CRAIG-BAMPTON METHOD FOR A TWO COMPONENT SYSTEM
Revision C

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Introduction

The Craig-Bampton method is method for reducing the size of a finite element model, particularly where two or more subsystems are connected. It combines the motion of boundary points with modes of the subsystem assuming the boundary points are held fixed.

The following tutorial provides an example for the Craig-Bampton fixed-interface method in Reference 1.

Governing Equation of Motion

The dynamic response of a system is modeled as

$$M\ddot{x} + K x = F \tag{1}$$

where

M	is the mass matrix
K	is the stiffness matrix
F	is the force vector
x	is the displacement vector

Matrix Partitioning

The partitioned mass and stiffness matrices for each subsystem or component are respectively

$$\begin{bmatrix} M_{ii} & M_{ib} \\ M_{bi} & M_{bb} \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} K_{ii} & K_{ib} \\ K_{bi} & K_{bb} \end{bmatrix}$$

The subscript i denotes an interior degree-of-freedom.

The subscript b denotes an interface boundary degree-of-freedom.

Normal Modes

The component fixed-interface normal modes are obtained by restraining all boundary degrees-of-freedom and solving the generalized eigenvalue problem:

$$\left\lfloor K_{ii} - \omega_j^2 M_{ii} \right\rfloor \{\phi_i\}_j = 0 \tag{2}$$

The complete set of N_i fixed-interface (flexible) normal modes is Φ_{ii}. The assembled modal matrix is

$$\underset{N_u \times N_i}{\Phi_i} \equiv \begin{bmatrix} \Phi_{ii} \\ 0_{bi} \end{bmatrix} \tag{3}$$

Next, the modes are normalized so that

$$\Phi_{ii}{}^T M_{ii} \Phi_{ii} = I_{ii} \tag{4}$$

$$\Phi_{ii}{}^T K_{ii} \Phi_{ii} = \Lambda_{ii} = \mathrm{diag}\left(\omega_j^2\right) \tag{5}$$

Constraint Modes

A constraint mode is defined as the static deformation of a structure when a unit displacement is applied to one coordinate of specified set of constraint coordinates, C, while the remaining coordinates of that set are restrained, and the remaining degrees-of-freedom of the structure are force-free.

The interface constraint mode matrix is calculated via

$$\begin{bmatrix} K_{ii} & K_{ib} \\ K_{bi} & K_{bb} \end{bmatrix} \begin{bmatrix} \Psi_{ib} \\ I_{bb} \end{bmatrix} = \begin{bmatrix} 0_{ib} \\ R_{bb} \end{bmatrix} \tag{6}$$

where

Ψ_{ib} is the interior partition of the constraint mode matrix

R contains the reaction forces on the component due to its connection to adjacent components at boundary degrees-of-freedom

The interface constraint mode matrix Ψ_c is

$$\underset{N_u \times N_b}{\Psi_c} \equiv \begin{bmatrix} \Psi_{ib} \\ I_{bb} \end{bmatrix} = \begin{bmatrix} -\left[K_{ii}{}^{-1} K_{ib}\right] \\ I_{bb} \end{bmatrix} \tag{7}$$

Note that the constraint modes are stiffness-orthogonal to all of the fixed-interface normal modes, that is

$$\Phi_i{}^T K \Psi_c = 0 \tag{8}$$

The displacement transformation of the Craig-Bampton Method uses both fixed-interface normal modes and interface constraint modes.

The physical coordinates $u^{(s)}$ can be represented as

$$u^{(s)} \equiv \begin{Bmatrix} u_i \\ u_b \end{Bmatrix}^{(s)} = \begin{bmatrix} \Phi_{ik} & \Psi_{ib} \\ 0 & I_{bb} \end{bmatrix}^{(s)} \begin{Bmatrix} p_k \\ p_b \end{Bmatrix}^{(s)} \tag{9}$$

where

p_k = interior generalized displacements

p_b = boundary generalized displacements

Φ_{ik} = interior partition of the matrix of kept fixed-interface modes

Ψ_{ib} = interior partition of the constraint mode matrix

The Craig-Bampton transformation matrix $\Psi_{CB}^{(s)}$ is

$$\Psi_{CB}^{(s)} = \begin{bmatrix} \Phi_{ik} & \Psi_{ib} \\ 0 & I_{bb} \end{bmatrix}^{(s)} \tag{10}$$

