

SHOCK RESPONSE SPECTRUM ANALYSIS VIA THE FINITE ELEMENT METHOD

Revision C

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November 19, 2010

Introduction

This report gives a method for determining the response of a multi-degree-of-freedom system to a base excitation shock, where the shock is defined in terms of a Shock Response Spectrum (SRS).

A finite element model is used to determine the normal modes and frequency response function of a sample structure. Commercial finite element analysis software is used for this purpose.

The following steps are done outside the finite element software by using programs written in C/C++. The source code for these programs is available from the author by request. See also Appendix C.

The impulse response function is calculated from the frequency response function via an inverse Fourier transform.

A time history is synthesized to satisfy the SRS. The response time history of the structure is then calculated via a convolution integral using the synthesized time history and the impulse response function.

This approach is referred to as the synthesis method in this report. An advantage of this method is that the impulse response function can be used for numerous time history inputs. There is no need to rerun the finite element analysis for each input case.

Sample Structure

Consider a circuit board made from G10 material. The modulus of elasticity is $2.00E+06$ lbf/in². The dimensions of the circuit board are 2 in x 4 in x 0.063 in. The board is fixed at each corner.

The board has a uniform mass distribution. The total mass is 0.115 lbm. This includes the G10 board and the electronic components. Assume that the electronic components do not add any stiffness.

The circuit board is assumed to have an amplification factor of $Q=10$ for all modes.

Normal Modes

The normal modes are analyzed via the finite element method using FEMAP and NE/Nastran software. The filename is SRS_plate_normal.nas.

The undeformed model is shown in Figure 1. The model consists of 1624 plate elements and 1711 nodes. The natural frequencies for the first 20 modes are given in Table 1. The mode shapes for the first twelve modes are given in Figures 2 through 13, respectively.

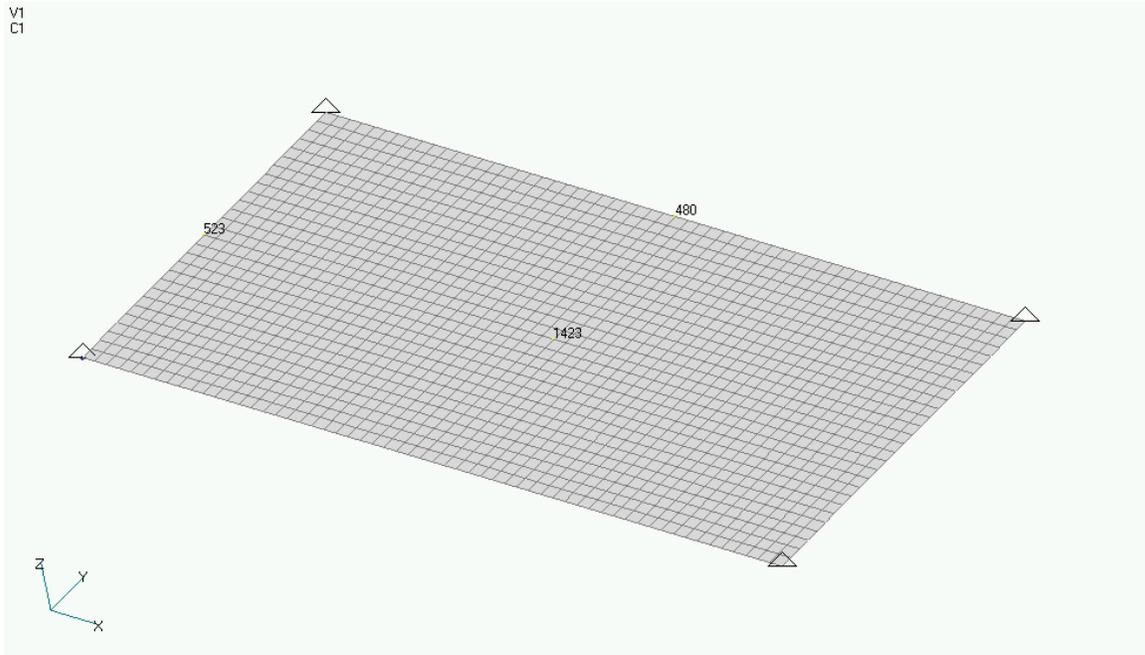


Figure 1. Finite Element Model of Circuit Board, Undeformed

The figure has node labels at three locations of interest. The node numbers are 480, 523, and 1423.

Table 1. Circuit Board Natural Frequencies			
Mode	fn (Hz)	Modal Participation Factor	Modal Effective Mass
1	134	1.546E-02	2.39E-04
2	350	2.289E-09	0.00E+00
3	410	1.613E-09	0.00E+00
4	660	6.376E-03	4.06E-05
5	779	-5.592E-14	0.00E+00
6	893	1.469E-08	0.00E+00
7	1116	2.073E-03	4.30E-06
8	1141	0.000E+00	0.00E+00
9	1322	1.804E-09	0.00E+00
10	1416	1.177E-03	1.39E-06
11	1687	1.149E-09	0.00E+00
12	1916	4.433E-15	0.00E+00
13	2142	-5.880E-10	0.00E+00
14	2348	3.373E-09	0.00E+00
15	2402	-2.177E-03	4.74E-06
16	2537	0.000E+00	0.00E+00
17	2717	0.000E+00	0.00E+00
18	2724	-1.254E-15	0.00E+00
19	2784	0.000E+00	0.00E+00
20	2805	-6.125E-10	0.00E+00

The modal participation factor does not have a unit.

The modal effective mass has a unit of lbf sec²/in.

Further information about these parameters is given in Reference 1.

