The Craig-Bampton Method

FEMCI Presentation
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<u>Topics:</u>

1) Background
2) Theory
3) Creating a C-B Model
4) Load Transformation Matrices
5) Verification

Appendix: Sample FLAME scripts

Background

- **Who is Craig Bampton?**

  "Coupling of Substructures for Dynamic Analysis"

  Roy R. Craig Jr. and Mervyn C. C. Bampton

  AIAA Journal

  Vol. 6, No. 7, July 1968

- **What is the Craig-Bampton Method?**

  – Method for reducing the size of a finite element model.

  – Combines motion of boundary points with modes of the structure assuming the boundary points are held fixed

  – Similar to other reduction schemes

  - {U} = [ϕ]{Ua} Where [ϕ] = -[Koo]-1[Koa] **Guyan Reduction**

    {Ua} = A-set points

  - {U} = [ϕ]{q} Where [ϕ] = Mode Shapes **Modal Decoupling**

    {q} = Modal dof's

  - {U} = [ϕ]{x_{cb}} Where [ϕ] = C-B Transformation **C-B Method**

    {x_{cb}} = C-B Dof's = boundary + modes

Background (Cont)

- Why is the C-B Method Used?
  - Allows problem size to be reduced
  - Accounts for both mass and stiffness (unlike Guyan reduction)
  - Problem size defined by frequency range
  - Allows for different boundary conditions at interface (unlike modal decoupling)
  - Example
    - Spacecraft Model: 10,000 DOF's
K,M = 10,000 x 10,000
10 Modes up to 50 Hz
Single Boundary grid at interface
    - C-B Reduction: to 50 Hz 16 DOF (6 i/f + 10 Modes)
K,M = 16 x 16

Craig-Bampton Theory

- Equation of motion (ignoring damping)

$$[M_{AA}]\{\ddot{u}_A\} + [K_{AA}]\{u_A\} = \{F(t)\} \qquad (1)$$

- The Craig-Bampton transform is defined as:

$$\{u_A\} = \left\{\begin{array}{c} u_b \\ \hline u_L \end{array}\right\} = \begin{bmatrix} I & 0 \\ f_R & f_L \end{bmatrix} \left\{\begin{array}{c} u_b \\ q \end{array}\right\} \qquad (2)$$

Where

C-B Transformation Matrix = ϕ_{cb}

u_b = boundary dof's

u_L = internal (leftover) dof's

f_R = Rigid body vector

f_L = Fixed base modeshapes

q = modal dof's

Craig-Bampton Theory (Cont.)

- Combining equations (1) & (2) and pre-multiplying by $[\phi_{cb}]^T$

$$\phi_{cb}^T \left[M_{AA} \right] \phi_{cb} \begin{Bmatrix} \ddot{u}_b \\ \ddot{q} \end{Bmatrix} + \phi_{cb}^T \left[K_{AA} \right] \phi_{cb} \begin{Bmatrix} u_b \\ q \end{Bmatrix} = \phi_{cb}^T \begin{Bmatrix} F_b \\ F_L \end{Bmatrix} \qquad (3)$$

- Define the C-B mass and stiffness matrices as

$$\left[M_{cb} \right] = \phi_{cb}^T \left[M_{AA} \right] \phi_{cb} = \begin{bmatrix} M_{bb} & M_{bq} \\ M_{qb} & M_{qq} \end{bmatrix} \qquad (4)$$

$$\left[K_{cb} \right] = \phi_{cb}^T \left[K_{AA} \right] \phi_{cb} = \begin{bmatrix} K_{bb} & 0 \\ 0 & K_{qq} \end{bmatrix} \qquad (5)$$

- Write equation (3) using equations (4) & (5)

$$\begin{bmatrix} M_{bb} & M_{bq} \\ M_{qb} & M_{qq} \end{bmatrix} \begin{Bmatrix} \ddot{u}_b \\ \ddot{q} \end{Bmatrix} + \begin{bmatrix} K_{bb} & 0 \\ 0 & K_{qq} \end{bmatrix} \begin{Bmatrix} u_b \\ q \end{Bmatrix} = \begin{Bmatrix} F_b \\ 0 \end{Bmatrix} \qquad (6)$$

where input forces are applied at the boundary only ($F_L = 0$)

Craig-Bampton Theory (Cont.)

