The Craig-Bampton Method

FEMCI Presentation
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Topics:
1) Background
2) Theory
3) Creating a C-B Model
4) Load Transformation Matrices
5) Verification

Appendix: Sample FLAME scripts
Background

• **Who is Craig Bampton?**
  “Coupling of Substructures for Dynamic Analysis”
  Roy R. Craig Jr. and Mervyn C. C. Bampton
  AIAA Journal
  Vol. 6, No. 7, July 1968

• **What is the Craig-Bampton Method?**
  – Method for reducing the size of a finite element model.
  – Combines motion of boundary points with modes of the structure assuming the boundary points are held fixed
  – Similar to other reduction schemes
  
  • \( \{U\} = [\phi]\{U_a\} \) \text{ Where } \[\phi\] = -[K_{oo}]^{-1}[K_{oa}] \text{ \hspace{1cm} Guyan Reduction} \\
  \{U_a\} = A\text{-set points} \\
  
  • \( \{U\} = [\phi]\{q\} \) \text{ Where } \[\phi\] = Mode Shapes \text{ \hspace{1cm} Modal Decoupling} \\
  \{q\} = Modal dof’s \\
  
  • \( \{U\} = [\phi]\{x_{cb}\} \) \text{ Where } \[\phi\] = C-B Transformation \text{ \hspace{1cm} C-B Method} \\
  \{x_{cb}\} = C-B Dof’s = boundary + modes
Background (Cont)

• Why is the C-B Method Used?
  – Allows problem size to be reduced
  – Accounts for both mass and stiffness (unlike Guyan reduction)
  – Problem size defined by frequency range
  – Allows for different boundary conditions at interface (unlike modal decoupling)
  – Example
    • Spacecraft Model: 10,000 DOF’s  
      K,M = 10,000 x 10,000  
      10 Modes up to 50 Hz  
      Single Boundary grid at interface
    • C-B Reduction: 16 DOF (6 i/f + 10 Modes)  
      to 50 Hz  
      K,M = 16 x 16
Craig-Bampton Theory

- Equation of motion (ignoring damping)

\[
[M] \{\ddot{u}\} + [K] \{u\} = \{F(t)\} \quad (1)
\]

- The Craig-Bampton transform is defined as:

\[
\{u\} = \begin{pmatrix} u_b \\ u_L \end{pmatrix} = \begin{bmatrix} I & 0 \end{bmatrix} \begin{pmatrix} u_b \\ q \end{pmatrix} \quad \text{(2)}
\]

Where
- \(u_b\) = boundary dof's
- \(u_L\) = internal (leftover) dof's
- \(\phi_R\) = Rigid body vector
- \(\phi_L\) = Fixed base modeshapes
- \(q\) = modal dof's

C-B Transformation Matrix = \(\phi_{cb}\)
Craig-Bampton Theory (Cont.)

- Combining equations (1) & (2) and pre-multiplying by $[\phi_{cb}]^T$

$$\Phi_{cb}^T [M_{AA}] \Phi_{cb} \begin{bmatrix} \ddot{u}_b \\ \ddot{q} \end{bmatrix} + \Phi_{cb}^T [K_{AA}] \Phi_{cb} \begin{bmatrix} u_b \\ q \end{bmatrix} = \Phi_{cb}^T \begin{bmatrix} F_b \\ F_l \end{bmatrix}$$

- Define the C-B mass and stiffness matrices as

$$[M_{cb}] = \Phi_{cb}^T [M_{AA}] \Phi_{cb} = \begin{bmatrix} M_{bb} & M_{bq} \\ M_{qb} & M_{qq} \end{bmatrix}$$

- Define the C-B mass and stiffness matrices as

$$[K_{cb}] = \Phi_{cb}^T [K_{AA}] \Phi_{cb} = \begin{bmatrix} K_{bb} & 0 \\ 0 & K_{qq} \end{bmatrix}$$

- Write equation (3) using equations (4) & (5)

$$\begin{bmatrix} M_{bb} & M_{bq} \\ M_{qb} & M_{qq} \end{bmatrix} \begin{bmatrix} \ddot{u}_b \\ \ddot{q} \end{bmatrix} + \begin{bmatrix} K_{bb} & 0 \\ 0 & K_{qq} \end{bmatrix} \begin{bmatrix} u_b \\ q \end{bmatrix} = \begin{bmatrix} F_b \\ 0 \end{bmatrix}$$

where input forces are applied at the boundary only ($F_l = 0$)
Craig-Bampton Theory (Cont.)