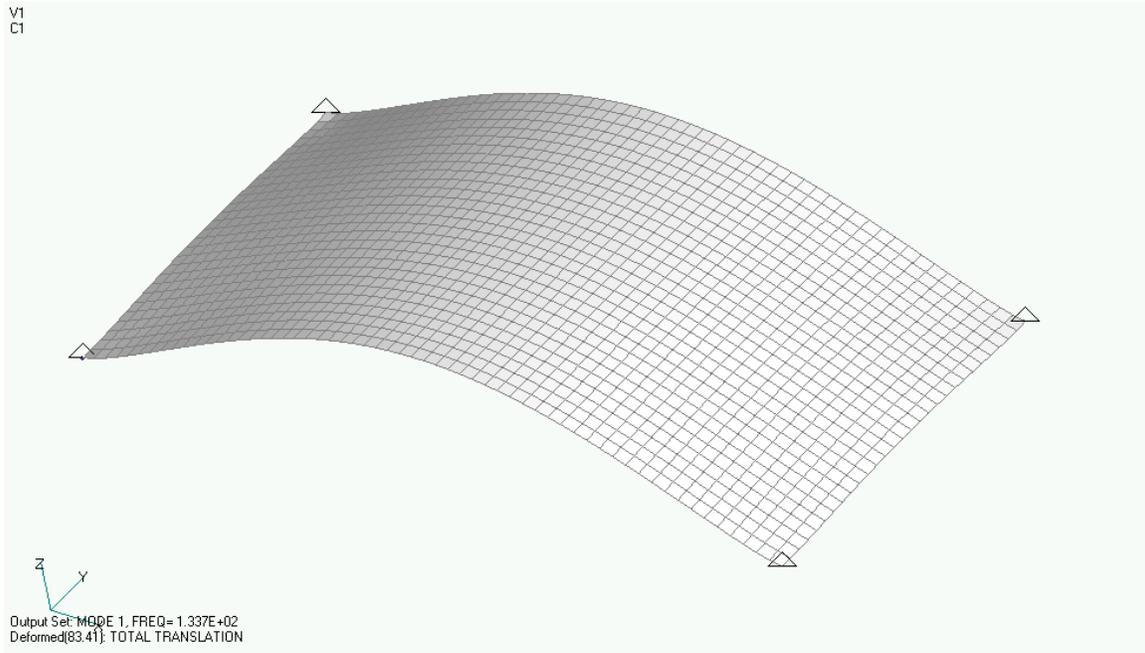


Figure 2. Mode 1

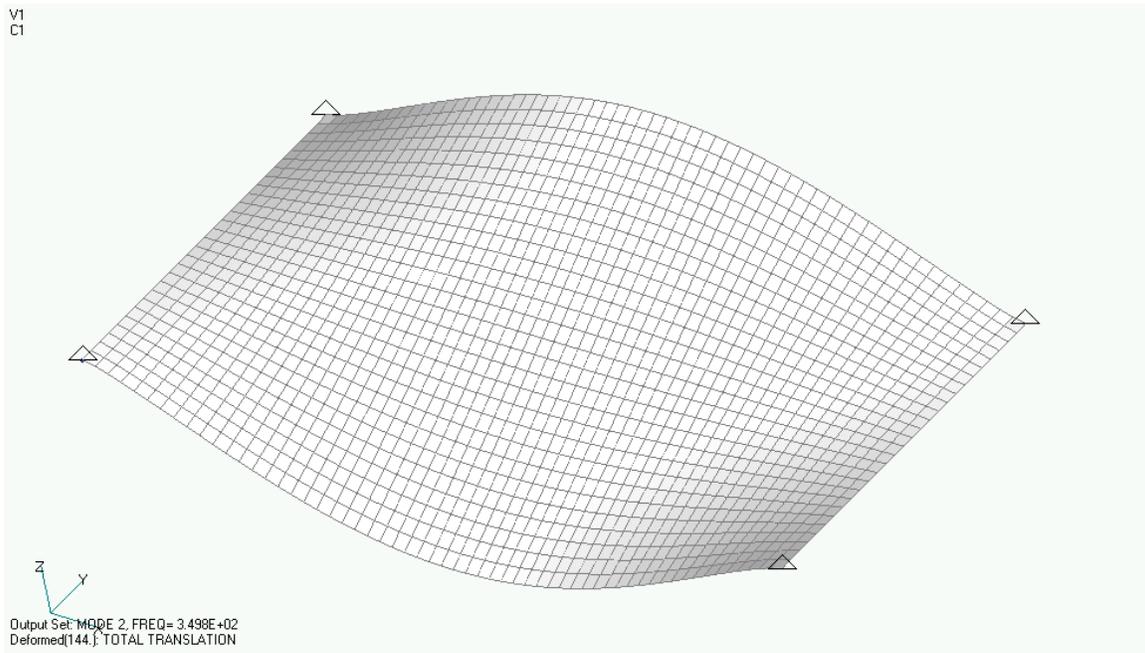


Figure 3. Mode 2

V1
C1

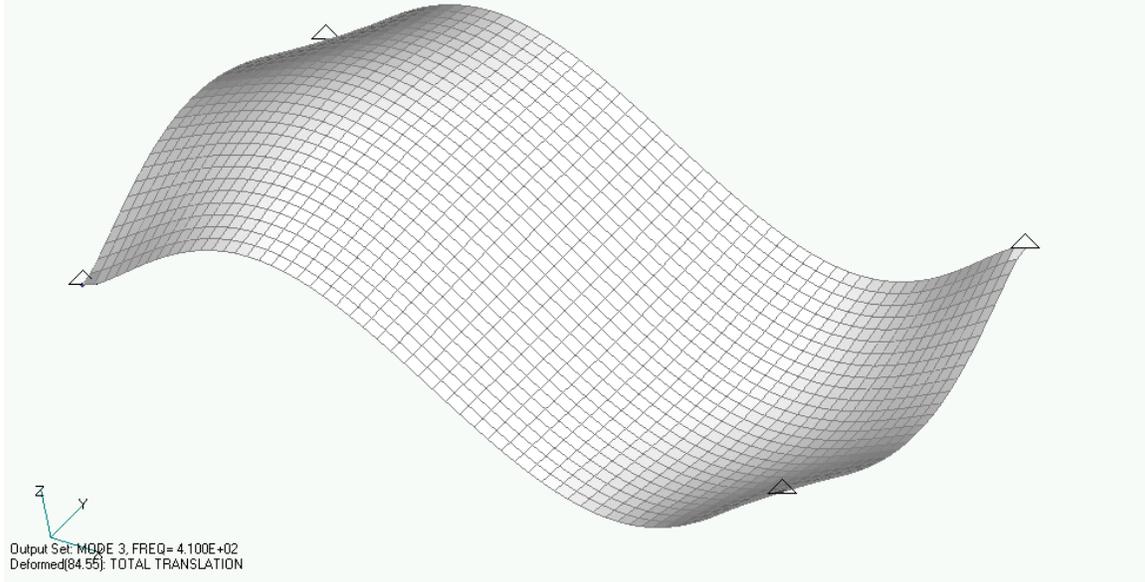


Figure 4. Mode 3

V1
C1

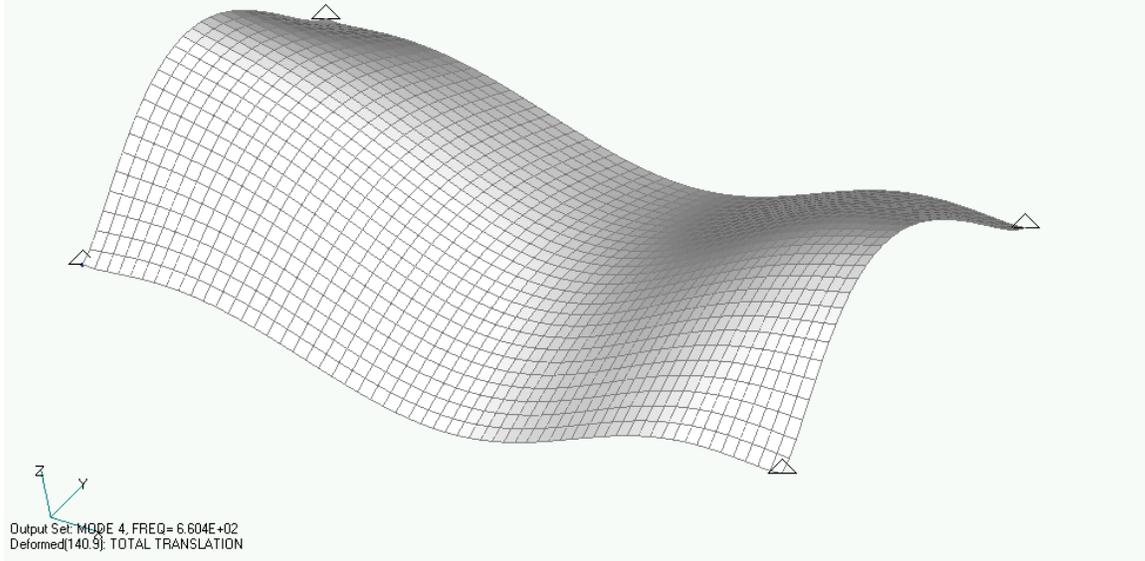


Figure 5. Mode 4

V1
C1

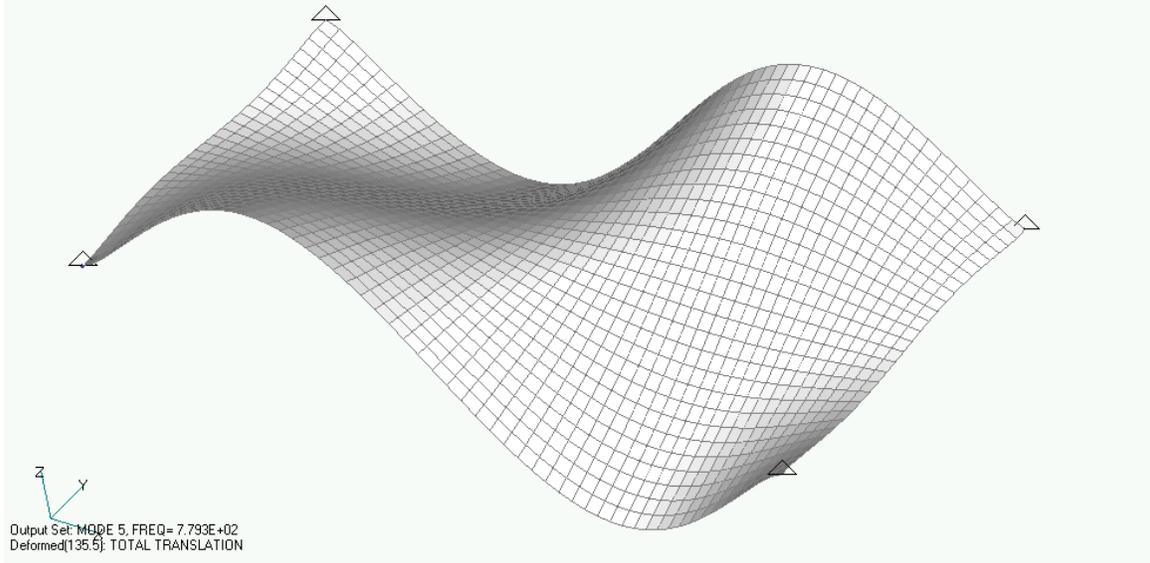


Figure 6. Mode 5

V1
C1

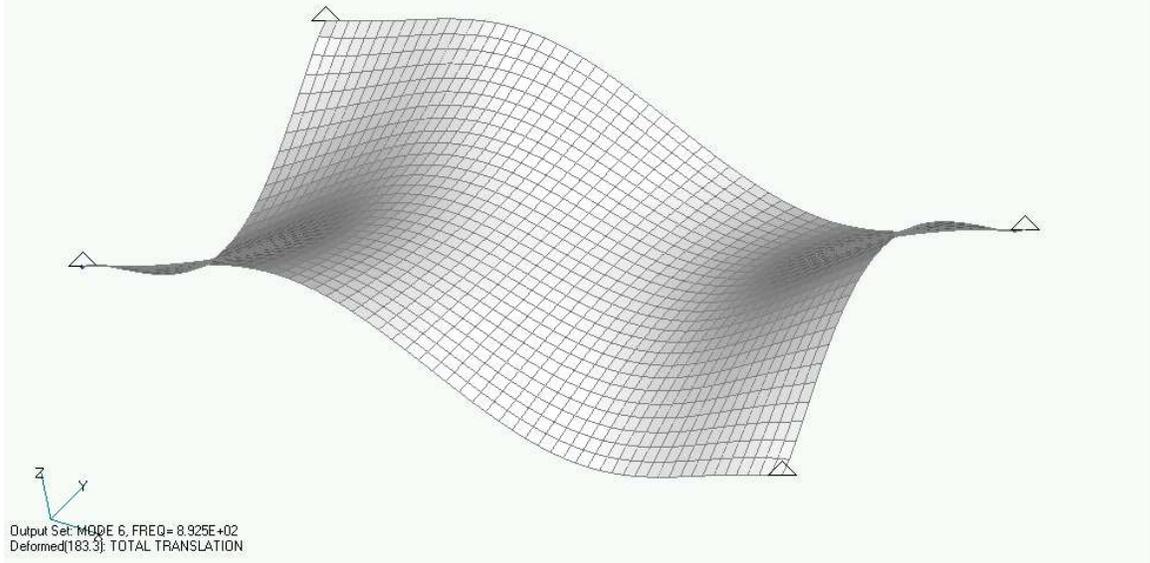


Figure 7. Mode 6

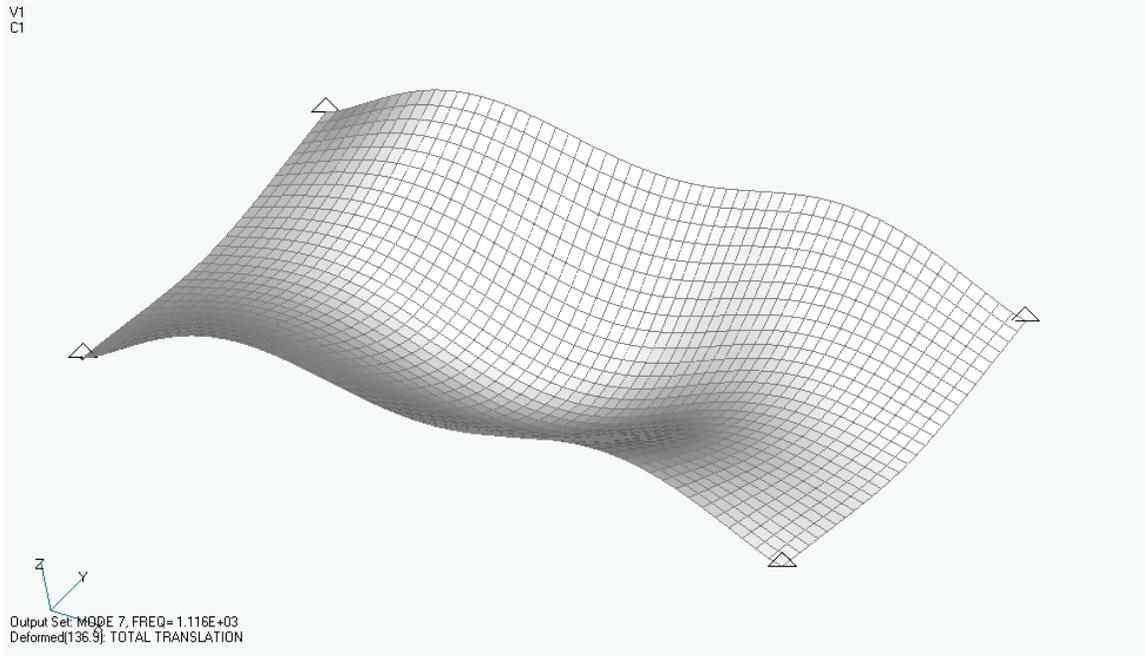


Figure 8. Mode 7

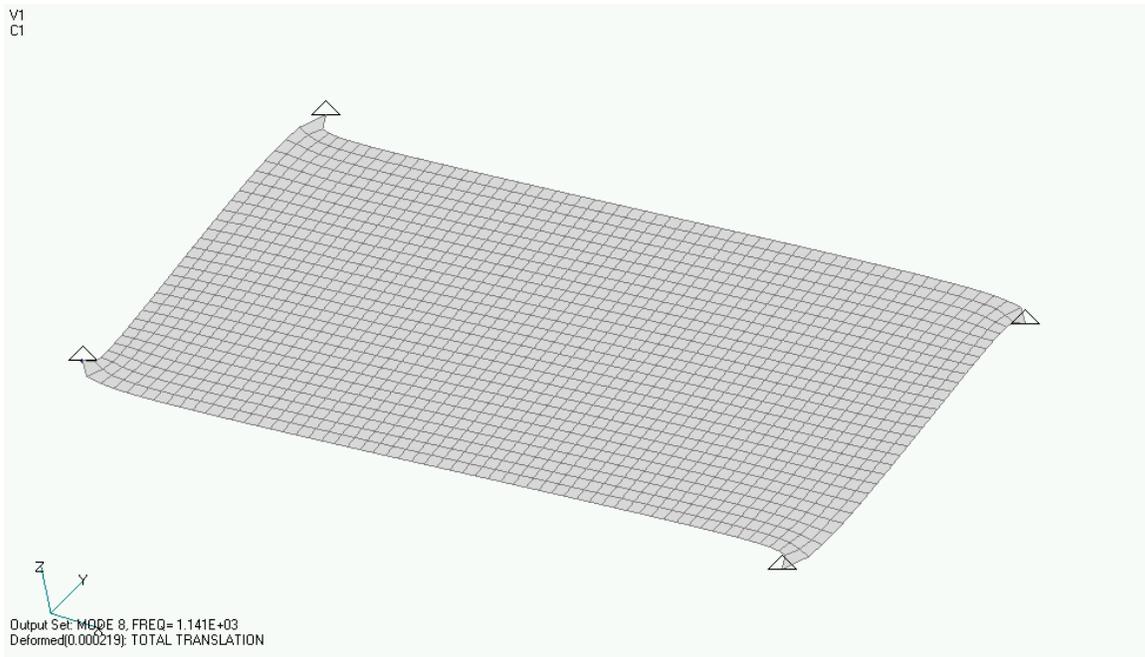


Figure 9. Mode 8

V1
C1

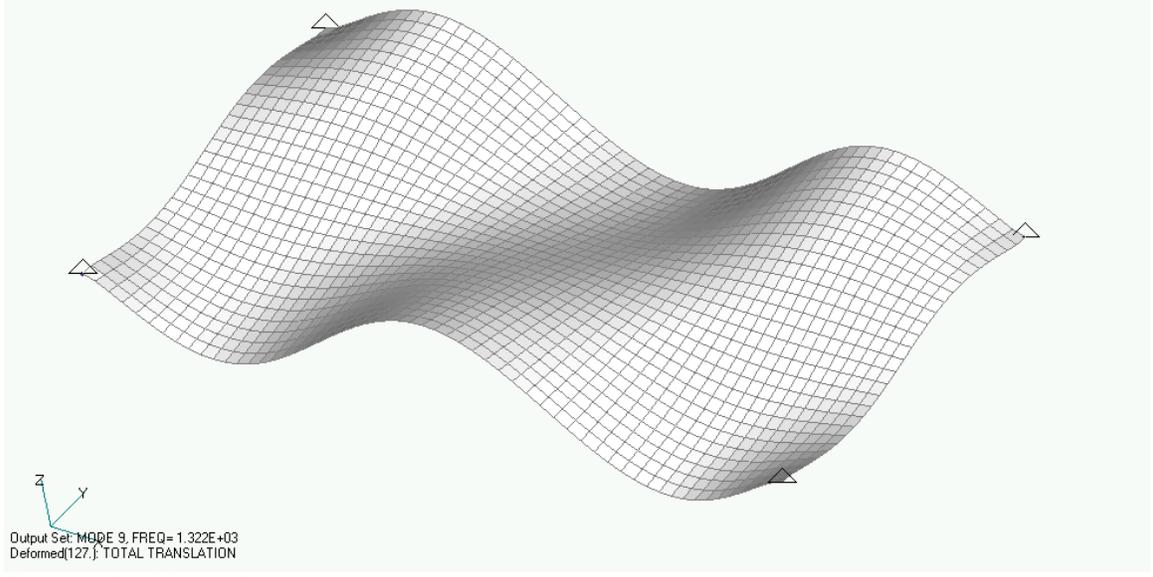


Figure 10. Mode 9

V1
C1

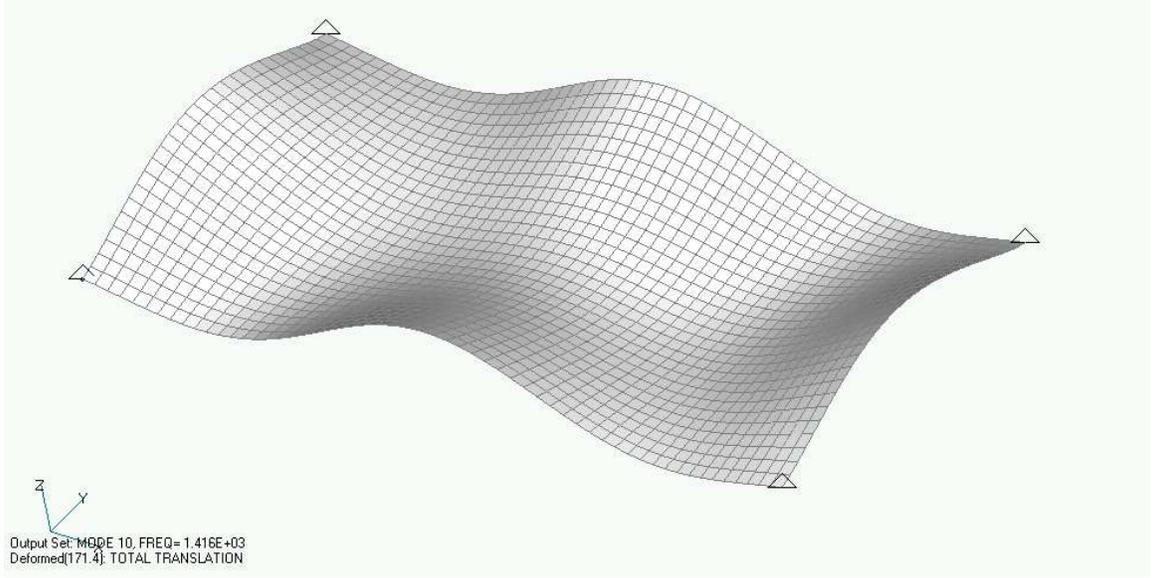


Figure 11. Mode 10

V1
C1

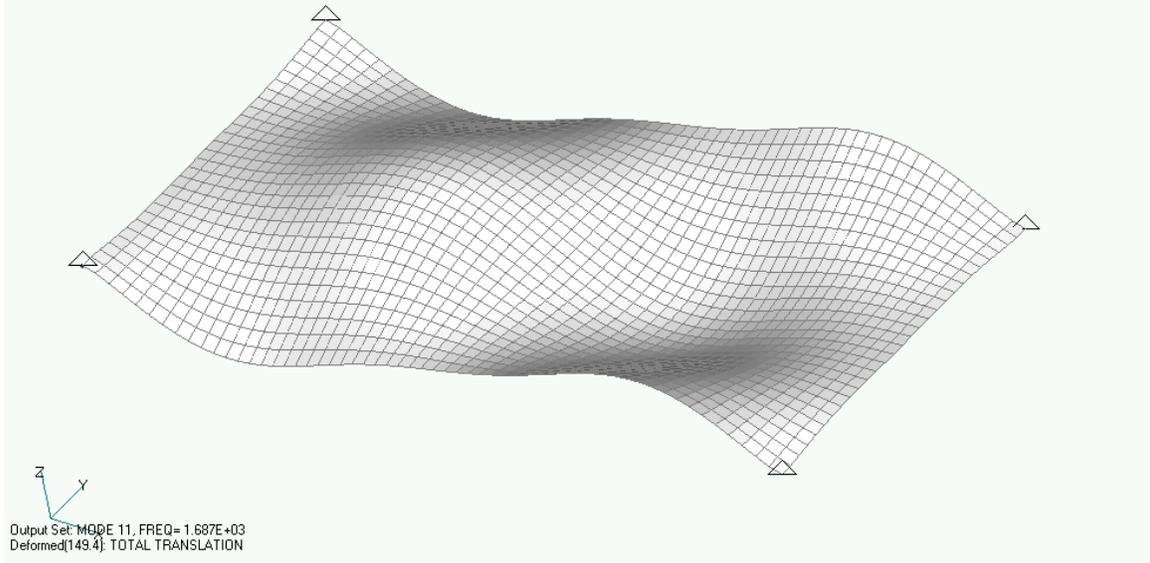


Figure 12. Mode 11

V1
C1

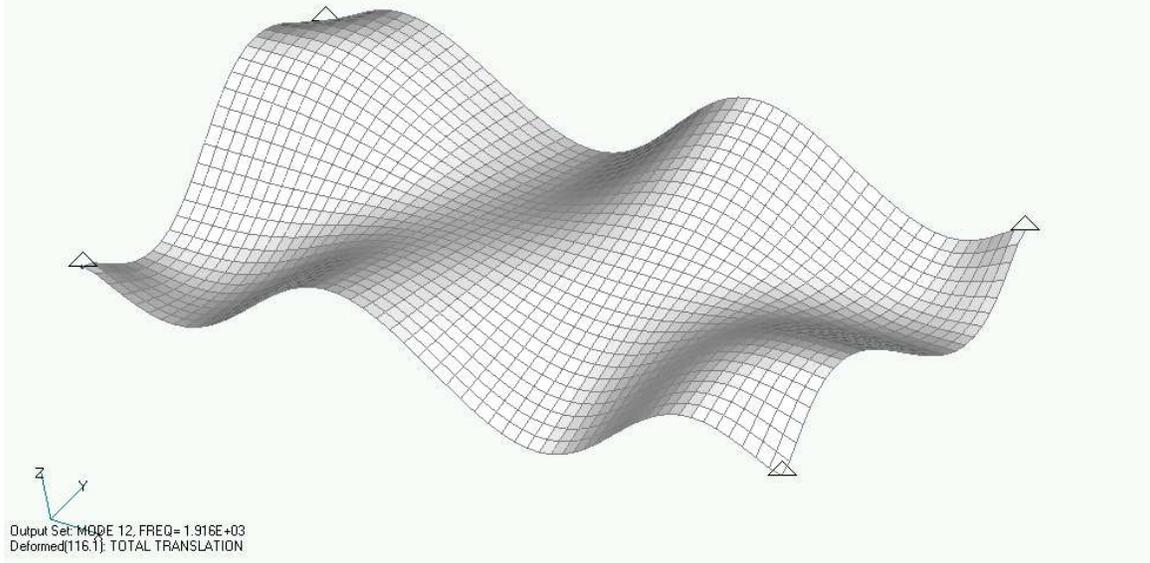


Figure 13. Mode 12

Frequency Response Functions

The next step is to perform a frequency response analysis. The resulting frequency response functions give the response to the base input. The response parameter may be either displacement or acceleration. The base input is in terms of acceleration for the sample problem.

This analysis is called “SOL SEMFREQ” in Nastran terminology. It is also referred to as a modal frequency analysis.

The filename is SRS_plate_frf_4k.nas.

The base function is a spectral function with a magnitude of unity, from 0 Hz to 4000 Hz. This represents acceleration. It is applied at the four corner nodes in the Z-axis, which is normal to the plane of the board.

The FEMAP software converts the base acceleration to an equivalent force. The force is applied to a base mass which is much greater than the circuit board mass. The four corner nodes are attached to the base mass via rigid link elements, as shown in Figure 14.

The force and mass values are adjusted so that the desired spectral acceleration is applied at the four corner nodes.

The resulting frequency response function magnitudes for the three nodes of interest are given in Figures 15 through 17, respectively.

