CHLADNI PATTERNS

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HISTORICAL BACKGROUND

Ernst Chladni (1756-1827) was a German physicist who performed experimental studies of vibrating plates. Specifically, he spread fine sand over metal or glass plates. He then excited the fundamental natural frequency of the plate by stroking a violin bow across one of its edges.

As a result, a standing wave formed in the plate. A standing wave has "anti-nodes" where the maximum displacement occurs. A standing wave also has "nodes" where no displacement occurs. For a vibrating plate, the nodes occur along "nodal lines."

In Chladni's experiment, the sand grains responded to the excitation by migrating to the nodal lines of the plates. The grains thus traced the nodal line pattern.

Sophie Germain (1776-1831) derived mathematical equations to describe Chladni's experiments. She published these equations in *Memoir on the Vibrations of Elastic Plates*.

Jules Lissajous (1822-1880) performed further vibration research using Chladni's test methods.

THEORY

Elastic plates have numerous natural frequencies. The lowest natural frequency is called the fundamental natural frequency. The fundamental frequency is usually the dominant frequency.

Each natural frequency has a corresponding "mode." The mode is defined in terms of its nodal line pattern. Each mode has a unique pattern.

Points on opposite sides of a nodal line vibrate "180 degrees out-of-phase."

Stroking a plate with a violin bow may excite several natural frequencies, thus complicating the experiment. Again, the response will usually be dominated by the fundamental mode.

The higher modes can be individually excited, to some extent, by stroking the plate at different edge locations. In addition, mechanical constraints can be added to hinder the formation of other modes.

TEST SETUP

The purpose of this experiment is to determine the natural frequencies and mode shapes for a common baking pan, using Chladni patterns. The pan had a thickness of 0.5 mm and a diameter of 0.229 m. The material was stainless steel. The bottom of the pan represented a circular plate.

Instead of a violin bow, an electromagnetic shaker was used to excite the bottom of the pan. Note that an electromagnetic shaker is similar to a loudspeaker. A sine function generator was used to drive the shaker, as shown in Figure 1. The frequency of the sine function was varied until mode shapes formed. Salt particles were used to trace the nodal lines for each mode shape.

The experimental results are shown in Figures 2 through 8. Note that considerable trial-and-error was required.

The circular plate boundary condition was irregular, but can be approximated as a clamped boundary along the outer circumference. The theoretical results are compared to the experimental results in Appendix A.