- **Important properties of the C-B mass and stiffness matrices**
  - Mbb = Bounday mass matrix => total mass properties translated to the boundary points

$$\left[M^{cg}\right] = f_{RB}^{cg^T}\left[M_{bb}\right] f_{RB}^{cg} \qquad (7)$$

  - Kbb = Interface stiffness matrix => stiffness associated with displacing one boundary dof while other are held fixed
    - If the boundary point is a single grid (i.e. non-redundant) then

$$\mathbf{K_{bb} = 0}$$

  - If the mode shapes have been mass normalized (typically they are) then

$$K_{qq} = \begin{bmatrix} \backslash & & 0 \\ & l & \\ 0 & & \backslash \end{bmatrix} \qquad l_i = k_i / m_i = w_i^2$$

$$M_{qq} = \begin{bmatrix} \backslash & & 0 \\ & I & \\ 0 & & \backslash \end{bmatrix} \qquad (8)$$

Craig-Bampton Theory (Cont.)

- We can finally write the dynamic equation of motion (including damping) using the C-B transform as

$$\begin{bmatrix} M_{bb} & M_{bq} \\ M_{qb} & I \end{bmatrix} \begin{Bmatrix} \ddot{u}_b \\ \ddot{q} \end{Bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 2\zeta\omega \end{bmatrix} \begin{Bmatrix} \dot{u}_b \\ \dot{q} \end{Bmatrix} + \begin{bmatrix} K_{bb} & 0 \\ 0 & \omega^2 \end{bmatrix} \begin{Bmatrix} u_b \\ q \end{Bmatrix} = \begin{Bmatrix} F_b \\ 0 \end{Bmatrix} \qquad (9)$$

where $2\zeta\omega$ = Modal damping (ζ=%critical)

- Summary of C-B Theory
  - C-B Mass and Stiffness Matrices fully define system
  - Dynamics problem solved using CB dof's
  - C-B boundary dofs provide location to apply BC's & Forces or to couple with another structure
  - CB transform is used to calculate physical responses from CB responses

How to Create a C-B Model

```
assign USER1=gi_v2_cb.kmnp,NEW,USE=OUTPUT4,TYPE=BINARY,reallocate  ←1) CB Output File
ID GLAST,inst
SOL 3                   ←2)  Normal Modes Solution
APP DISP
TIME 5
$
INCLUDE '/home/sag721/dmap/uai/cb_v118b.dmp'     ←3)  C-B DMAP
CEND
$
TITLE = GLAST SI Instrument
SUBTITLE = Craig-Bampton Run
ECHO = NONE
METHOD = 1
$SPC = 998
$POST SDRC
DISP(NOPRINT)     =ALL
$SPCFORCES(NOPRINT)=ALL
$MPCFORCES(NOPRINT)=ALL
$
AUTOSPC = YES
BEGIN BULK
$
PARAM,GRDPNT,0
PARAM,WTMASS,2.59e-3
PARAM,USETPRT,0         ←4) Print G-set & R-set internal order
$
EIGRL,1,-0.1,70.0       ←5) Define frequency range
$
$ Instrument Interface at S/C
SUPORT      800290      123456      ←6) Boundary Defined on suport cards
SUPORT      800291      123456            8 boundary points x 6 dof's
SUPORT      800292      123456            = 48 physical boundary points
SUPORT      800293      123456
SUPORT      800294      123456
SUPORT      800295      123456
SUPORT      800296      123456
SUPORT      800297      123456
$
INCLUDE 'glast_inst_v2.blk'         ←7) Don't forget the rest of your bulk data
$
ENDDATA
```

How to Create a C-B Model (Cont.)

- What is created?
  - file (.kmnp) which contains CB stiffness and mass matrices (k,m), net CG ltm (n), and the CB transformation matrix (phig)
  - .kmnp file is in NASTRAN binary output4 format
  - K&M size is [CB dofs (boundary + modal) x CB dofs]
  - phig size is [G-set rows x CB dofs]
  - Net CG LTM recovers CG accelerations and I/F Forces, Size is [6+boundary dofs x CB dofs]
- How do you use this?
  - Solve dynamics problem for CB dof response using the K & M matrices
  - Transform CB responses using phig to get physical responses

Load Transformation Matrices (LTMs)

- LTM is a generic term referring to the matrix used to transform from CB dofs to physical dofs (also referred to at OTMs, ATMs, DTMs…)
- In its simplest form, the LTM is simply the phig matrix

$$\left\{ \ddot{U}_G \right\} = \left[f_{cb} \right] \left\{ \begin{matrix} \ddot{u}_b \\ \ddot{q} \end{matrix} \right\} \qquad (10)$$

(Only the rows corresponding to the physical dofs of interest are needed)

- There are other useful LTMs that can be created
  - I/F forces
  - Net CG accelerations
  - Stress & force LTMs

LTM's (Cont.)

