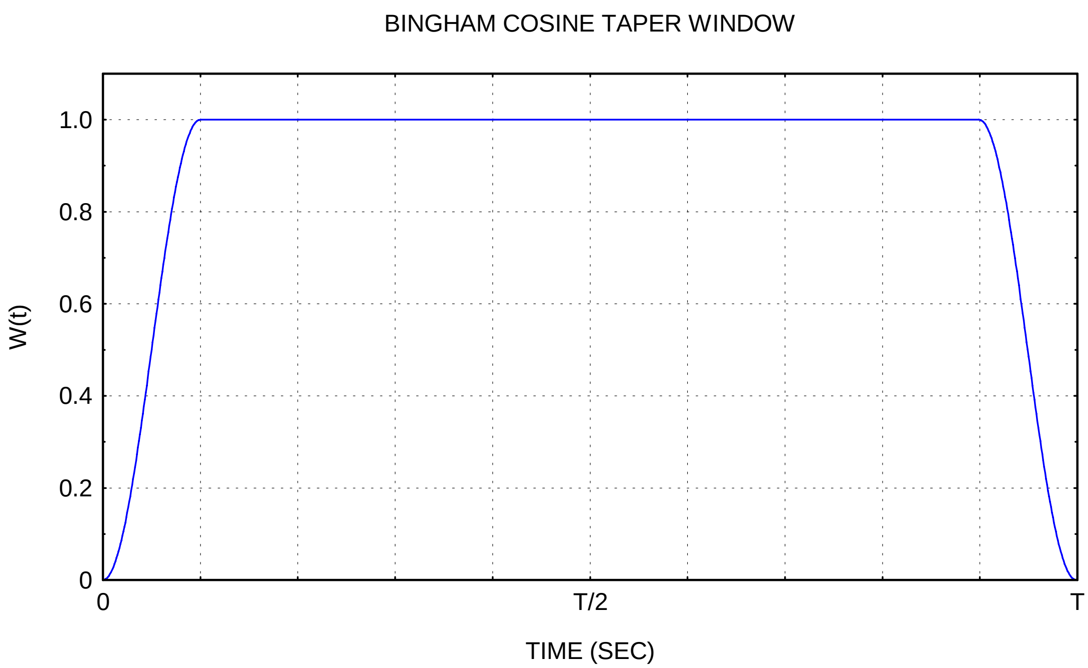

Figure 1.

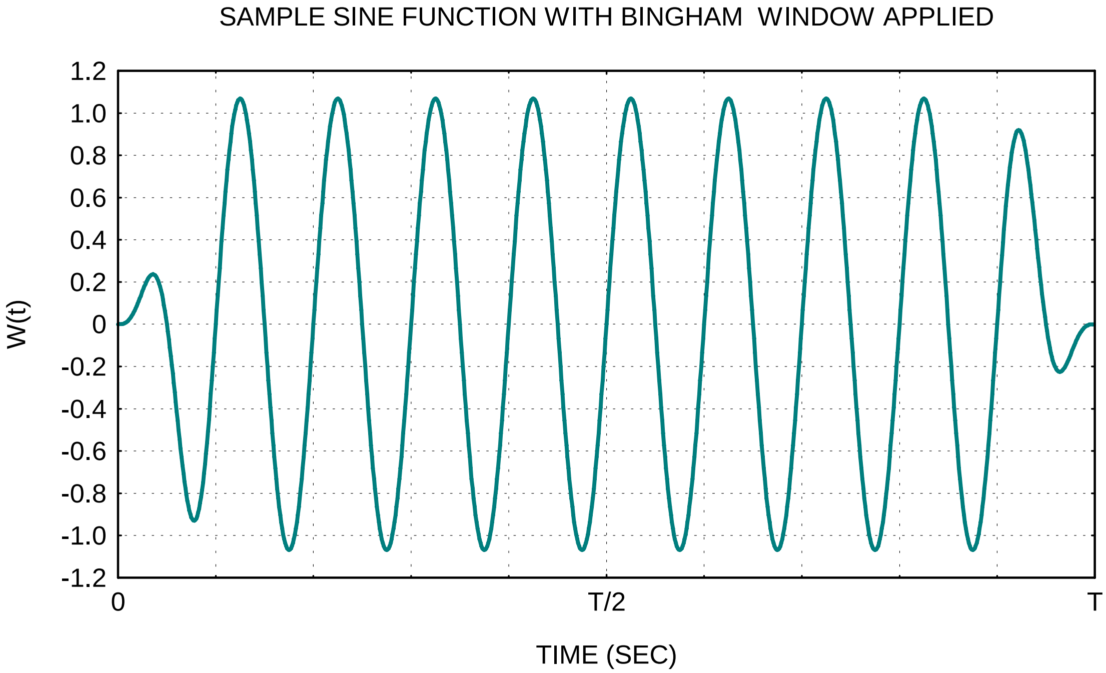

Figure 2.