<u>Reduced Component Matrices</u>

The reduced component mass matrix for system s is

$$\hat{M}_{CB}^{(s)} = \left\{\Psi_{CB}^{(s)}\right\}^{T} \left\{M^{(s)}\right\}\left\{\Psi_{CB}^{(s)}\right\} \tag{11}$$

$$\hat{M}_{CB}^{(s)} = \begin{bmatrix} \Phi_{ik} & 0 \\ \Psi_{ib} & I_{bb} \end{bmatrix}^{(s)} \begin{bmatrix} M_{ii} & M_{ib} \\ M_{bi} & M_{bb} \end{bmatrix}^{(s)} \begin{bmatrix} \Phi_{ik} & \Psi_{ib} \\ 0 & I_{bb} \end{bmatrix}^{(s)} \tag{12}$$

$$\hat{M}_{CB}^{(s)} = \begin{bmatrix} \Phi_{ik}{}^{T} & 0 \\ \Psi_{ib}{}^{T} & I_{bb} \end{bmatrix}^{(s)} \begin{bmatrix} M_{ii}\Phi_{ik} & M_{ii}\Psi_{ib} + I_{bb}M_{ib} \\ M_{bi}\Phi_{ik} & M_{bi}\Psi_{ib} + I_{bb}M_{bb} \end{bmatrix}^{(s)} \tag{13}$$

$$\hat{M}_{CB}^{(s)} = \begin{bmatrix} \Phi_{ik}{}^{T}M_{ii}\Phi_{ik} & \Phi_{ik}{}^{T}\left(M_{ii}\Psi_{ib} + I_{bb}M_{ib}\right) \\ \left(\Psi_{ib}{}^{T}M_{ii} + I_{bb}M_{bi}\right)\Phi_{ik} & I_{bb}\left(M_{bi}\Psi_{ib} + I_{bb}M_{bb}\right) \end{bmatrix}^{(s)} \tag{14}$$

$$\hat{M}_{CB}^{(s)} = \begin{bmatrix} \hat{I}_{kk} & \hat{M}_{kb} \\ \hat{M}_{bk} & \hat{M}_{bb} \end{bmatrix}^{(s)} \tag{15}$$

The reduced stiffness matrix for system s is

$$\hat{K}_{CB}^{(s)} = \left\{\Psi_{CB}^{(s)}\right\}^{T}\left\{K^{(s)}\right\}\left\{\Psi_{CB}^{(s)}\right\} \tag{16}$$

$$\hat{K}_{CB}^{(s)} = \begin{bmatrix}\Phi_{ik} & 0\\ \Psi_{ib} & I_{bb}\end{bmatrix}^{(s)}\begin{bmatrix}K_{ii} & K_{ib}\\ K_{bi} & K_{bb}\end{bmatrix}^{(s)}\begin{bmatrix}\Phi_{ik} & \Psi_{ib}\\ 0 & I_{bb}\end{bmatrix}^{(s)} \tag{17}$$

$$\hat{K}_{CB}^{(s)} = \begin{bmatrix}\Phi_{ik}{}^{T} & 0\\ \Psi_{ib}{}^{T} & I_{bb}\end{bmatrix}^{(s)}\begin{bmatrix}K_{ii}\Phi_{ik} & K_{ii}\Psi_{ib} + I_{bb}K_{ib}\\ K_{bi}\Phi_{ik} & K_{bi}\Psi_{ib} + I_{bb}K_{bb}\end{bmatrix}^{(s)} \tag{18}$$

$$\hat{K}_{CB}^{(s)} = \begin{bmatrix}\Phi_{ik}{}^{T}K_{ii}\Phi_{ik} & \Phi_{ik}{}^{T}\left(K_{ii}\Psi_{ib} + I_{bb}K_{ib}\right)\\ \left(\Psi_{ib}{}^{T}K_{ii} + I_{bb}K_{bi}\right)\Phi_{ik} & I_{bb}\left(K_{bi}\Psi_{ib} + I_{bb}K_{bb}\right)\end{bmatrix}^{(s)} \tag{19}$$