- **Important properties of the C-B mass and stiffness matrices**
  - $M_{bb}$ = Boundary mass matrix => total mass properties translated to the boundary points
    \[
    [M^c_g] = \Phi^{c_g} [M_{bb}] \Phi^{c_g}_{RB} \tag{7}
    \]
  - $K_{bb}$ = Interface stiffness matrix => stiffness associated with displacing one boundary dof while other are held fixed
    - If the boundary point is a single grid (i.e. non-redundant) then $K_{bb} = 0$
    - If the mode shapes have been mass normalized (typically they are) then
      \[
      \begin{align*}
      K_{qq} &= \begin{bmatrix}
      \lambda & 0 \\
      0 & \lambda
      \end{bmatrix} \\
      \lambda_i &= k_i / m_i = \omega_i^2
      \\
      M_{qq} &= \begin{bmatrix}
      0 & 1 \\
      1 & 0
      \end{bmatrix}
      \end{align*}
      \tag{8}
      \]
Craig-Bampton Theory (Cont.)

• We can finally write the dynamic equation of motion (including damping) using the C-B transform as

\[
\begin{pmatrix}
M_{bb} & M_{bq} \\
M_{qb} & I
\end{pmatrix}
\begin{Bmatrix}
\ddot{\mathbf{u}}_b \\
\dot{\mathbf{q}}
\end{Bmatrix}
+ \begin{pmatrix}
0 & 0 \\
0 & 2\zeta \omega
\end{pmatrix}
\begin{Bmatrix}
\dot{\mathbf{u}}_b \\
\dot{\mathbf{q}}
\end{Bmatrix}
+ \begin{pmatrix}
K_{bb} & 0 \\
0 & \omega^2
\end{pmatrix}
\begin{Bmatrix}
\mathbf{u}_b \\
\mathbf{q}
\end{Bmatrix}
= \begin{Bmatrix}
\mathbf{F}_b \\
0
\end{Bmatrix}
\]  

(9)

where \(2\zeta \omega\) = Modal damping \((\zeta = \%\text{critical})\)

• Summary of C-B Theory

– C-B Mass and Stiffness Matrices fully define system
– Dynamics problem solved using CB dof’s
– C-B boundary dofs provide location to apply BC’s & Forces or to couple with another structure
– CB transform is used to calculate physical responses from CB responses
How to Create a C-B Model

assign USER1=gi_v2_cb.kmnp,NEW,USE=OUTPUT4,TYPE=BINARY,reallocate

1) CB Output File
ID GLAST,inst
SOL 3
APP DISP
TIME 5

2) Normal Modes Solution
INCLUDE '/home/sag721/dmap/uai/cb_v118b.dmp'

3) C-B DMAP
CEND
TITLE = GLAST SI Instrument
SUBTITLE = Craig-Bampton Run
ECHO = NONE
METHOD = 1
$SFC = 999
$POST SDRC
DISP(NOPRINT) =ALL
$SPCFORCES(NOPRINT) =ALL
$MPCFORCES(NOPRINT) =ALL

4) Print G-set & R-set internal order
EIGRL,1,-0.1,70.0

5) Define frequency range
$ Instrument Interface at S/C
SUPORT 800290 123456
SUPORT 800291 123456
SUPORT 800292 123456
SUPORT 800293 123456
SUPORT 800294 123456
SUPORT 800295 123456
SUPORT 800296 123456
SUPORT 800297 123456

INCLUDE 'glast_inst_v2.blk'

6) Boundary Defined on suport cards
8 boundary points x 6 dof’s = 48 physical boundary points

7) Don’t forget the rest of your bulk data

ENDDATA
How to Create a C-B Model (Cont.)

• What is created?
  – file (.kmnp) which contains CB stiffness and mass matrices (k,m), net CG ltm (n), and the CB transformation matrix (phig)
  – .kmnp file is in NASTRAN binary output4 format
  – K&M size is [CB dofs (boundary + modal) x CB dofs]
  – phig size is [G-set rows x CB dofs]
  – Net CG LTM recovers CG accelerations and I/F Forces, Size is [6+boundary dofs x CB dofs]

• How do you use this?
  – Solve dynamics problem for CB dof response using the K & M matrices
  – Transform CB responses using phig to get physical responses
Load Transformation Matrices (LTM)

• LTM is a generic term referring to the matrix used to transform from CB dofs to physical dofs (also referred to at OTMs, ATMs, DTMs…)

• In its simplest form, the LTM is simply the phig matrix

\[
\{\ddot{U}_G\} = [\phi_{cbG}] \begin{bmatrix} \dddot{u} \\ \dddot{q} \end{bmatrix}
\]  

(10)

(Only the rows corresponding to the physical dofs of interest are needed)

• There are other useful LTMs that can be created
  – I/F forces
  – Net CG accelerations
  – Stress & force LTMs
LTM’s (Cont.)

