5. FREQUENCY RESPONSE FUNCTION MEASUREMENTS

5.1 Introduction

For current approaches to experimental modal analysis, the frequency response function is the most important measurement to be made. When estimating frequency response functions, a measurement model is needed that will allow the frequency response function to be estimated from measured input and output data in the presence of noise (errors). Some of the errors are:

- Digital Signal Processing Errors (Leakage, Aliasing)
- Noise
    - Equipment problem (Power supply noise)
    - Cabling problems (RFI,EMI)
    - Rattles, cable motion
- Calibration (operator error)
    - Complete system calibration
    - Transducer calibration

Since the frequency response function can be expressed in terms of system properties of mass, stiffness, and damping, it is reasonable to conclude that in most realistic structures, the frequency response functions are considered to be constants just like mass, stiffness, and damping. This concept means that when formulating the frequency response function using H_1, H_2, or H_v algorithms, the estimate of frequency response is intrinsically unique, as long as the system is linear and the noise can be minimized or eliminated. The estimate of frequency response is valid whether the input is stationary, non-stationary, or deterministic. Therefore, several important points to remember before estimating frequency response functions are:

- The system (with the boundary conditions for that test) determines the frequency response functions for the given input/output locations.
- It is important to eliminate or at least minimize all errors (aliasing, leakage, noise, calibration, etc.) when collecting data.

- If all noise terms are identically zero, the assumption concerning the source/location of the noise does not matter ($H_1 = H_2 = H_v = H_s = H$). Therefore, concentrate on eliminating the source of the noise.
- Since modal parameters are computed from estimated frequency response functions, the modal parameters are only as accurate as the estimated frequency response functions.

There are at least four different testing configurations that can be compared. These different testing conditions are largely a function of the number of acquisition channels or excitation sources that are available to the test engineer. In general, the best testing situation is the multiple input/multiple output configuration (MIMO) since the data is collected in the shortest possible time with the fewest changes in the test conditions.

- Single input/single output. (SISO)
    - Only option if 2 channel data acquisition system.
    - Longest testing time. Roving inputs. Roving outputs.
    - Time invariance problems between measurements.
- Single input/multiple output. (SIMO)
    - Multiple channel system (3 or more). (One ADC channel for each response signal to be measured plus one ADC channel for an input signal.)
    - Shorter testing time than SISO. Transducers not necessarily moved.
    - Consistent frequency and damping for data acquired simultaneously.
    - Time invariance problems between measurements from different inputs.
- Multiple input/single output. (MISO)
    - Multiple channel system required (3 or more.). (One ADC channel for each input signal to be measured plus one ADC channel for a response signal.)
    - Long testing time. Roving response transducer.
    - More than one input location per measurement cycle.
    - Detects repeated roots. Maxwell reciprocity checks are possible.

    - Time invariance problems between measurements from different responses.
- Multiple input/multiple output. (MIMO)
    - Multiple channel system (up to 512 channels). Increased set-up time. Large amount of data to be stored and organized.
    - Shortest testing time.
    - Consistent frequency and damping for all data acquired simultaneously.
    - Detects repeated roots. Maxwell reciprocity checks are possible.
    - Best overall testing scheme.

5.2 Frequency Response Function Estimation

Frequency response functions are normally used to describe the input-output (force-response) relationships of any system. Most often, the system is assumed to be linear and time invariant although this is not necessary. In the cases where assumptions of linearity and time invariance are not valid, the measurement of frequency response functions are also dependent upon the independent variables of time and input. In this way, a conditional frequency response function is measured as a function of other independent variables in addition to frequency. Note that the different possible formulations listed in Table 5-1 can all be considered frequency response functions since each of these formulations can be numerically manipulated (synthetic differentiation, integration, etc.) into the equivalent displacement over force relationship. This assumes that initial conditions can be ignored.

Receptance	$\frac{Acceleration}{Force}$
Effective Mass	$\frac{Force}{Acceleration}$
Mobility	$\frac{Velocity}{Force}$
Impedance	$\frac{Force}{Velocity}$
Dynamic Compliance	$\frac{Displacement}{Force}$
Dynamic Stiffness	$\frac{Force}{Displacement}$

TABLE 5-1. Frequency Response Function Formulations

The estimation of the frequency response function depends upon the transformation of data from the time to frequency domain. The Fourier transform is used for this computation. Unfortunately, though, the integral Fourier transform definition requires time histories from negative infinity to positive infinity. Since this is not possible experimentally, the computation is performed digitally using a ***fast Fourier transform (FFT)*** algorithm which is based upon only a limited time history. In this way the theoretical advantages of the Fourier transform can be implemented in a digital computation scheme. The frequency response function(s) satisfy the following single and multiple input relationships:

Single Input Relationship

$$X_p = H_{pq}\, F_q \tag{5.1}$$

Multiple Input Relationship

$$\begin{bmatrix} X_1 \\ X_2 \\ . \\ . \\ X_p \end{bmatrix}_{N_o \times 1} = \begin{bmatrix} H_{11} & . & . & . & . & . & . & . & H_{1q} \\ H_{21} & & & & & & & & . \\ . & & & & & & & & . \\ . & & & & & & & & . \\ H_{p1} & . & . & . & . & . & . & . & H_{pq} \end{bmatrix}_{N_o \times N_i} \begin{bmatrix} F_1 \\ F_2 \\ . \\ . \\ F_q \end{bmatrix}_{N_i \times 1} \tag{5.2}$$

An example of a two input, two output case for Equation (5.2) is shown in Equation (5.3) and Figure 5-1.

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} \tag{5.3}$$

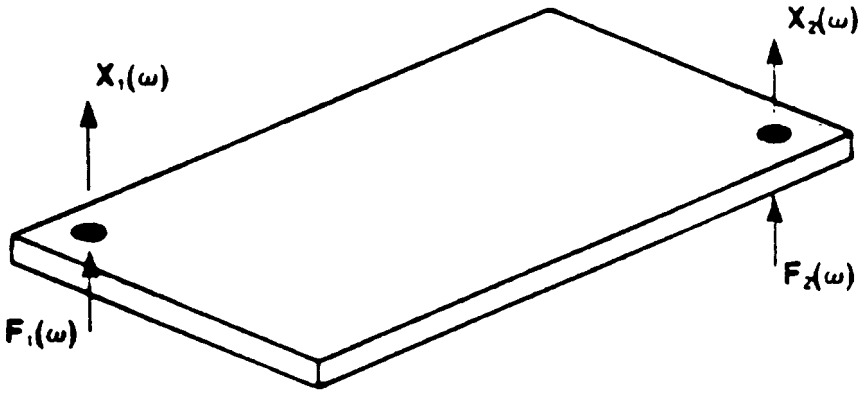

Figure 5-1. Two Input, Two Output FRF Concept

5.2.1 Noise/Error Minimization

The most reasonable, and most common, approach to the estimation of frequency response functions is by use of ***least squares*** (LS) or ***total least squares*** (TLS) techniques [1-3,6-7] . This is a standard technique for estimating parameters in the presence of noise. Least squares methods minimize the square of the magnitude error and, thus, compute the *best* estimate of the

magnitude of the frequency response function but have little effect on the phase of the frequency response function. The primary difference in the algorithms used to estimate frequency response functions is in the assumption of where the noise enters the measurement problem. The different assumptions of the source of the error is noted graphically in Figure 5-2.

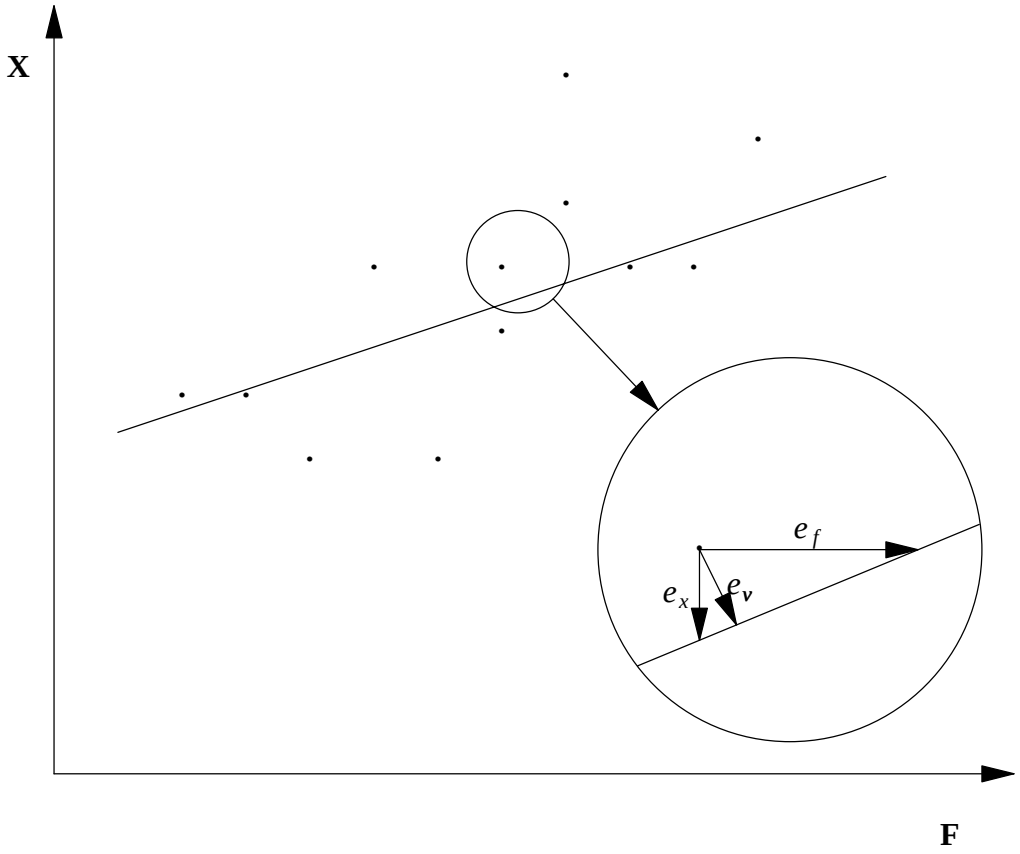

Figure 5-2. Least Squares Concept

Three algorithms, referred to as the H_1, H_2, and H_v algorithms, are commonly available for estimating frequency response functions. Table 5-2 summarizes this characteristic for the three methods that are widely used.

Frequency Response Function Models			
Technique	Solution	Assumed Location of Noise	
	Method	Force Inputs	Response
H_1	LS	no noise	noise
H_2	LS	noise	no noise
H_v	TLS	noise	noise

TABLE 5-2. Summary of Frequency Response Function Estimation Models

Consider the case of N_i inputs and N_o outputs measured during a modal test. Based upon the assumed location of the noise entering the estimation process, Eqs. (5.4) through (5.6) represent the corresponding model for the H_1, H_2, and H_v estimation procedures.

H_1 ***Technique***

$$[H]_{N_o \times N_i} \{F\}_{N_i \times 1} = \{X\}_{N_o \times 1} - \{\eta\}_{N_o \times 1} \tag{5.4}$$

H_2 ***Technique***

$$[H]_{N_o \times N_i} \ \{ \{F\}_{N_i \times 1} - \{v\}_{N_i \times 1}\} = \{X\}_{N_o \times 1} \tag{5.5}$$

H_v ***Technique***

$$[H]_{N_o \times N_i} \ \{ \{F\}_{N_i \times 1} - \{v\}_{N_i \times 1}\} = \{X\}_{N_o \times 1} - \{\eta\}_{N_o \times 1} \tag{5.6}$$

Note that in all methods, the inversion of a matrix is involved. Therefore, the inputs (references) that are used must not be fully correlated so that the inverse will exist. Extensive evaluation tools (using eigenvalue decomposition) have been developed in order to detect and avoid this condition [8].

5.2.2 Single Input FRF Estimation

Figure 5.3 represents the model of the measurement situation for a single input, single output frequency response function measurement.

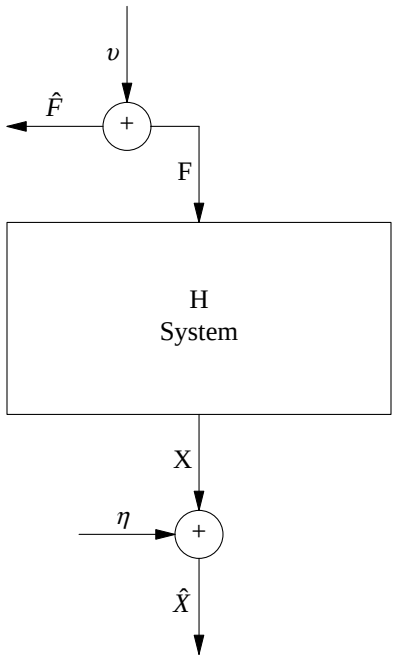

Figure 5-3. System Model: Single Input

With reference to Figure 5.3 for a case involving only one input and one output (input location q and response location p), the equation that is used to represent the input-output relationship is:

$$\hat{X}_p - \eta_p = H_{pq}(\hat{F}_q - \upsilon_q) \tag{5.7}$$

where:

- $F = \hat{F} - \upsilon$ = Actual input
- $X = \hat{X} - \eta$ = Actual output

- $\hat{X}$ = Spectrum of the $p-th$ output, measured
- $\hat{F}$ = Spectrum of the $q-th$ input, measured
- H = Frequency response function
- υ = Spectrum of the noise part of the input
- η = Spectrum of the noise part of the output
- X = Spectrum of the $p-th$ output, theoretical
- F = Spectrum of the $q-th$ input, theoretical

If $\upsilon = \eta = 0$, the theoretical (expected) frequency response function of the system is estimated. If $\eta \neq 0$ and/or $\upsilon \neq 0$, a least squares method is used to estimate a *best* frequency response function, in the presence of noise.

In order to develop an estimation of the frequency response function, a number of averages N_{avg} is used to minimize the random errors (variance). This can be easily accomplished through use of intermediate measurement of the auto and cross power spectrums. The estimate of the auto and cross power spectrums for the model in Figure 5.3 can be defined as follows. Note that each function is a function of frequency.

Cross Power Spectra

$$GXF_{pq} = \sum_{1}^{N_{avg}} X_p \, F_q^* \tag{5.8}$$

$$GFX_{qp} = \sum_{1}^{N_{avg}} F_q \, X_p^* \tag{5.9}$$

Auto Power Spectra

$$GFF_{qq} = \sum_{1}^{N_{avg}} F_q \, F_q^* \tag{5.10}$$

$$GXX_{pp} = \sum_{1}^{N_{avg}} X_p \, X_p^* \tag{5.11}$$

where:

- F^* = Complex conjugate of $F(\omega)$
- X^* = Complex conjugate of $X(\omega)$

H_1 Algorithm: Minimize Noise on Output (η)

The most common formulation of the frequency response function, often referred to as the H_1 algorithm, tends to minimize the noise on the output. This formulation is shown in Eq. (5.12).

$$H_{pq} = \frac{GXF_{pq}}{GFF_{qq}} \tag{5.12}$$

H_2 Algorithm: Minimize Noise on Input (υ)

Another formulation of the frequency response function, often referred to as the H_2 algorithm, tends to minimize the noise on the input. This formulation is shown in Eq. (5.13).

$$H_{pq} = \frac{GXX_{pp}}{GFX_{qp}} \tag{5.13}$$

In the H_2 formulation, an auto power spectrum is divided by a cross power spectrum. This can be a problem since the cross power spectrum can theoretically be zero at one or more frequencies. In both formulations, the phase information is preserved in the cross-power spectrum term.

H_v Algorithm: Minimize Noise on Input and Output (η and υ)

The solution for H_{pq} using the H_v algorithm is found by the eigenvalue decomposition of a matrix of power spectrums. For the single input case, the following matrix involving the auto and cross power spectrums can be defined:

$$[GFFX_p] = \begin{bmatrix} GFF_{qq} & GFX_{qp} \\ GXF_{pq} & GXX_{pp} \end{bmatrix}_{2 \times 2} \tag{5.14}$$

The solution for H_{pq} is found by the eigenvalue decomposition of the $[GFFX]$ matrix as follows:

$$[GFFX_p] = [V] \lceil \Lambda \rfloor [V]^H \tag{5.15}$$

where:

- $\lceil \Lambda \rfloor$ = *diagonal matrix of eigenvalues*

Solution for the H_{pq} matrix is found from the eigenvector associated with the smallest (minimum) eigenvalue (λ_1). The size of the eigenvalue problem is second order resulting in finding the roots of a quadratic equation. This eigenvalue solution must be repeated for each frequency and that the complete solution process must be repeated for each response point X_p.

Alternately, the solution for H_{pq} is found by the eigenvalue decomposition of the following matrix of auto and cross power spectrums:

$$[GXFF_p] = \begin{bmatrix} GXX_{pp} & GXF_{pq}{}^H \\ GFX_{qp} & GFF_{qq} \end{bmatrix}_{2 \times 2} \tag{5.16}$$

$$[GXFF_p] = [V] \lceil \Lambda \rfloor [V]^H \tag{5.17}$$

where:

- $\lceil \Lambda \rfloor$ = *diagonal matrix of eigenvalues*

The solution for H_{pq} is again found from the eigenvector associated with the smallest (minimum)

eigenvalue (λ_1).

The frequency response function is found from the normalized eigenvector associated with the smallest eigenvalue. If [$GFFX_p$] is used, the eigenvector associated with the smallest eigenvalue must be normalized as follows:

$$\{V\}_{\lambda_{\min}} = \begin{Bmatrix} H_{pq} \\ -1 \end{Bmatrix} \tag{5.18}$$

If [$GXFF_p$] is used, the eigenvector associated with the smallest eigenvalue must be normalized as follows:

$$\{V\}_{\lambda_{\min}} = \begin{Bmatrix} -1 \\ H_{pq} \end{Bmatrix} \tag{5.19}$$

One important consideration of the three formulations for frequency response function estimation is the behavior of each formulation in the presence of a bias error such as leakage. In all cases, the estimate differs from the expected value particularly in the region of a resonance (magnitude maxima) or anti-resonance (magnitude minima). For example, H_1 tends to underestimate the value at resonance while H_2 tends to overestimate the value at resonance. The H_v algorithm gives an answer that is always bounded by the H_1 and H_2 values. The different approaches are based upon minimizing the magnitude of the error but have no effect on the phase characteristics.

In addition to the attractiveness of H_1, H_2 and H_v in terms of the minimization of the error, the availability of auto and cross power spectra allows the determination of other important functions. The quantity γ_{pq}^2 is called the scalar or ***ordinary coherence function*** and is a frequency dependent, real value between zero and one. The ordinary coherence function indicates the degree of causality in a frequency response function. If the coherence is equal to one at any specific frequency, the system is said to have perfect causality at that frequency. In other words, the measured response power is caused totally by the measured input power (or by sources which are coherent with the measured input power). A coherence value less than unity at any frequency indicates that the measured response power is greater than that due to the measured input. This is due to some extraneous noise also contributing to the output power. It

should be emphasized, however, that low coherence does not necessarily imply poor estimates of the frequency response function, but simply means that more averaging is needed for a reliable result. The ordinary coherence function is computed as follows:

$$COH_{pq} = \gamma^2_{pq} = \frac{|\, GXF_{pq}\,|^2}{GFF_{qq}\, GXX_{pp}} = \frac{GXF_{pq}\, GFX_{qp}}{GFF_{qq}\, GXX_{pp}} \tag{5.20}$$

When the coherence is zero, the output is caused totally by sources other than the measured input. In general, then, the coherence can be a measure of the degree of noise contamination in a measurement. Thus, with more averaging, the estimate of coherence may contain less variance, therefore giving a better estimate of the noise energy in a measured signal. This is not the case, though, if the low coherence is due to bias errors such as nonlinearities, multiple inputs or leakage. A typical ordinary coherence function is shown in Fig. 5.4 together with the corresponding frequency response function magnitude. In Fig. 5.4, the frequencies where the coherence is lowest is often the same frequencies where the frequency response function is at a maxima in magnitude or at a minima in magnitude. This is often an indication of leakage since the frequency response function is most sensitive to the leakage error at the lightly damped peaks corresponding to the maxima. At the minima, where there is little response from the system, the leakage error, even though it is small, may still be significant.

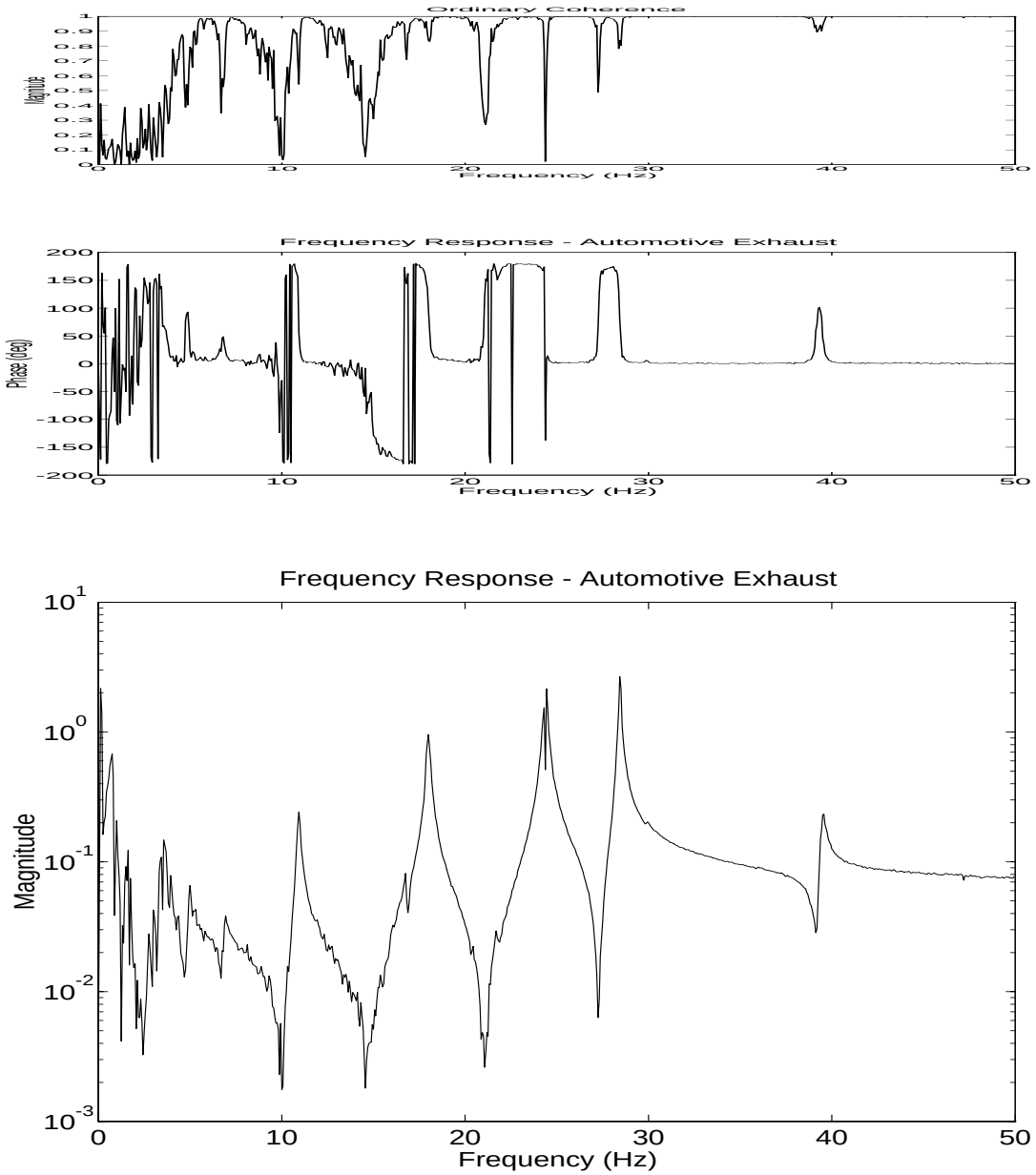

Figure 5-4. FRF and Corresponding Ordinary Coherence Function

In all of these cases, the estimated coherence function will approach, in the limit, the expected value of coherence at each frequency, dependent upon the type of noise present in the structure and measurement system. Note that, with more averaging, the estimated value of coherence will

not increase; the estimated value of coherence always approaches the expected value from the upper side. This is described in Figure 5-4. Note that a high value of coherence (0.9) after 16 averages has approximately the same possible variance of the frequency response function as a low value of coherence (0.1) after 256 averages.