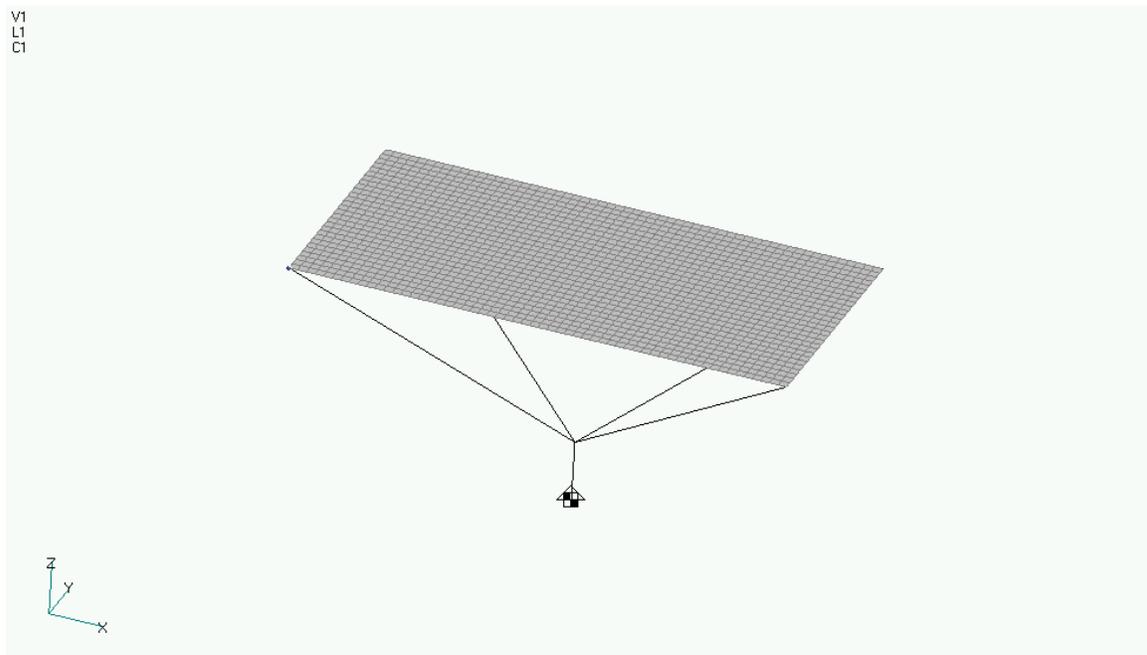


Figure 14. Mode with Base Mass and Rigid Links

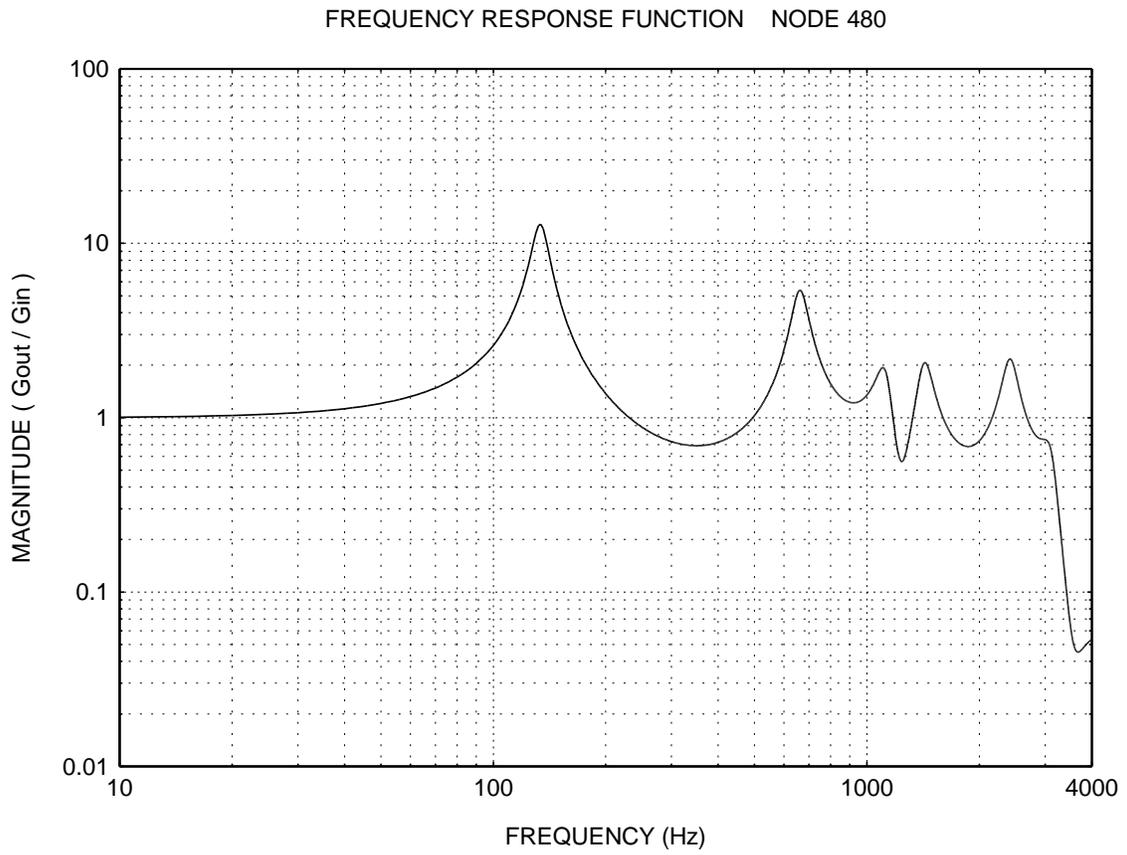


Figure 15. Edge, Midpoint along Length

FREQUENCY RESPONSE FUNCTION NODE 523

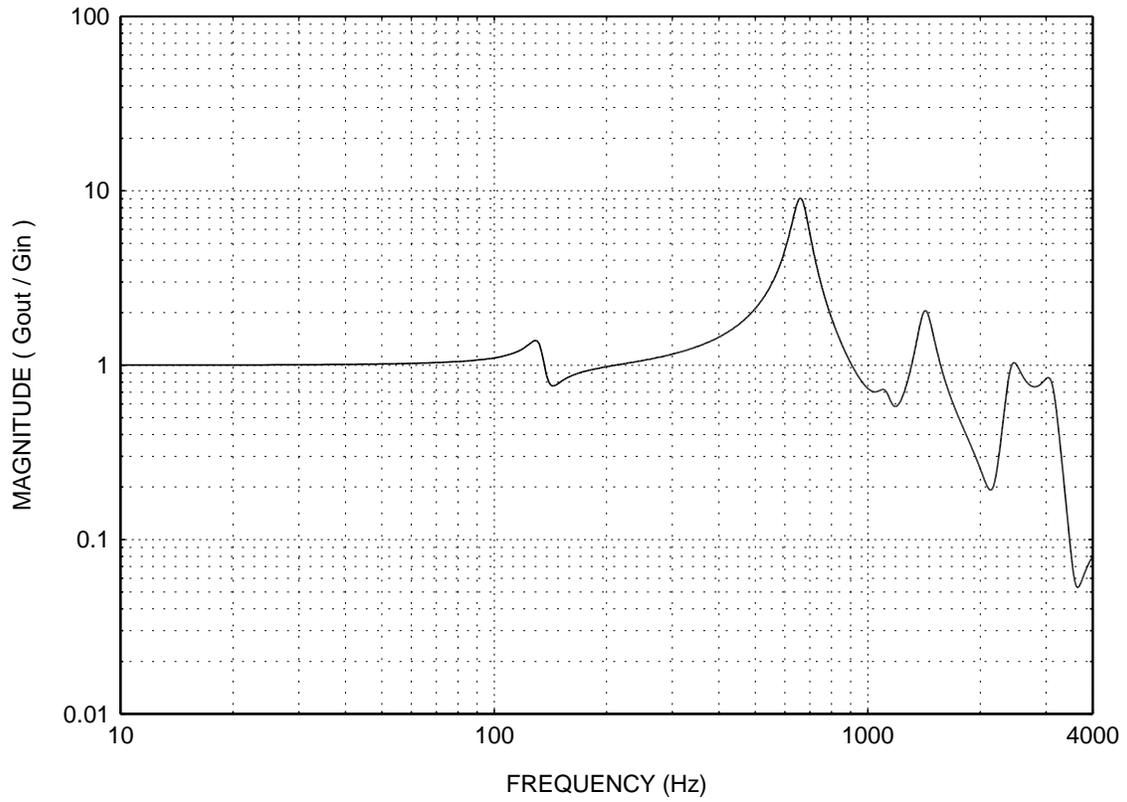


Figure 16. Edge, Midpoint along Width

FREQUENCY RESPONSE FUNCTION NODE 1423

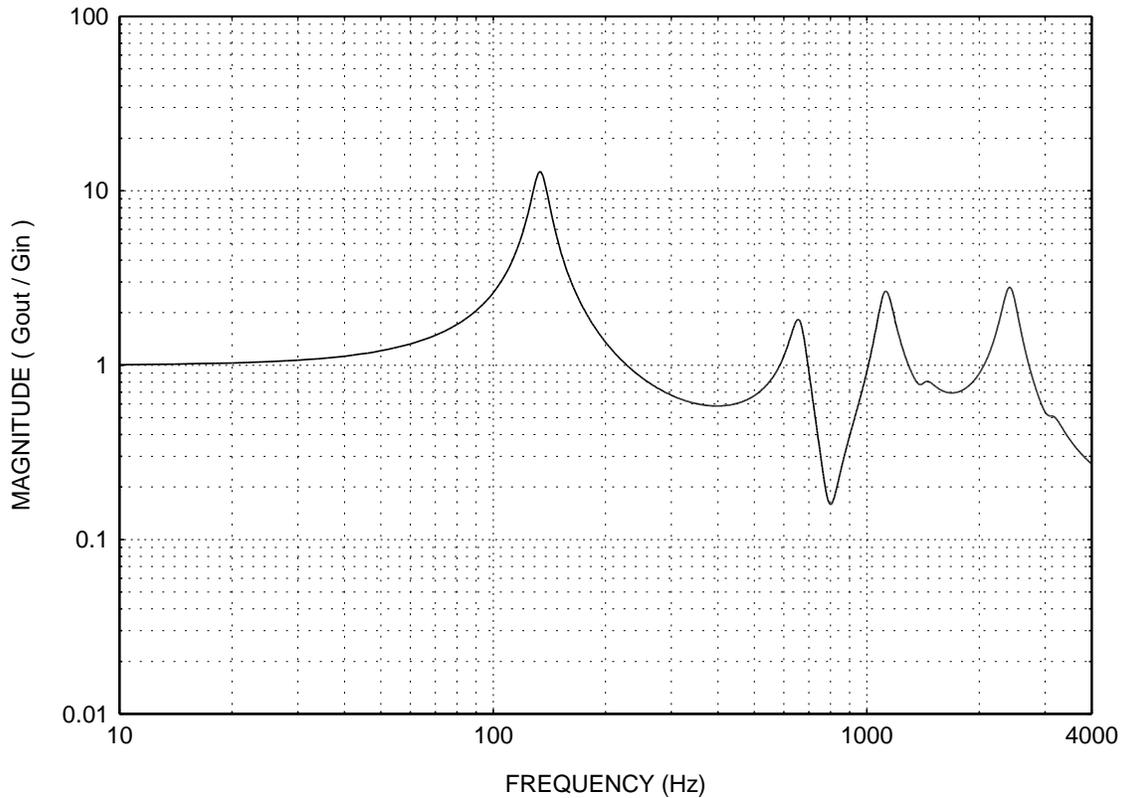


Figure 17. Center of Board

Impulse Response Functions

The impulse response function for each node is calculated by taking an inverse Fourier transform of the complex frequency response function. The results for the three nodes of interest are shown in Figures 18 through 20, respectively.

Note that the impulse response functions presented in this report consist of discrete coordinate pairs. Each response function must be divided by the total number of coordinate points. This is done during the convolution integration in this analysis.

Thus, the plotted impulse response functions are not yet normalized by the number of coordinate points.