A compensation factor must be applied to the windowed signal so that the RMS value remains the same as the original signal.

Derive the compensation factor.

Consider a sample sine function.

$$y(t) = A \sin(2\pi f\ t) \quad (2)$$

Assume for simplicity that the duration is such that

$$f = 5m/T\ , \quad \text{where } m = 1, 2, 3, \ldots.. \quad (3)$$

Let $y_{h,rms}$ be the rms value of the signal with the Bingham windowed applied.

$$y_{h,rms}^2 = \frac{1}{T}\int_0^T \{w(t)y(t)\}^2\,dt \tag{4}$$

$$w(t) = \begin{cases} 0.5\left(1-\cos\left(10\pi t/T\right)\right), & 0 \le t \le T/10 \\ 1, & T/10 \le t \le 9T/10 \\ 0.5\left(1+\cos\left(10\pi(t-(9T/10))/T\right)\right), & 9T/10 \le t \le T \end{cases} \tag{5}$$

$$\left[y_{h,rms}^2\right][T] = \int_0^{T/10} \{0.5(1-\cos(10\pi t/T)A\sin(2\pi f\,t))\}^2\,dt + \int_{T/10}^{9T/10} \{A\sin(2\pi f\,t)\}^2\,dt + \int_{9T/10}^{T} \{0.5(1+\cos(10\pi(t-(9T/10))/T)A\sin(2\pi f\,t)\}^2\,dt \tag{6}$$

$$\left[y_{h,rms}^2\right][T] = B + C + D \tag{7}$$

$$B = \int_0^{T/10} \{0.5(1-\cos(10\pi t/T)A\sin(2\pi f\,t))\}^2\,dt \tag{8}$$

$$C = \int_{T/10}^{9T/10} \{A\sin(2\pi f\,t)\}^2\,dt \tag{9}$$

$$D = \int_{9T/10}^{T} \{0.5(1+\cos(10\pi(t-(9T/10))/T)A\sin(2\pi f\,t)\}^2\,dt \tag{10}$$

$$B = \int_0^{T/10} \{0.5(1-\cos(10\pi t/T))A\sin(2\pi f\, t)\}^2\, dt \tag{11}$$

$$B = A^2 \int_0^{T/10} \{0.5\,(1-\cos(10\pi t/T))\sin(2\pi f\, t)\}^2\, dt \tag{12}$$

$$f = 5m/T\,, \quad \text{where } m = 1, 2, 3, \ldots.. \tag{13}$$

$$B = \frac{A^2}{4} \int_0^{T/10} \{\,(1-\cos(10\pi t/T))\sin(10\pi m\, t/T)\}^2\, dt \tag{14}$$

$$B = \frac{A^2}{4} \int_0^{T/10} \{\,\sin(10\pi m\, t/T) - \cos(10\pi t/T)\sin(10\pi m\, t/T)\}^2\, dt \tag{15}$$

$$B = \frac{A^2}{4} \int_0^{T/10} \left\{\,\sin(10\pi m\, t/T) - \frac{1}{2}\sin(10\pi(1+m)t/T) + \frac{1}{2}\sin(10\pi(1-m)t/T)\right\}^2 dt \tag{16}$$

$$
\begin{aligned}
B = \; & \frac{A^2}{4}\int_0^{T/10} \sin^2(10\pi m\, t/T)dt \\
& + \frac{A^2}{16}\int_0^{T/10} \sin^2(10\pi(1+m)t/T)dt \\
& + \frac{A^2}{16}\int_0^{T/10} \sin^2(10\pi(1-m)t/T)dt \\
& - \frac{A^2}{4}\int_0^{T/10} \sin(10\pi m\, t/T)\sin(10\pi(1+m)t/T)dt \\
& - \frac{A^2}{4}\int_0^{T/10} \sin(10\pi(1+m)t/T)\sin(10\pi(1-m)t/T)dt \\
& + \frac{A^2}{16}\int_0^{T/10} \sin(10\pi m\, t/T)\sin(10\pi(1-m)t/T)dt
\end{aligned}
\tag{17}
$$

$$
\begin{aligned}
B = \; & \frac{A^2}{8}\int_0^{T/10} [1-\cos(20\pi m\, t/T)]dt \\
& + \frac{A^2}{32}\int_0^{T/10} [1-\cos(20\pi\,(1+m)\,t/T)]dt \\
& + \frac{A^2}{32}\int_0^{T/10} [1-\cos(20\pi\,(1-m)\,t/T)]dt \\
& - \frac{A^2}{8}\int_0^{T/10} \{-\cos(10\pi(1+2m)\,t/T)+\cos(10\pi mt/T)\}dt \\
& - \frac{A^2}{8}\int_0^{T/10} \{-\cos(10\pi m\, t/T)+\cos(10\pi(2m)\,t/T)\}dt \\
& + \frac{A^2}{16}\int_0^{T/10} \{-\cos(10\pi(2m\, t)/T)+\cos(10\pi(1+2m)\,t/T)\}dt
\end{aligned}
\tag{18}
$$