Again,

$$\Psi_{ib} = -K_{ii}{}^{-1}K_{ib} \tag{20}$$

Thus, the off-diagonal terms are each zero.

$$\hat{K}_{CB}^{(s)} = \begin{bmatrix}\Lambda_{kk} & 0_{kb}\\ 0_{bk} & \hat{K}_{bb}\end{bmatrix}^{(s)} \tag{21}$$

The reduced force vector for system s is

$$\hat{F}_{CB}^{(s)} = \left\{\Psi_{CB}^{(s)}\right\}^T \left\{F^{(s)}\right\} = \left\{\Psi_{CB}^{(s)}\right\}^T \begin{bmatrix} F_i \\ F_b \end{bmatrix}^{(s)} \tag{22}$$

Assembled Global Matrices

The following assembled mass matrix is formed.

$$\hat{M}_{CB} = \begin{bmatrix} \hat{I}_{k_1 k_1} & 0_{k_1 k_2} & \hat{M}_{k_1 b}^{(1)} \\ 0_{k_2 k_1} & \hat{I}_{k_2 k_2} & \hat{M}_{k_2 b}^{(2)} \\ \hat{M}_{b k_1}^{(1)} & \hat{M}_{b k_2}^{(2)} & \hat{M}_{bb}^{(1)} + \hat{M}_{bb}^{(2)} \end{bmatrix} \tag{23}$$

Again, the subscript b denotes an interface boundary degrees-of-freedom.

The numerical subscripts denote non-interface degrees-of-freedom.

The following assembled stiffness matrix is formed.

$$\hat{K}_{CB} = \begin{bmatrix} \Lambda_{k_1 k_1}^{(1)} & 0_{k_1 k_2} & 0_{k_1 b} \\ 0_{k_2 k_1} & \Lambda_{k_2 k_2}^{(2)} & 0_{k_2 b} \\ 0_{bk_1} & 0_{bk_2} & \hat{K}_{bb}^{(1)} + \hat{K}_{bb}^{(2)} \end{bmatrix} \tag{24}$$

$$\hat{M}_{CB} \begin{Bmatrix} \ddot{p}_k \\ \ddot{p}_b \end{Bmatrix} + \hat{K}_{CB} \begin{Bmatrix} p_k \\ p_b \end{Bmatrix} = \hat{F}_{CB} \tag{25}$$

Examples

Examples are given in Appendices A and B.

References

1. R. Craig & A. Kurdila, Fundamentals of Structural Dynamics, Second Edition, Wiley, New Jersey, 2006.
2. T. Irvine, Component Mode Synthesis, Fixed-Interface Model, Revision A, Vibrationdata, 2010.