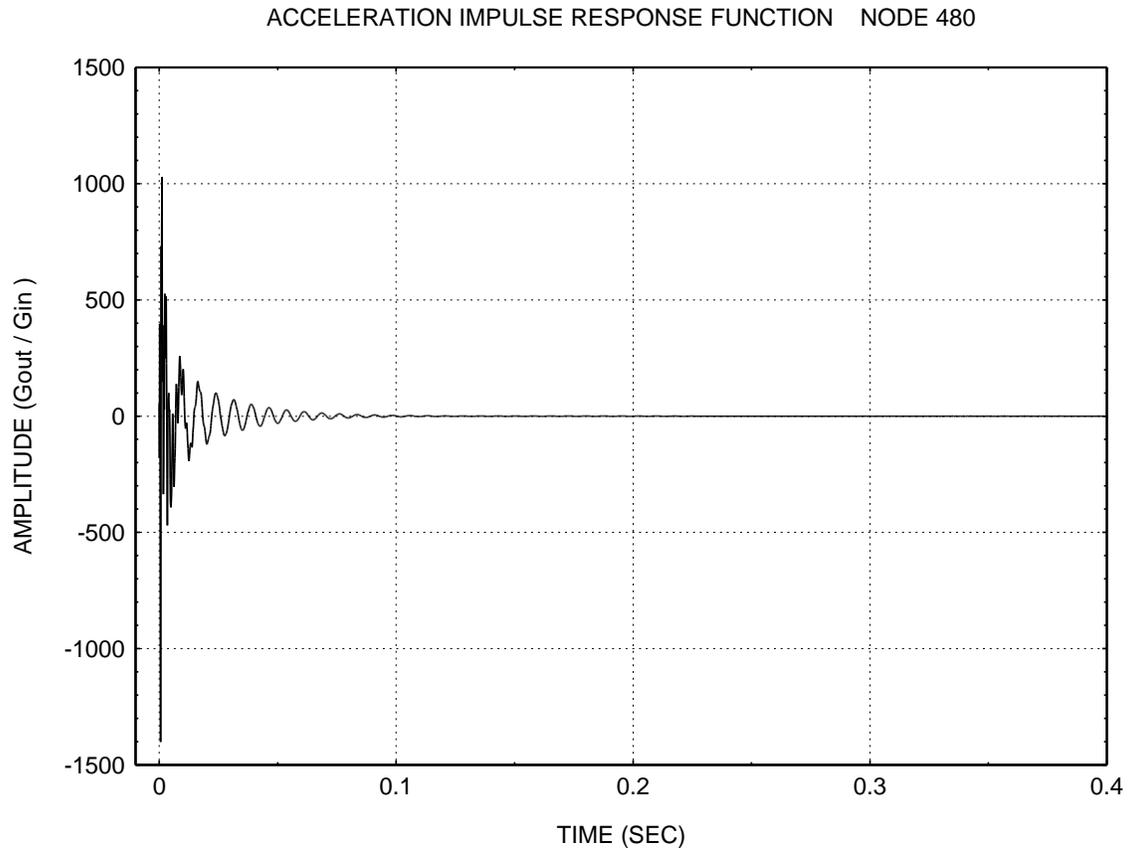


Figure 18. Edge, Midpoint along Length

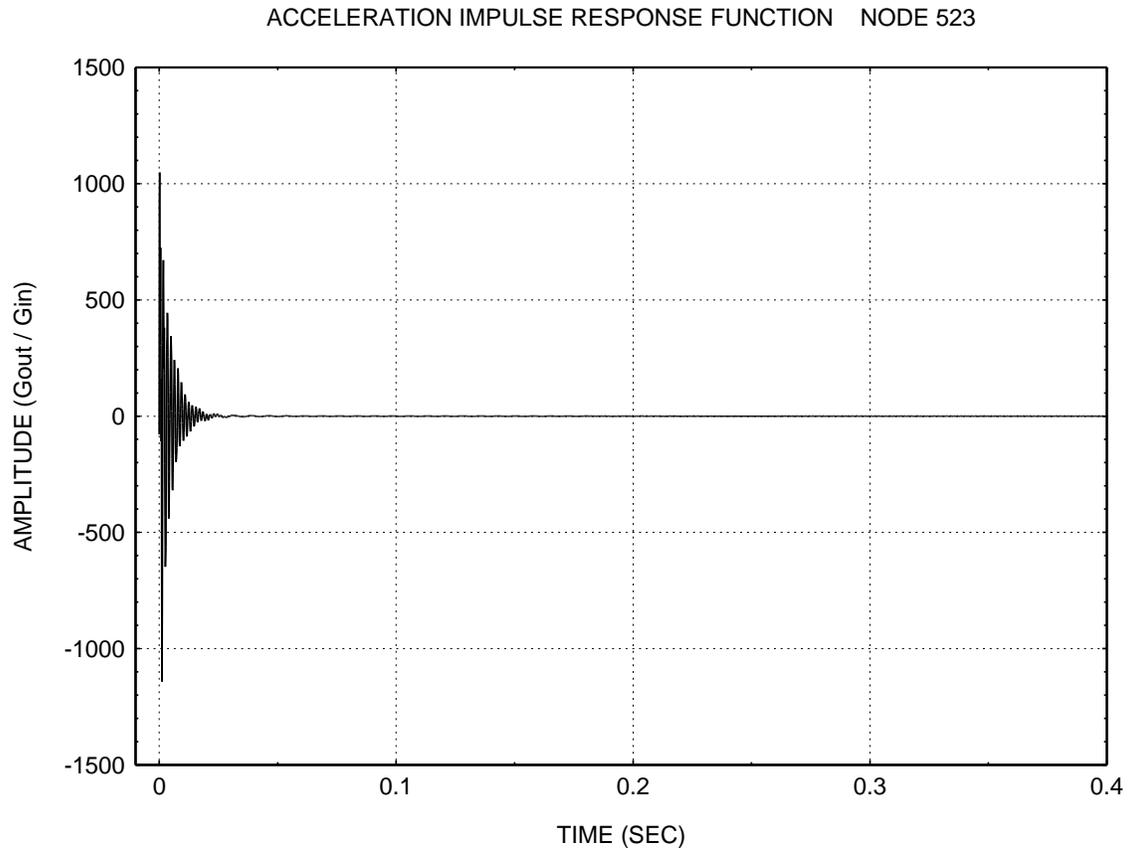


Figure 19. Edge, Midpoint along Width

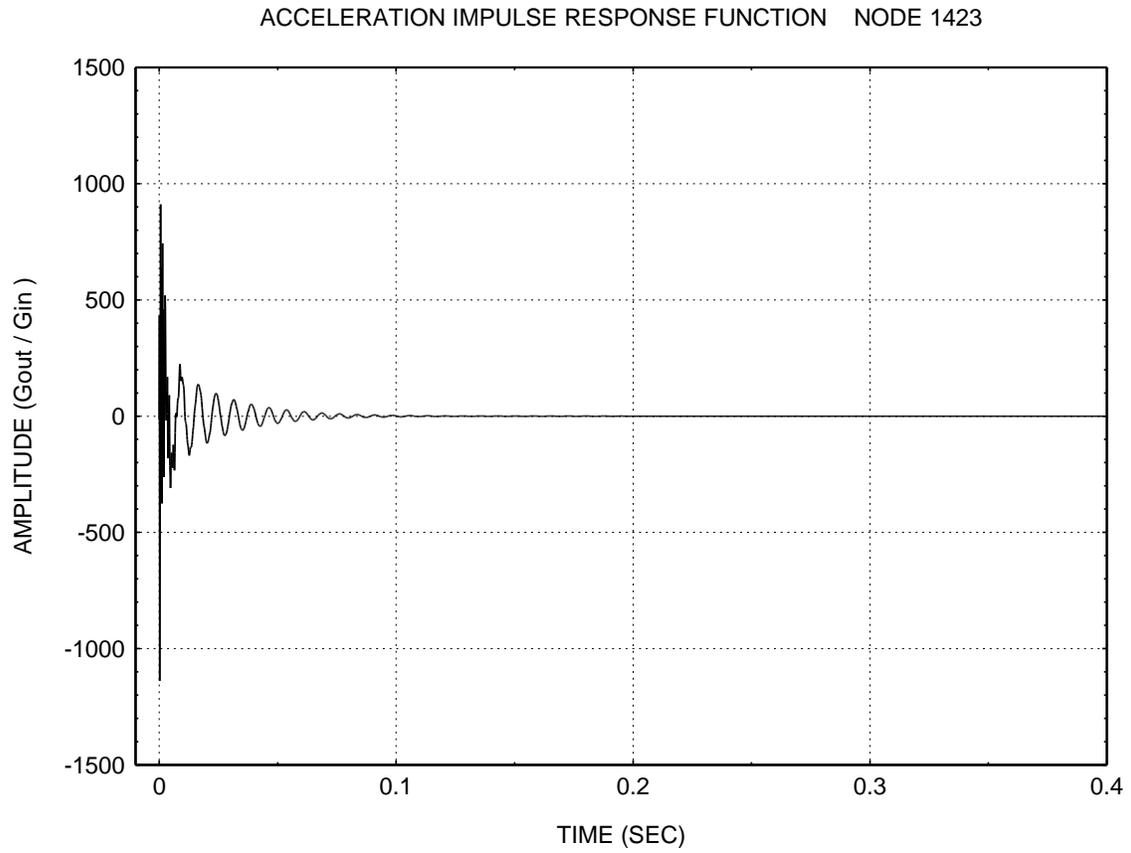


Figure 20. Center of Board

SRS Synthesis

The SRS specification for the sample problem is given in Table 2. This is the same level as MIL-STD-810E, Method 516.4, Crash Hazard for Ground Equipment. The test can be performed using a shaker table.

Table 2. SRS Q=10	
Natural Frequency (Hz)	Peak Acceleration (G)
10	9.4
80	75
2000	75

A time history is synthesized to satisfy the specification, using the wavelet method in Reference 2.

The resulting time history is given in Figure 21. The corresponding positive and negative spectra closely match the specification as shown in Figure 22.

The time history is not unique, however.

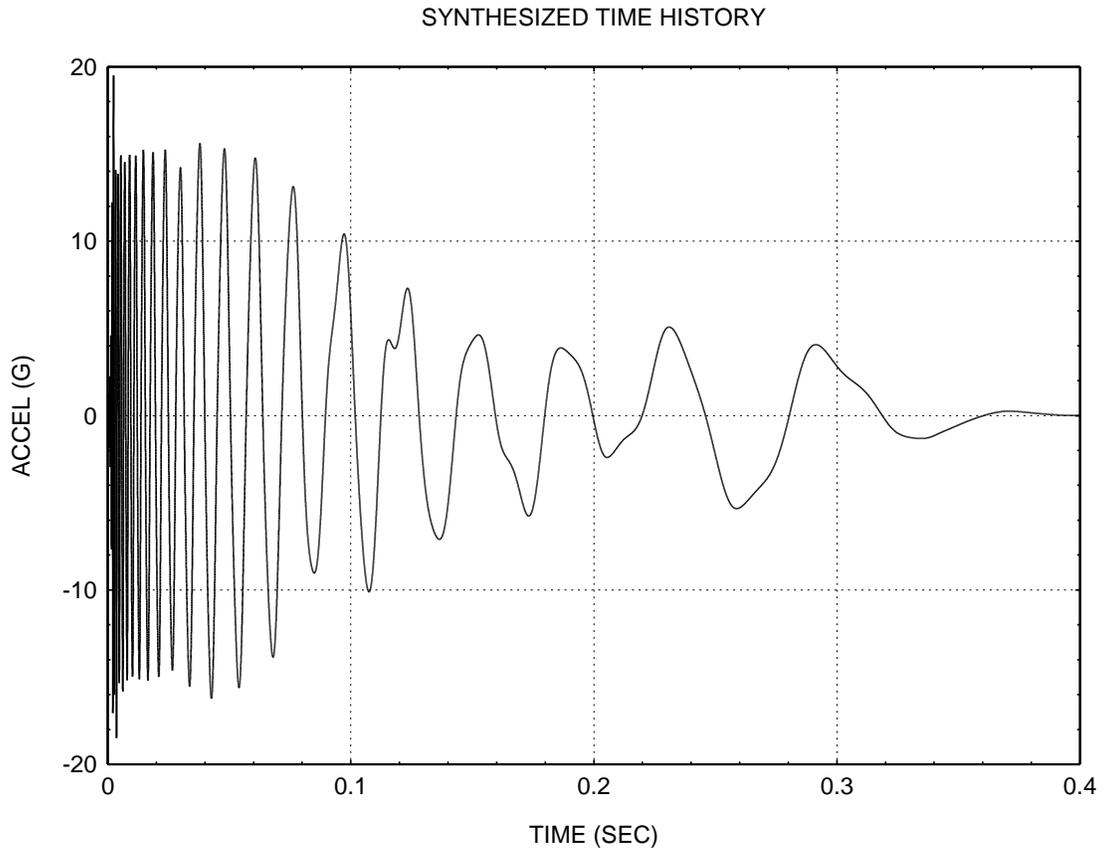


Figure 21. Synthesized Time History to Satisfy SRS Specification

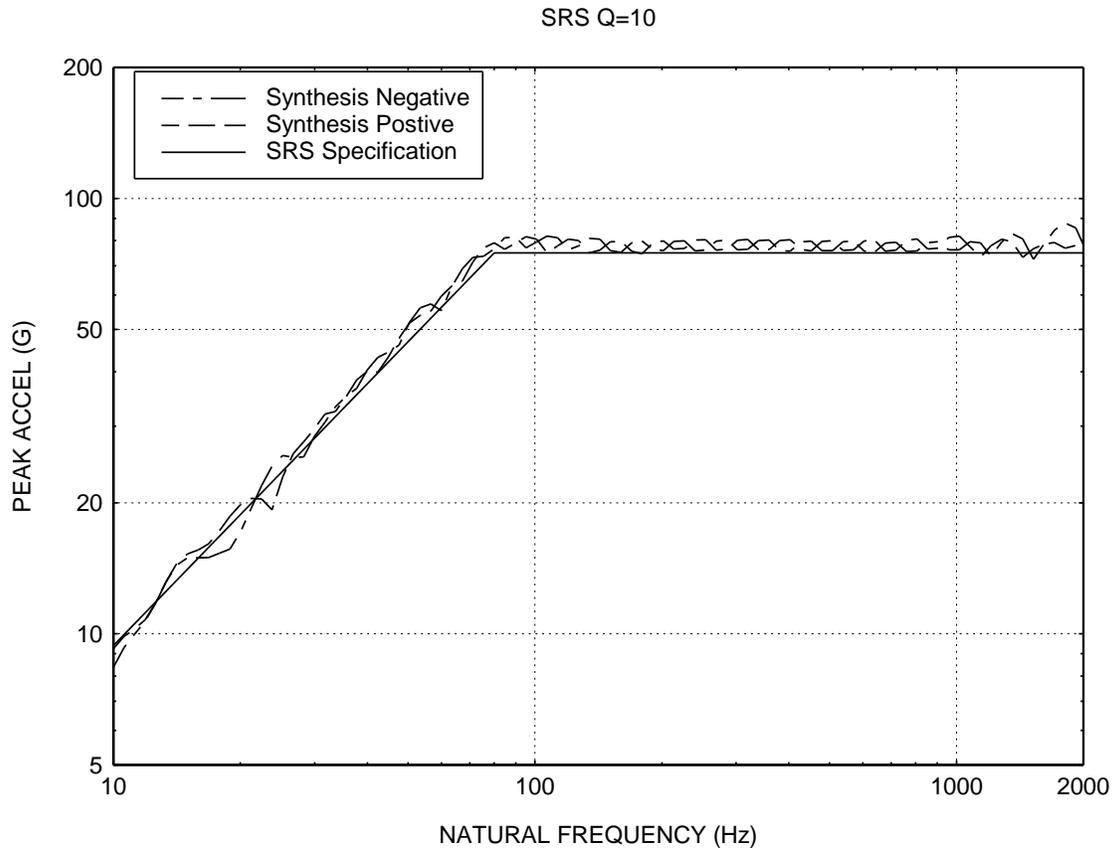


Figure 22. Comparison of Spectra

Acceleration Response Time Histories

The response time history is calculate using the synthesized time history and the appropriate impulse response function via a convolution integral.

The acceleration responses for the three nodes of interest are shown in Figures 23 through 25, respectively. The peak values are summarized in Table 3.

Table 3. Acceleration Results		
Node	Location	Peak Absolute Accel (G)
480	Edge, Midpoint along Length	99.0
523	Edge, Midpoint along Width	69.5
1423	Center of Board	99.7