$$
\begin{aligned}
B = \;& \frac{A^2}{8}\left[t - \frac{T}{20\pi m}\sin(20\pi m\,t/T)\right]\Bigg|_0^{T/10} \\
&+ \frac{A^2}{32}\left[t - \frac{T}{20\pi(1+m)}\sin(20\pi\,(1+m)\,t/T)\right]\Bigg|_0^{T/10} \\
&+ \frac{A^2}{32}\left[t - \frac{T}{20\pi(1-m)}\sin(20\pi\,(1-m)\,t/T)\right]\Bigg|_0^{T/10} \\
&- \frac{A^2}{8}\left[-\frac{T}{10\pi(1+2m)}\sin(10\pi(1+2m)\,t/T) + \frac{T}{10\pi m}\sin(10\pi mt/T)\right]\Bigg|_0^{T/10} \\
&- \frac{A^2}{8}\left[-\frac{T}{10\pi m}\sin(10\pi m\,t/T) + \frac{T}{10\pi(2m)}\sin(10\pi(2m)\,t/T)\right]\Bigg|_0^{T/10} \\
&+ \frac{A^2}{16}\left[-\frac{T}{10\pi(2m)}\sin(10\pi(2m\,t)/T) + \frac{T}{10\pi(1+2m)}\sin(10\pi(1+2m)\,t/T)\right]\Bigg|_0^{T/10}
\end{aligned}
\tag{19}
$$

$$
B = \frac{A^2}{8}\left[\frac{1}{10}\right] + \frac{A^2}{32}\left[\frac{1}{10}\right] + \frac{A^2}{32}\left[\frac{1}{10}\right] \tag{20}
$$

$$
B = \left[\frac{1}{10}\right]\left[\frac{A^2}{8} + \frac{A^2}{32} + \frac{A^2}{32}\right] \tag{21}
$$

$$
B = \frac{3A^2}{160} \tag{22}
$$

$$
D = \frac{3A^2}{160} \quad \text{by symmetry} \tag{23}
$$

$$C = \int_{T/10}^{9T/10} \{A \sin(2\pi f\, t)\}^2\, dt \tag{24}$$

$$C = A^2 \int_{T/10}^{9T/10} \sin^2(2\pi f\, t) dt \tag{25}$$

$$C = A^2 \int_{T/10}^{9T/10} 0.5[1 - \cos(4\pi f\, t)] dt \tag{26}$$

$$C = \frac{A^2}{2} \int_{T/10}^{9T/10} [1 - \cos(4\pi f\, t)] dt \tag{27}$$

f = 5m/T ,   where m =1, 2, 3, ….. (28)

$$C = \frac{A^2}{2} \int_{T/10}^{9T/10} [1 - \cos(20\pi m\, t / T)] dt \tag{29}$$

$$C = \frac{A^2}{2} \left[t - \frac{T}{20\pi m} \sin(20\pi m\, t / T) \right] \Bigg|_{T/10}^{9T/10} \tag{30}$$

$$C = \frac{A^2}{2} \left[\frac{8T}{10} - \frac{T}{20\pi m} \sin(18\pi m) + \frac{T}{20\pi m} \sin(2\pi m) \right] \tag{31}$$

$$C = \frac{A^2}{2} \left[\frac{8T}{10} \right] \tag{32}$$

$$C = \frac{2}{5} A^2 \tag{33}$$

$$y_{h,rms}{}^2 = B + C + D \tag{34}$$

$$y_{h,rms}{}^2 = \frac{3A^2}{160} + \frac{2A^2}{5} + \frac{3A^2}{160} \tag{35}$$

$$y_{h,rms}{}^2 = \frac{7}{16} A^2 \tag{36}$$

$$y_{h,rms} = \sqrt{\frac{7}{16}} A \tag{37}$$

The rms value of the original data is

$$y_{,rms} = \frac{1}{\sqrt{2}} A \tag{38}$$

Define a compensation factor as

$$\alpha = \frac{y_{,rms}}{y_{h,rms}} \tag{39}$$

$$\alpha = \frac{\frac{1}{\sqrt{2}}}{\sqrt{\frac{7}{16}}} \tag{40}$$

$$\alpha = \sqrt{\frac{16}{14}} \tag{41}$$

$$\alpha = \sqrt{\frac{8}{7}} \tag{42}$$