APPENDIX A

Example

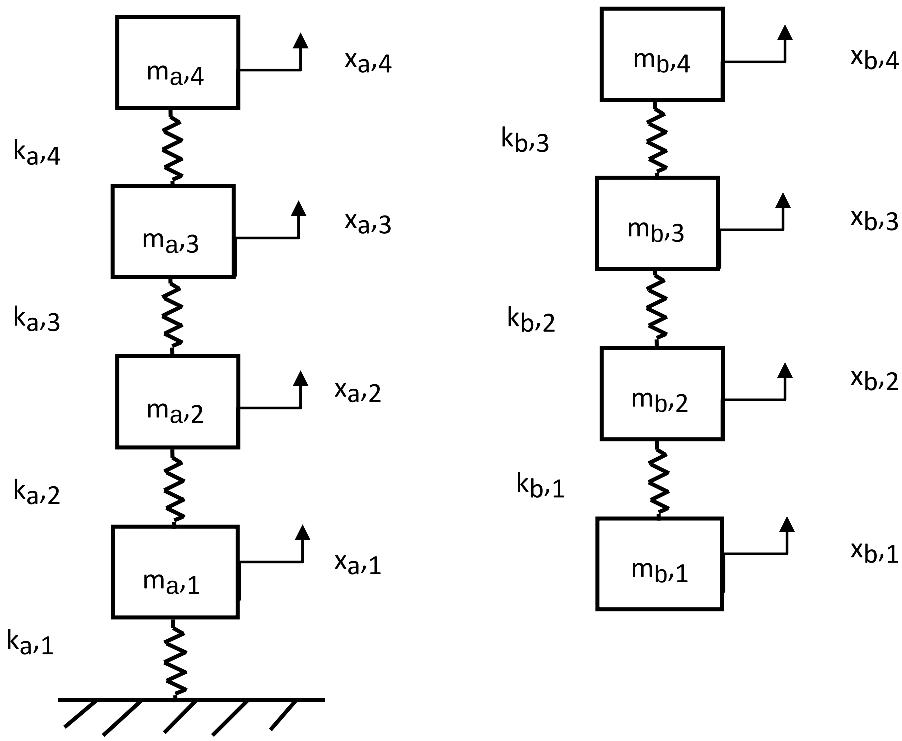

Figure A-1.

Form two separate models as an intermediate step. The system on the left represents a launch vehicle on a pad.

The system on the right represents a spacecraft that is to be mounted on top of the launch vehicle.

Note that mass $m_{b,1}$ is to be connected to $m_{a,4}$ via a rigid link.

The following values are used for the model.

English units: stiffness (lbf/in), mass (lbf sec^2/in)

ka1	900,000
ka2	600,000
ka3	500,000
ka4	420,000

ma1	150
ma2	125
ma3	100
ma4	100

kb1	100,000
kb2	90,000
kb3	80,000

mb1	10
mb2	8
mb3	6
mb4	5

<u>Complete Launch Vehicle & Spacecraft Model, Unreduced</u>

```
>> mass_stiffness_assembly

 mass_stiffness_assembly.m  ver 1.1  Feb 16, 2010
 by Tom Irvine
 Assemble mass and stiffness matrices using transformation matrices.

 Enter total dof
 7

 Enter number of systems
 2

 Enter system 1 mass matrix name
 MLV

 Enter system 1 stiffness matrix name
 KLV

 Enter system 1 transformation matrix name
 ta

 Enter system 2 mass matrix name
 MSC

 Enter system 2 stiffness matrix name
 KSC

 Enter system 2 transformation matrix name
 tb

MG =

   150      0      0      0      0      0      0
```

```
     0   125     0     0     0     0     0
     0     0   100     0     0     0     0
     0     0     0   110     0     0     0
     0     0     0     0     8     0     0
     0     0     0     0     0     6     0
     0     0     0     0     0     0     5


KG =

     1500000     -600000           0           0           0           0           0
     -600000     1100000     -500000           0           0           0           0
           0     -500000      920000     -420000           0           0           0
           0           0     -420000      520000     -100000           0           0
           0           0           0     -100000      190000      -90000           0
           0           0           0           0      -90000      170000      -80000
           0           0           0           0           0      -80000       80000

 Natural Frequencies (Hz)
     4.04
    8.981
    11.32
    16.51
    20.03
    23.11
    33.48

  Modes Shapes (column format)

ModeShapes =

    0.0143   -0.0211    0.0368   -0.0553    0.0400    0.0010    0.0000
    0.0334   -0.0360    0.0455    0.0106   -0.0583   -0.0027   -0.0001
    0.0510   -0.0252   -0.0016    0.0612    0.0544    0.0070    0.0005
    0.0641    0.0068   -0.0558   -0.0354   -0.0166   -0.0167   -0.0041
    0.0737    0.1174    0.0269   -0.0218   -0.0258    0.2706    0.1752
    0.0801    0.2071    0.1066    0.0141    0.0003    0.0825   -0.3146
    0.0835    0.2586    0.1559    0.0432    0.0293   -0.2596    0.1782
```

The transformation matrices for the assembly were

```
>> ta

ta =

     1     0     0     0     0     0     0
     0     1     0     0     0     0     0
     0     0     1     0     0     0     0
     0     0     0     1     0     0     0

>> tb

tb =

     0     0     0     1     0     0     0
     0     0     0     0     1     0     0
     0     0     0     0     0     1     0
     0     0     0     0     0     0     1
```