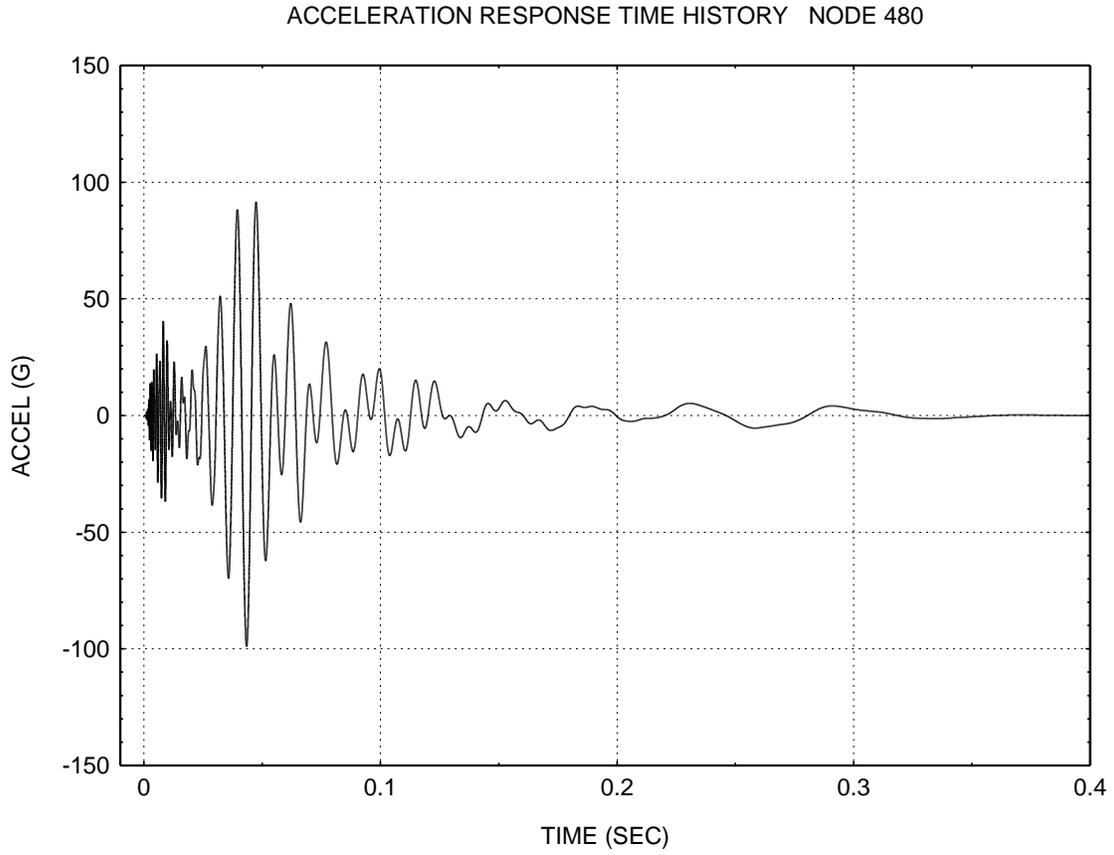


Figure 23. Edge, Midpoint along Length

ACCELERATION RESPONSE TIME HISTORY NODE 523

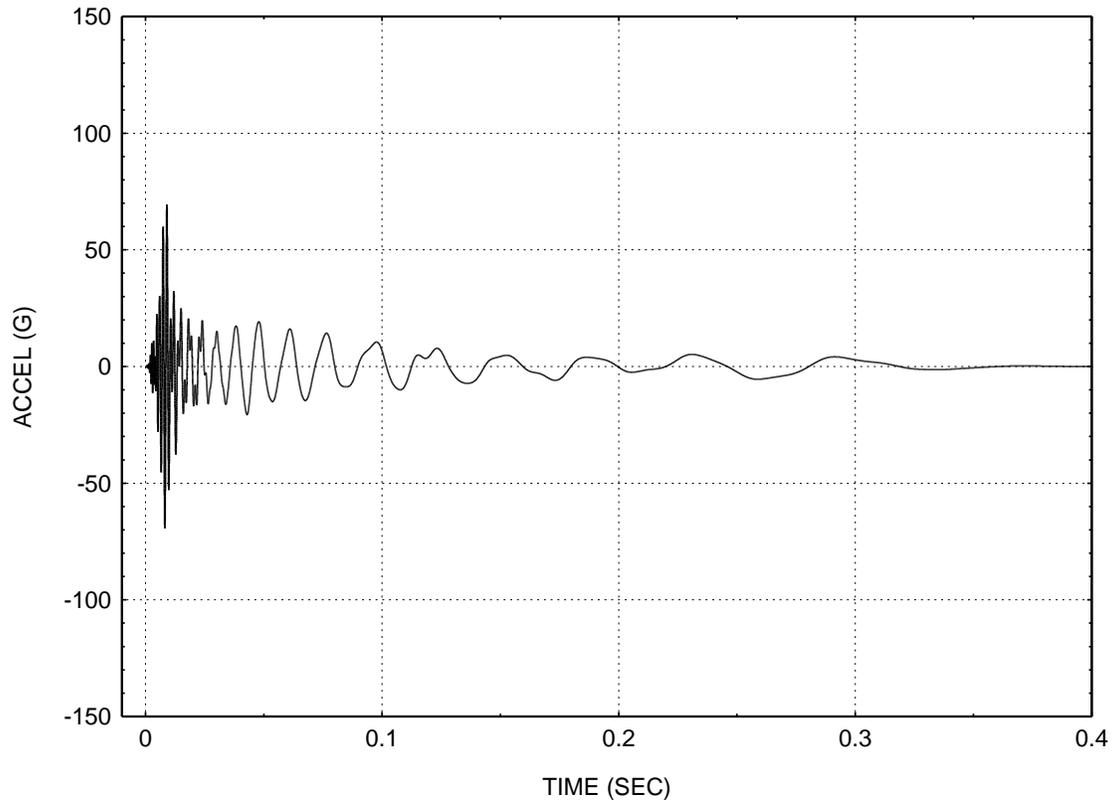


Figure 24. Edge, Midpoint along Width

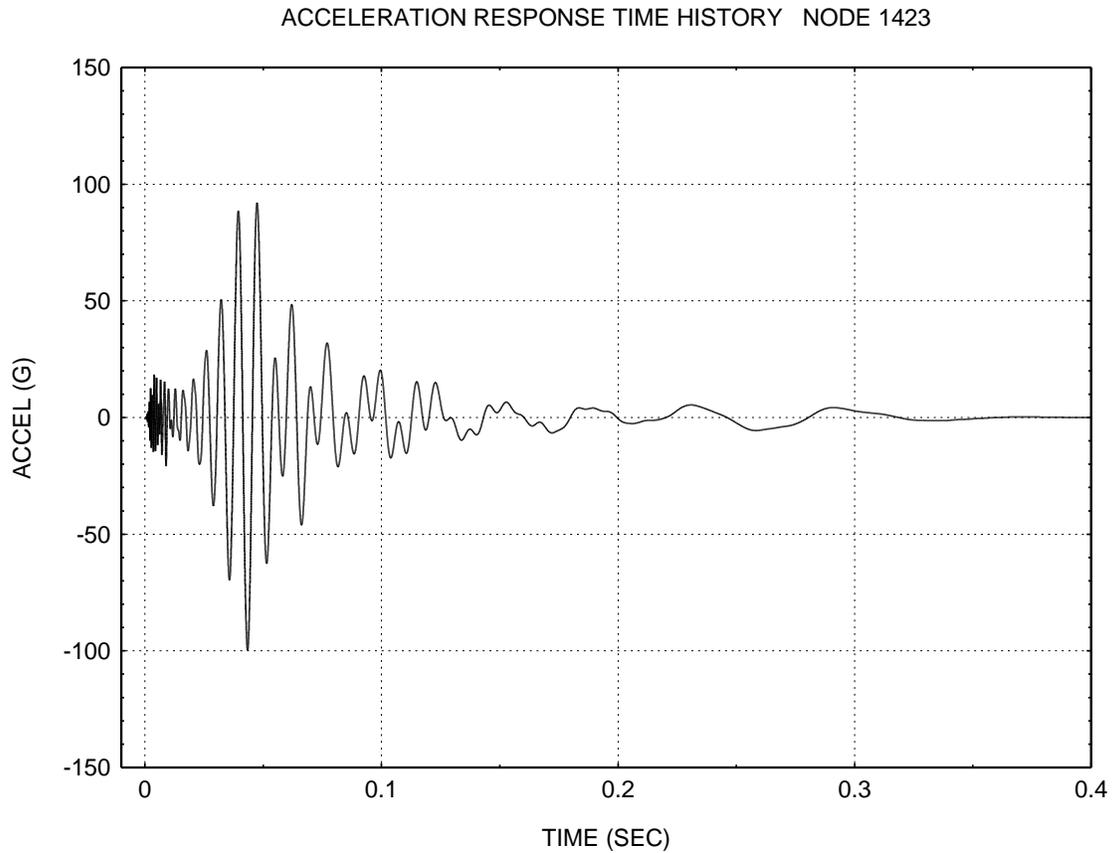


Figure 25. Center of Board

This analysis is repeated for relative displacement in Appendix A.

The results are compared with a separate method in Appendix B.

References

1. T. Irvine, Effective Modal Mass & Modal Participation Factors, Vibrationdata, 2003.
2. T. Irvine, Shock Response Spectrum Testing for Commercial Products, Rev C, Vibrationdata, 1999.
3. T. Irvine, Shock Response of Multi-degree-of-freedom Systems, Rev C, Vibrationdata, 2003.

APPENDIX A

RELATIVE DISPLACEMENT ANALYSIS

Frequency Response Functions

The following frequency response functions relate the relative displacement to the base input acceleration.