• I/F Force LTM (created by CB dmap)

\[
\text{I/F Force} = \begin{bmatrix} M_{bb} & M_{bg} & K_{bb} \end{bmatrix} \begin{bmatrix} q \\ u_b \end{bmatrix} \tag{11}
\]

(If boundary is non-redundant, then \(K_{bb}=0\))

• Net CG LTM (created by CB dmap)

\[
\text{Net CG Accel} = \left( \begin{bmatrix} \phi_{rb}^{CG} \end{bmatrix}^T \begin{bmatrix} M_{bb} & M_{bg} \end{bmatrix} \begin{bmatrix} \phi_{rb}^{CG} \end{bmatrix}^{-1} \begin{bmatrix} M_{bb} & M_{bg} \end{bmatrix} \begin{bmatrix} K_{bb} \end{bmatrix} \right) \begin{bmatrix} \ddot{x}_b \\ \ddot{\phi}_{rb}^{CG} \end{bmatrix} \tag{12}
\]

where \(\begin{bmatrix} \phi_{rb}^{CG} \end{bmatrix}^T \begin{bmatrix} M_{bb} & M_{bg} \end{bmatrix} \begin{bmatrix} \phi_{rb}^{CG} \end{bmatrix}^{-1}\) = mass matrix about cg (6x6)

\([\phi_{rb}^{CG}] = \text{rigid body transform from I/F to CG (bdof x6)}\)
LTM’s (Cont.)

• **PHIZ LTM**
  
  - Allows physical displacements to be calculated from CB accelerations
    \[
    \{X_c\} = [PHIZ] \begin{bmatrix} \ddot{x}_b \\ \ddot{q} \end{bmatrix}
    \] (13)
  
  - Same as modal acceleration approach in NASTRAN
  - Useful in calculating relative displacements between DOF’s
  - Also used to calculate stresses and forces which are a function of displacements
  - Calculated from C-B dmap using => param,phzout,1
LTM’s (Cont.)

- LTM’s can be created using FLAME, MATLAB or using DMAP
- LTM’s can (and usually do) contain multiple types of responses
  \[
  LTM = \begin{bmatrix}
  [\text{Net CG}] \\
  [I / F \text{ Force}] \\
  [\text{Accel}] \\
  [\text{Element Forces}]
  \end{bmatrix}
  \]  \hspace{1cm} (14)
- LTM’s can be used to recover responses for nested C-B models
  \[
  \{X^{cb1}\} = [\phi_{cb1}^{cb1}] \begin{bmatrix}
  \ddot{x}_b^{cb0} \\
  \dot{q}_{cb0}
  \end{bmatrix}
  \]  \hspace{1cm} (15)
  where \( \phi_{cb0}^{cb1} \) = row partition of the CB1 Dofs from the CB0 PHIG Matrix
- Creating LTMs - See appendix for a sample FLAME script for creating an LTM
Checking CB Models & LTM’s

• C-B Models and LTMs should be verified to make sure that they have been created correctly (especially for complicated LTM’s or nested C-B models)

• CB Mass and stiffness matrices can be checked by computing free-free and fixed-base modes

• CB boundary Mass matrix can be transformed to CG and compared with NASTRAN GPWG
Checking C-B Models and LTM’s (Cont.)

- LTMS can be checked by applying unit acceleration at the boundary

\[
\{X_{resp}\} = \begin{bmatrix} b \\ q \\ \phi^b_{RB} \\ 0 \end{bmatrix}
\]

where \( \phi^b_{RB} \) = Boundary rigid body vector (b x 6)

- Each response column represents acceleration in a single direction
- Accelerations should be in correct directions
- Forces should recover weight or correct moments
- Unit acceleration applied to PHIZ can be checked by gravity run with physical model and comparing displacements

- See appendix for sample FLAME scripts to check a CB model and LTM
Appendix
Sample FLAME Scripts

cb_chk.fla  ==>  Checking CB K&M Matrices
etm_ltma.fla ==>  LTM creation
etm_chk.fla  ==>  Checking an LTM