<u>System A, Launch Vehicle, CB Matrix</u>

```
>> Craig_Bampton

 Craig_Bampton.m  ver 1.0  April 30, 2013

 by Tom Irvine

 Enter the units system
 1=English  2=metric
 1

 Assume symmetric mass and stiffness matrices.

 Select input mass unit
  1=lbm  2=lbf sec^2/in
 2

 stiffness unit = lbf/in

 Select file input method
   1=file preloaded into Matlab
   2=Excel file
1

 Mass Matrix
```

```
 Enter the matrix name:  massa

 Stiffness Matrix
 Enter the matrix name:  stiffnessa

 The mass matrix is

m =

   150     0     0     0
     0   125     0     0
     0     0   100     0
     0     0     0   100


 The stiffness matrix is

k =

     1500000     -600000           0           0
     -600000     1100000     -500000           0
           0     -500000      920000     -420000
           0           0     -420000      420000


 Enter number of boundary dof 1
 Enter boundary dof 1: 4

 ** Fixed Interface Flexible Natural Frequencies & Modes **

Natural Frequencies
 No.      f(Hz)
1.        8.5873
2.        15.598
3.        19.804

  Modes Shapes (column format)

ModeShapes =

    0.0367   -0.0577    0.0445
    0.0651   -0.0057   -0.0611
    0.0518    0.0704    0.0486


 Enter number of modes to keep 3
```

Craig-Bampton Transformation Matrix

```
CBTM =

    0.0367   -0.0577    0.0445    0.1552
    0.0651   -0.0057   -0.0611    0.3880
    0.0518    0.0704    0.0486    0.6674
         0         0         0    1.0000
```

Partitioned Matrices

```
m_partition =

   150     0     0     0
     0   125     0     0
     0     0   100     0
     0     0     0   100


k_partition =

     1500000     -600000           0           0
     -600000     1100000     -500000           0
           0     -500000      920000     -420000
           0           0     -420000      420000
```

Transformed matrices (reduced component matrices)

```
mq =

    1.0000    0.0000    0.0000    7.4670
    0.0000    1.0000    0.0000    3.0796
         0    0.0000    1.0000    1.3181
    7.4670    3.0796    1.3181  166.9772


kq =

   1.0e+05 *

    0.0291    0.0000   -0.0000         0
    0.0000    0.0960    0.0000    0.0000
   -0.0000    0.0000    0.1548   -0.0000
    0.0000    0.0000   -0.0000    1.3969
```

order vector

```
ngw =
     1     2     3     4
```

System B, Spacecraft, CB Matrix

```
>> Craig_Bampton

 Craig_Bampton.m  ver 1.0  April 30, 2013

 by Tom Irvine

 Enter the units system
 1=English  2=metric
 1

 Assume symmetric mass and stiffness matrices.

 Select input mass unit
  1=lbm  2=lbf sec^2/in
 2

 stiffness unit = lbf/in

 Select file input method
   1=file preloaded into Matlab
   2=Excel file
1

 Mass Matrix
 Enter the matrix name:  massb

 Stiffness Matrix
 Enter the matrix name:  stiffnessb

 The mass matrix is

m =

    10     0     0     0
     0     8     0     0
     0     0     6     0
     0     0     0     5


 The stiffness matrix is

k =

      100000     -100000           0           0
     -100000      190000      -90000           0
           0      -90000      170000      -80000
```

```
          0           0      -80000       80000


 Enter number of boundary dof 1
 Enter boundary dof 1: 1

 ** Fixed Interface Flexible Natural Frequencies & Modes **

 Natural Frequencies
 No.      f(Hz)
1.        9.1344
2.        22.854
3.        33.449

  Modes Shapes (column format)

ModeShapes =

    0.1360    0.2762    0.1739
    0.2473    0.0769   -0.3156
    0.3114   -0.2662    0.1792


 Enter number of modes to keep 3

 Craig-Bampton Transformation Matrix

CBTM =

    0.1360    0.2762    0.1739    1.0000
    0.2473    0.0769   -0.3156    1.0000
    0.3114   -0.2662    0.1792    1.0000
         0         0         0    1.0000


 Partitioned Matrices

m_partition =

     8     0     0     0
     0     6     0     0
     0     0     5     0
     0     0     0    10


k_partition =

      190000      -90000           0     -100000
      -90000      170000      -80000           0
           0      -80000       80000           0
     -100000           0           0      100000
```

```
 Transformed matrices (reduced component matrices)

mq =

    1.0000   -0.0000    0.0000    4.1293
         0    1.0000   -0.0000    1.3394
    0.0000   -0.0000    1.0000    0.3936
    4.1293    1.3394    0.3936   29.0000


kq =

   1.0e+04 *

    0.3294    0.0000    0.0000    0.0000
    0.0000    2.0619    0.0000    0.0000
    0.0000    0.0000    4.4170    0.0000
    0.0000    0.0000    0.0000    0.0000


 order vector

ngw =

     2     3     4     1
```