90% confidence limits on the measurement of the amplitude $|H|$ and phase ϕ of transfer functions, as a function of the measured value of coherence and the number of averages.

Measured value of coherence function	Number of Averages				
	16	32	64	128	256
0.2	+ 5.2 - 14.6 (± 54)	+ 3.8 - 7.1 (± 34)	+ 2.8 - 4.2 (± 23)	+ 2.1 - 2.7 (± 16)	+ 1.5 - 1.8 (± 11)
0.3	+ 4.2 - 8.4 (± 38)	+ 3.1 - 4.8 (± 25)	+ 2.2 - 3.0 (± 17)	+ 1.6 - 2.0 (± 12)	+ 1.2 - 1.4 (± 8)
0.4	+ 3.5 - 6.0 (± 30)	+ 2.6 - 3.6 (± 20)	+ 1.8 - 2.3 (± 14)	+ 1.3 - 1.6 (± 10)	+ 1.0 - 1.1 (± 7)
0.5	+ 3.0 - 4.5 (± 24)	+ 2.1 - 2.8 (± 16)	+ 1.5 - 1.9 (± 11)	+ 1.1 - 1.3 (± 8)	+ 0.8 - 0.9 (± 5)
0.6	+ 2.5 - 3.5 (± 19)	+ 1.8 - 2.2 (± 13)	+ 1.3 - 1.5 (± 9)	+ 0.9 - 1.0 (± 6)	+ 0.7 - 0.7 (± 4)
0.7	+ 2.1 - 2.7 (± 15)	+ 1.5 - 1.7 (± 10)	+ 1.0 - 1.2 (± 7)	+ 0.7 - 0.8 (± 5)	+ 0.5 - 0.6 (± 4)
0.8	+ 1.6 - 2.0 (± 12)	+ 1.1 - 1.3 (± 8)	+ 0.8 - 0.9 (± 6)	+ 0.6 - 0.6 (± 4)	+ 0.4 -0.4 (± 3)
0.9	+ 1.1 - 1.3 (± 8)	+ 0.8 - 0.8 (± 5)	+ 0.5 - 0.6 (± 4)	+ 0.4 - 0.4 (± 3)	+ 0.3 - 0.3 (± 2)

For each entry, the first two digits are the upper and lower bounds on $|H|$, in dB.
Digits in parentheses are the bounds on ϕ, in degrees.

Figure 5-5. Ordinary Coherence Relationship - Averaging

Two special cases of low coherence are worth particular mention. The first situation occurs when a *leakage error* occurs in one or both of the input and output measurements. This causes the coherence in the area of the peaks of the frequency response to be less than unity. This error can

be reduced by the use of weighting functions or by cyclic averaging. The second situation occurs when a significant *propagation time delay* occurs between the input and output as may be the case with acoustic measurements. If a propagation delay of length t is compared to a sample function length of T, a low estimate of coherence will be estimated as a function of the ratio t/T. This propagation delay causes a bias error in the frequency response and should be removed prior to computation if possible.

5.2.3 Multiple Input FRF Estimation

Multiple input estimation of frequency response functions is desirable for several reasons. The principal advantage is the increase in the accuracy of estimates of the frequency response functions. During single input excitation of a system, there may exist large differences in the amplitudes of vibratory motion at various locations because of the dissipation of the excitation power within the structure. This is especially true when the structure has heavy damping. Small nonlinearities in the structure will consequently cause errors in the measurement of the response. With multiple input excitation, the vibratory amplitudes across the structure typically will be more uniform, with a consequent decrease in the effect of nonlinearities.

A second reason for improved accuracy is the increase in consistency of the frequency response functions compared to the single input method. When a number of exciter systems are used, the elements from columns of the frequency response function matrix corresponding to those exciter locations are being determined simultaneously. With the single input method, each column is determined independently, and it is possible for small errors of measurement due to nonlinearities and time dependent system characteristics to cause a change in resonance frequencies, damping, or mode shapes among the measurements in the several columns. This is particularly important for the polyreference modal parameter estimation algorithms that use frequency response functions from multiple columns or rows of the frequency response function matrix simultaneously.

An additional, significant advantage of the multiple input excitation is a reduction of the test time. In general, using multiple input estimation of frequency response functions, frequency response functions are obtained for all input locations in approximately the same time as required for acquiring a set of frequency response functions for one of the input locations, using a single

input estimation method.

Another potential advantage of the simultaneous measurement of a number of columns or rows of the frequency response function matrix is the ability to use a linear combination of frequency response functions in the same row of the matrix in order to enhance specific modes of the system. This technique is analogous to the forced normal mode excitation experimental modal analysis in which a structure is excited by a forcing vector which is proportional to the modal vector of interest. For this analysis, the coefficients of a preliminary experimental modal analysis are used to weight the frequency response functions, so that the sum emphasizes the modal vector that is sought. The revised set of conditioned frequency response functions is analyzed to improve the accuracy of the modal vector. A simple example of this approach for a structure with approximate geometrical symmetry would be to excite at two symmetric locations. The sum of the two frequency response functions at a specific response location should enhance the symmetric modes. Likewise, the difference of the two functions should enhance the antisymmetric modes.

Multiple Input versus Single Input

Advantages

- Better energy distribution reduces nonlinearities at input location.
- Better energy distribution excites the structure more evenly.
- Data collected simultaneously has consistent frequency and damping information which is consistent with parameter estimation algorithms.
- Advances in hardware/software has kept data collection time the same for single input/multiple output. More measurements per measurement cycle.
- Multiple input data permits the detection of repeated or closely spaced roots.

Disadvantages:

- Inputs must not be correlated.
- More equipment required.

The theoretical basis of multiple-input frequency response function analysis is well documented in a number of sources [1-3,18-27]. While much had been written about multiple input theory, the application of multiple input theory to experimental modal analysis apparently had not been seriously investigated prior to 1980 [18-27]. It also needs to be noted that this application of multiple input-output theory represents a very special case of multiple-input, multiple-output data analysis. For this case, everything about the inputs is known or can be controlled. The number of inputs, the location of the inputs, and the characteristics of the inputs are controlled by the test procedure. For the general case, none of these characteristics may be known.

Consider the case of N_i inputs and N_o outputs measured during a modal test on a dynamic

system as shown in Figure 5-6. The model assumed for the dynamics is:

$$\hat{X}_p - \eta_p = \sum_{q=1}^{N_i} H_{pq} * (\hat{F}_q - \upsilon_q) \tag{5.21}$$

where:

- $F = \hat{F} - \upsilon$ = Actual input
- $X = \hat{X} - \eta$ = Actual output
- $\hat{X}_p$ = Spectrum of the $p - th$ output, measured
- $\hat{F}_q$ = Spectrum of the $q - th$ input, measured
- H_{pq} = Frequency response function of output p with respect to input q
- υ_q = Spectrum of the noise part of the input
- η_p = Spectrum of the noise part of the output
- X_p = Spectrum of the $p - th$ output, theoretical
- F_q = Spectrum of the $q - th$ input, theoretical

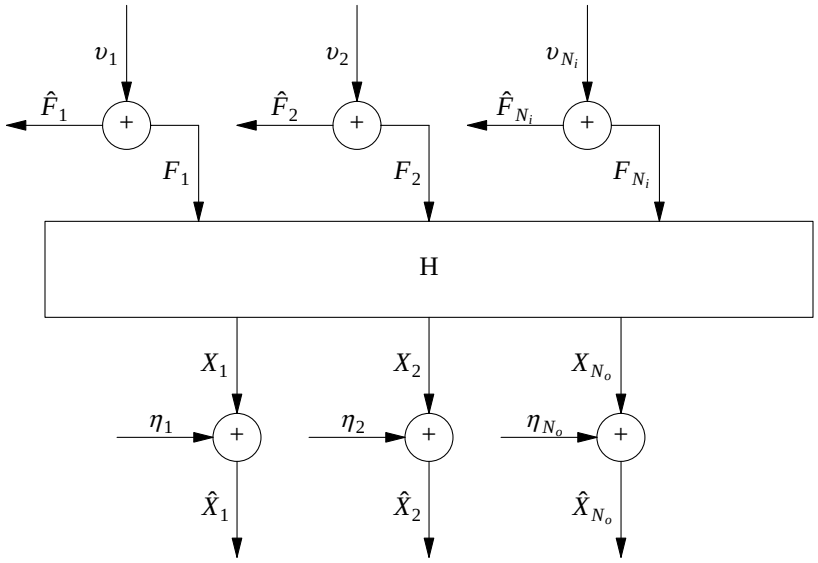

Figure 5-6. System Model: Multiple Inputs

In order to develop an estimation of the frequency response function for the multiple input case, a number of averages N_{avg} will be used to minimize the random errors (variance). This can be easily accomplished through use of intermediate measurment of the auto and cross power spectrums as defined in Equations (5.8) through (5.11). Additional matrices, constructed from the auto and cross power spectrums need to be defined as follows. Note that each function and, therefore, each resulting matrix is a function of frequency.

Input/Output Cross Spectra Matrix

$$[GXF] = \{X\}\{F\}^H = \begin{Bmatrix} X_1 \\ X_2 \\ . \\ . \\ X_{N_o} \end{Bmatrix} \; [F_1^* \;\; F_2^* \;\; . \;\; . F_{N_i}^*] = \begin{bmatrix} GXF_{11} & . & . & . & GXF_{1N_i} \\ . & & . & & . \\ . & & . & & . \\ GXF_{N_o1} & . & . & . & GXF_{N_oN_i} \end{bmatrix} \tag{5.22}$$

Input Cross Spectra Matrix

$$[GFF] = \{F\}\{F\}^H = \begin{Bmatrix} F_1 \\ F_2 \\ . \\ . \\ F_{N_i} \end{Bmatrix} \; [F_1^* \;\; F_2^* \;\; . \;\; . \;\; F_{N_i}^*] = \begin{bmatrix} GFF_{11} & . & . & . & GFF_{1N_i} \\ . & & . & & . \\ . & & . & & . \\ GFF_{N_i1} & . & . & . & GFF_{N_iN_i} \end{bmatrix} \tag{5.23}$$

The frequency response functions can now be estimated for the three algorithms as follows:

H_1 ***Algorithm: Minimize Noise on Output (η)***

$$[H]_{N_o \times N_i} \{F\}_{N_i \times 1} = \{X\}_{N_o \times 1} - \{\eta\}_{N_o \times 1} \tag{5.24}$$

$$[H] \{F\} \{F\}^H = \{X\} \{F\}^H - \{\eta\} \{F\}^H \tag{5.25}$$

$$[H]_{N_o \times N_i} \{F\}_{N_i \times 1} \{F\}^H_{1 \times N_i} = \{X\}_{N_o \times 1} \{F\}^H_{1 \times N_i} \tag{5.26}$$

The above relationship can be concisely stated as:

$$[H][GFF] = [GXF] \tag{5.27}$$

$$[H] = [GXF][GFF]^{-1} \tag{5.28}$$

where:

- $[\ ]^H$ = Complex conjugate transpose (Hermitian Matrix)

In the experimental procedure, the input and response signals are measured, and the averaged cross spectra and auto spectra necessary to create the $[GXF]$ and $[GFF]$ matrices are computed. If the computation of ordinary, multiple, or partial coherence functions will be required, then the diagonal elements of the output cross spectrum matrix $[GXX]$ must be computed also.

Equation (5.27) is valid regardless of whether the various inputs are correlated. Unfortunately, there are a number of situations where the input cross spectrum matrix $[GFF]$ may be singular for specific frequencies or frequency intervals. When this happens, the inverse of $[GFF]$ will not exist and Equation (5.28) cannot be used to solve for the frequency response function at those frequencies or in those frequency intervals. A computational procedure that solves Equation (5.28) for $[H]$ should therefore monitor the rank the matrix $[GFF]$ that is to be inverted, and desirably provide direction on how to alter the input signals or use the available data when a problem exists. The current approach for evaluating whether the inputs are sufficiently uncorrelated at each frequency involves determining the principal/virtual forces using principal component analysis [8]. This will be covered in a later section.

H_2 *Algorithm: Minimize Noise on Input (v)*

$$[H]_{N_o \times N_i} \{ \{F\}_{N_i \times 1} - \{v\}_{N_i \times 1} \} = \{X\}_{N_o \times 1} \tag{5.29}$$

$$[H] \{ \{F\} - \{v\} \} \{X\}^H = \{X\} \{X\}^H \tag{5.30}$$

$$[H]_{N_o \times N_i} \{F\}_{N_i \times 1} \{X\}^H_{1 \times N_o} = \{X\}_{N_o \times 1} \{X\}^H_{1 \times N_o} \tag{5.31}$$

One problem with using the H_2 algorithm is that the solution for $[H]$ can only be found directly using an inverse when the number of inputs N_i and number of outputs N_o are equal. Then:

$$[\,H\,]\,[\,GFX\,] = [\,GXX\,] \tag{5.32}$$

$$[\,H\,] = [\,GXX\,]\,[\,GFX\,]^{-1} \tag{5.33}$$

H_v *Algorithm: Minimize Noise on Input and Output (υ and η)*

$$[H]_{N_o \times N_i}\ \{\,\{F\}_{N_i \times 1} - \{\upsilon\}_{N_i \times 1}\} = \{X\}_{N_o \times 1} - \{\eta\}_{N_o \times 1} \tag{5.34}$$

$$[H]\,\{\,\{F\} - \{\upsilon\}\,\} = \{X\} - \{\eta\} \tag{5.35}$$

The solution for $[H]$ is found by the eigenvalue decomposition of one of the following two matrices:

$$[GFFX_p] = \begin{bmatrix} [GFF] & [GXF_p]^H \\ [GXF_p] & [GXX_p] \end{bmatrix}_{(N_i+1)\times(N_i+1)} \tag{5.36}$$

$$[GXFF_p] = \begin{bmatrix} [GXX_p] & [GXF_p] \\ [GXF_p]^H & [GFF] \end{bmatrix}_{(N_i+1)\times(N_i+1)} \tag{5.37}$$

Therefore, the eigenvalue decomposition would be:

$$[GFFX_p] = [V]\ \lceil\ \Lambda\ \rfloor\,[V]^H \tag{5.38}$$

Or:

$$[GXFF_p] = [V]\ \lceil\ \Lambda\ \rfloor\,[V]^H \tag{5.39}$$

where:

- $\lceil\ \Lambda\ \rfloor$ = *diagonal matrix of eigenvalues*

Solution for the $p-th$ row of the $[H]$ matrix is found from the eigenvector associated with the smallest (minimum) eigenvalue. Note that the size of the eigenvalue problem is $N_i + 1$ and that the eigenvalue solution must be repeated for each frequency. Note also that the complete solution process must be repeated for each response point X_p.

The frequency response functions associated with a single output p and all inputs is found by normalizing the eigenvector associated with the smallest eigenvalue. If $[GFFX_p]$ is used, the eigenvector associated with the smallest eigenvalue must be normalized as follows:

$$\{V\}_{\lambda_{\min}} = \begin{Bmatrix} H_{p1} \\ H_{p2} \\ . \\ . \\ H_{pN_i} \\ -1 \end{Bmatrix} \tag{5.40}$$

If $[GXFF_p]$ is used, the eigenvector associated with the smallest eigenvalue must be normalized as follows:

$$\{V\}_{\lambda_{\min}} = \begin{Bmatrix} -1 \\ H_{p1} \\ H_{p2} \\ . \\ . \\ H_{pN_i} \end{Bmatrix} \tag{5.41}$$

The concept of the coherence function, as defined for single-input measurement, needs to be expanded to include the variety of relationships that are possible for multiple inputs. ***Ordinary coherence*** is defined in this general sense as the correlation coefficient describing the linear relationship between any two spectra. This is consistent with the ordinary coherence function that is defined for single input, single output measurements. Great care must be taken in the interpretation of ordinary coherence when more than one input is present. The ordinary coherence of an output with respect to an input can be much less than unity even though the

linear relationship between inputs and outputs is valid, because of the influence of the other inputs.

The ordinary coherence function can be formulated in terms of the elements of the matrices defined previously. The ordinary coherence function between the p^{th} output and the q^{th} input can be computed from the following formula:

Ordinary Coherence Function

$$COH_{pq} = \gamma_{pq}^{\ 2} = \frac{|GXF_{pq}|^2}{GFF_{qq}\ GXX_{pp}} \tag{5.42}$$

where:

- GXX_{pp} = *Auto power spectrum of the output p*
- GFF_{qq} = *Auto power spectrum of the input q*
- GXF_{pq} = *Cross power spectrum between output p and input q*

Partial coherence is defined as the ordinary coherence between a conditioned output and another conditioned output, between a conditioned input and another conditioned input, or between a conditioned input and a conditioned output. The output and input are conditioned by removing contributions from other input(s). The removal of the effects of the other input(s) is formulated on a linear least squares basis. The order of removal of the inputs during "conditioning" has a definite effect upon the partial coherence if some of the input(s) are mutually correlated. There will be a partial coherence function for every input/output, input/input and output/output combination for all permutations of conditioning. The usefulness of partial coherence, especially between inputs, for experimental modal analysis is of limited value.

Multiple coherence is defined as the correlation coefficient describing the linear relationship between an output and all known inputs. There is a multiple coherence function for every output. Multiple coherence can be used to evaluate the importance of unknown contributions to each output. These unknown contributions can be measurement noise, nonlinearities, or unknown inputs. Particularly, as in the evaluation of ordinary coherence, a low value of multiple

coherence near a resonance will often mean that the "leakage" error is present in the frequency response function. Unlike the ordinary coherence function, a low value of multiple coherence is not expected at antiresonances. The antiresonances for different response locations occur at the same frequency. Though one response signal may have a poor signal-to-noise ratio at its antiresonance, other inputs will not at the same frequency.

The formulation of the equations for the multiple coherence functions can be simplified from the normal computational approach to the following equation.

Multiple Coherence Function

$$MCOH_p = \sum_{q=1}^{N_i} \sum_{t=1}^{N_i} \frac{H_{pq}\, GFF_{qt}\, H_{pt}^*}{GXX_{pp}} \tag{5.43}$$

where:

- H_{pq} = Frequency response function for output p and input q
- H_{pt} = Frequency response function for output p and input t
- GFF_{qt} = Cross power spectrum between output q and output t
- GXX_{pp} = Auto power spectrum of output p

If the multiple coherence of the $p-th$ output is near unity, then the $p-th$ output is well predicted from the set of inputs using the least squares frequency response functions.

Example: H_1 Technique: Two Inputs/One Output Case

To begin to understand the size of the problem involved, start with the two input, one output case.

$$\hat{X}_p - \eta_p = H_{p1}\, F_1 + H_{p2}\, F_2 \tag{5.44}$$

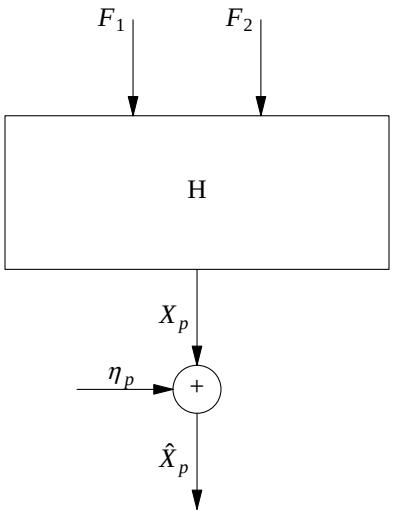

Figure 5-7. Two Input, One Output Model

If more than one output is measured, the equations become:

$$\{X_p\}\,[F_1^*\ F_2^*] = [H_{p1}\ \ H_{p2}] \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} [F_1^*\ F_2^*] \tag{5.45}$$

Therefore, for input locations 1 and 2, each output is used with the two inputs to compute two frequency response functions. Therefore, there will be $2 \times N_o$ frequency response functions to be computed.

$$\begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \\ H_{31} & H_{32} \\ . & . \\ . & . \\ H_{N_o1} & H_{N_o2} \end{bmatrix} = \begin{bmatrix} GXF_{11} & GXF_{12} \\ GXF_{21} & GXF_{22} \\ GXF_{31} & GXF_{32} \\ . & . \\ . & . \\ GXF_{N_o1} & GXF_{N_o2} \end{bmatrix} \begin{bmatrix} GFF_{11} & GFF_{12} \\ GFF_{21} & GFF_{22} \end{bmatrix}^{-1} \tag{5.46}$$

For each output location, one formulation of the equations to be solved can be developed by replacing the inverse of the [*GFF*] matrix with the equivalent adjoint of the [*GFF*] matrix divided by the determinant of the [*GFF*] matrix. In this way, it is clear that the frequency response functions can be found as long as the determinant of the [*GFF*] matrix is not zero.

$$H_{p1} = \frac{GXF_{p1}\, GFF_{22} - GXF_{p2}\, GFF_{21}}{\det[GFF]} \tag{5.47}$$

$$H_{p2} = \frac{GXF_{p2}\, GFF_{11} - GXF_{p1}\, GFF_{12}}{\det[GFF]} \tag{5.48}$$

where:

- $\det[GFF]$ = Determinant of $[GFF]$ matrix
- $\det[GFF] = GFF_{11}\, GFF_{22} - GFF_{21}\, GFF_{12}$

For the two input, one output case several possible coherence functions can be formulated. While the ordinary coherence between the output and each input can be formulated, these coherence functions may not provide useful information due to the possible interaction between the two forces.

Ordinary Coherence (Output p and Input 1)

$$COH_{p1} = \frac{|GXF_{p1}|^2}{GFF_{11}\, GXX_{pp}} \tag{5.49}$$

Ordinary Coherence (Output p and Input 2)

$$COH_{p2} = \frac{|GXF_{p2}|^2}{GFF_{22}\, GXX_{pp}} \tag{5.50}$$

The ordinary coherence between the two inputs is a useful function since this is a measure of whether the forces are correlated. If the forces are perfectly correlated at a frequency, the inverse of the $[GFF]$ matrix will not exist and the frequency response functions cannot be estimated at that frequency. In this case, the ordinary coherence between the two forces cannot be unity, although values from 0.0 to 0.99 are theoretically acceptable. The limit is determined by the accuracy of the measured data and the numerical precision of the computation.

Ordinary Coherence (Input 1 and Input 2)

$$COH_{12} = \frac{|GFF_{12}|^2}{GFF_{11}\ GFF_{22}} \tag{5.51}$$

Multiple coherence is always a good measure of whether the output response is caused by the combination of the measured inputs. Multiple coherence is is used in multiple input situations in the same way that ordinary coherence is used in the single input situations.

Multiple Coherence

$$MCOH_p = \sum_{q=1}^{2} \sum_{t=1}^{2} \frac{H_{pq}\ GFF_{qt}\ H_{pt}^*}{GXX_{pp}} \tag{5.52}$$

Summary of Methods

H_1 Technique:

- Underestimates amplitude at resonances. Causes damping to be overestimated.
- Underestimates amplitude at anti-resonances.

H_2 Technique:

- Overestimates amplitude at resonances. Causes damping to be underestimated.
- Overestimates amplitude at anti-resonances.

H_v Technique:

- Best estimate of amplitude at resonances. Causes damping to be estimated best.
- Best estimate of amplitude at anti-resonances.
- Phase characteristics not altered.