- I/F Force LTM (created by CB dmap)

$$\text{I/F Force} = \begin{bmatrix} M_{bb} & M_{bq} & K_{bb} \end{bmatrix} \begin{Bmatrix} \ddot{u}_b \\ q \\ u_b \end{Bmatrix} \qquad (11)$$

(If boundary is non-redundant, then Kbb=0)

- Net CG LTM (created by CB dmap)

$$\text{Net CG Accel} = \left(\left[f_{rb}^{CG} \right]^T \left[M_{bb} \right] \left[f_{rb}^{CG} \right] \right)^{-1} \left[f_{rb}^{CG} \right]^T \begin{bmatrix} M_{bb} & M_{bq} & K_{bb} \end{bmatrix} \begin{Bmatrix} \ddot{x}_b \\ \ddot{q} \\ x_b \end{Bmatrix} \qquad (12)$$

where $\left(\left[f_{rb}^{CG} \right]^T \left[M_{bb} \right] \left[f_{rb}^{CG} \right] \right) = \text{mass matrix about cg (6x6)}$

$\left[f_{rb}^{CG} \right] = \text{rigid body transform from I/F to CG (bdof x6)}$

LTM's (Cont.)

- PHIZ LTM
  - Allows physical displacements to be calculated from CB accelerations

$$\{X_G\} = [PHIZ]\begin{Bmatrix} \ddot{x}_b \\ \ddot{q} \end{Bmatrix} \qquad (13)$$

  - Same as modal acceleration approach in NASTRAN
  - Useful in calculating relative displacements between DOF's
  - Also used to calculate stresses and forces which are a function of displacements
  - Calculated from C-B dmap using => param,phzout,1

LTM's (Cont.)

- LTM's can be created using FLAME, MATLAB or using DMAP
- LTM's can (and usually do) contain multiple types of responses

$$LTM = \begin{bmatrix} [Net\,CG] \\ [I/F\ Force] \\ [Accel] \\ \begin{bmatrix} Element \\ Forces \end{bmatrix} \end{bmatrix} \qquad (14)$$

- LTM's can be used to recover responses for nested C-B models

$$\left\{X^{cb1}\right\} = \left[f_{cb1}\right]\left[f_{cb0}^{cb1}\right]\begin{Bmatrix} \ddot{x}_b^{cb0} \\ \ddot{q}^{cb0} \end{Bmatrix} \qquad (15)$$

where f_{cb0}^{cb1} = row partition of the CB1 Dofs from the CB0 PHIG Matrix

- Creating LTMs - See appendix for a sample FLAME script for creating an LTM

Checking CB Models & LTM’s

- C-B Models and LTMs should be verified to make sure that they have been created correctly (especially for complicated LTM’s or nested C-B models)
- CB Mass and stiffness matrices can be checked by computing free-free and fixed-base modes
- CB boundary Mass matrix can be transformed to CG and compared with NASTRAN GPWG

Checking C-B Models and LTM's (Cont.)

- LTMS can be checked by applying unit acceleration at the boundary

$$\{X_{resp}\} = \begin{array}{c} LTM \\ \left[\begin{array}{c|c} b & q \\ & \end{array}\right] \end{array} \begin{Bmatrix} f_{RB}^{b} \\ 0 \end{Bmatrix} \qquad (16)$$

where f_{RB}^{b} = Boundary rigid body vector (b x 6)

  - Each response column represents acceleration in a single direction
  - Accelerations should be in correct directions
  - Forces should recover weight or correct moments
  - Unit acceleration applied to PHIZ can be checked by gravity run with physical model and comparing displacements

- See appendix for sample FLAME scripts to check a CB model and LTM

Appendix
Sample FLAME Scripts

cb_chk.fla	==>	Checking CB K&M Matrices
etm_ltma.fla	==>	LTM creation
etm_chk.fla	==>	Checking an LTM