Combined CB System

```
>> mass_stiffness_assembly

 mass_stiffness_assembly.m  ver 1.1  Feb 16, 2010
 by Tom Irvine
 Assemble mass and stiffness matrices using transformation matrices.

 Enter total dof
 7

 Enter number of systems
 2

 Enter system 1 mass matrix name
 mqa

 Enter system 1 stiffness matrix name
 kqa

 Enter system 1 transformation matrix name
 tqa

 Enter system 2 mass matrix name
 mqb

 Enter system 2 stiffness matrix name
```

```
 kqb

 Enter system 2 transformation matrix name
 tqb

MG =

    1.0000    0.0000    0.0000         0         0         0    7.4670
    0.0000    1.0000    0.0000         0         0         0    3.0796
         0    0.0000    1.0000         0         0         0    1.3181
         0         0         0    1.0000   -0.0000    0.0000    4.1293
         0         0         0         0    1.0000   -0.0000    1.3394
         0         0         0    0.0000   -0.0000    1.0000    0.3936
    7.4670    3.0796    1.3181    4.1293    1.3394    0.3936  195.9772


KG =

  1.0e+005 *

    0.0291    0.0000   -0.0000         0         0         0   -0.0000
    0.0000    0.0960    0.0000         0         0         0   -0.0000
   -0.0000    0.0000    0.1548         0         0         0   -0.0000
         0         0         0    0.0329    0.0000    0.0000    0.0000
         0         0         0    0.0000    0.2062    0.0000   -0.0000
         0         0         0    0.0000    0.0000    0.4417   -0.0000
    0.0000   -0.0000   -0.0000    0.0000   -0.0000   -0.0000    1.3969


Natural Frequencies (Hz)
     4.04
    8.981
    11.32
    16.51
    20.03
    23.11
    33.48

  Modes Shapes (column format)

ModeShapes =

    0.1361   -0.5909    0.9813    0.3624    0.1520    0.1444    0.0326
    0.0142    0.0104   -0.1909    1.0116    0.1301    0.0943    0.0160
    0.0037    0.0023   -0.0356   -0.1065    0.9981    0.0827    0.0083
    0.0644    0.8098    0.6605    0.2107    0.0866    0.0816    0.0182
    0.0028    0.0017   -0.0243   -0.0518   -0.0736    1.0037    0.0102
    0.0004    0.0002   -0.0028   -0.0045   -0.0037   -0.0060    1.0007
    0.0641    0.0068   -0.0558   -0.0354   -0.0166   -0.0167   -0.0041
```

The transformation matrices for the assembly were

```
>> tqa

tqa =

     1     0     0     0     0     0     0
     0     1     0     0     0     0     0
     0     0     1     0     0     0     0
     0     0     0     0     0     0     1

>> tqb

tqb =

     0     0     0     1     0     0     0
     0     0     0     0     1     0     0
     0     0     0     0     0     1     0
     0     0     0     0     0     0     1
```

Note that the interface is set at degree-of-freedom number 7.

Summary

The natural frequencies match.

Table A-1. Natural Frequencies		
Mode	Full Model fn (Hz)	Combined CB Systems fn (Hz)
1	4.04	4.04
2	8.98	8.98
3	11.32	11.32
4	16.51	16.51
5	20.03	20.03
6	23.11	23.11
7	33.48	33.48

APPENDIX B

Example, Part II

Repeat the example from Appendix A but only include the first fixed interface mode from the spacecraft.

System A, Launch Vehicle, CB Matrix

The matrices are the same as in Appendix A.