5.2.3.1 Multiple Input Force Analysis/Evaluation

Of the variety of situations that can cause difficulties in the computation of the frequency response functions, the highest potential for trouble is the case of coherent inputs. If two of the inputs are fully coherent at one of the analysis frequencies, then there are no unique frequency response functions associated with those inputs at that analysis frequency. Unfortunately, there

are a number of situations where the input cross spectrum matrix [*GFF*] may be singular at specific frequencies or frequency intervals. When this happens, the inverse of [*GFF*] will not exist and Equation (5.28) cannot be used to solve for the frequency response function at those frequencies or in those frequency intervals. First, one of the input autospectra may be zero in amplitude over some frequency interval. When this occurs, then all of the cross spectra in the same row and column in the input cross spectrum matrix [*GFF*] will also be zero over that frequency interval. Consequently, the input cross spectrum matrix [*GFF*] will be singular over that frequency interval. Second, two or more of the input signals may be fully coherent over some frequency interval. Although the signals used as inputs to the exciter systems must be uncorrelated random inputs, the response of the structure at resonance, combined with the inability to completely isolate the exciter systems from this response results in the ordinary or conditioned partial coherence functions with values other than zero, particularly, at the system poles. For example, for the two input case, as long as the coherence function between the inputs is not unity at these frequencies, Equation (5.28) can be solved uniquely for the frequency response functions. Note that the auto and cross spectra involved in the calculation of the multiple input case for the estimation of frequency response functions should be computed from analog time data that has been digitized *simultaneously*. If data is not processed in this manner, many more averages are required to reduce the variance on each individual auto and cross spectrum and the efficiency of the multiple input approach to the estimation of frequency response functions will not be as attractive. Finally, numerical problems, which cause the computation of the inverse to be inexact, may be present. This can happen when an autospectrum is near zero in amplitude, when the cross spectra have large dynamic range with respect to the precision of the computer, or when the matrix is ill-conditioned because of nearly redundant input signals.

Due to the form of the equations that must be solved to compute frequency response functions in the presence of multiple inputs, special care must be taken to assure that the input spectrum matrix is not singular. Therefore, techniques have been investigated to evaluate the form of the input spectrum matrix before taking any data. Singular, in this case, implies that:

- Input forces may not be coherent at any frequency.
- Independent, uncorrelated noise sources must be used. (Random, Random Transient, Periodic Random)

- The impedance of the structure at the input locations may tend to correlate the inputs at resonance.

- There are no zero's in the input spectrum matrix.

Ordinary and Partial Coherence Functions

The historical approach that was used to try to evaluate the correlation between the forces utilized ordinary and partial coherence functions. The ordinary coherence function measures the degree of linear dependence (or correlation) between the spectra of two signals. The partial coherence function measures the degree of linear dependence between the spectra of two signals, after eliminating in a least squares sense, the contribution of some other signals. Both functions can be used in systematic procedure to verify that the forces are not correlated or that the input cross spectra matrix [*GFF*] is not singular. For cases involving more than two inputs, this approach is very difficult and requires considerable judgement. In reality, only the ordinary coherence function, for the case of two inputs, is still used.

$$COH_{ik} = \frac{| GFF_{ik} |^2}{GFF_{ii}\, GFF_{kk}} \tag{5.53}$$

where:

- GFF_{ik} = Cross power spectrum between inputs i and k
- GFF_{ii} = Auto power spectrum of input i
- GFF_{kk} = Auto power spectrum of input k

Principal/Virtual Input Forces (Virtual Forces)

The current approach used to determine correlated inputs involves utilizing principal component analysis to determine the number of contributing forces to the [*GFF*] matrix. In this approach, the matrix that must be evaluated is:

$$[GFF] = \begin{bmatrix} GFF_{11} & . & . & . & GFF_{1N_i} \\ . & & & & . \\ . & & & & . \\ . & & & & . \\ GFF_{N_i1} & . & . & . & GFF_{N_iN_i} \end{bmatrix} \tag{5.54}$$

where:

- $GFF_{ik} = GFF_{ki}^{*}$ (Hermitian Matrix)
- $GFF_{ik} = \sum F_i F_k^{*}$
- GFF is the power spectrum of a given input.

Principal component analysis involves an eigenvalue decomposition of the [GFF] matrix [8]. Since the eigenvectors of such a decomposition are unitary, the eigenvalues should all be of approximately the same size if each of the inputs is contributing. If one of the eigenvalues is much smaller at a particular frequency, one of the inputs is not present or one of the inputs is correlated with the other input(s).

$$[\ GFF\ ] = [\ V\ ]\ [\ \Lambda\ ]\ [\ V\ ]^{H} \tag{5.55}$$

Since the eigenvectors of such a decomposition are unitary, the eigenvalues should all be of approximately the same size if each of the inputs is contributing. If one of the eigenvalues is much smaller at a particular frequency, one of the inputs is not present or one of the inputs is correlated with the other input(s). [Λ] represents the eigenvalues of the [GFF] matrix. If any of the eigenvalues of the [GFF] matrix are zero or insignificant, then the [GFF] matrix is singular. Therefore, for a three input test, the [GFF] matrix should have three eigenvalues. (The number of eigenvalues is the number of uncorrelated inputs). This concept is shown graphically in Figure 5-8 for the auto power spectra for a three input case. It is difficult to determine if the inputs are mutually correlated from these plots. Figure 5-9 shows the principal force plots for the same case. At the frequencies where the third principal/virtual force drops (lowest curve), this indicates that the inputs are mutually correlated at those frequencies. This is not apparent from Figure 5-8.

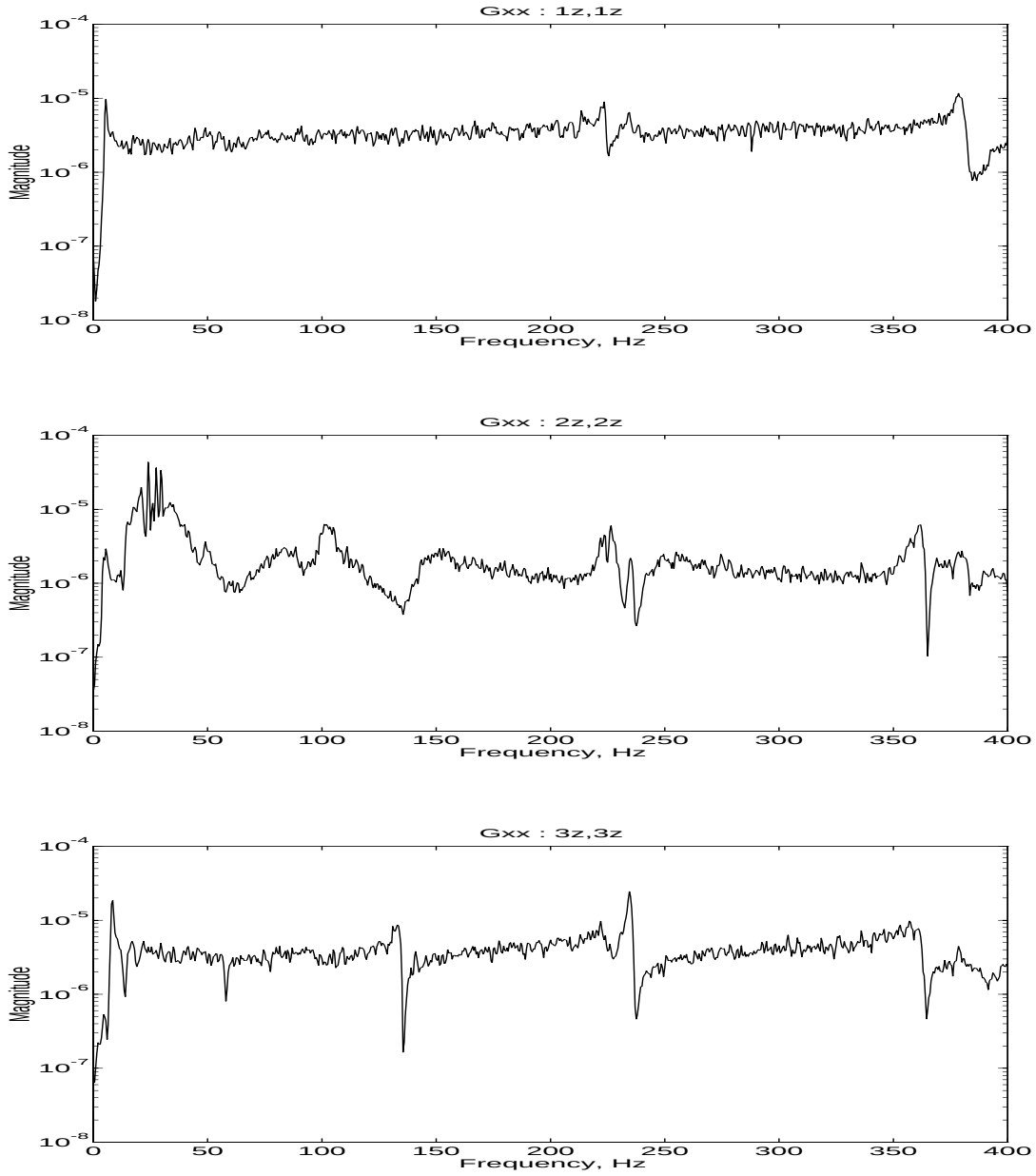

Figure 5-8. AutoPower Spectrum of Input Forces

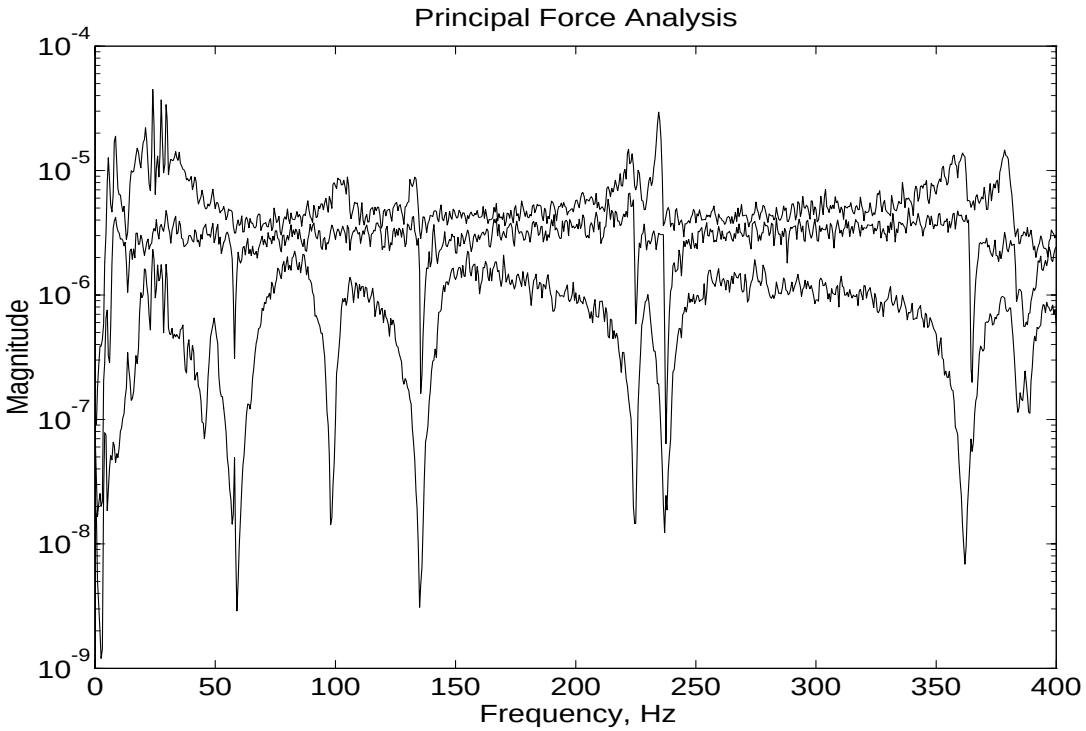

Figure 5-9. Principal (Virtual) Force Spectrum

Optimum Number Of Inputs

The location and number of inputs has a direct effect on the quality of frequency response functions that are estimated. This is an area that has not been researched completely and is still being reviewed. It is clear that beyond some number of inputs, the return from the investment of more equipment, in the form of inputs, is not warranted. Some considerations are:

- Two at symmetric locations. Frequency response functions can be added or subtracted to enhance in phase or out of phase modes.
- To excite as many modes as possible in one test configuration.

- Two vertical and one horizontal on a car.
- One on each wing and one on each horizontal stabilizer, all symmetric, on an aircraft structure.

- To excite "operating" conditions.

5.3 Excitation

Excitation includes any form of input that is used to create a response in a mechanical system. This can include environmental or operational inputs as well as the controlled force input(s) that are used in a vibration or modal analysis test. In general, the following discussion will be limited to the force inputs that can be measured and/or controlled in some rigorous way. With respect to frequency response function measurements to be used in experimental modal analysis, the excitation normally is applied using shakers or with impact devices (hammers). For those excitation signals that require the use of a shaker, Figure 5-10 shows a typical test configuration; Figure 5-11 shows a typical test configuration when an impact form of excitation is to be used.

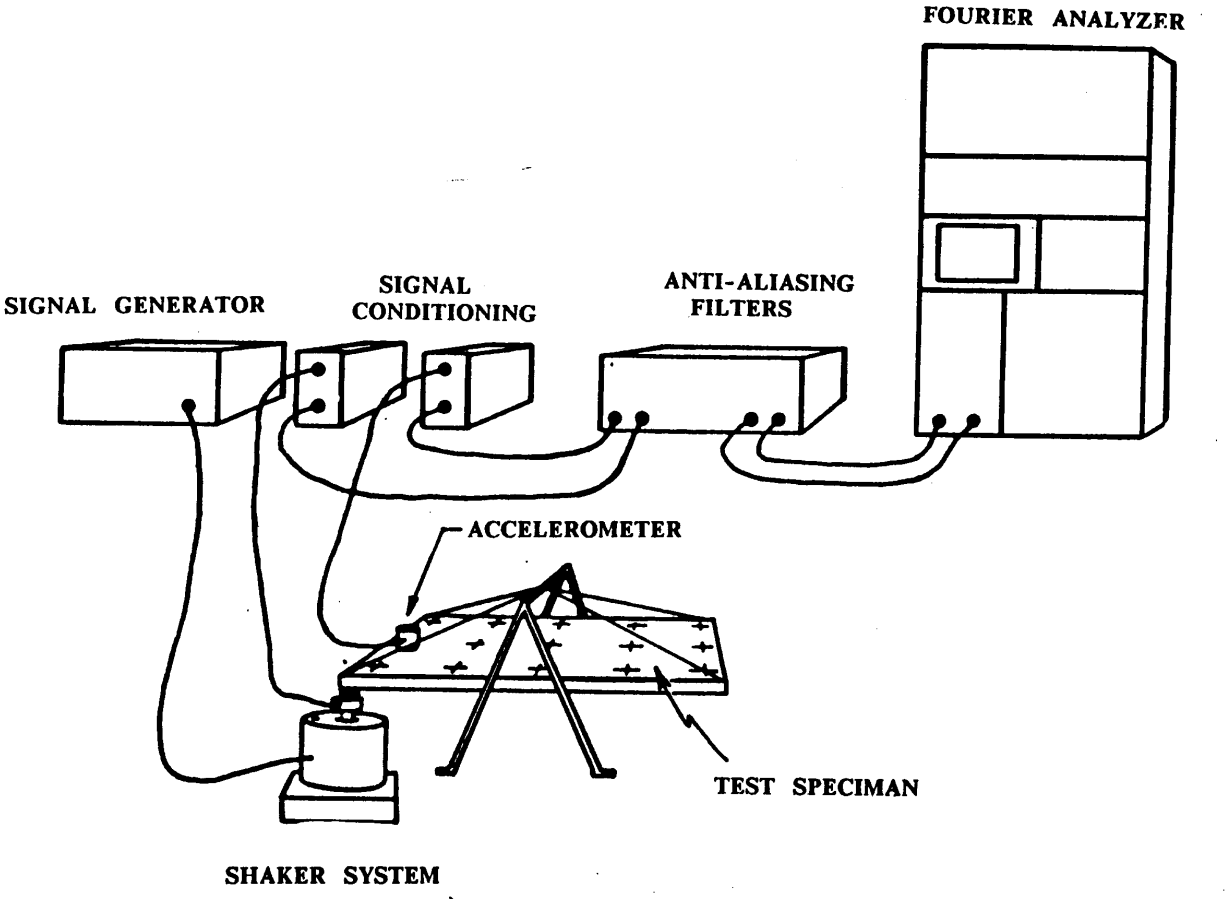

Figure 5-10. Typical Test Configuration: Shaker

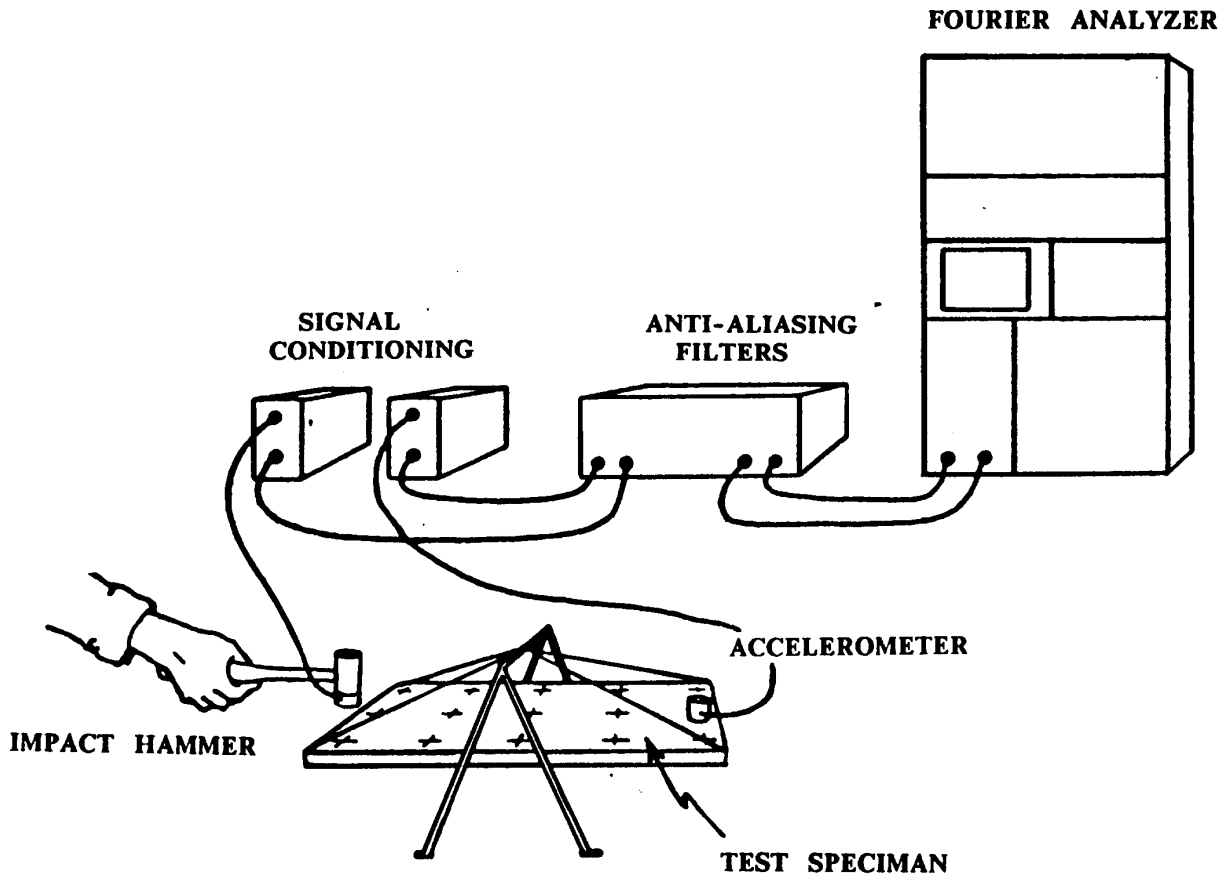

Figure 5-11. Typical Test Configuration: Impact Hammer

Single and multiple input estimation of frequency response functions (FRFs) via shaker excitation has become the mainstay of most mechanical structure measurements, particularly in the automotive and aircraft industries. While there are appropriate occasions for the use of deterministic excitation signals (sinusoids), the majority of these measurements are made using broadband (random) excitation signals. These signals work well for moderate to heavily damped mechanical structures which exhibit linear characteristics. When the mechanical structures are very lightly damped, care must be taken to minimize the leakage error so that accurate frequency response function (FRF) data can be estimated in the vicinity of the modal frequencies of the system. Frequently, when random excitation methods are compared to deterministic methods (sinusoids), the comparisons are questionable since proper procedures for eliminating the leakage error have not been followed.

Historically, a number of random excitation signals have been utilized, together with appropriate

digital signal processing techniques [1-5], to obtain accurate FRF data. The most common random signal that is used in this situation is the pure random signal together with a Hann window. This signal is normally generated by the data acquisition system utilizing built-in random signal generator(s) or via external random signal generator(s). While this approach does not eliminate the source of leakage and the effect of applying the Hann window must be considered, this approach is normally considered as a baseline random excitation method for estimating FRF measurements since this method is available with almost any data acquisition system.

Other forms of random signals (pseudo random, periodic random, burst random, etc.) utilize more control or frequency shaping of the excitation signal(s) and generally require digital-to-analog (DAC) converter(s). For this reason, some of these alternate methods are infrequently available and therefore not used. This is unfortunate since these methods often yield a superior FRF measurement in less total testing time.

When FRFs are measured on lightly damped systems, great care must be taken to eliminate the leakage error. Regardless of the type of excitation signal hardware involved (random signal generator or DAC), there are random excitation methods that can nearly eliminate the leakage error. In some cases, one approach will be superior on the basis of minimizing the total test time but on the basis of accurate, leakage-free FRFs, one of the methods will always work if test time can be sacrificed. Note that these alternate forms of random excitation focus on eliminating the source of leakage by customizing the random signal to match the requirements of fast Fourier transform (FFT) that is used in converting from the time to frequency domain. The FFT requires that the time domain signal must either be totally observed in the observation period (T) or be periodic in the observation period (T). For leakage free FRF measurements, all of the input and output signals must match one of these two requirements. Burst random excitation is an attempt to match the first requirement; pseudo and periodic random excitations are attempts to match the second requirement.

5.3.1 Excitation Assumptions

The primary assumption concerning the excitation of a linear structure is that the excitation is observable. Whenever the excitation is measured, this assumption simply implies that the measured characteristic properly describes the actual input characteristics. For the case of multiple inputs, the different inputs must often be uncorrelated for the computational procedures

to yield a solution. In most cases this means only that the multiple inputs must not be perfectly correlated at any frequency. As long as the excitation is measured, the validity of these limited assumptions can be evaluated.

Currently, there are a number of techniques that can be used to estimate modal characteristics from response measurements with no measurement of the excitation. If this approach is used, the excitation assumptions are much more imposing. Obviously, if the excitation is not measured, estimates of modal scaling (modal mass, modal A, residues, etc.) cannot be generated. Even under the assumption that the estimation of these parameters is not required, all of these techniques have one further restriction: an assumption has to be made concerning the characteristics of the excitation of the system. Usually, one assumes that the autospectrum of the excitation signal is sufficiently smooth over the frequency interval of interest.