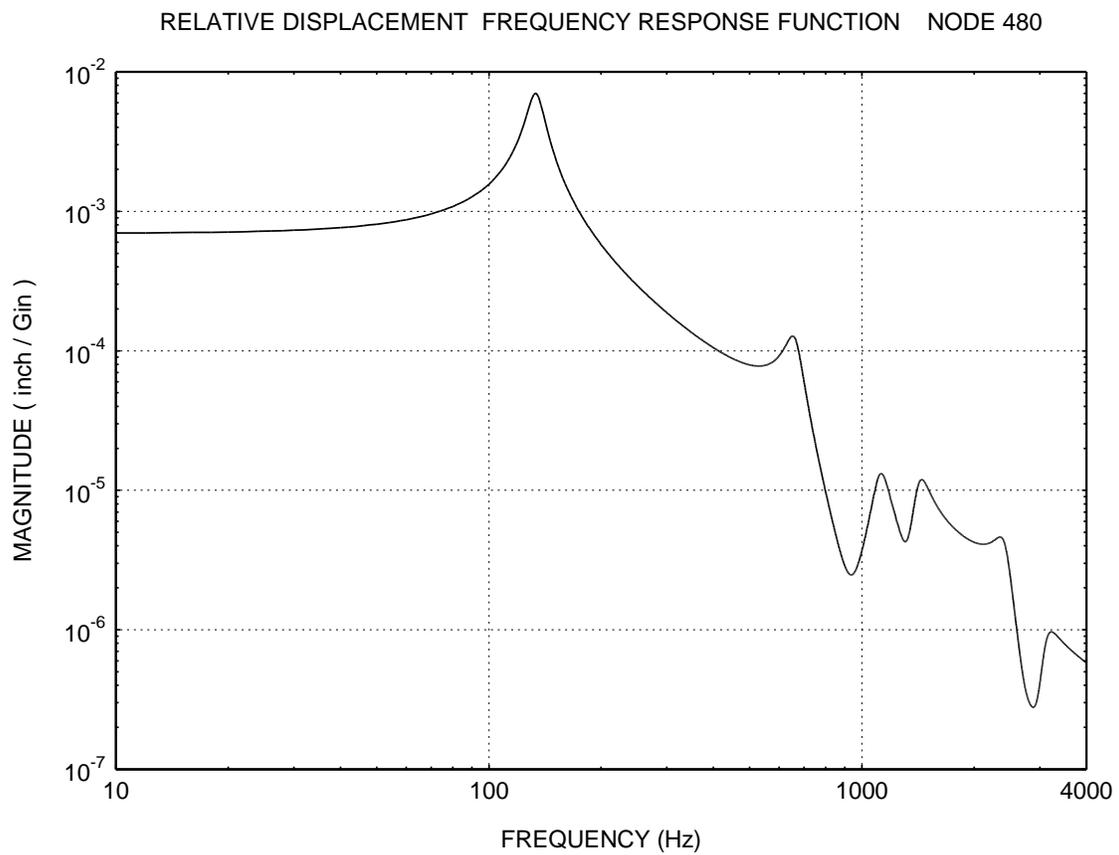


Figure A-1. Edge, Midpoint along Length

RELATIVE DISPLACEMENT FREQUENCY RESPONSE FUNCTION NODE 523

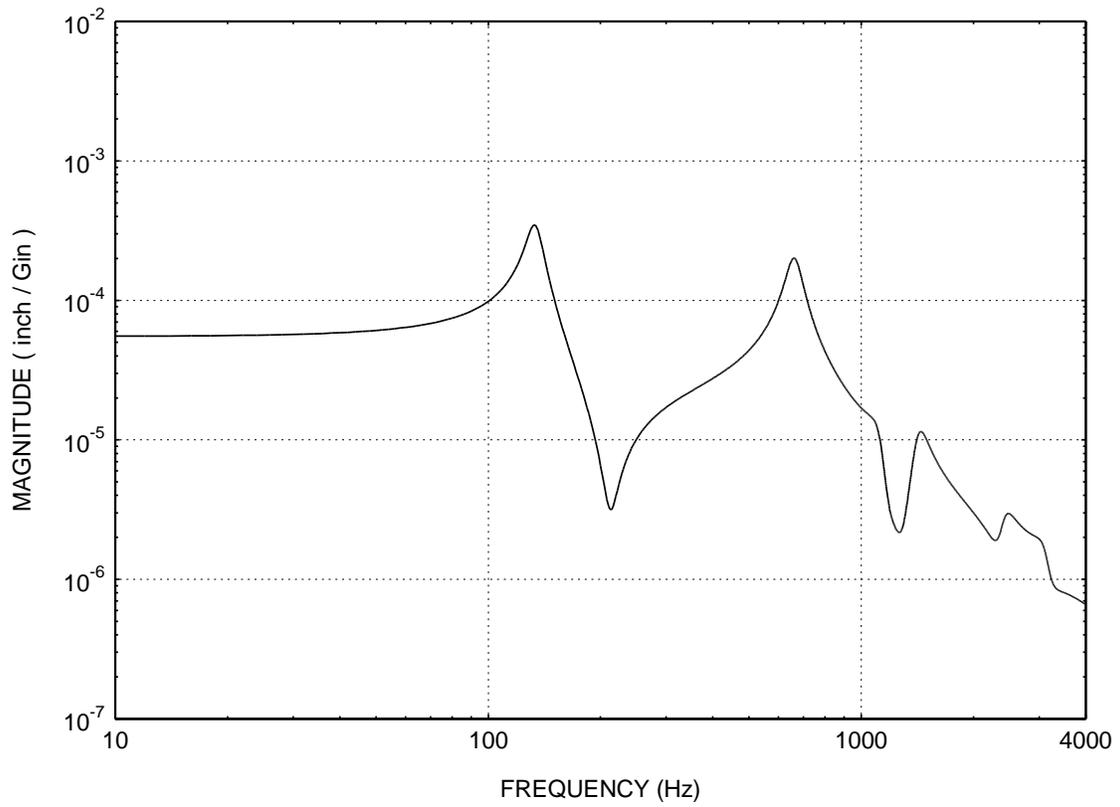


Figure A-2. Edge, Midpoint along Width

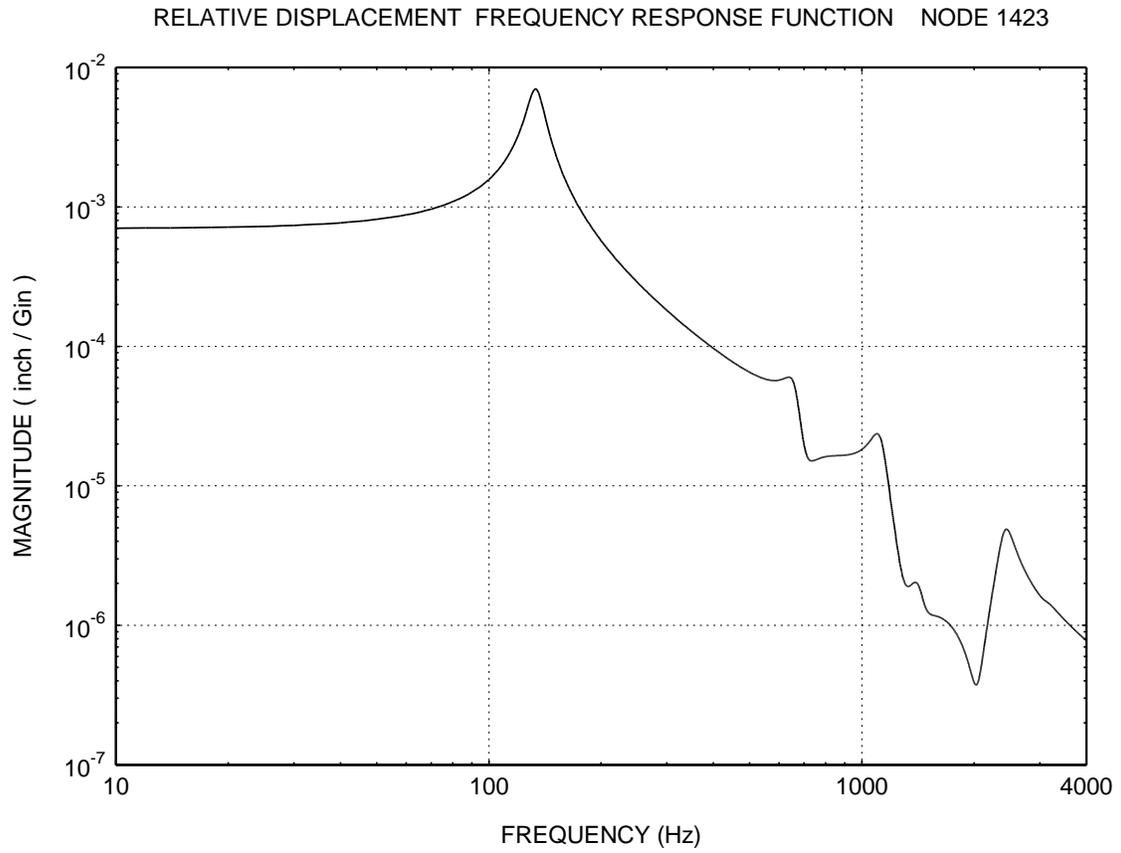


Figure A-3. Center of Board

Impulse Response Functions

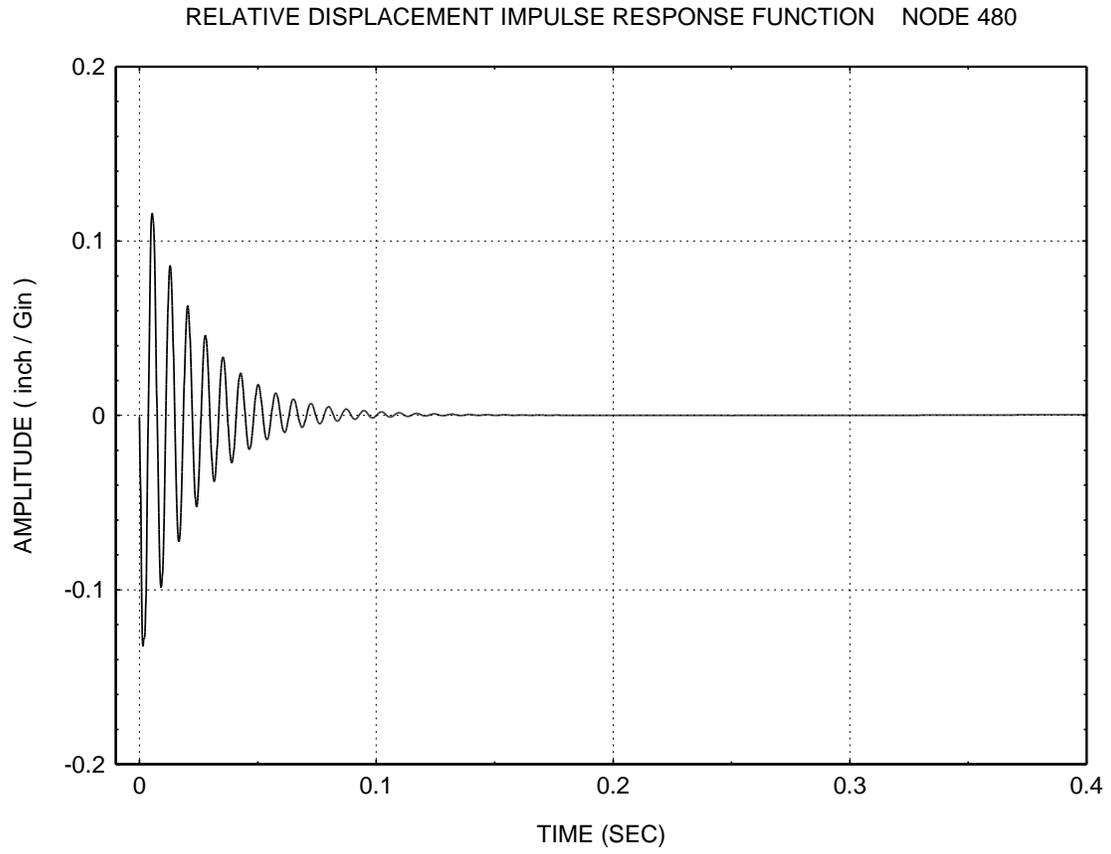


Figure A-4. Edge, Midpoint along Length

RELATIVE DISPLACEMENT IMPULSE RESPONSE FUNCTION NODE 523

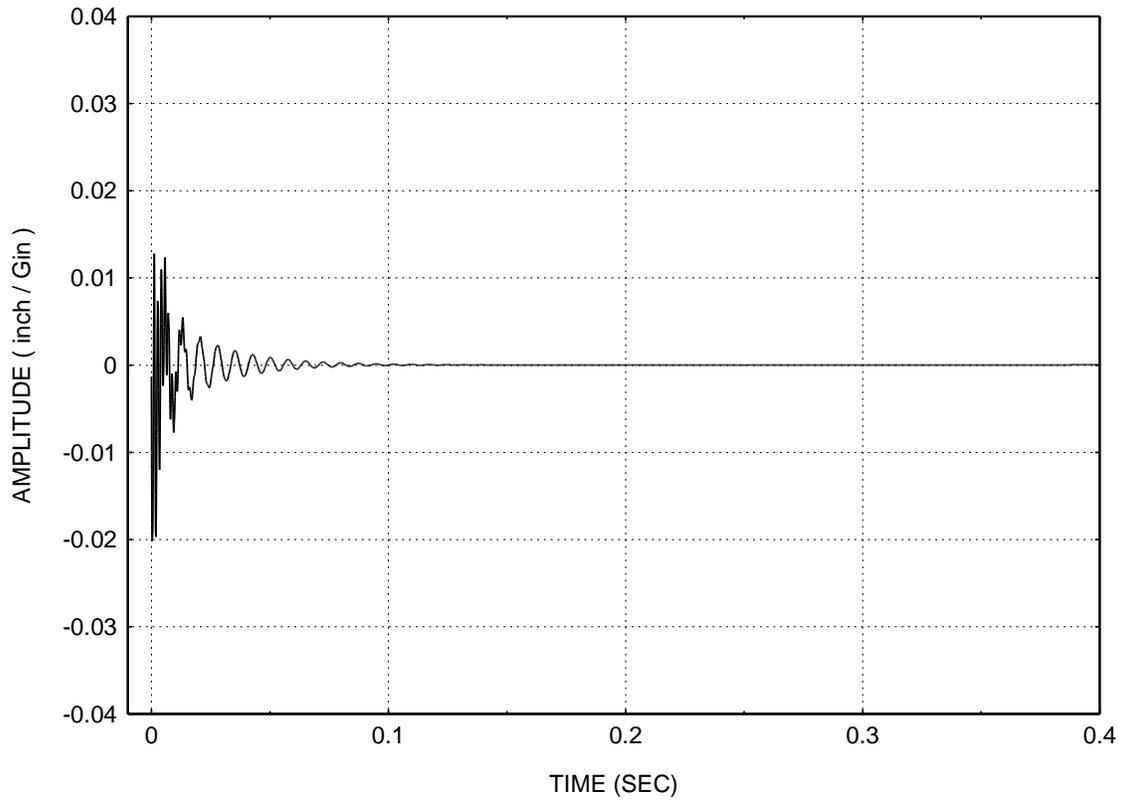


Figure A-5. Edge, Midpoint along Width

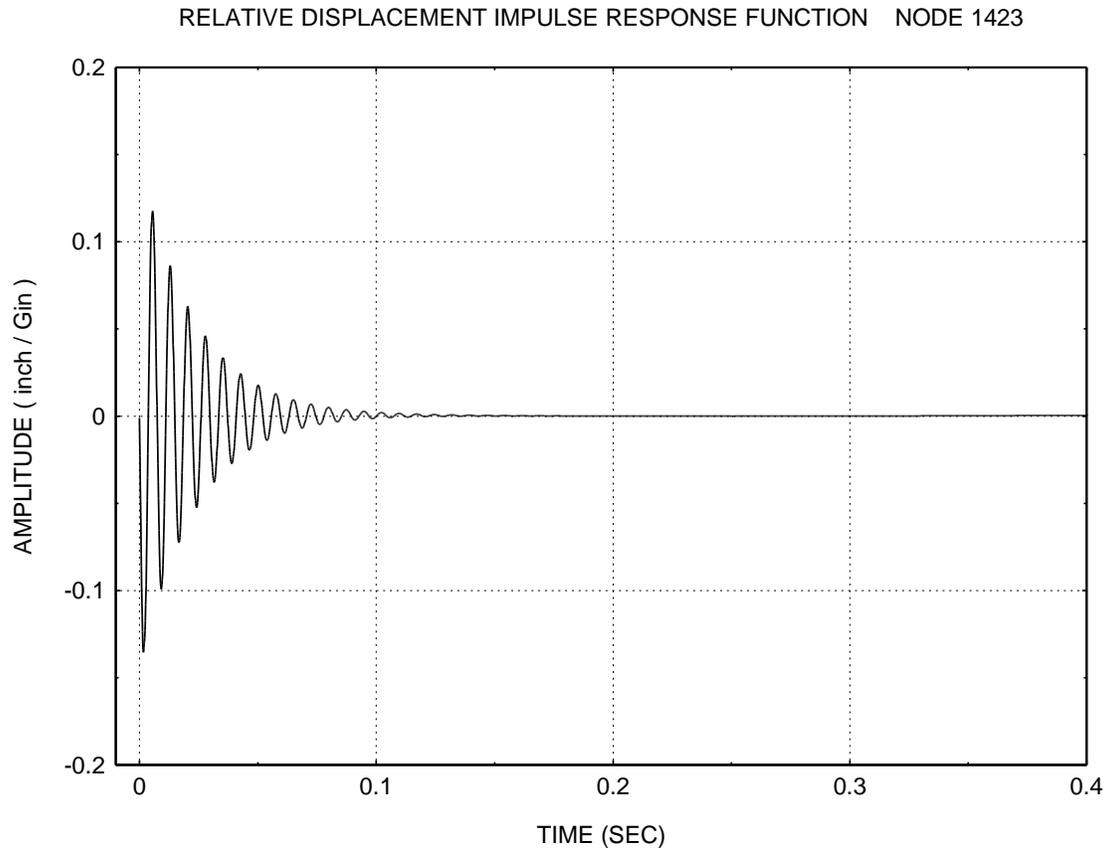


Figure A-6. Center of Board

Response Time Histories

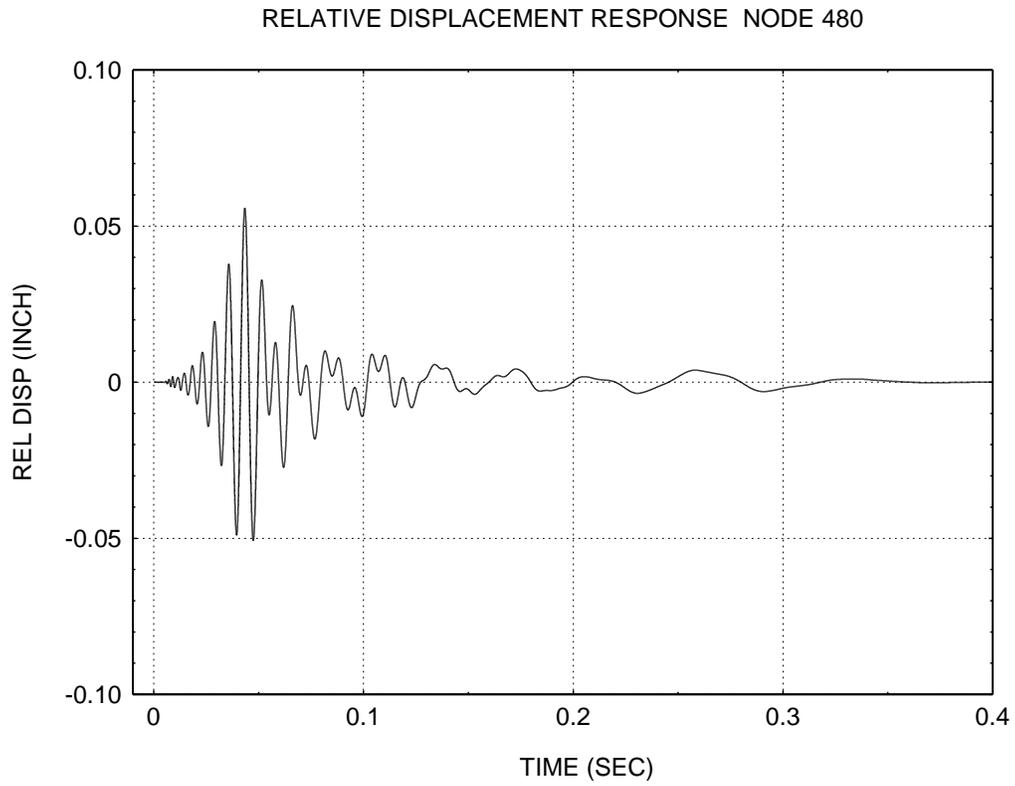


Figure A-7. Edge, Midpoint along Length

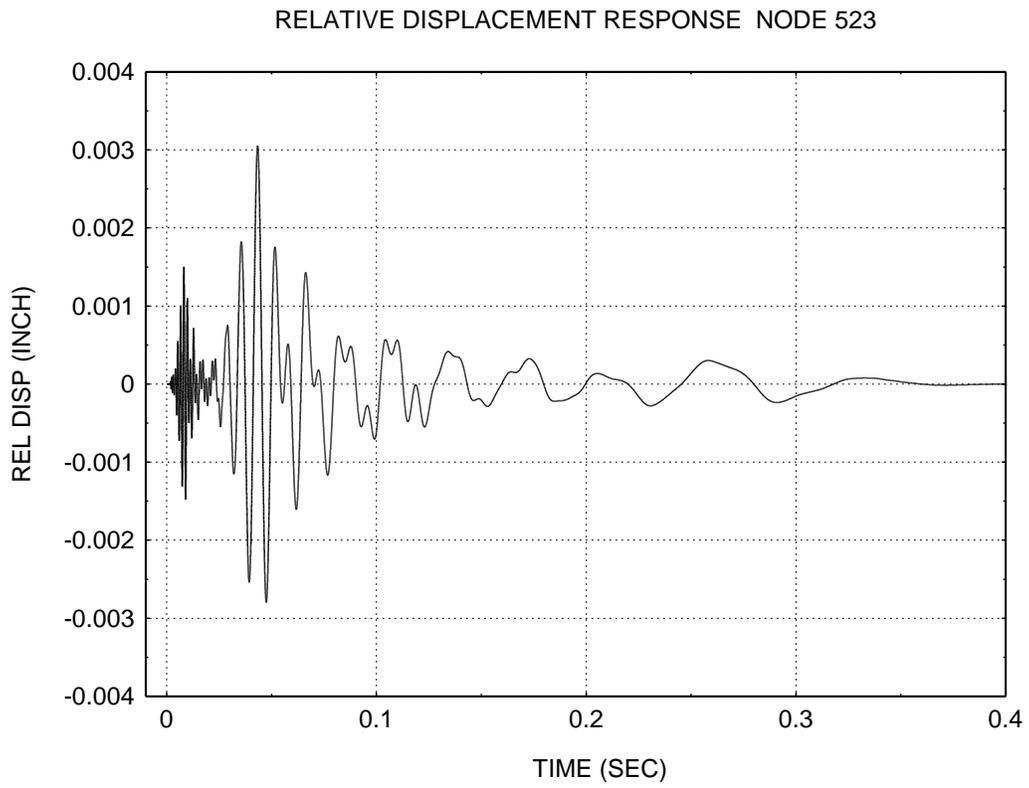


Figure A-8. Edge, Midpoint along Width

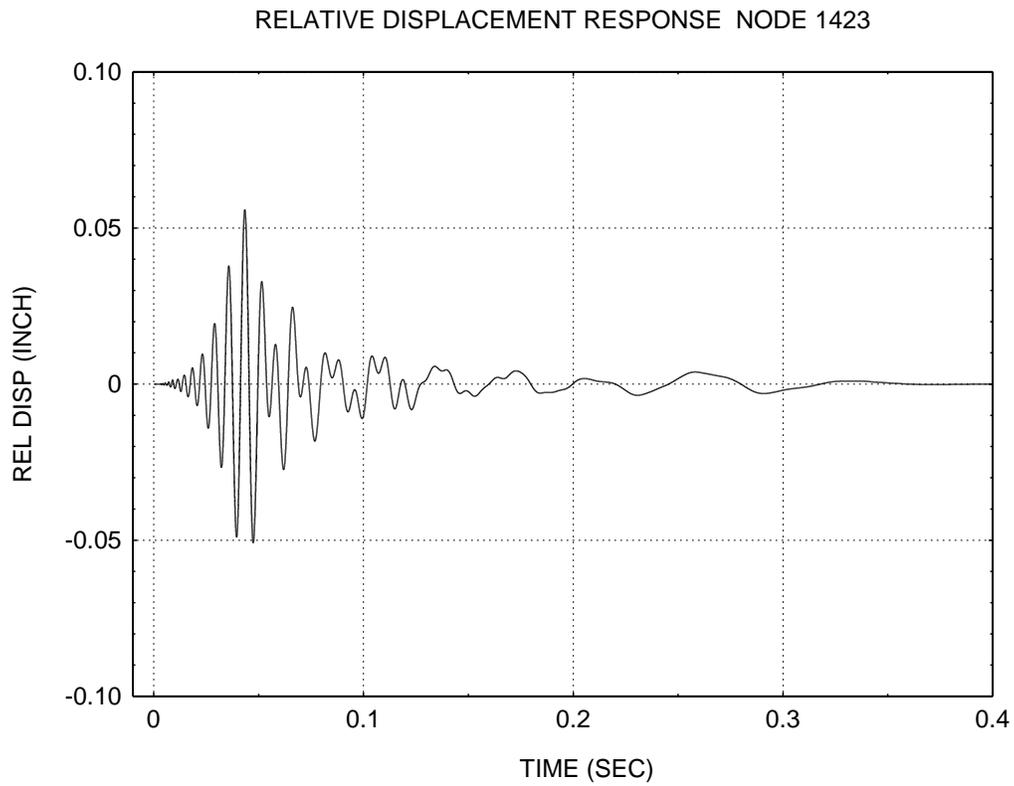


Figure A-9. Center of Board

Table 3. Relative Displacement Results		
Node	Location	Peak Absolute Relative Displacement (inch)
480	Edge, Midpoint along Length	0.0562
523	Edge, Midpoint along Width	0.0031
1423	Center of Board	0.0564

APPENDIX B

Square Root of the Sum of the Squares

The relative displacement results from Appendix A are compared with the results from the Square Root of the Sum of the Squares (SRSS). The SRSS is an approximation method, given in Reference 3.

The SRSS equation for the relative displacement z_i of node i is

$$(z_i)_{\max} \approx \sqrt{\sum_{j=1}^N [\Gamma_j \hat{q}_{ij} D_{j,\max}]^2} \quad (\text{B-1})$$

where

Γ_j is the modal participation factor for mode j

\hat{q}_{ij} is the mass-normalized eigenvector coefficient for node i and mode j

D_j is the relative displacement of mode j regarded as a single-degree-of-freedom system

The eigenvectors and participation factors are taken from the finite element analysis. The participation factors Γ_j are shown in Table 1.

The relative displacement values D_j are approximated by dividing the SRS acceleration by ω_n^2 at the corresponding natural frequency. Refer to Table 2 for the SRS specification.

Now model the circuit board with three natural frequencies, at 134, 660, and 1116 Hz.

These frequencies are chosen based on the frequency response functions and on the modal effective mass values. Refer to Table 1 and to Figures A-1 through A-3.

The SRSS parameters are given in Tables B-1 through B-3.

The SRSS results are given in Table B-4.

Table B-1. Parameters for the f1 = 134 Hz Case				
Node	Location	Γ_j	\hat{q}_{ij}	D_j (inch)
480	Edge, Midpoint along Length	1.546E-02	8.33E+01	0.0408
523	Edge, Midpoint along Width	1.546E-02	4.10E+00	0.0408
1423	Center of Board	1.546E-02	8.34E+01	0.0408

Table B-2. Parameters for the f2 = 660 Hz Case				
Node	Location	Γ_j	\hat{q}_{ij}	D_j (inch)
480	Edge, Midpoint along Length	6.376E-03	-8.48E+01	0.0017
523	Edge, Midpoint along Width	6.376E-03	1.41E+02	0.0017
1423	Center of Board	6.376E-03	-2.71E+01	0.0017

Table B-3. Parameters for the f3 = 1116 Hz Case				
Node	Location	Γ_j	\hat{q}_{ij}	D_j (inch)
480	Edge, Midpoint along Length	2.073E-03	8.07E+01	0.0006