System B, Spacecraft, CB Matrix

```
>> Craig_Bampton

 Craig_Bampton.m  ver 1.0  April 30, 2013

 by Tom Irvine

 Enter the units system
 1=English  2=metric
 1

 Assume symmetric mass and stiffness matrices.

 Select input mass unit
  1=lbm  2=lbf sec^2/in
 2

 stiffness unit = lbf/in

 Select file input method
   1=file preloaded into Matlab
   2=Excel file
1

 Mass Matrix
 Enter the matrix name:  massb

 Stiffness Matrix
 Enter the matrix name:  stiffnessb
```

```
The mass matrix is

m =

    10     0     0     0
     0     8     0     0
     0     0     6     0
     0     0     0     5


 The stiffness matrix is

k =

      100000     -100000           0           0
     -100000      190000      -90000           0
           0      -90000      170000      -80000
           0           0      -80000       80000


 Enter number of boundary dof 1
 Enter boundary dof 1: 1

 ** Fixed Interface Flexible Natural Frequencies & Modes **

 Natural Frequencies
 No.      f(Hz)
1.        9.1344
2.        22.854
3.        33.449

  Modes Shapes (column format)

ModeShapes =

    0.1360    0.2762    0.1739
    0.2473    0.0769   -0.3156
    0.3114   -0.2662    0.1792


 Enter number of modes to keep 1

 Craig-Bampton Transformation Matrix

CBTM =

    0.1360    1.0000
    0.2473    1.0000
    0.3114    1.0000
         0    1.0000
```

```
 Partitioned Matrices

m_partition =

     8     0     0     0
     0     6     0     0
     0     0     5     0
     0     0     0    10


k_partition =

      190000      -90000           0     -100000
      -90000      170000      -80000           0
           0      -80000       80000           0
     -100000           0           0      100000

 Transformed matrices (reduced component matrices)

mq =

    1.0000    4.1293
    4.1293   29.0000


kq =

   1.0e+03 *

    3.2940    0.0000
    0.0000    0.0000
```

Combined CB System

```
>> mass_stiffness_assembly

 mass_stiffness_assembly.m  ver 1.1  Feb 16, 2010
 by Tom Irvine
 Assemble mass and stiffness matrices using transformation matrices.

 Enter total dof
 5

 Enter number of systems
 2

 Enter system 1 mass matrix name
 mqa
```

```
 Enter system 1 stiffness matrix name
 kqa

 Enter system 1 transformation matrix name
 tqaa

 Enter system 2 mass matrix name
 mqbb

 Enter system 2 stiffness matrix name
 kqbb

 Enter system 2 transformation matrix name
 tqbb

MG =

    1.0000    0.0000    0.0000         0    7.4670
    0.0000    1.0000    0.0000         0    3.0796
         0    0.0000    1.0000         0    1.3181
         0         0         0    1.0000    4.1293
    7.4670    3.0796    1.3181    4.1293  195.9772


KG =

   1.0e+05 *

    0.0291    0.0000   -0.0000         0         0
    0.0000    0.0960    0.0000         0    0.0000
   -0.0000    0.0000    0.1548         0   -0.0000
         0         0         0    0.0329    0.0000
    0.0000    0.0000   -0.0000    0.0000    1.3969

 Natural Frequencies
 No.      f(Hz)
1.        4.0405
2.        8.9806
3.        11.328
4.        16.535
5.        20.043

  Modes Shapes (column format)

ModeShapes =

    0.1362   -0.5906    0.9830    0.3706    0.1646
    0.0142    0.0103   -0.1925    1.0137    0.1405
    0.0037    0.0023   -0.0359   -0.1099    1.0011
    0.0644    0.8100    0.6610    0.2154    0.0938
    0.0641    0.0068   -0.0560   -0.0362   -0.0180
```

The transformation matrices for the assembly were

```
>> tqaa

tqaa =

     1     0     0     0     0
     0     1     0     0     0
     0     0     1     0     0
     0     0     0     0     1

>> tqbb

tqbb =

     0     0     0     1     0
     0     0     0     0     1
```

Note that the interface is set at degree-of-freedom number 5.

Table B-1. Natural Frequencies

Mode	Full Model fn (Hz)	Combined CB Systems, with One Fixed-Interface Mode for the Spacecraft fn (Hz)
1	4.04	4.04
2	8.98	8.98
3	11.32	11.33
4	16.51	16.54
5	20.03	20.04
6	23.11	-
7	33.48	-