In particular, the following assumptions about the excitation signal can be used:

- The excitation is impulsive. The autospectrum of a short pulse (time duration much smaller than the period of the greatest frequency of interest) is nearly uniform, or constant in amplitude, and largely independent of the shape of the pulse.
- The excitation is white noise. White noise has an autospectrum that is uniform over the bandwidth of the signal.
- The excitation signal is a step. A step signal has an autospectrum that decreases in amplitude in proportion to the reciprocal of frequency. The step signal can be viewed as the integral of an impulsive signal.
- There is no excitation. This is called the free response or free decay situation. The structure is excited to a condition of nonzero displacement, or nonzero velocity, or both. Then the excitation is removed, and the response is measured during free decay. This kind of response can be modeled as the response of the structure to an excitation signal that is a linear combination of impulsive and step signals.

When the excitation autospectrum is uniform, the autospectrum of the response signal is proportional to the square of the modulus of the frequency response function. Using the notation of a pole-zero model, the poles of the response spectrum are the poles of the frequency response, which are the parameters of the system resonances. If the autospectrum is not uniform, then the

excitation spectrum can be modeled as an analytic function, to a precision comparable to typical experimental error in the measurement of spectra. In this model, the excitation spectrum has poles that account for the nonuniformity of the spectrum amplitude. The response signal, therefore, can be modeled by a spectrum that contains zeros at the zeros of the excitation and the zeros of the frequency response, and contains poles at the poles of the excitation and at the poles of the frequency response. It is obviously important that the force spectrum should have no poles or zeros which coincide with poles of the frequency response.

For transient inputs, such as an impact or step relaxation, the assumption of smooth excitation spectra is generally true, but for operating inputs or inputs generated by an exciter system, care must be taken to insure the input force spectrum is smooth. This is especially true for tests performed using a hydraulic or an electro-mechanical exciter, because the system being analyzed may "load" the exciter system (the structure’s impedance is so low that the desired force level cannot be achieved within the constraint of small motion), and this causes a nonuniformity in the input force spectrum.

To determine the characteristics of the system from the response, it is necessary that the response have the same poles as the frequency response, or that the analysis process corrects for the zeros and poles of the excitation. If the force input spectrum has a zero in the frequency range of interest, the pole location measured from the response spectrum will not match that of the frequency response. This potential problem is demonstrated in Figure 5-12 for the typical case of shaker excitation. The top figure is the magnitude of the frequency response function. The middle figure is the auto power spectrum of the input and the lower figure is the auto power spectrum of the response. Note that the estimates of modal parameters that would be derived from the auto power spectrum of the response would be quite different from those derived from the frequency response function.

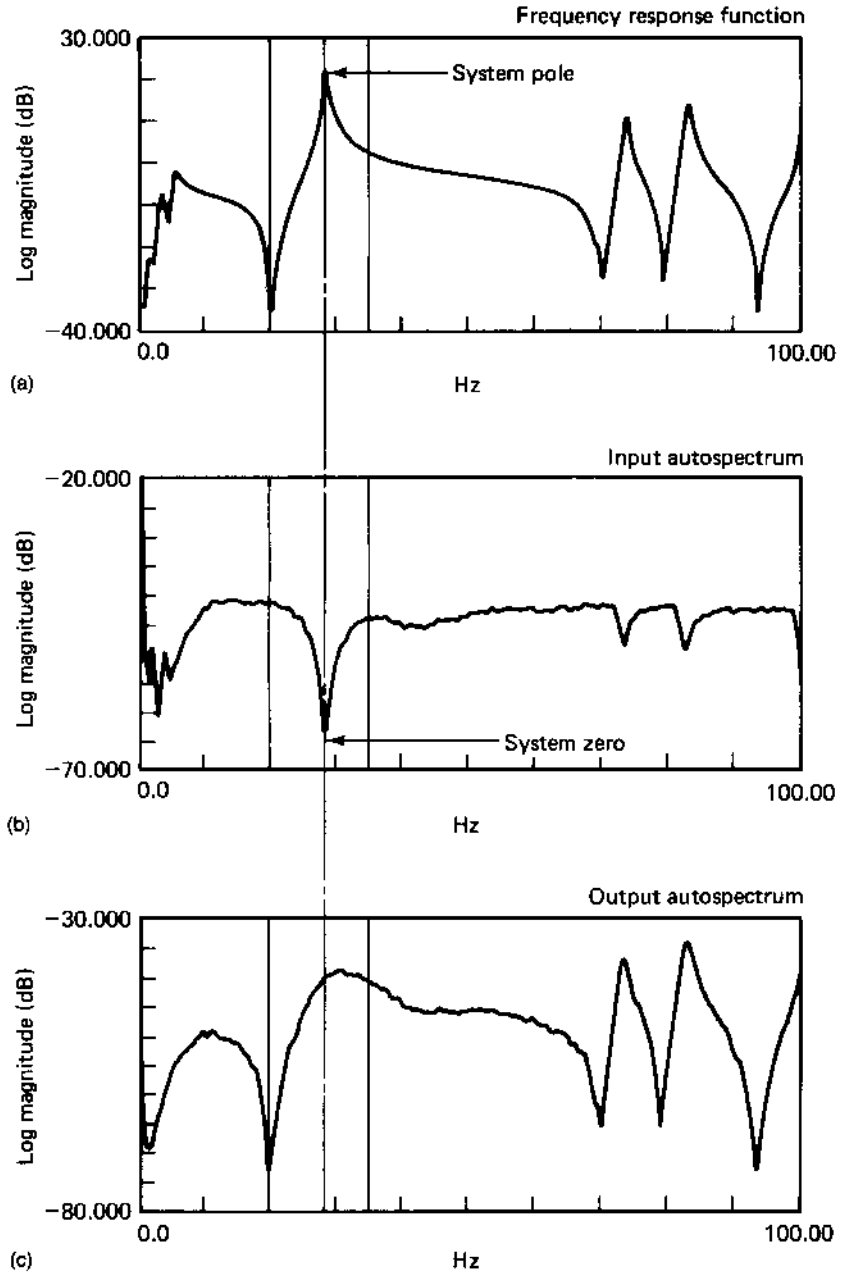

Figure 5-12. Input Spectrum Example

Presently, there is a great deal of interest in determining modal parameters from measured response data taken on operating systems (for example: turbulent flow over an airfoil, road inputs to automobiles, and environmental inputs to proposed large space structures). For these cases,

care must be taken not to confuse poles that are system resonances with those that exist in the output spectrum due to inputs.

In general, the poles of the response include those of the frequency response and of the input spectrum. Therefore, if the force is not measured, it is not possible without some prior knowledge about the input to determine if the poles of the response are truly system characteristics. If no poles or zeros exist in the force spectrum in the frequency range of interest, then any poles in the response in this range must be a result of the system characteristics. Obviously, when the excitation can be measured, it is prudent to do so.

5.3.2 Terminology and Nomenclature

Historically, a number of terminology and nomenclature issues have not been rigorously defined when excitation methods have been described.

The following terminology is important to the explanation of different excitation methods together with the associated digital signal processing requirements.

Signal Type - Signal type refers to the basic form of the signal, such as random, impact, sinusoidal or chirp.

Frequency Shaping - Frequency shaping refers to any frequency domain constraint or characteristic that is applied to the specific signal type. With respect to random excitation, a common frequency shaping is pseudo random. Other frequency shaping is commonly applied to sinusoids and chirps via the rate at which the change of frequency and/or amplitude occurs. Impact excitation is commonly frequency shaped by controlling the tip characteristic of the hammer.

Delay Blocks - The number of contiguous blocks of excitation that take place without the associated input and output data being acquired are referred to as the delay blocks (N_d). This is normally associated with a excitation technique that is periodic in nature. The delay blocks are needed in order to give the transient response to any start or change in the periodic excitation to decay out of the response signal(s) so that both the input(s) and output(s) are periodic with respect to any observation period (T). It is this requirement that makes swept sinusoidal excitation methods (analog swept or digitally stepped) so time consuming, particularly on lightly

damped systems. Each delay block is equal in length to the observation period (T) and the number of delay blocks is normally chosen as an integer. The number of delay blocks does not have to be an integer for all excitation methods but, for the purposes of this paper and in common usage, is normally chosen as an integer. The delay blocks are not recorded and are not used in the estimation of the FRFs.

Capture Blocks - The number of capture blocks refers to the number of contiguous blocks of time data (excitation (input) and response (output)) that are recorded or captured for each average (N_c). The number of capture blocks is also the number of cyclic averages that will be used to reduce the leakage error. Each group of contiguous capture blocks (N_c) is used as the time domain data contributing to one power spectral average that contributes to the estimate of the FRF measurements.

Window Function - The window function refers to the digital signal processing, time domain window that is applied to the capture blocks. The application of the window function to the capture blocks is on the basis of the group of contiguous capture blocks not on each capture block individually.

Average (Ensemble) - The average or ensemble refers to the total collection of contiguous time blocks that contribute to each power spectral average. The total time of each average is equal to the sum of the number of delay blocks (N_d) plus the number of capture blocks (N_c) times the observation period (T) which is the same for all delay and capture blocks. The number of averages (N_{avg}) refers to the number of these contiguous collections of time blocks and is, therefore, the same as the number of power spectral averages. The number of capture blocks can also be thought of as the number of cyclic averages (N_c). Cyclic signal averaging is often used with excitation characteristics in order to better match the time domain input and output signals to the requirements of the FFT prior to the application of the FFT. Cyclic signal averaging essentially digitally comb filters the time domain data to reduce the amount of information in the data that is not periodic with the observation period (T). This type of averaging reduces the effects of the leakage error. As long as the N_c successive blocks of data are contiguous, the blocks of time domain data can be averaged together, with or without windows, to achieve the benefit of leakage reduction [9-10].

Periodic - If the excitation signal is repeated for each delay and capture block, the signal is referred to as periodic. This classification is consistent with the definition of a periodic function and includes typical examples of sinusoids and chirps as well as a random signal that is repeated on the basis of the observation period (T). The periodic classification does not define whether

the same signal is repeated for each successive group of contiguous delay and capture blocks.

Burst Length - Burst length is the percentage (0 to 100%) of the average or ensemble time that the excitation signal is present. Burst length is normally adjusted in order to achieve a signal that is a totally observed transient. The decay of the signal is a function of the system damping and the characteristics of the excitation hardware. Burst length can be defined as the percentage of the total number of contiguous delay and capture blocks or of a percentage of just the capture blocks. For the purpose of this paper, the burst length refers to the percentage of the total number of contiguous delay and capture blocks.

Power Spectral Averages - The number of power spectral averages (N_{avg} or N_a) is the number of auto and cross spectra that are averaged together to estimate the FRF measurements. The actual amount of test time contributing to each power spectral average is a function of the number of contiguous delay and capture blocks. The purpose of power spectral averages is to eliminate the noise that is random with respect to the averaging procedure in order to reduce the variance on the resulting FRF estimate. This type of averaging does not reduce the effects of bias errors like the leakage error.

In order to clarify the preceding terminology, Figure 5-13 is a schematic representation of the number of contiguous blocks of time domain data contributing to one power spectral average. In this example, the two blocks marked "D" represent delay blocks and the four blocks marked "C" represent capture blocks. The total time for each power spectral average is, therefore, six contiguous blocks of time data ($6 \times T$ seconds of data).

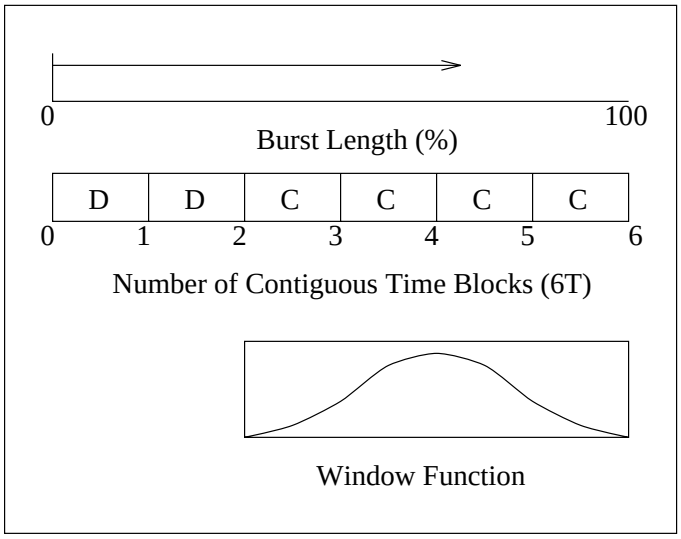

Figure 5-13. Total Contiguous Time Per Power Spectral Average (Ensemble)

5.3.3 Classification of Excitation

Inputs which can be used to excite a system in order to determine frequency response functions belong to one of two classifications. The first classification is that of a random signal. Signals of this form can only be defined by their statistical properties over some time period. Any subset of the total time period is unique and no explicit mathematical relationship can be formulated to describe the signal. Random signals can be further classified as stationary or non-stationary. Stationary random signals are a special case where the statistical properties of the random signals do not vary with respect to translations with time. Finally, stationary random signals can be classified as ergodic or non-ergodic. A stationary random signal is ergodic when a time average on any particular subset of the signal is the same for any arbitrary subset of the random signal. All random signals which are commonly used as input signals fall into the category of ergodic, stationary random signals.

The second classification of inputs which can be used to excite a system in order to determine frequency response functions is that of a deterministic signal. Signals of this form can be represented in an explicit mathematical relationship. Deterministic signals are further divided into periodic and non-periodic classifications. The most common inputs in the periodic deterministic signal designation are sinusoidal in nature while the most common inputs in the

non-periodic deterministic designation are transient in form.

The choice of input to be used to excite a system in order to determine frequency response functions depends upon the characteristics of the system, upon the characteristics of the parameter estimation, and upon the expected utilization of the data. The characterization of the system is primarily concerned with the linearity of the system. As long as the system is linear, all input forms should give the same expected value. Naturally, though, all real systems have some degree of nonlinearity. Deterministic input signals result in frequency response functions that are dependent upon the signal level and type. A set of frequency response functions for different signal levels can be used to document the nonlinear characteristics of the system. Random input signals, in the presence of nonlinearities, result in a frequency response function that represents the best linear representation of the nonlinear characteristics for a given level of random signal input. For small nonlinearities, use of a random input will not differ greatly from the use of a deterministic input.

The characterization of the parameter estimation is primarily concerned with the type of mathematical model being used to represent the frequency response function. Generally, the model is a linear summation based upon the modal parameters of the system. Unless the mathematical representation of all nonlinearities is known, the parameter estimation process cannot properly weight the frequency response function data to include nonlinear effects. For this reason, random input signals are prevalently used to obtain the best linear estimate of the frequency response function when a parameter estimation process using a linear model is to be utilized.

The expected utilization of the data is concerned with the degree of detailed information required by any post-processing task. For experimental modal analysis, this can range from implicit modal vectors needed for trouble-shooting to explicit modal vectors used in an orthogonality check. As more detail is required, input signals, both random and deterministic, will need to match the system characteristics and parameter estimation characteristics more closely. In all possible uses of frequency response function data, the conflicting requirements of the need for accuracy, equipment availability, testing time, and testing cost will normally reduce the possible choices of input signal.

With respect to the reduction of the variance and bias errors of the frequency response function,

random or deterministic signals can be utilized most effectively if the signals are periodic with respect to the sample period or totally observable with respect to the sample period. If either of these criteria are satisfied, regardless of signal type, the predominant bias error, leakage, will be eliminated. If these criteria are not satisfied, the leakage error may become significant. In either case, the variance error will be a function of the signal-to-noise ratio and the amount of averaging.

5.3.4 Random Excitation Methods

Inputs which can be used to excite a system in order to determine frequency response functions (FRFs) belong to one of two classifications, random or deterministic [6-8]. Random signals are widely utilized for general single-input and multiple-input shaker testing when evaluating structures that are essentially linear. Signals of this form can only be defined by their statistical properties over some time period. Any subset of the total time period is unique and no explicit mathematical relationship can be formulated to describe the signal. Random signals can be further classified as stationary or non-stationary. Stationary random signals are a special case where the statistical properties of the random signals do not vary with respect to translations with time. Finally, stationary random signals can be classified as ergodic or non-ergodic. A stationary random signal is ergodic when a time average on any particular subset of the signal is the same for any arbitrary subset of the random signal. All random signals which are commonly used as input signals fall into the category of ergodic, stationary random signals. Deterministic signals can be characterized directly by mathematical formula and the characteristic of the excitation signal can be computed for any instance in time. While this is true for the theoretical signal sent to the exciter, it is only approximately true for the actual excitation signal due to the amplifier/shaker/structure interaction that is a function of the impedances of these electro-mechanical systems. Deterministic signals can, nevertheless, be controlled more precisely and are frequently utilized in the characterization of nonlinear systems for this reason. The random classification of excitation signals is the only signal type discussed in this paper.

The choice of input to be used to excite a system in order to determine frequency response functions depends upon the characteristics of the system, upon the characteristics of the modal parameter estimation, and upon the expected utilization of the data. The characterization of the system is primarily concerned with the linearity of the system. As long as the system is linear, all input forms should give the same expected value. Naturally, though, all real systems have some degree of nonlinearity. Deterministic input signals result in frequency response functions

that are dependent upon the signal level and type. A set of frequency response functions for different signal levels can be used to document the nonlinear characteristics of the system. Random input signals, in the presence of nonlinearities, result in a frequency response function that represents the best linear representation of the nonlinear characteristics for a given RMS level of random signal input. For systems with small nonlinearities, use of a random input will not differ greatly from the use of a deterministic input.

The characterization of the modal parameter estimation is primarily concerned with the type of mathematical model being used to represent the frequency response function. Generally, the model is a linear summation based upon the modal parameters of the system. Unless the mathematical representation of all nonlinearities is known, the parameter estimation process cannot properly weight the frequency response function data to include nonlinear effects. For this reason, random input signals are prevalently used to obtain the best linear estimate of the frequency response function when a parameter estimation process using a linear model is to be utilized.

The expected utilization of the data is concerned with the degree of detailed information required by any post-processing task. For experimental modal analysis, this can range from implicit modal vectors needed for trouble-shooting to explicit modal vectors used in an orthogonality check. As more detail is required, input signals, both random and deterministic, will need to match the system characteristics and parameter estimation characteristics more closely. In all possible uses of frequency response function data, the conflicting requirements of the need for accuracy, equipment availability, testing time, and testing cost will normally reduce the possible choices of input signal.

With respect to the reduction of the variance and bias errors of the frequency response function, random or deterministic signals can be utilized most effectively if the signals are periodic with respect to the sample period or totally observable with respect to the sample period. If either of these criteria are satisfied, regardless of signal type, the predominant bias error, leakage, will be minimized. If these criteria are not satisfied, the leakage error may become significant. In either case, the variance error will be a function of the signal-to-noise ratio and the amount of averaging.

Many signals are appropriate for use in experimental modal analysis. Some of the most commonly used random signals, used with single and multiple input shaker testing, are described in the following sections.

Pure Random - The pure random signal is an ergodic, stationary random signal which has a Gaussian probability distribution. In general, the frequency content of the signal contains energy at all frequencies (not just integer multiples of the FFT frequency increment ($\Delta f = 1/T$)). This characteristic is shown in Figure 5-14. This is undesirable since the frequency information between the FFT frequencies is the cause of the leakage error. The pure random signal may be filtered ($F_{\min}$ to $F_{\max}$) to include only information in a frequency band of interest. The measured input spectrum of the pure random signal, as with all random signals, will be altered by any impedance mismatch between the system and the exciter. The number of power spectral averages used in the pure random excitation approach is a function of the reduction of the variance error and the need to have a significant number of averages to be certain that all frequencies have been adequately excited.

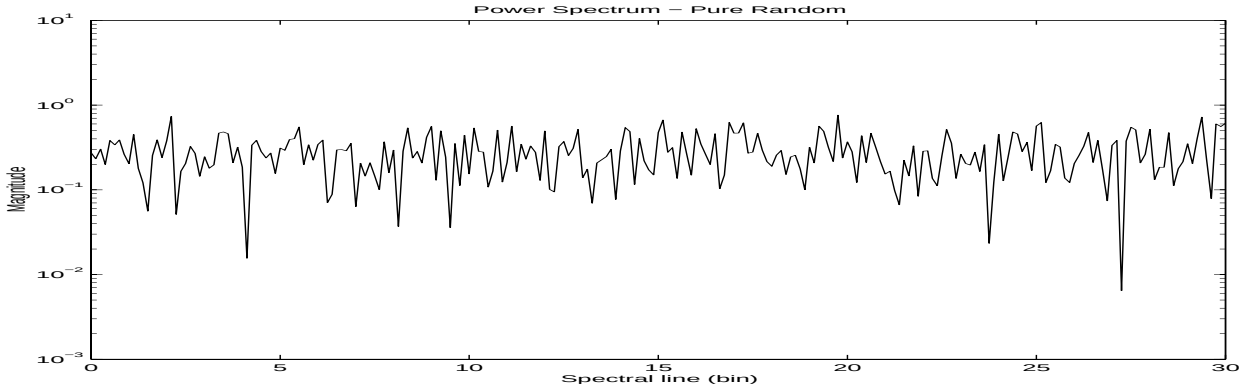

Figure 5-14. Signal Energy Content - Pure Random

Pseudo Random - The pseudo random signal is an ergodic, stationary random signal consisting of energy content only at integer multiples of the FFT frequency increment (Δf). The frequency spectrum of this signal is shaped to have a constant amplitude with random phase. This characteristic is shown in Figure 5-15. If sufficient delay time is allowed in the measurement procedure for any transient response to the initiation of the signal to decay (number of delay blocks), the resultant input and output histories are periodic with respect to the sample period. The number of power spectral averages used in the pseudo random excitation approach is a function of the reduction of the variance error. In a noise free environment, only one average (per input) may be necessary.

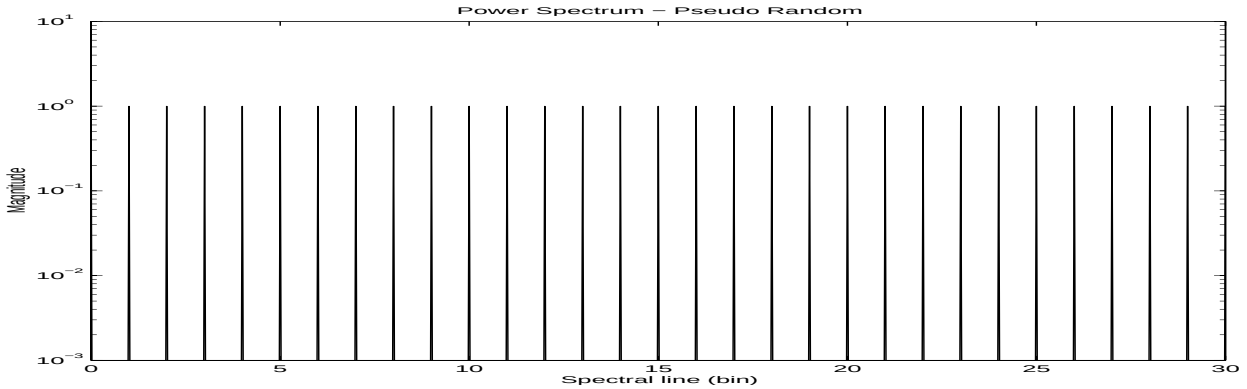

Figure 5-15. Signal Energy Content - Pseudo Random

Periodic Random **-** The periodic random signal is an ergodic, stationary random signal consisting only of integer multiples of the FFT frequency increment. The frequency spectrum of this signal has random amplitude and random phase distribution. This characteristic is shown in Figure 5-16. For each average, input signal(s) are created with random amplitude and random phase. The system is excited with these input(s) in a repetitive cycle until the transient response to the change in excitation signal decays (number of delay blocks). The input and response histories should then be periodic with respect to the observation time (T) and are recorded as one power spectral average in the total process. With each new average, a new history, random with respect to previous input signals, is generated so that the resulting measurement will be completely randomized. The number of power spectral averages used in the periodic random excitation approach is a function of the reduction of the variance error and the need to have a significant number of averages to be certain that all frequencies have been adequately excited.