523	Edge, Midpoint along Width	2.073E-03	-4.21E+01	0.0006
1423	Center of Board	2.073E-03	-1.28E+02	0.0006

Table B-4. Relative Displacement Results				
Node	Location	SRSS (inch)	Synthesis Method (inch)	Difference
480	Edge, Midpoint along Length	0.0526	0.0562	6.4%
523	Edge, Midpoint along Width	0.0030	0.0031	3.2%
1423	Center of Board	0.0526	0.0564	6.7%

The synthesis method results are taken from Appendix A. The difference is with respect to the synthesis results. The synthesis results are considered as nearly exact for the synthesized pulse.

Some of the error is due to the fact that the synthesized spectra tended to be slightly higher than the SRS specification as shown in Figure 22.

APPENDIX C

Web Pages

The source and executable codes for the following pre and post-processing programs are taken from the following pages:

<http://www.vibrationdata.com/StructuralFE.htm>
<http://www.vibrationdata.com/signal.htm>
<http://www.vibrationdata.com/SRS.htm>

FRF Generation

The FEA solver was: NEiNastran version 9.0.1.183

The input file was: srs_plate_frf_4k.nas.

The response file was: srs_plate_frf_4k.out

The response file was input to program: ne_postprocess_all.exe

This program generated a series of output files for each node of interest.

Node 1423 was at the center of the plate.

The output file of interest for this node was: 1423_complex_accelH_double.zz

The output file is a complex, double-sided frequency response function. The amplitude dimension is (G out / G in) as a function of frequency (Hz). Double-sided means that the function's upper frequency is equal to the sample rate, which is twice the Nyquist frequency.

The first z in the filename extension identifies the input axis. The second z identifies the response axis.

Base Input Synthesis

The synthesized base input time history is: synthesis.txt (Figure 21)

The time history was generated outside of the FEA software.

As an aside, two examples of synthesis programs are:

1. damped_sine_syn.exe
2. wavelet_synth.exe

Acceleration Response

The 1423_complex_accelH_double.zz and synthesis.txt files were then applied to program: blast.exe

This program calculates a response time history from an input time history and a transfer function where the transfer function is a complex Fourier transform. It uses the convolution method.

The output file was: 1423_accel.out (Figure 25)

Relative Displacement Response

The relative displacement FRF file was: 1423_complex_rdH_double.zz

The amplitude dimension of the FRF is (inch out / G in).

The 1423_complex_rdH_double.zz and synthesis.txt files were then applied to program: blast.exe

The resulting relative displacement time history had a peak value of 0.056 inches as reported in Table B-4.

Alternate Method

The synthesized time history can be applied directly to the FEA model for a "modal transient analysis."

The following program can be used to prepare the base input time history in a format suitable for Nastran-type programs: ne_tabled2.exe

Note that some FEA programs may have FRF capability but not modal transient.