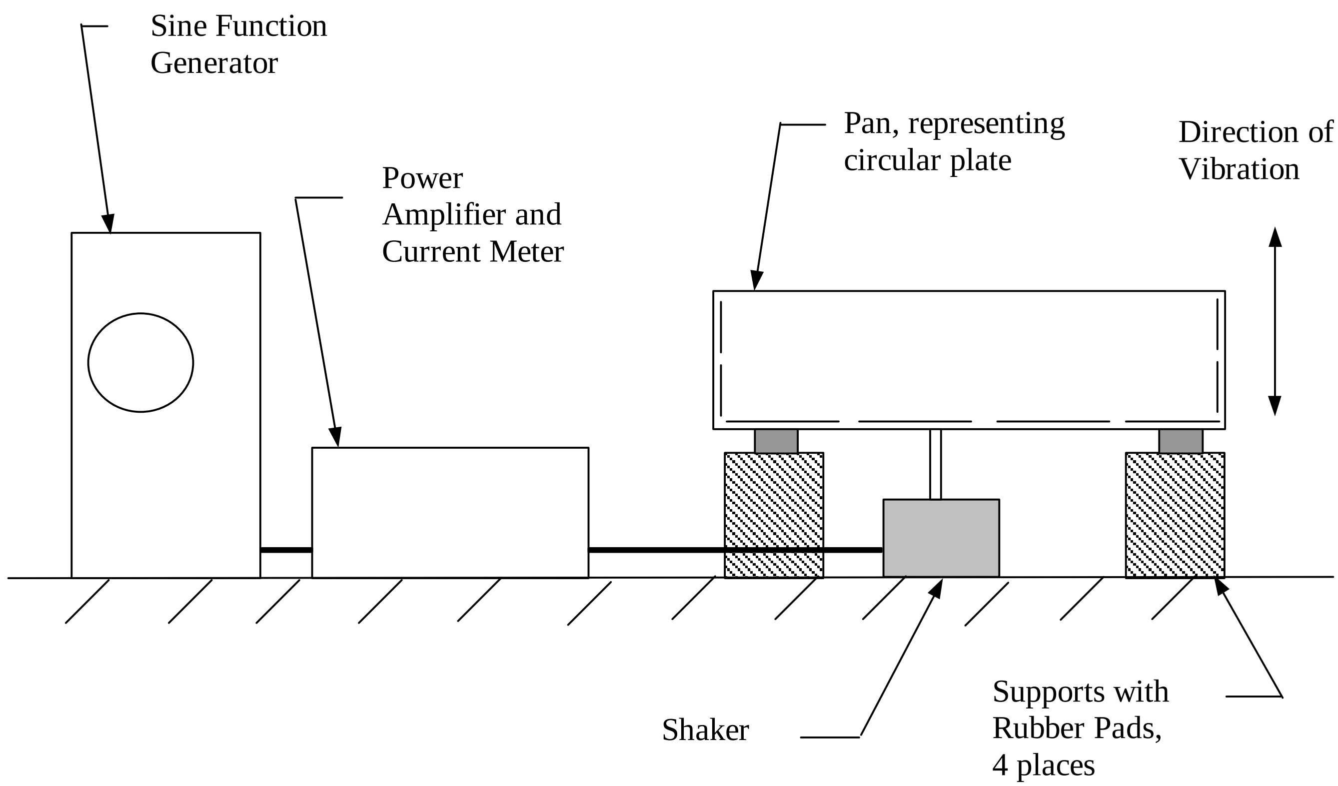

Figure 1.

RESULTS

Figure 2. First Bending Mode, "Oil Can Mode," 110 Hz

Figure 3. Second Bending Mode, One Diameteral Nodal Line, 230 Hz

Figure 4. Third Bending Mode, Two Diameteral Nodal Lines, 400 Hz

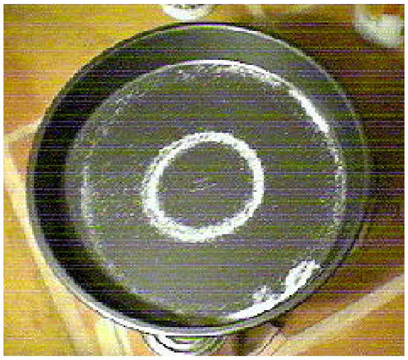

Figure 5. Fourth Bending Mode, One Circular Nodal Line, 495 Hz

Figure 6. Fifth Bending Mode, Three Diameteral Nodal Lines, 600 Hz

Figure 7. Sixth Bending Mode, One Circular and One Diameteral Nodal Line, 720 Hz

Figure 8. Eighth Bending Mode, One Circular and Two Diameteral Nodal Lines, 980 Hz

Note: The seventh bending mode, which should have four diameteral nodal lines, was not recorded.

<u>Reference</u>

1. Arthur W. Leissa, Vibration of Plates, NASA SP-160, National Aeronautics and Space Administration, Washington D.C., 1969.

APPENDIX A

The plate dimensions and properties are given in Table A-1.

Table A-1. Plate Parameters	
Parameter	Value
Boundary Condition	Fixed
Diameter	0.229 m (9 inch)
Radius	0.115 m
Thickness	0.00051 m
Skin Elasticity	205 (10^9) N/m^2
Mass Density	7700 kg/m^3
Poisson's Ratio	0.3

The following theoretical equations are taken from Reference 1.

The plate stiffness factor D is given by

$$D = \frac{Eh^3}{12\left(1-\mu^2\right)} \tag{A-1}$$

where

E = elastic modulus
h = plate thickness
μ = Poisson's ratio

$$D = \frac{\left\lfloor 205(10^9)\ \ \text{N/m\^{}2} \right\rfloor [0.00051\ \text{m}]^3}{12\left[1-(0.3)^2\right]} \tag{A-2}$$

$$D = 2.490\ \text{Nm} \tag{A-3a}$$

$$D = 2.490\ \text{kg m}^2/\text{sec}^2 \tag{A-3b}$$

The natural frequency f_n is

$$f_n = \frac{\lambda^2}{2\pi\, a^2}\sqrt{\frac{D}{\rho h}} \tag{A-4}$$

where

λ^2 is a constant which depends on the boundary condition,

ρ is the mass density,

a is the radius.

$$f_n = \frac{\lambda^2}{2\pi\,[0.115\ \text{m}]^2}\sqrt{\frac{2.490\ \text{kg m}^2/\text{sec}^2}{[7700\ \text{kg/m\^{}3}][0.00051\ \text{m}]}} \tag{A-5}$$

$$f_n = \lambda^2\ [\ 9.583\ ] \tag{A-6}$$

A comparison between the theoretical and experimental natural frequencies is given in Table A-2.

Table A-2. First Four Bending Mode Frequencies			
Mode	λ^2	Theoretical fn (Hz)	Experimental fn (Hz)
1	10.22	98	110
2	21.26	203	230
3	34.88	334	400
4	39.77	381	495

The λ^2 values are taken from Reference 1.

Again, the theoretical natural frequencies assumed a clamped boundary condition. In reality, the boundary condition was complex.

The frequency agreement for the first two modes is good. Thereafter, the agreement is poor.

Note that the rubber pads in Figure 1 acted as springs. A spring-mass mode occurred below 50 Hz, but is not shown in this report.