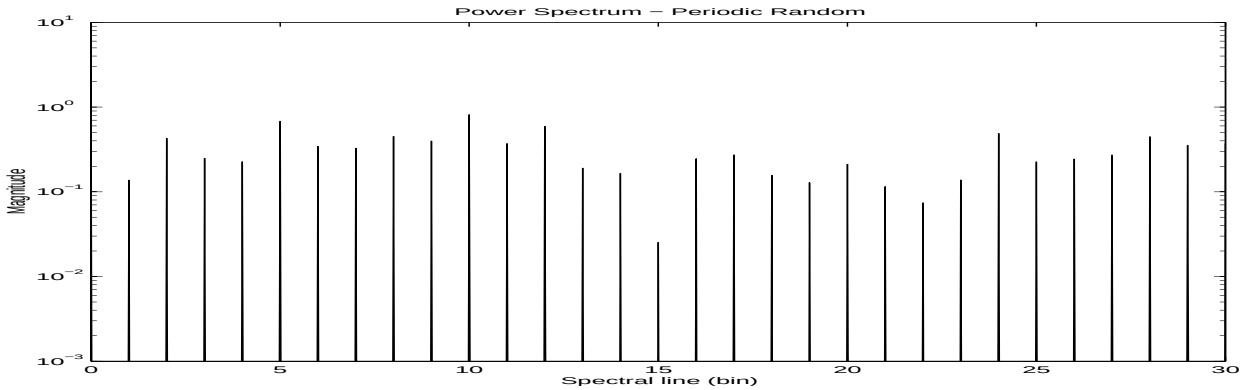

Figure 5-16. Signal Energy Content - Periodic Random

Burst Random (Random Transient) - The burst random signal is neither a completely transient deterministic signal nor a completely ergodic, stationary random signal but contains properties of both signal types. The frequency spectrum of this signal has random amplitude and random phase distribution and contains energy throughout the frequency spectrum. This characteristic is shown in Figure 5-17. The difference between this signal and the random signal is that the random transient history is truncated to zero after some percentage of the observation time (T). Normally, an acceptable percentage is fifty to eighty percent. The measurement procedure duplicates the random procedure but without the need to utilize a window to reduce the leakage problem as long as both the input and output decays to zero in the observation time (*T*).

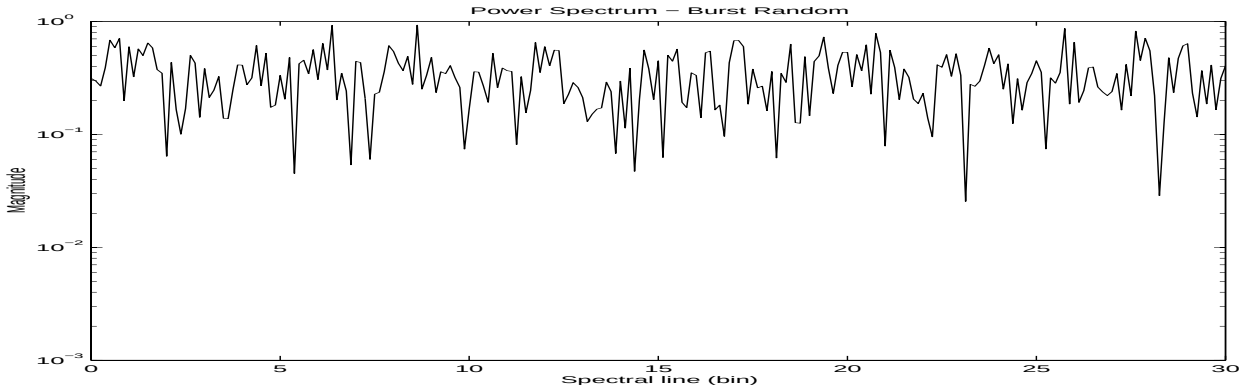

Figure 5-17. Signal Energy Content - Burst Random

The burst length (0-100%) is chosen so that the response history decays to zero within the observation time (T). For moderate to heavily damped systems, the response history will decay to zero very quickly due to the damping provided by the system being tested. These systems do not cause a leakage error in the first place.

For lightly damped cases, burst random will force the response to decay to zero in the observation time (*T*) primarily due to the exciter system characteristics. Exciter systems, particularly electromagnetic, attempt to match the excitation signal to some physical characteristic of the exciter. Typically, this means that the displacement, velocity or acceleration of the armature of the shaker will attempt to match the excitation signal. (Note that this is normally an open loop control process; no attempt is made to exactly match the excitation signal.) Electromagnetic shaker systems work either in a voltage or current feedback configuration in order to control the shaker according to the desired input signal. ***Voltage feedback*** refers to the type of amplifier in the exciter system that attempt to match the voltage

supplied to the shaker to the excitation signal. This effectively means that the displacement of the armature will follow the excitation signal. Therefore, if a zero voltage signal is sent to the exciter system, the exciter will attempt to prevent the armature from moving. This damping force, provided by the exciter/amplifier system, is often overlooked in the analysis of the characteristics of this signal type. Since this measured input, although not part of the generated signal, includes the variation of the input during the decay of the response history, the input and response histories are totally observable within the sample period and the system damping that will be computed from the measured FRF data is unaffected.

Current feedback refers to the type of amplifier in the exciter system that attempt to match the current supplied to the shaker to the excitation signal. This effectively means that the acceleration of the armature will follow the excitation signal. Therefore, if a zero voltage signal is sent to the exciter system, the exciter will allow the armature to move, preventing any force to be applied by the exciter system. The characteristic of a voltage feedback exciter system for a burst random exciation is shown in the following figures. Note the difference between the desired burst random signal and the actual force measured.

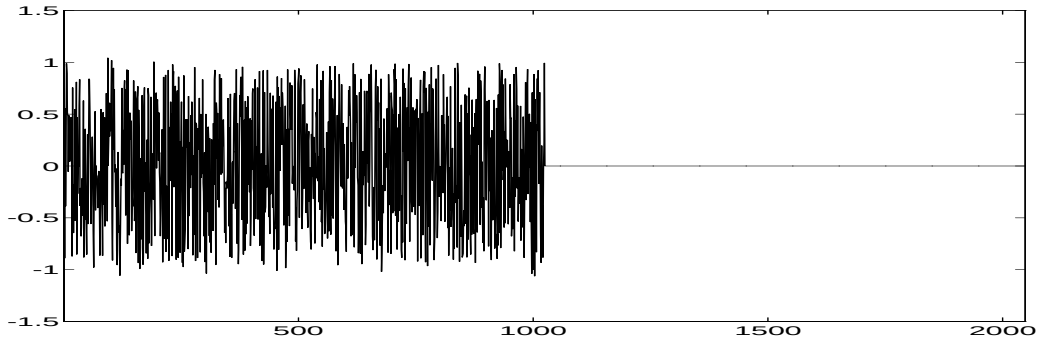

Figure 5-18. Burst Random - Signal to Shaker

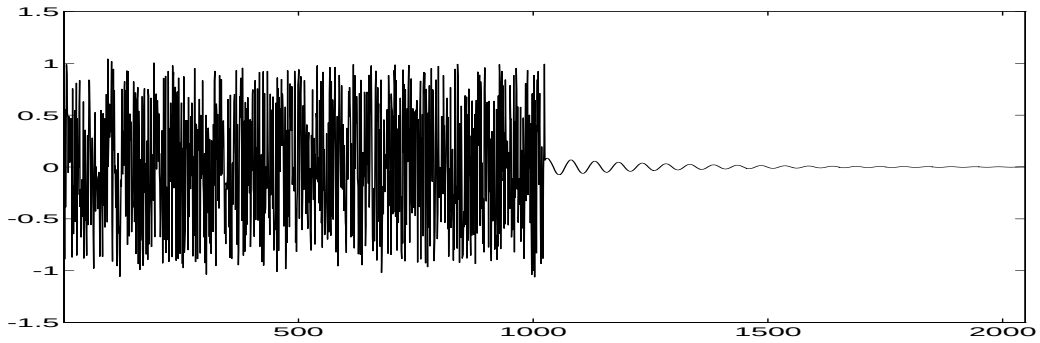

Figure 5-19. Burst Random - Signal from Load Cell (Voltage Feedback)

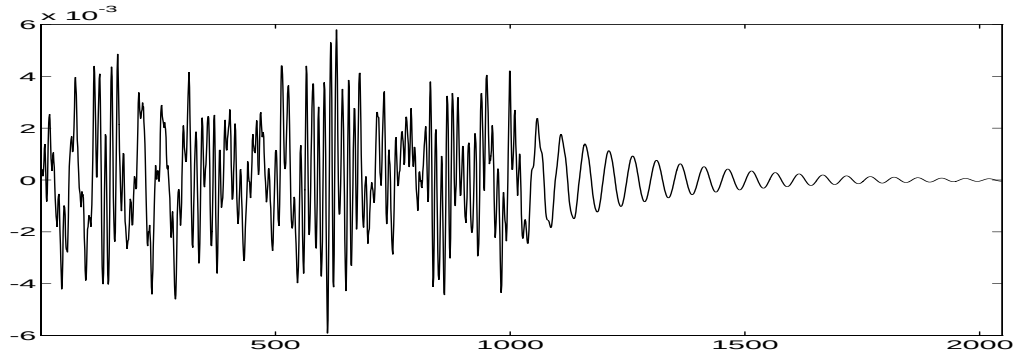

Figure 5-20. Burst Random - Signal from Accelerometer

For very lightly damped systems, the burst length may have to be shortened below 20 percent. This may yield an unacceptable signal to noise ratio (SNR). The number of power spectral averages used in the burst random excitation approach is a function of the reduction of the variance error and the need to have a significant number of averages to be certain that all frequencies have been adequately excited. plus the exciter/amplifier system trying to maintain the input at zero (voltage feedback amplifier in the excitation system).

Slow Random - The slow random signal is an ergodic, stationary random signal consisting only of integer multiples of the FFT frequency increment. This signal behaves just like the pseudo random signal but without the frequency shaping of the amplitude. The slow random signal is generated by cyclic averaging a random signal in order to produce digitally comb filtered excitation signal(s) with the proper characteristics.

MOOZ Random - The MOOZ random signal is an ergodic, stationary random signal consisting only of integer multiples of the FFT frequency increment frequency band limited to the frequency band of a ZOOM fast Fourier transform (FFT)($F_{\min}$ to $F_{\max}$). The MOOZ (ZOOM spelled backwards) random signal requires synchronization between the data acquisition and the digital-to-analog converter (DAC). The MOOZ random signal is essentially a slow random excitation signal adjusted to accommodate the frequencies of a ZOOM FFT.

The relationship between delay blocks and averages for some of the most commonly used random excitation methods are summarized in Table 5-3.

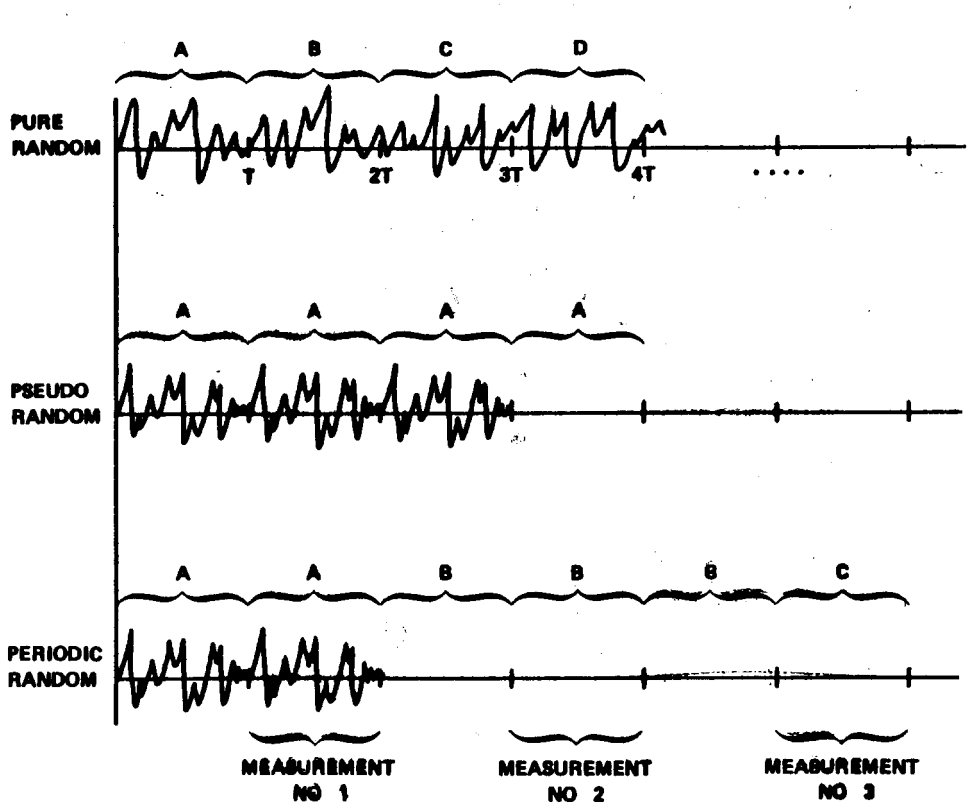

TABLE 5-3. Excitation Characteristics

Hybrid Random Excitation Methods

Several random excitation methods have recently been demonstrated that are hybrid methods involving combinations of burst random and pseudo random, burst random and periodic random together with cyclic averaging.

Burst Pseudo Random - Figure 5-21 shows the energy content of a hybrid excitation method that combines pseudo random with burst random. This excitation signal would be combined with cyclic averaging.

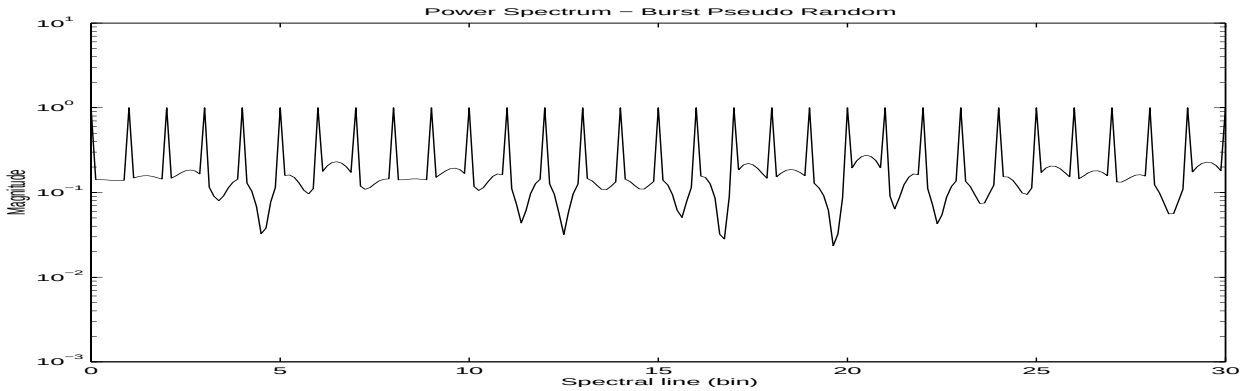

Figure 5-21. Signal Energy Content - Burst Pseudo Random

Burst Periodic Random - Figure 5-22 shows the energy content of a hybrid excitation method that combines periodic random with burst random. This excitation signal would be combined with cyclic averaging.

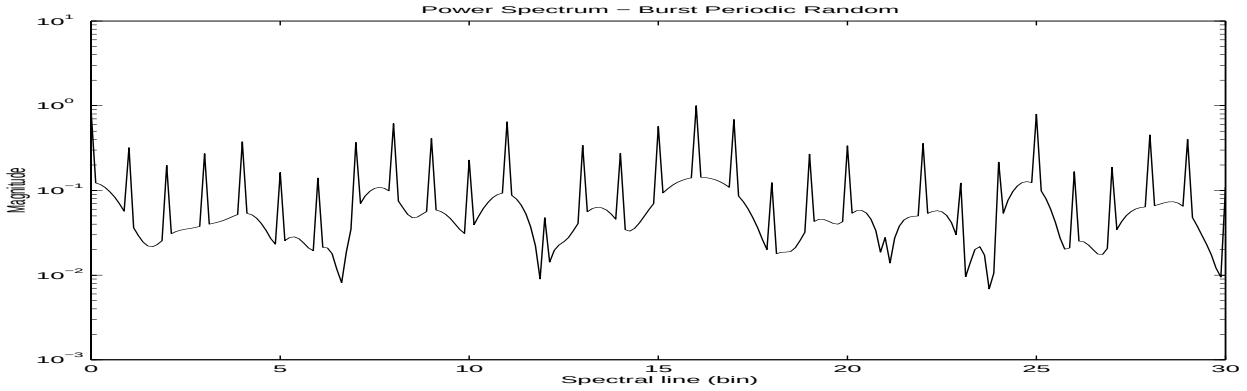

Figure 5-22. Signal Energy Content - Burst Periodic Random

5.3.5 Deterministic Excitation Methods

Slow Swept Sine **-** The slow swept sine signal is a periodic deterministic signal with a frequency that is an integer multiple of the FFT frequency increment. Sufficient time is allowed in the measurement procedure for any transient response to the changes in frequency to decay so that the resultant input and response histories will be periodic with respect to the sample period. Therefore, the total time needed to compute an entire frequency response function will be a function of the number of frequency increments required and the system damping.

Periodic Chirp **-** The periodic chirp is a deterministic signal where a sinusoid is rapidly swept from $F_{\min}$ to $F_{\max}$ within a single observation period (T). This signal is then repeated in a periodic fashion. While this signal is not random in characteristic, it is often included in discussions of random excitation since it has similar properties as pseudo random.

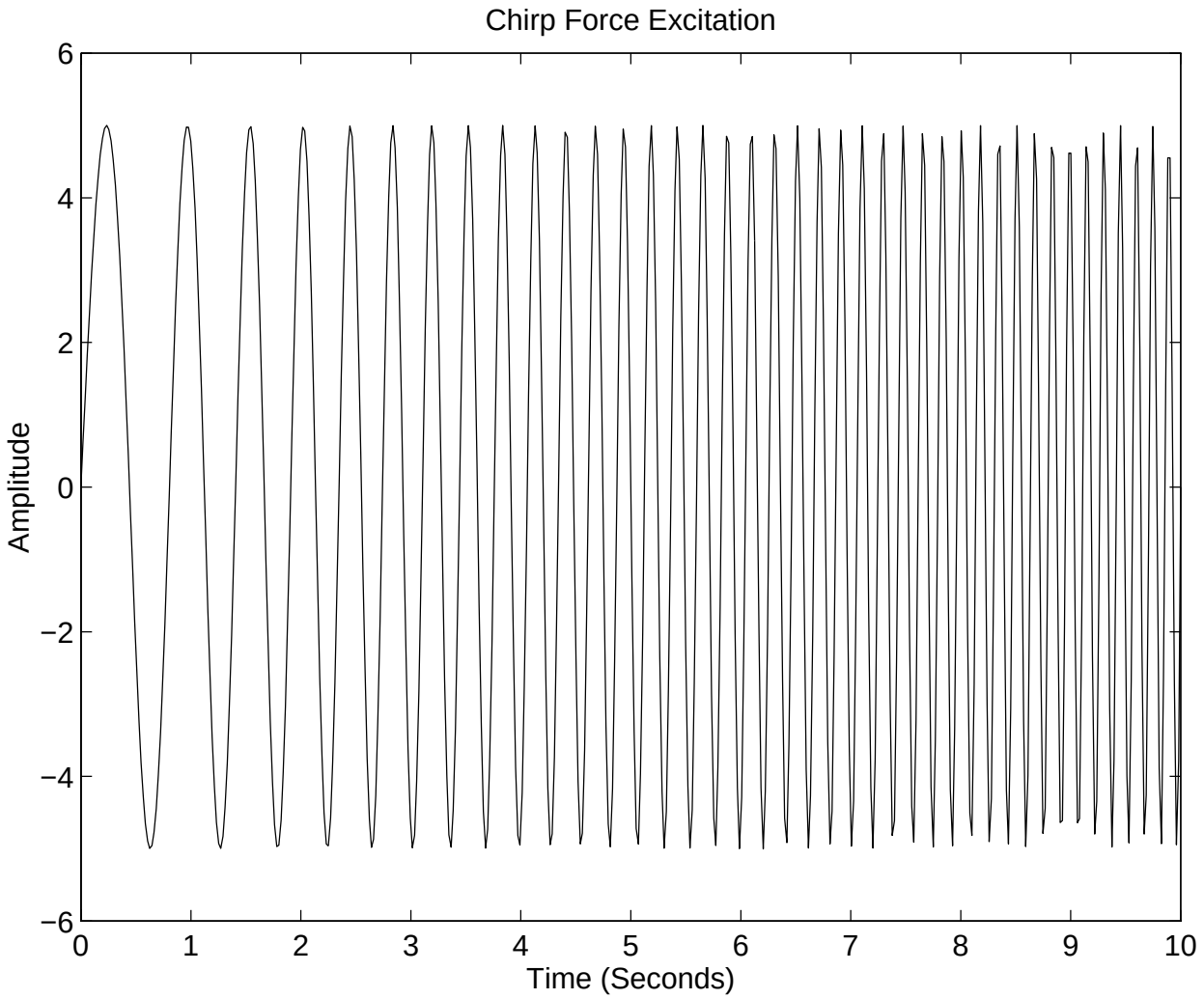

Figure 5-23. Typical Chirp Signal - Time Domain

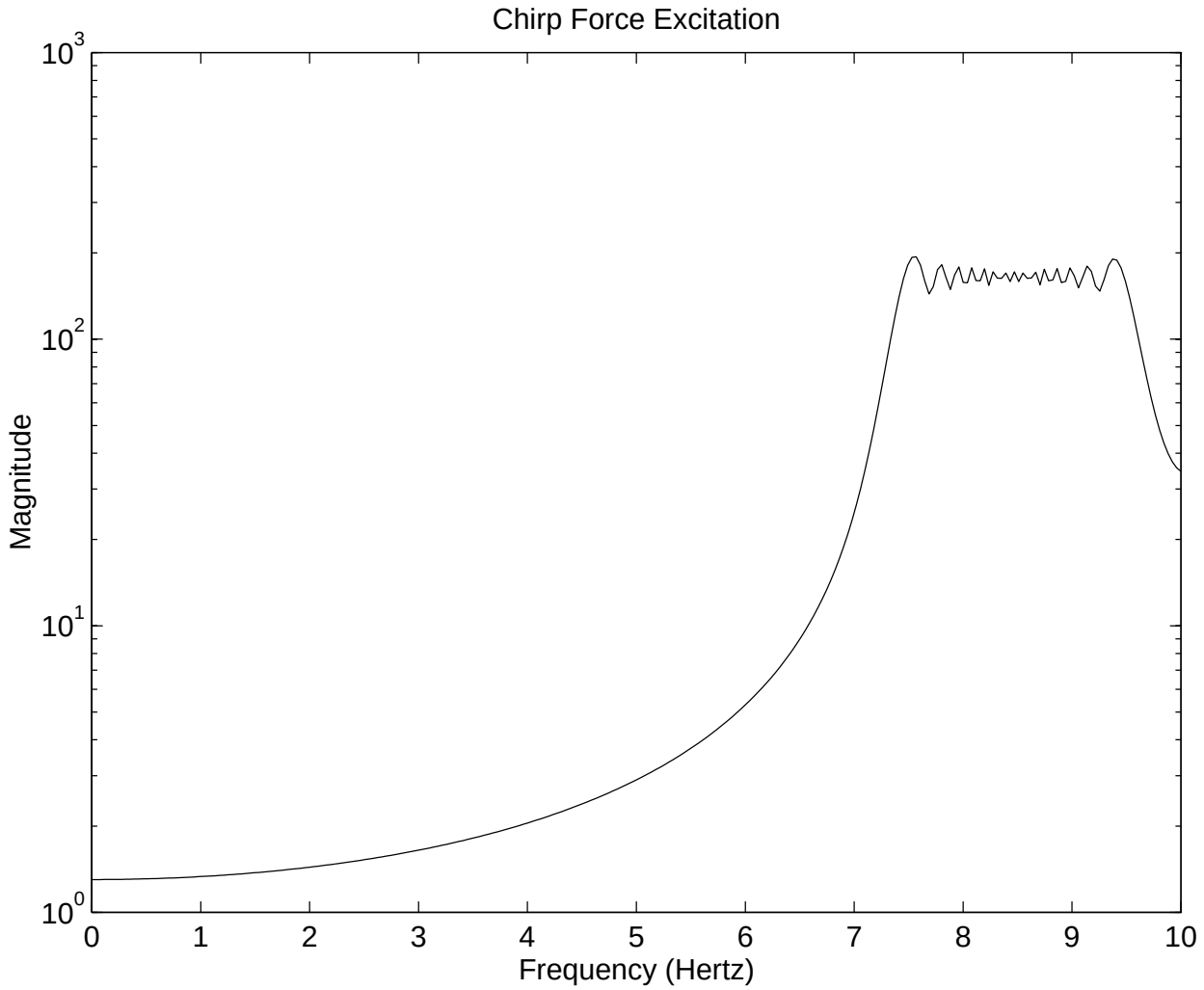

Figure 5-24. Typical Chirp Signal - Frequency Domain

Impact (Impulse) **-** The impact signal is a transient deterministic signal which is formed by applying an input pulse to a system lasting only a very small part of the sample period. The width, height, and shape of this pulse will determine the usable spectrum of the impact. Briefly, the width of the pulse will determine the frequency spectrum while the height and shape of the pulse will control the level of the spectrum. Impact signals have proven to be quite popular due to the freedom of applying the input with some form of an instrumented hammer. While the concept is straight forward, the effective utilization of an impact signal is very involved [33,34,37].

Step Relaxation - The step relaxation signal is a transient deterministic signal which is formed by releasing a previously applied static input. The sample period begins at the instant that the release occurs. This signal is normally generated by the application of a static force through a cable. The cable is then cut or allowed to release through a shear pin arrangement [39] .

Table 5-4 summarizes the advantages and disadvantages for the most commonly used excitation signals.

Excitation Signal Characteristics								
	Steady State Sine	Pure Random	Pseudo Random	Periodic Random	Fast Sine	Impact	Burst Sine	Burst Random
Minimize Leakage	No	No	Yes	Yes	Yes	Yes	Yes	Yes
Signal-to-Noise Ratio	Very High	Fair	Fair	Fair	High	Low	High	Fair
RMS-to-Peak Ratio	High	Fair	Fair	Fair	High	Low	High	Fair
Test Measurement Time	Very Long	Good	Very Short	Fair	Fair	Very Short	Very Short	Very Short
Controlled Frequency Content	Yes	Yes *	Yes *	Yes *	Yes *	No	Yes *	Yes *
Controlled Amplitude Content	Yes	No	Yes *	No	Yes *	No	Yes *	No
Removes Distortion	No	Yes	No	Yes	No	No	No	Yes
Characterize Nonlinearity	Yes	No	No	No	Yes	No	Yes	No

* Special Hardware Required

TABLE 5-4. Summary of Excitation Signals

5.3.6 Excitation Example - H-Frame

The following example presents a single FRF measurement on an H-frame test structure in a test lab environment as a representative example. The configuration of the test involved two shaker locations (inputs) and eight response accelerometers (outputs). The test results are representative of all data taken on the H-frame structure. This H-frame test structure is very lightly damped and has been the subject of many previous studies.

For all FRF measurement cases, the same test configuration was used. Sensors were installed and left in place; no additions or changes were made to the test configuration other than altering the excitation, averaging and digital signal processing parameters. Therefore, any changes in the FRF measurements are assumed to be due to the change in measurement technique and not due to a test set-up variation. The test results were repeated to be certain that the results are representative.

All FRF measurements are estimated using the H_1 estimation algorithm using 1024 spectral (frequency) lines of information. The frequency bandwidth is from 0 to 250 Hertz for the 1024 spectral lines; only the first 80 % of the spectral lines (0 to 200 Hertz) are displayed in order to exclude the data affected by the anti-aliasing filters.

The FRF data is plotted with phase above log magnitude. The log magnitude portion of the plot also contains the relevant multiple coherence plotted on a linear scale in the background. The log magnitude scaling is annotated on the left side of the plot and the multiple coherence scaling is annotated on the right side of the plot.

Fourteen representative cases were measured on this structure. The relevant excitation and digital signal processing characteristics of each case are shown in Table 5-5.

Case	Signal Type	Frequency Shaping	Periodic Function	Burst Length	Window Function	N_d	N_c	N_{avg}	Total Blocks
Case 1	Random	No	No	No	Hann	0	1	20	20
Case 2	Random	No	No	No	Hann	0	5	4	20
Case 3	Random	No	No	Yes (75%)	Uniform	0	5	4	20
Case 4	Random	Pseudo	No	No	Uniform	4	1	4	20
Case 5	Random	No	Yes	No	Uniform	4	1	4	20
Case 6	Random	Pseudo	No	No	Uniform	3	1	5	20
Case 7	Random	No	Yes	No	Uniform	3	1	5	20
Case 8	Random	Pseudo	No	Yes (75%)	Uniform	0	5	4	20
Case 9	Random	No	Yes	Yes (75%)	Uniform	0	5	4	20
Case 10	Random	No	No	Yes (75%)	Uniform	0	8	12	20
Case 11	Random	No	No	No	Hann	0	1	96	96
Case 12	Random	No	No	No	Hann	0	8	12	96
Case 13	Random	Pseudo	No	No	Uniform	3	2	4	20
Case 14	Random	No	Yes	No	Uniform	3	2	4	20

TABLE 5-5. Test Cases - Excitation/Averaging/DSP Parameters

Case 1 (Figure 5-25) is considered a baseline case since this a very popular method for making a FRF measurement and it can be easily made on all data acquisition equipment. However, it is clear that in this measurement situation, there is a significant drop in the multiple coherence function at frequencies consistent with the peaks in the FRF measurement. This characteristic drop in multiple (or ordinary) coherence is often an indication of a leakage problem. This can be confirmed if a leakage reduction method reduces or eliminates the problem when the measurement is repeated. In all subsequent cases, the test configuration was not altered in any way - data was acquired simply using different excitation, averaging and digital signal processing combinations.

Case 2 (Figure 5-26) demonstrates an improvement over Case 1 when the same total measurement time is used but cyclic averaging is used to reduce the leakage error. Case 3 (Figure 5-27) further demonstrates that burst random with cyclic averaging improves the measurement further. Again the total measurement time remains the same.

Cases 4 through 7 (Figures 5-28 through 5-31) demonstrate the quality of FRF measurements

that can be achieved with pseudo and periodic random excitation methods with very few power spectral averages.

Cases 8 and 9 (Figures (5-32 through 5-33) are hybrid techniques involving the combination of burst random with pseudo and periodic random excitation together with cyclic averaging.

Case 10 (Figure 5-34) demonstrates that Case 3 can be marginally improved with more averages, both cyclic and power spectral averages. However, Case 11 (Figure 5-35) demonstrates that Case 1 (Random with Hann Window) cannot be improved by adding power spectral averages. This is a popular misconception that adding power spectral averages will improve the FRF estimate. This is clearly not true for this case.

Case 12 (Figure 36) demonstrates that additional cyclic averages, together with power spectral averages, is an improvement over Case 2 but the improvement is not significant considering the additional measurement time.

Finally, Cases 13 and 14 (Figures 5-37 through 5-38) demonstrate that, when pseudo and periodic random excitation is coupled with cyclic averaging, a nearly perfect FRF measurement results. Note also that in almost every case where high quality FRF measurements have been been achieved, window functions are not required so correction for the window characteristics is unnecessary.

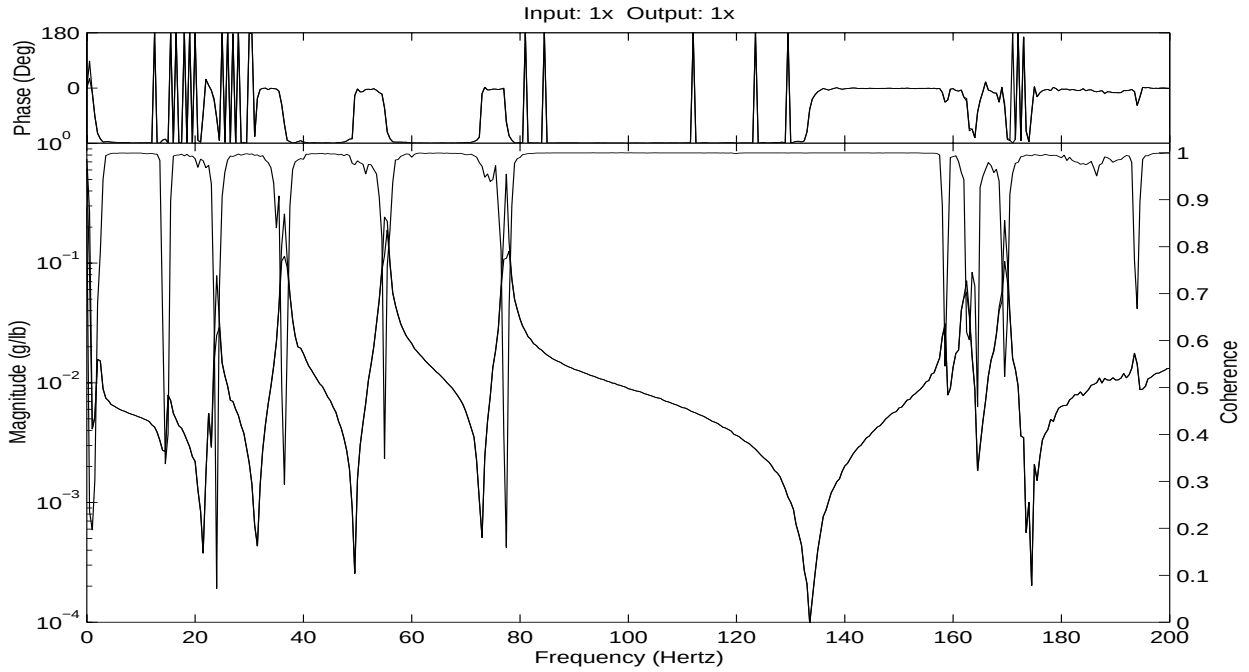

Figure 5-25. Case 1: Random Excitation with Hann Window

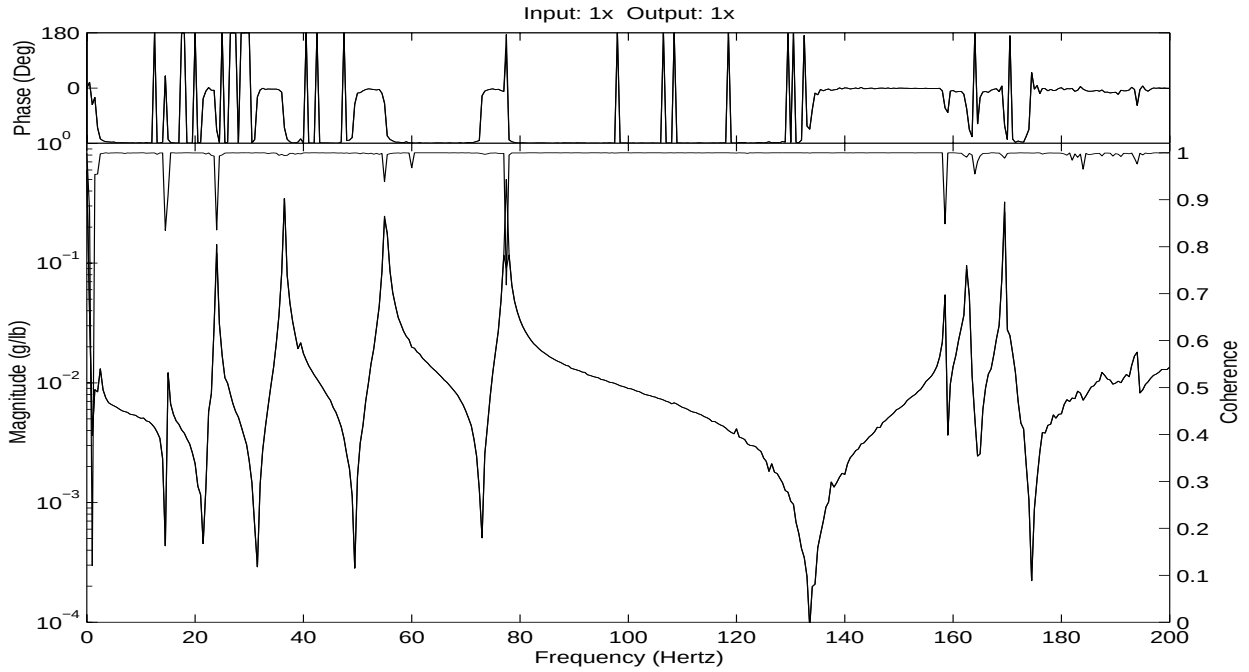

Figure 5-26. Case 2: Random Excitation with Hann Window and Cyclic Averaging

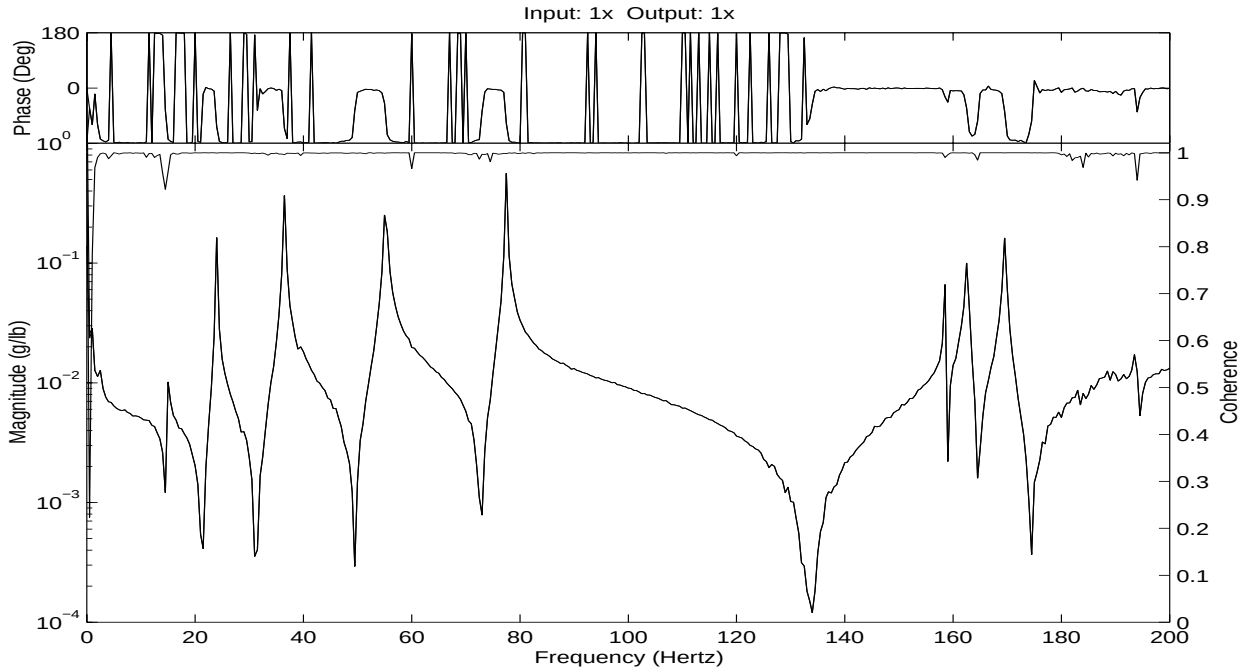

Figure 5-27. Case 3: Burst Random Excitation with Cyclic Averaging

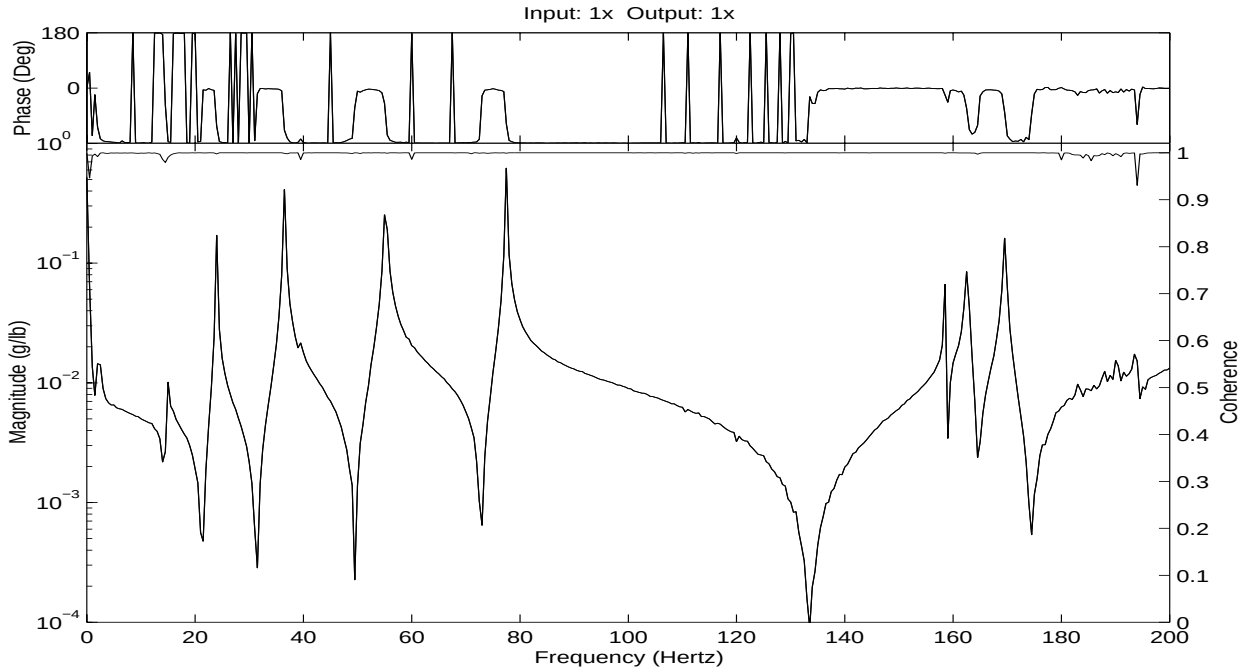

Figure 5-28. Case 4: Pseudo Random Excitation

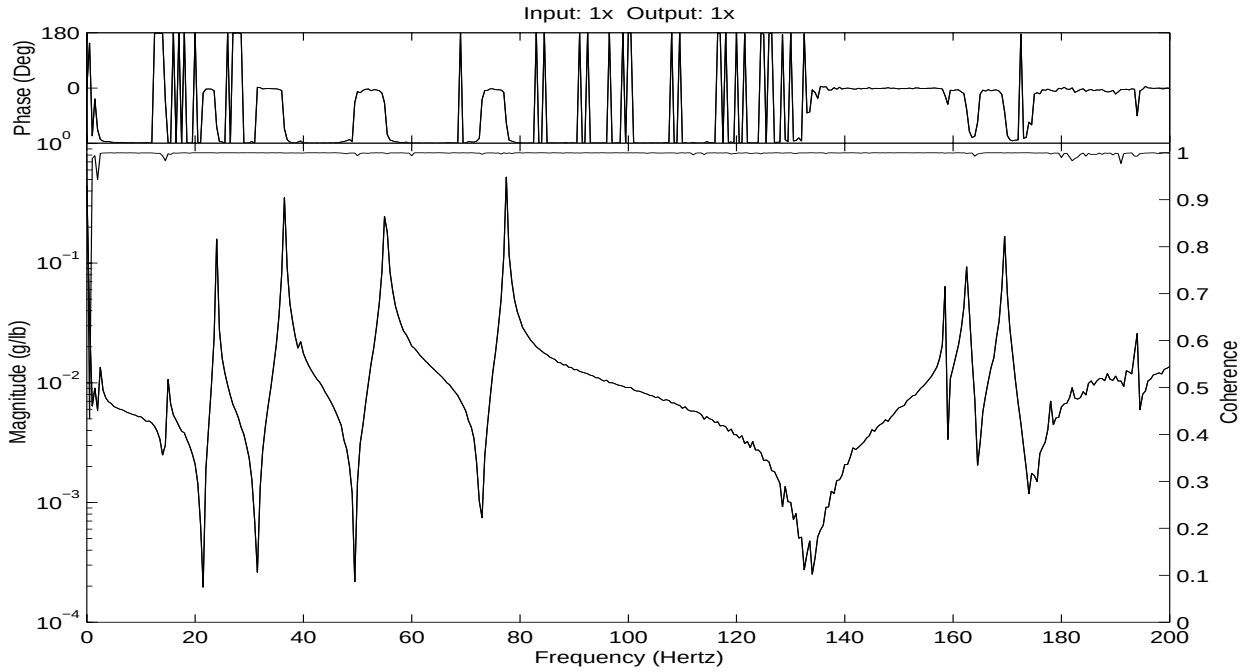

Figure 5-29. Case 5: Periodic Random Excitation

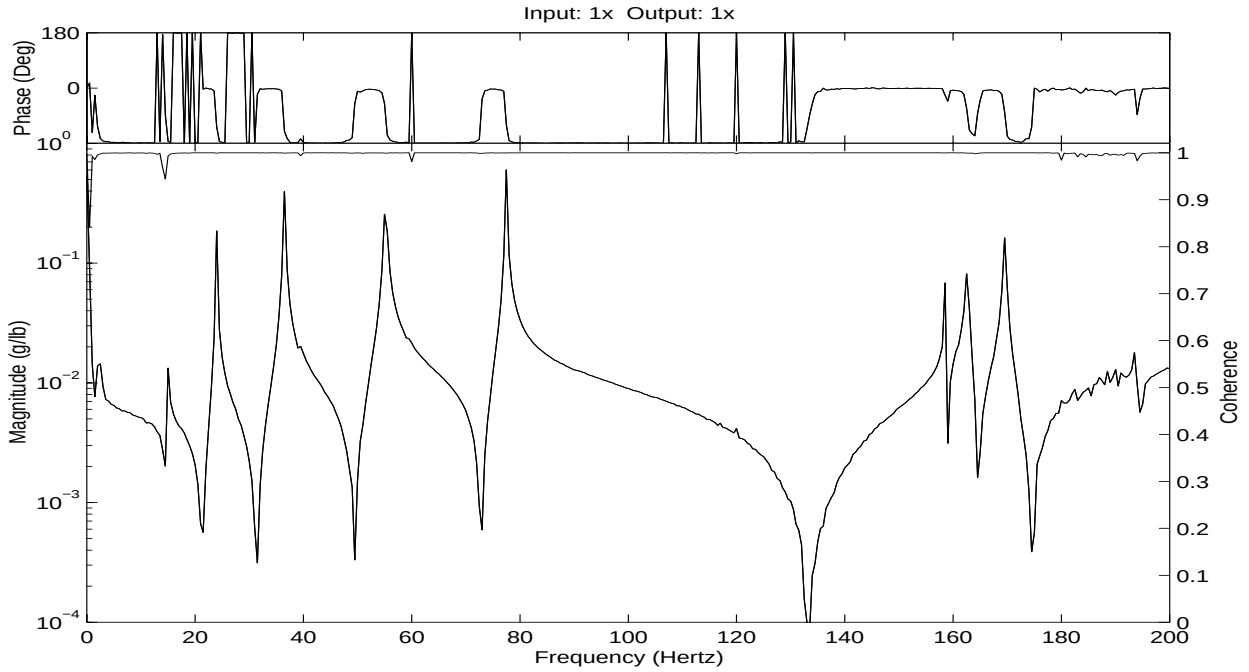

Figure 5-30. Case 6: Pseudo Random Excitation

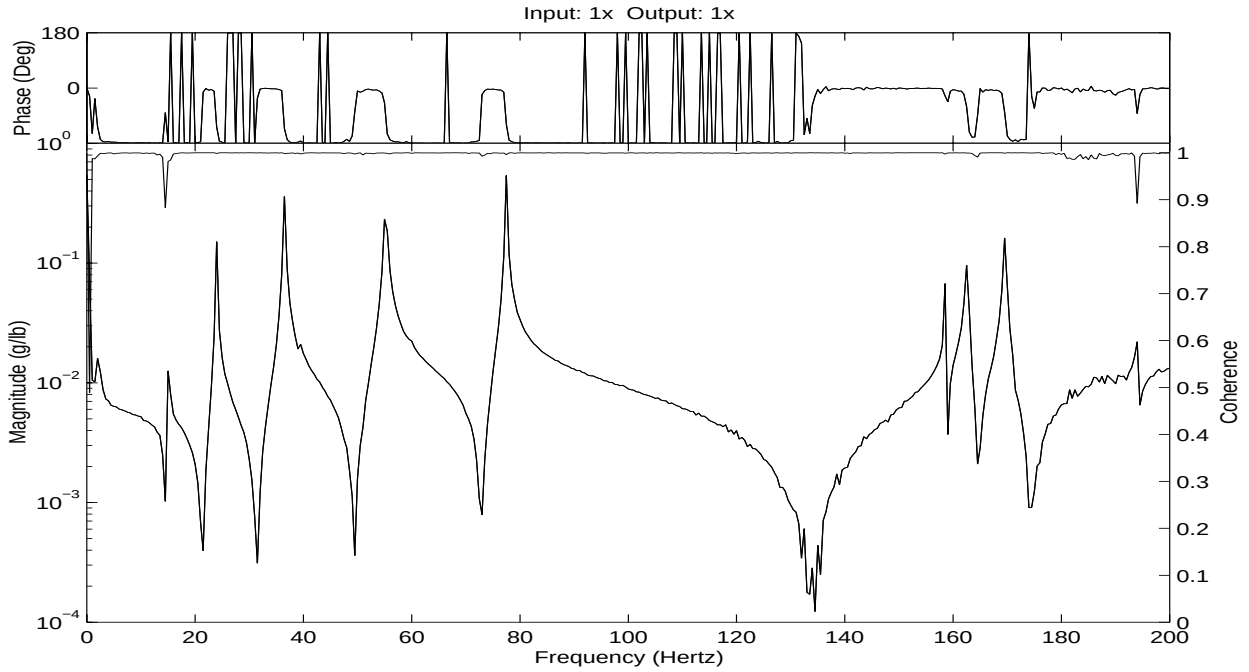

Figure 5-31. Case 7: Periodic Random Excitation

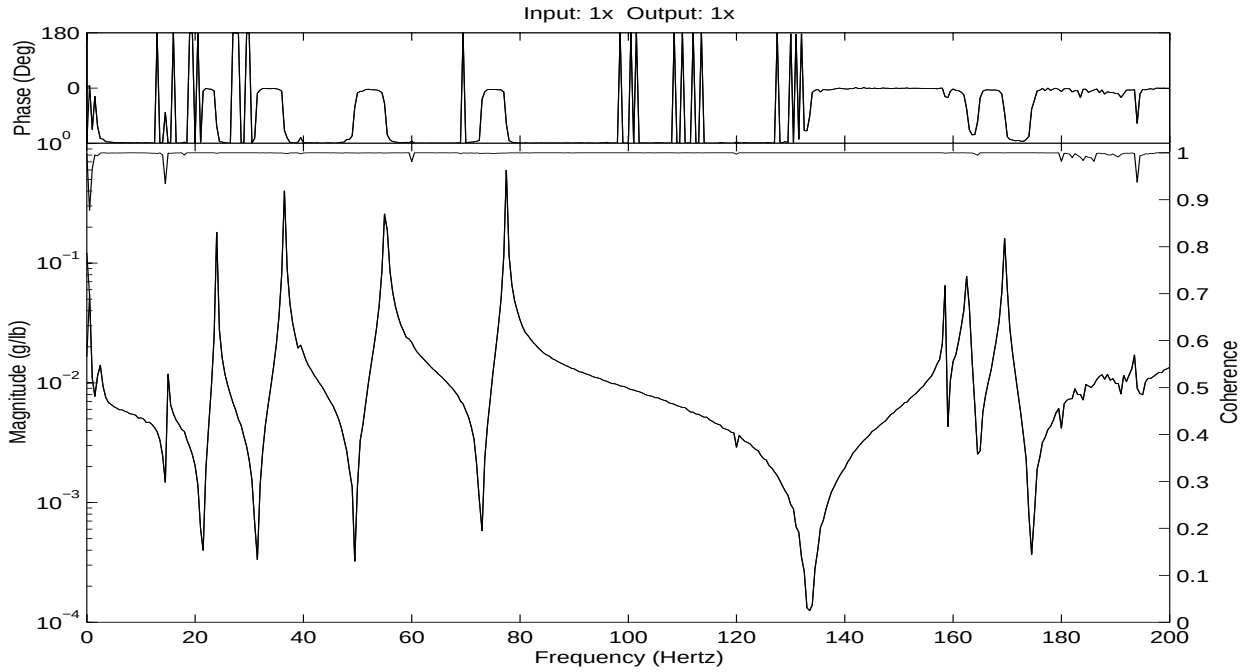

Figure 5-32. Case 8: Burst Pseudo Random Excitation with Cyclic Averaging

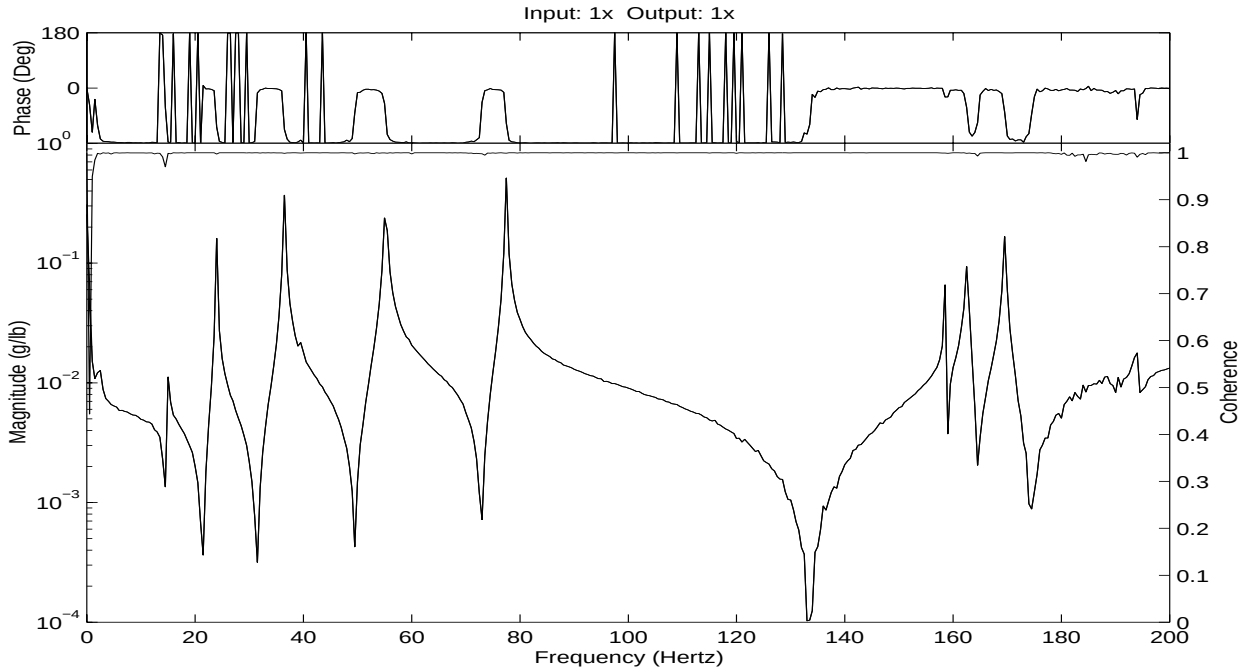

Figure 5-33. Case 9: Burst Periodic Random Excitation with Cyclic Averaging

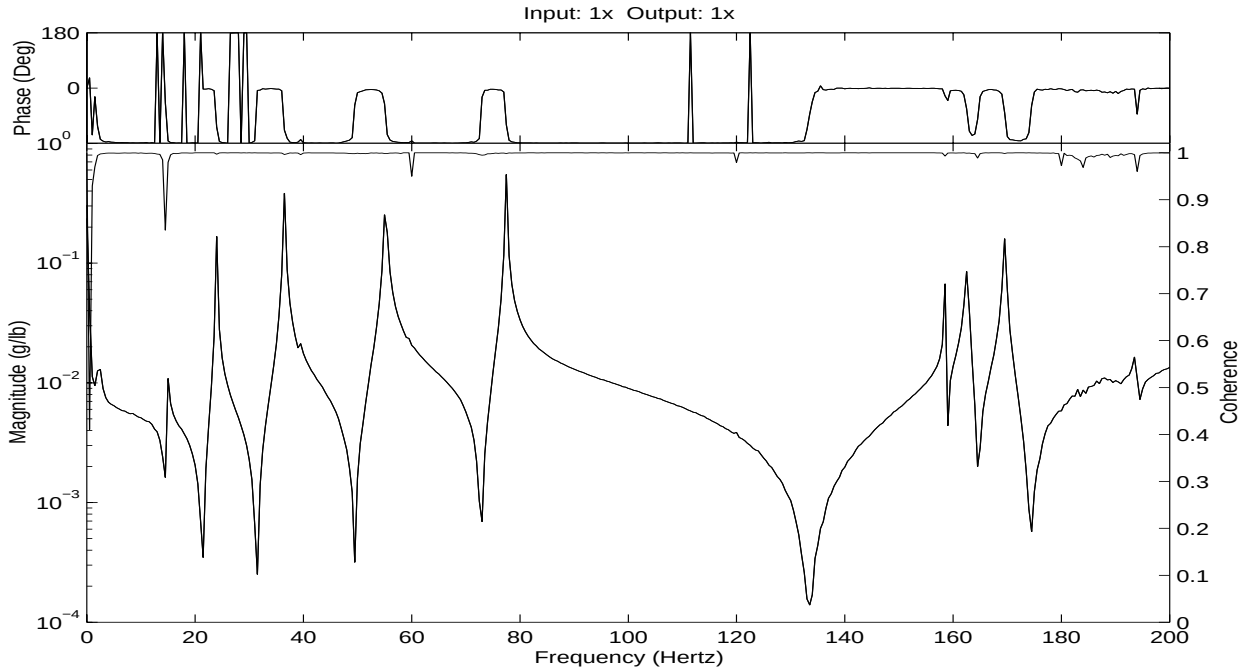

Figure 5-34. Case 10: Burst Random Excitation with Cyclic Averaging

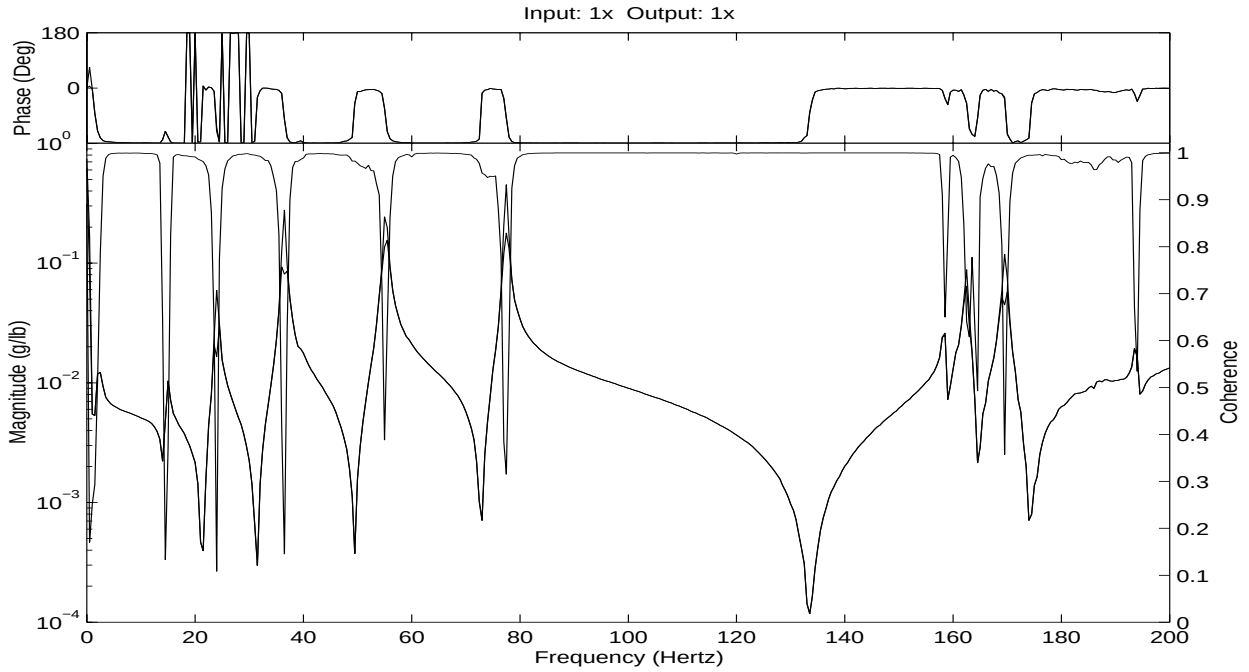

Figure 5-35. Case 11: Random Excitation with Hann Window

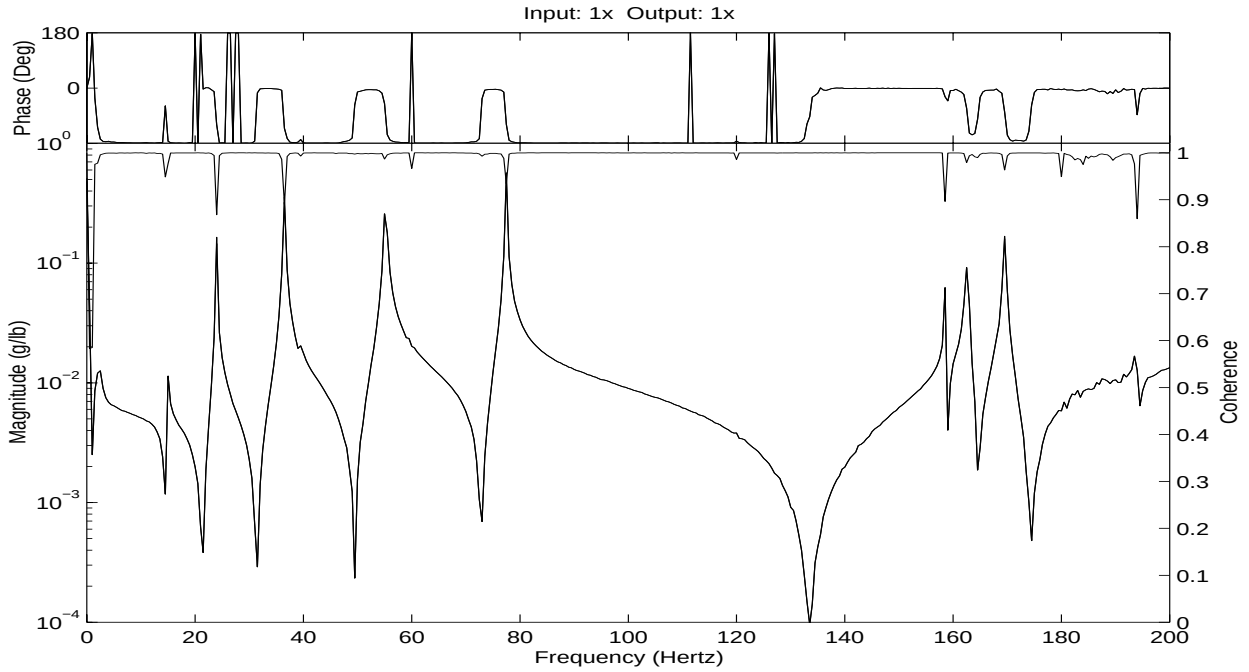

Figure 5-36. Case 12: Random Excitation with Hann Window and Cyclic Averaging

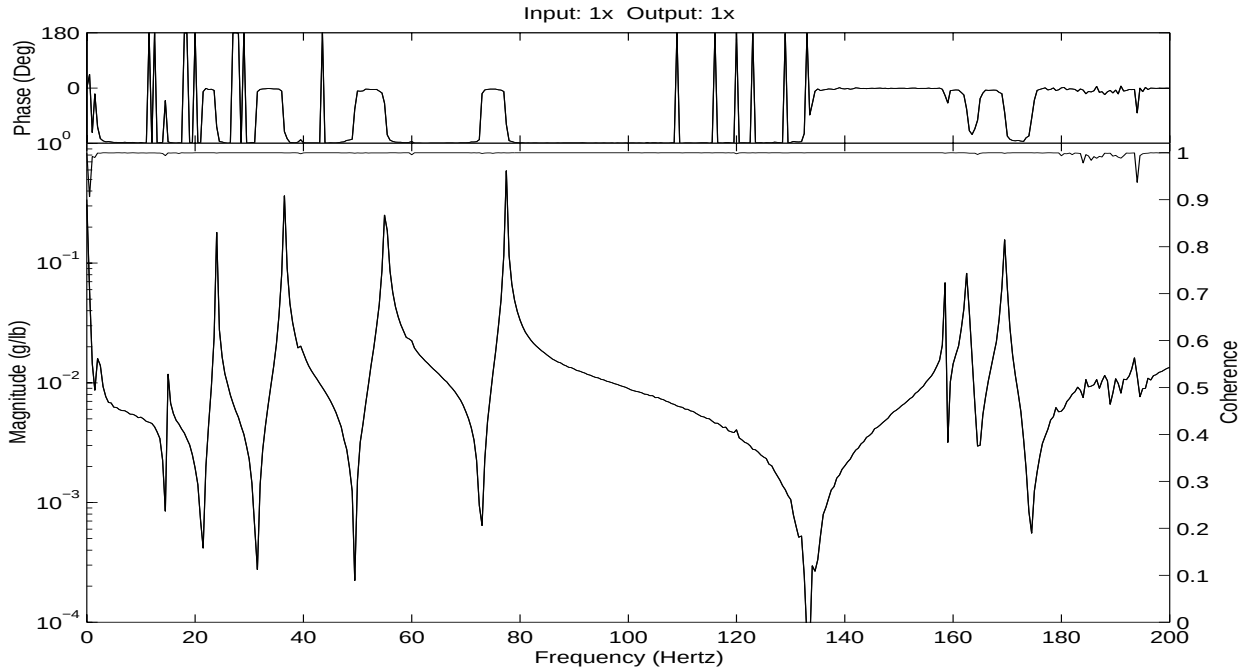

Figure 5-37. Case 13: Pseudo Random Excitation with Cyclic Averaging

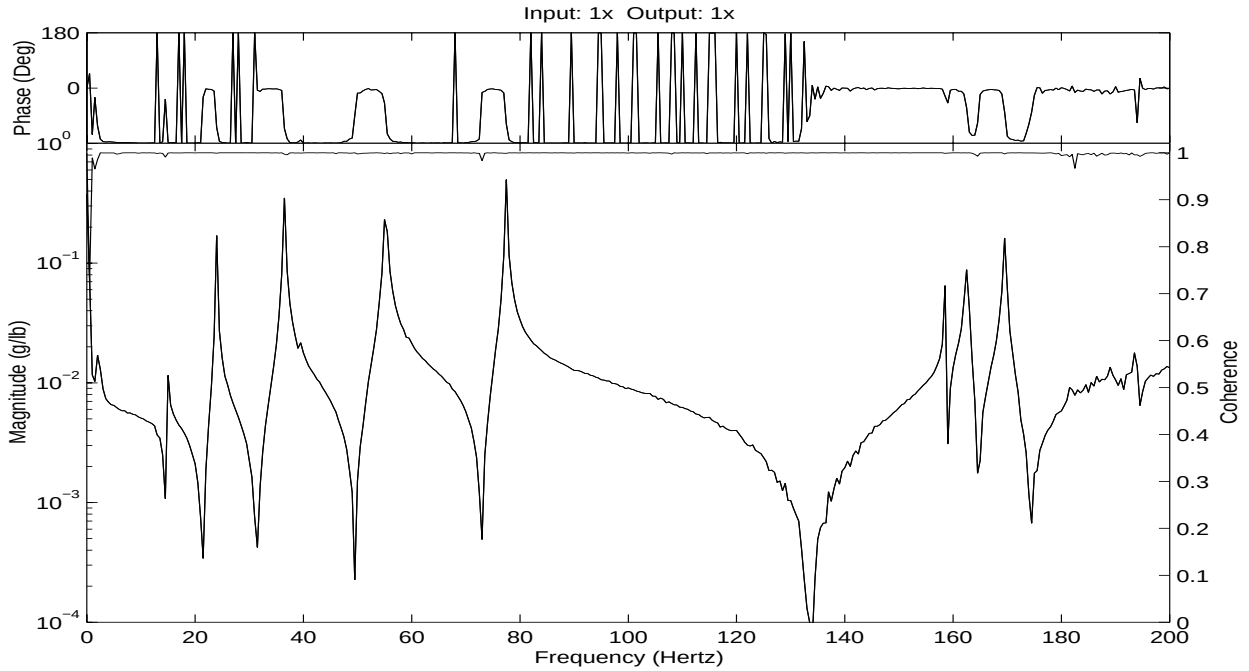

Figure 5-38. Case 14: Periodic Random Excitation with Cyclic Averaging

It is clear that in many of the measurement cases, the multiple coherence can be improved dramatically using simple excitation, averaging and digital signal processing methods. Note that, as the multiple coherence improves, dramatic changes in the FRF magnitude accompany the improvement (factors of 2 to more than 10). When estimating modal parameters, the frequency and mode shape would probably be estimated reasonably in all cases. However, the damping and modal scaling would be distorted (over estimating damping and under estimating modal scaling). Using these results for model prediction or FE correction would bias the predicted results.

The most important conclusion that can be drawn from the results of this measurement exercise on a lightly damped mechanical system is that accurate data is an indirect function of measurement time or number of averages but is a direct function of measurement technique. The leakage problem associated with utilizing fast Fourier transform (FFT) methodology to estimate frequency response functions on a mechanical system with light damping is a serious problem that can be managed with proper measurement techniques, like periodic and pseudo random excitation or cyclic averaging with burst random excitation. Hybrid techniques demonstrated in this paper clearly show that a number of measurement techniques are acceptable but some commonly used techniques are clearly unacceptable.

It is also important to note that while ordinary/multiple coherence can indicate a variety of input/output problems, a drop in the ordinary/multiple coherence function, at the same frequency as a lightly damped peak in the frequency response function, is often a direct indicator of a leakage problem. Frequently, comparisons are made between results obtained with narrowband (sinusoid) excitation and broadband (random) excitation when the ordinary/multiple coherence function clearly indicates a potential leakage problem. It is important that good measurement technique be an integral part of such comparisons.

5.3.7 Impact Excitation

Impact testing is an attempt to match the input and output data to the requirement of the discrete or fast Fourier transform that the data be a totally observed transient in the observation time (T). While the impact is almost always totally observable, the response for lightly damped systems may not be. Special windows are often used for impact testing that accommodate the characteristics of the transient input and the response of the system to a transient input.

Impact excitation is widely used due to the minimal equipment required, portability and low cost of the impact devices and broad applicability to both small, medium and large size structures. However, impact testing also suffers from limitations imposed by the human control of the impact. Repeatability and consistency of the impact (force and direction) cannot be guaranteed, particularly as the test becomes long and repetitious. Care must be taken to ensure that the impact and response is not too small, not too large (overload) and that there is only one impact per observation period.

When impact testing is used, windows are generally required on both the force and response data in order to minimize different errors. The force window is used to eliminate the signal coming from the impact device after the short duration impact is over. This eliminates electrical noise and spurious output from the hammer during data acquisition that is caused by motion of the impact device that does not put force into the system. The response (exponential) window is used to force the response closer to zero by the end of the observation period (T) and should be used carefully. If the response is already near zero at time T, no response window should be added. To be theoretically correct and to allow for the effects of this response window to be accounted for, the decay rate of the exponential must be recorded and the same window should also be applied to the input data, in addition to the force window.

Force Window

Force windows are used to improve the signal-to-noise problem caused by the noise on the input channel measured after the impact is completed. Note that the exponential window used on the response should also be applied to the input in addition to the force window.

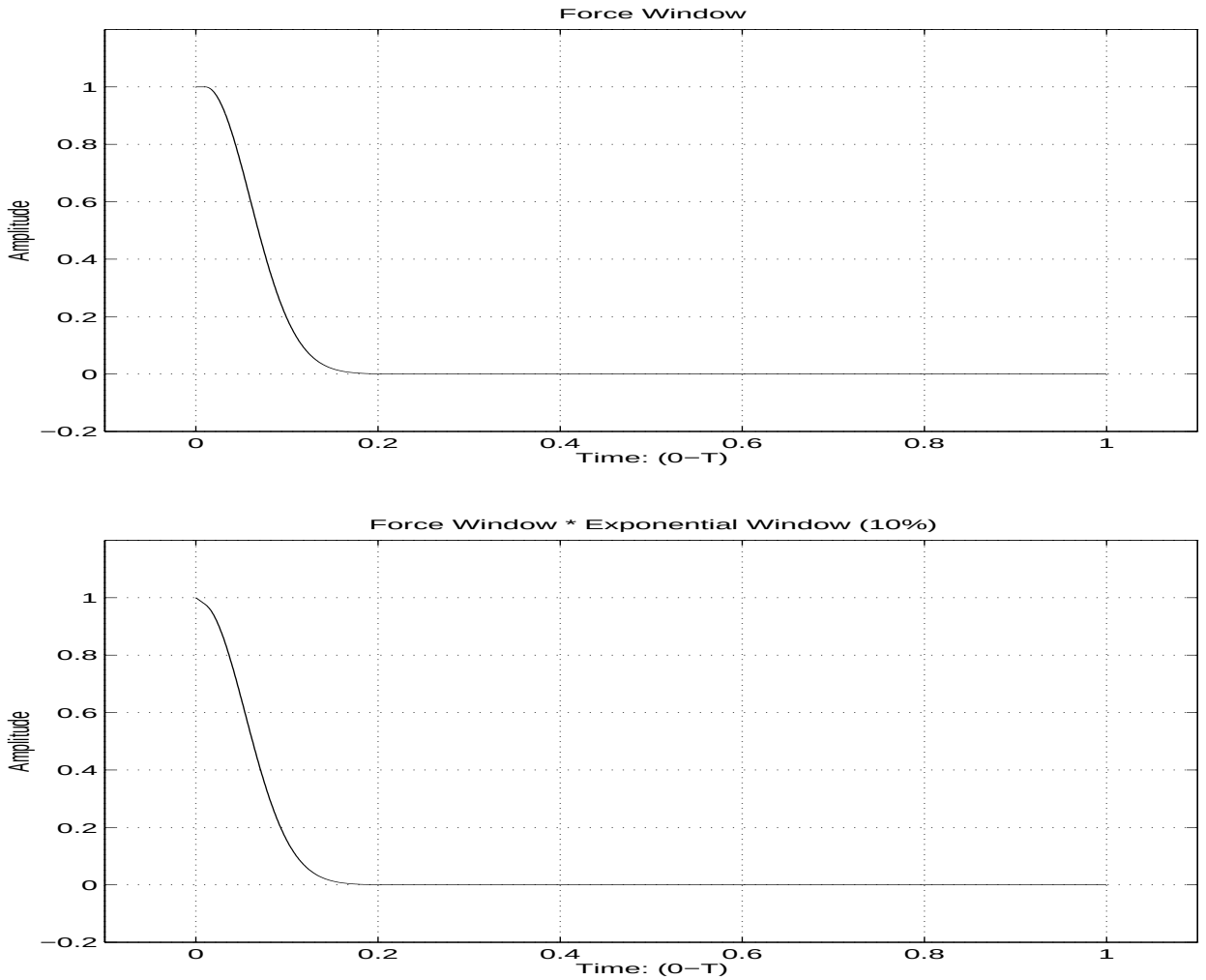

Figure 5-39. Typical Force Windows

Response (Exponential) Windows

Response (exponential) windows are used to minimize the leakage error for lightly damped systems by attenuating the response so that it decays to zero within the observation period. Normally, for lightly damped systems, a window that attenuates to 1-5 percent at the end of the response is appropriate. For heavily damped systems, a window that is similar to the decay of the system will attenuate any noise.

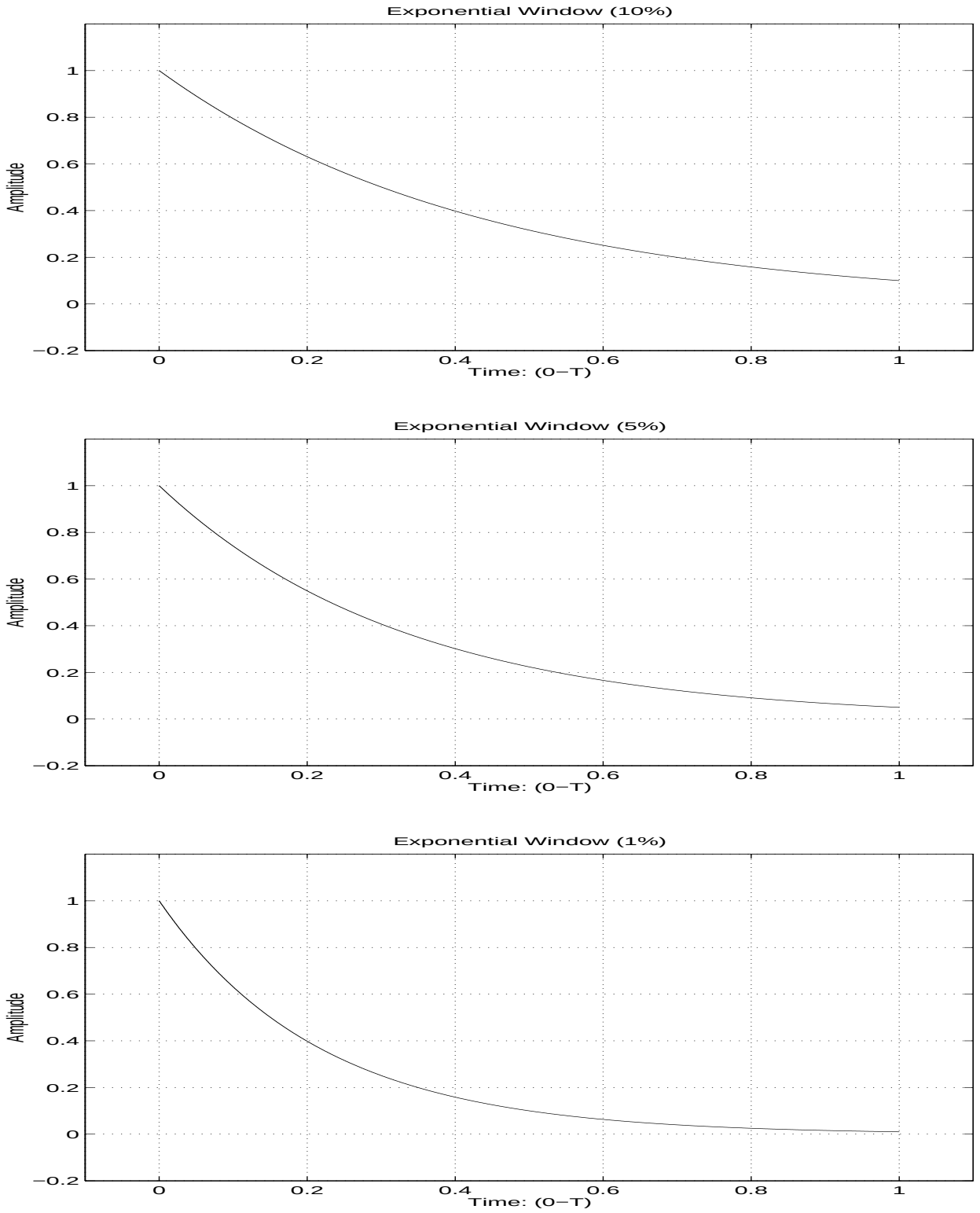

Figure 5-40. Typical Response Windows

Response (Exponential) Windows Correction

The windows that are added to the force and response signals must be corrected. Primarily, the response (exponential) window may add significant damping to the resultant frequency response function. This can only be corrected after the modal damping for each mode is found.

- $h_{pq}(t) = \sum_{r=1}^{2N} A_{pqr}\ e^{\lambda_r t}$
- $e^{\beta t} h_{pq}(t) = e^{\beta t} \sum_{r=1}^{2N} A_{pqr}\ e^{\lambda_r t}$
- $e^{\beta t} h_{pq}(t) = \sum_{r=1}^{2N} A_{pqr}\ e^{\beta t} e^{\lambda_r t}$
- $e^{\beta t} h_{pq}(t) = \sum_{r=1}^{2N} A_{pqr}\ e^{(\lambda_r + \beta)t} = \sum_{r=1}^{2N} A_{pqr}\ e^{\hat{\lambda}_r t}$
- $\hat{\lambda}_r = \hat{\sigma}_r + j\ \hat{\omega}_r = (\sigma_r + \beta) + j\ \omega_r$
- $\hat{\sigma}_r = \sigma_r + \beta$
- $\sigma_r = \hat{\sigma}_r - \beta$
- $\omega_r = \hat{\omega}_r$

5.4 Structural Testing Conditions

The test condition for any modal analysis test involves several environmental factors as well as appropriate boundary conditions. First of all, the temperature, humidity, vacuum, and gravity effects must be properly considered to match with previous analysis models or to allow the experimentally determined model to properly reflect the system.

In addition to the environmental concerns, the boundary conditions of the system under test are very important. Traditionally, modal analysis tests have been performed under the assumption that the test boundary conditions can be made to conform to one of four conditions:

- Free-free boundary conditions (Impedance is zero).
- Fixed boundary conditions (Impedance is infinite)
- Operating boundary conditions (Impedance is correct).
- Arbitrary boundary conditions (Impedance is known).

It should be obvious that, except in very special situations, none of these boundary conditions can be practically achieved. Instead, practical guidelines are normally used to evaluate the appropriateness of the chosen boundary conditions. For example, if a free-free boundary is chosen, the desired frequency of the highest rigid body mode should be a factor of ten below the first deformation mode of the system under test. Likewise, for the fixed boundary test, the desired interface stiffness should be a factor of ten greater than the local stiffness of the system under test. While either of these practical guidelines can be achieved for small test objects, a large class of flight vehicle systems can not be acceptably tested in either configuration. Arguments have been made that the impedance of a support system can be defined (via test and /or analysis) and the effects of such a support system can be eliminated from the measured data. This technique is theoretically sound but, due to significant dynamics in the support system and limited measurement dynamics, this approach has not been uniformly applicable.

In response to this problem, many alternate structural testing concepts have been proposed and are under current evaluation. Active, or combinations of active and passive, suspension systems are being evaluated, particularly for application to very flexible space structures. Active inert gas suspension systems have been used in the past for the testing of smaller commercial and military aircraft and, in general, such approaches are formulated to better match the requirements of a free-free boundary condition.

Another alternate test procedure is to define a series of relatively conventional tests with various boundary conditions. The various boundary conditions are chosen in such a way that each perturbed boundary condition can be accurately modeled (for example, the addition of a large mass at interface boundaries). Therefore, as the experimental model is acquired for each configuration and used to validate and correct the associated analytical model, the underlying model will be validated and corrected accordingly. This procedure has the added benefit of adding the influence of modes of vibration that would normally occur above the maximum frequency of the test into the validation of the model. For example, the inertial effect of the

addition of a mass at an interface will cause a downward shift in frequency of any mode that is active at the interface (modes that are not affected by the interface dynamics will not be shifted). Since this shift is measured and the analytical model can accurately define the dynamics of the added mass, any inaccuracy in the analytical prediction of the frequency shifts as well as the corresponding effects on the modal vectors will be due to the lack of fidelity of the underlying analytical model.

Recently, other researchers have proposed multiple configurations of test conditions as a methodology of utilizing practical test configurations in the testing of flight vehicle systems. In a related research area, work is progressing on using constrained testing together with direct parameter estimation methods to define the characteristics of the unconstrained structure. In this test procedure, the excitation forces and the constraint forces are measured together with appropriate response information. The direct parameter estimation method produces a general matrix model that describes the unconstrained (free-free) structural system. All of these newer methods will increase the cost (time, financial, technical) of performing structural tests with the attendant incremental increase in the accuracy of the test results.

5.5 Practical Measurement Considerations

There are several factors that contribute to the quality of actual measured frequency response function estimates. Some of the most common sources of error are due to measurement mistakes. With a proper measurement approach, most of this type of error, such as overloading the input, extraneous signal pick-up via ground loops or strong electric or magnetic fields nearby, etc., can be avoided. Violation of test assumptions are often the source of another inaccuracy and can be viewed as a measurement mistake. For example, frequency response and coherence functions have been defined as parameters of a linear system. Nonlinearities will generally shift energy from one frequency to many new frequencies, in a way which may be difficult to recognize. The result will be a distortion in the estimates of the system parameters, which may not be apparent unless the excitation is changed. One way to reduce the effect of nonlinearities is to randomize these contributions by choosing a randomly different input signal for each of the n measurements. Subsequent averaging will reduce these contributions in the same manner that random noise is reduced. Another example involves control of the system input. One of the most obvious requirements is to excite the system with energy at all frequencies for which

measurements are expected. It is important to be sure that the input signal spectrum does not have "holes" where little energy exist. Otherwise, coherence will be very low, and the variance on the frequency response function will be large.

Assuming that the system is linear, the excitation is proper, and obvious measurement mistakes are avoided, some amount of error (noise) will be present in the measurement process. Five different approaches can be used to reduce the error involved in frequency response function measurements in current fast Fourier transform (FFT) analyzers. First of all, the use of ***different frequency response function estimation algorithms*** (H_v compared to H_1) will reduce the effect of the leakage error on the estimation of the frequency response function computation. The use of ***averaging*** can significantly reduce errors of both variance and bias and is probably the most general technique in the reduction of errors in frequency response function measurement. ***Selective excitation*** is often used to verify nonlinearities or randomize characteristics. In this way, bias errors due to system sources can be reduced or controlled. The ***increase of frequency resolution*** through the zoom fast Fourier transform can improve the frequency response function estimate primarily by reduction of the leakage bias error due to the use of a longer time sample. The zoom fast Fourier transform by itself is a linear process and does not involve any specific error reduction characteristics compared to a baseband fast Fourier transform(FFT). Finally, the ***use of weighting functions(windows)*** is widespread and much has been written about their value [1-3,40-41]. Primarily, weighting functions compensate for the bias error (leakage) caused by the analysis procedure.

5.6 References

[1] Bendat, J.S.; Piersol, A.G., **Random Data: Analysis and Measurement Procedures**, John Wiley and Sons, Inc., New York, 1971, 407 pp.

[2] Bendat, J. S., Piersol, A. G., **Engineering Applications of Correlation and Spectral Analysis**, John Wiley and Sons, Inc., New York, 1980, 302 pp.

[3] Otnes, R.K., Enochson, L., **Digital Time Series Analysis**, John Wiley and Sons, Inc., New York, 1972, 467 pp.

[4] Hsu, H.P., **Fourier Analysis**, Simon and Schuster, 1970, 274 pp.

[5] Dally, J.W.; Riley, W.F.; McConnell, K.G., **Instrumentation for Engineering Measurements**, John Wiley & Sons, Inc., New York, 1984.

[6] Strang, G., **Linear Algebra and Its Applications, Third Edition**, Harcourt Brace Jovanovich Publishers, San Diego, 1988, 505 pp.

[7] Lawson, C.L., Hanson, R.J., **Solving Least Squares Problems**, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1974, 340 pp.

[8] Jolliffe, I.T., **Principal Component Analysis** Springer-Verlag New York, Inc., 1986, 271 pp.

[9] Akaike, H., "On the Statistic Estimation of the Frequency Response Function of a System Having Multiple Inputs", Annals of the Institute of Statistical Mathematics, Volume 17, Number 2, 1965.

[10] Carter, G.C.; Knapp, C.H.; Nuttall, A.H., "Estimation of the Magnitude-Squared Coherence Function Via Overlapped Fast Fourier Transform Processing", IEEE Transactions on Audio and Electroacoustics, August 1973, pp. 337-344.

[11] Wada, B. K., "Modal Test - Measurement and Analysis Requirements", SAE Paper Number 751066, 1975, 17 pp.

[12] Ramsey, K. A.; Richardson, M., "Making Effective Transfer Function Measurements for Modal Analysis", Hewlett-Packard Co., 1975.

[13] Ramsey, K., "Effective Measurements for Structural Dynamics Testing: Part I", Sound and Vibration, November, 1975.

[14] Ramsey, K., "Effective Measurements for Structural Dynamics Testing: Part II", Sound and Vibration, April, 1976.

[15] Potter, R. W., "Matrix Formulation of Multiple and Partial Coherence", Journal of the Acoustic Society of America, Volume 66, Number 3, March 1977, pp. 776-781.

[16] Dodds, C.J., Robson, J.D., "Partial Coherence in Multivariate Random Processing", Journal of Sound and Vibration, Volume 42, Number 2, 1975, pp. 243-249.

[17] Ewins, D.J., Griffin, J., "A State-of-the-Art Assessment of Mobility Measurement Techniques - Results for the Mid-Range Structures", Journal of Sound and Vibration, Volume 78, Number 2, 1981, pp. 197-222.

[18] Allemang, R. J., "Investigation of Some Multiple Input/Output Frequency Response Function Experimental Modal Analysis Techniques", Doctor of Philosophy Dissertation, University of Cincinnati, Mechanical Engineering Department, 1980, 358 pp.

[19] Allemang, R.J., Rost, R.W., Brown, D.L., "Dual Input Estimation of Frequency Response Functions for Experimental Modal Analysis of Aircraft Structures", Proceedings, International Modal Analysis Conference, pp.333-340, 1982.

[20] Carbon, G.D., Brown, D.L., Allemang, R.J., "Application of Dual Input Excitation Techniques to the Modal Testing of Commercial Aircraft", Proceedings, International Modal Analysis Conference, pp.559-565, 1982.

[21] Allemang, R.J., Brown, D.L., Rost, R.W., "Dual Input Estimation of Frequency Response Functions for Experimental Modal Analysis of Automotive Structures", SAE Paper No. 820193.

[22] Allemang, R.J., Rost, R.W., Brown, D.L., "Dual Input Estimation of Frequency Response Functions for Experimental Modal Analysis of Aircraft Structures", Proceedings, International Modal Analysis Conference, pp.333-340, 1982.

[23] Allemang, R.J., Brown, D.L., Rost, R.W., "Multiple Input Estimation of Frequency Response Functions for Experimental Modal Analysis", U.S. Air Force Report Number AFATL-TR-84-15, 1984, 185 pp.

[24] Allemang, R.J., Brown, D.L., Rost, R.W., "Measurement Techniques for Experimental Modal Analysis", *Experimental Modal Analysis and Dynamic Component Synthesis*, USAF Technical Report, Contract Number F33615-83-C-3218, AFWAL-TR-87-3069, Volume 2, 1987.

[25] Cobb, R.E., Mitchell, L.D., "A Method for the Unbiased Estimate of System FRFs in the presence of Multiple-Correlated Inputs", *Journal of Analytical and Experimental Modal Analysis*, Volume 3, Number 4, 1988, pp. 123-128.

[26] Mitchell, L.D., Cobb, R.E., Deel, J.C., Luk, Y.W., "An Unbiased Frequency Response Function Estimator", *Journal of Analytical and Experimental Modal Analysis*, Volume 3, Number 1, 1988, pp. 12-19.

[27] Vold, H., Crowley, J., Rocklin, G. "A Comparison of H_1, H_2, H_v, Frequency Response Functions," Proceedings, International Modal Analysis Conference, 1985, pp.272-278.

[28] Brown, D.L., Carbon, G., Zimmerman, R.D., "Survey of Excitation Techniques Applicable to the Testing of Automotive Structures", SAE Paper No. 770029, 1977.

[29] Allemang, R.J., Phillips, A.W., "A New Excitation Method: Combining Burst Random Excitation with Cyclic Averaging", Proceedings, International Modal Analysis Conference, pp. 891-899, 1998.

[30] "Using Cyclic Averaging with Impact Testing", William A. Fladung, Andrew T. Zucker, Allyn W. Phillips, Randall J. Allemang Proceedings, International Modal Analysis Conference, 7 pp., 1999.

[31] "Frequency Resolution Effects on FRF Estimation: Cyclic Averaging vs. Large Block Size", Allyn W. Phillips, Andrew T. Zucker, Randall J. Allemang Proceedings, International Modal Analysis Conference, 4 pp., 1999.

[32] "A Comparison of MIMO-FRF Excitation/Averaging Techniques on Heavily and Lightly Damped Structures", Phillips, A.W., Zucker, A.T., Allemang, R.J., Proceedings, International Modal Analysis Conference, 10 pp., 1999.

[33] "An Overview of MIMO-FRF Excitation/Averaging Techniques", Phillips, A.W., Allemang, R.J., Zucker, A.T., Proceedings, International Conference on Noise and Vibration Engineering, Volume 1, Katholieke Universiteit Leuven, Belgium, 12 pp., 1998.

[34] Halvorsen, W.G., Brown, D.L., "Impulse Technique for Structural Frequency Response Testing", Sound and Vibration Magazine, November, 1977, pp. 8-21.

[35] Imes, R.S., Jennings, W.P., Olsen, N.L., "The Use of Transient Testing Techniques in Boeing YC-14 Flutter Clearance Program", AIAA Paper Number 78-0505, 1978.

[36] Allemang, R.J., Rost, R.W., Brown, D.L., "Multiple Input Estimation of Frequency Response Functions: Excitation Considerations", ASME Paper Number 83-DET-73, 1983, 11 pp.

[37] Van Karsen, C., "A Survey of Excitation Techniques for Frequency Response Function Measurement", Master of Science Thesis, University of Cincinnati, 1987, 81 pp.

[38] Mitchell, L.K.; Elliott, K.B., "How to Design Stingers for Vibration Testing of Structures", Sound and Vibration Magazine, April 1984, pp. 14-18.

[39] Stenger, G., "Step Relaxation Method for Structural Dynamic Excitation", Master of Science Thesis, Department of Mechanical Engineering, University of Cincinnati, 1979, 54 pp.

[40] Coleman, A.D., Driskill, T.C., Anderson, J.B., Brown, D.L., "A Mass Additive Technique for Modal Testing as Applied to the Space Shuttle ASTRO-1 Payload", Proceedings, International Modal Analysis Conference, 1988, pp. 154-159.

[41] Kuo, Chin-Po, Wada, B.K., "Multiple Boundary Condition Test (MBCT): Identification With Mode Shapes", NASA JPL Report, NPO-17574, 1989.